

Chapter 4 Problem 22

Alan Arnholt

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Problem 22

Let $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} F$ with corresponding pdf $f(x) = 3x^2, 0 \leq x \leq 1$.

(a) Find the pdf for X_{\min} .

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- (a) Find the pdf for X_{\min} .
- (b) Find the pdf for X_{\max} .

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- (a) Find the pdf for X_{\min} .
- (b) Find the pdf for X_{\max} .
- (c) If $n = 10$, find the probability that the largest value, X_{\max} , is greater than 0.92

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Let $X_1, X_2, \dots, X_n \stackrel{iid}{\sim} F$ with corresponding pdf $f(x) = 3x^2, 0 \leq x \leq 1$.

- (a) Find the pdf for X_{\min} .
- (b) Find the pdf for X_{\max} .
- (c) If $n = 10$, find the probability that the largest value, X_{\max} , is greater than 0.92
- (d) If $n = 10$, find the expected value of X_{\max} .

(a) The pdf for X_{\min} is:

$$f_{\min}(x) = n(1 - F(X))^{n-1} f(x) \quad (1)$$

Since $F(x) = \int_0^x 3t^2 dt = x^3, 0 \leq x \leq 1$, it follows that

$$f_{\min}(x) = n(1 - x^3)^{n-1} 3x^2, 0 \leq x \leq 1$$

(b) The pdf for X_{\max} is:

$$f_{\max}(x) = nF^{n-1}(x)f(x) \quad (2)$$

Since $F(x) = \int_0^x 3t^2 dt = x^3, 0 \leq x \leq 1$, it follows that

$$f_{\max}(x) = n(x^3)^{n-1} 3x^2, 0 \leq x \leq 1$$

$$P(X_{\max} > 0.92)$$

(c) Using (2) gives:

$$f_{\max}(x) = 10 (x^3)^{10-1} 3x^2 = 30x^{29}, 0 \leq x \leq 1$$

$$P(X_{\max} > 0.92) = \int_{0.92}^1 30x^{29} dx$$

```
f <- function(x){30*x^29}  
ans <- integrate(f, 0.92, 1)$value  
ans
```

```
[1] 0.918
```

$$P(X_{\max} > 0.92) = \int_{0.92}^1 30x^{29} dx = 0.918.$$

(d)

$$E[X_{\max}] = \int_0^1 x_{\max} f_{\max}(x) = \int_0^1 30x^{30}$$

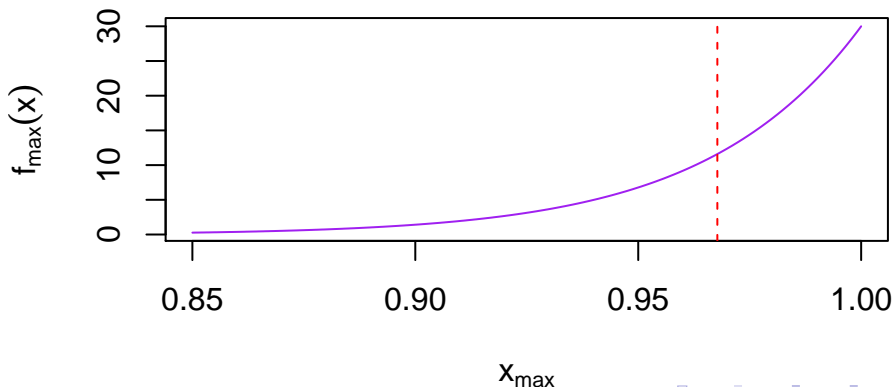
```
f1 <- function(x){30*x^30}  
ans <- integrate(f1, 0, 1)$value  
ans
```

```
[1] 0.9677
```

$$E[X_{\max}] = 0.9677$$

What does the pdf look like?

```
curve(f, 0.85, 1, n = 500, col = "purple",  
      xlab = expression(x[max]),  
      ylab = expression(f[max](x)))  
abline(v = ans, lty = "dashed", col = "red")
```



What does the pdf look like with ggplot2?

```
library(ggplot2)
ggplot(data = data.frame(x = c(0.85, 1)), aes(x = x)) +
  stat_function(fun = f, color = "purple") +
  theme_bw() +
  geom_vline(xinter = ans, lty = "dashed", color = "red") +
  labs(x = expression(x[max]), y = expression(f[max](x)))
```

