Quadratics: Nature of roots 1

Rules for quadratic $ax^2 + bx + c$

$b^2 - 4ac > 0$	two distinct real roots
$b^2 - 4ac = 0$	one, repeated, real roots
$b^2 - 4ac < 0$	no real roots

Examples:

(1) $f(x) = 2x^2 + 7x + 6$

$a = 2, b = 7, c = 6$
$b^2 - 4ac = 7^2 - (4 \times 2 \times 6) = 49 - 48 = 1$
$b^2 - 4ac > 0$, so $f(x)$ has two real roots

(2) $g(x) = x^2 - 6x + 9$

$a = 1, b = -6, c = 9$
$b^2 - 4ac = (-6)^2 - (4 \times 1 \times 9) = 36 - 36 = 0$
$b^2 - 4ac = 0$, so $g(x)$ has one repeated real root

(3) $h(x) = x^2 + 9$

$a = 1, b = 0, c = 9$
$b^2 - 4ac = 0^2 - (4 \times 1 \times 9) = -36$
$b^2 - 4ac < 0$, so $h(x)$ has no real roots

Exercises:

(4) $2x^2 + 3x + 1$

(5) $4x^2 + 4x + 1$

(6) $x^2 + 3x + 2$

(7) $4x^2 - 9$

(8) $x^2 + 2x + 5$

(9) $-x^2 + 2x - 5$

(10) $4x^2 + 4x + 1$

(11) $2x^2 + 3x - 9$

(12) $x^2 - \frac{4}{3}x + \frac{4}{9}$

Next level:

(13) For what value(s) of c does the quadratic $p(x) = x^2 + 4x + c$ have one repeated real root?

(14) For what positive value(s) of b does the quadratic $q(x) = x^2 + bx + 9$ have two distinct real roots?