Samping Distributions

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Contents

1 Uniform Distribution

1.1 Definition

A random variable X has a uniform distribution on the interval $[a, b]$ (for $a < b$) if its **pdf** is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

If $X \sim \text{Unif}(a, b)$, then

$$E(X) = \frac{a+b}{2} \tag{2}$$

$$V(X) = \frac{(b-a)^2}{12} \tag{3}$$

1.2 Example

Suppose $X \sim \text{Unif}(10, 20)$.

a. Find the $E(X)$ and $V(X)$ using the definitions of $E(X)$ and $V(X)$.

```
# part a.
xfx <- function(x){x/10}
EX <- integrate(xfx, 10, 20)$value
EX
```

```
[1] 15
```

```
x2fx  <- function(x){(x - EX)^2/10}
VX <- integrate(x2fx, 10, 20)$value
VX
```

```
[1] 8.333333
```

b. Find the $E(X)$ and $V(X)$ using the short cut formulas in (2) and (3).

$E(X) = \frac{a+b}{2} = \frac{10+20}{2} = 15$, and $V(X) = \frac{(b-a)^2}{12} = \frac{(20-10)^2}{12} = \frac{100}{12} = 8.3333333$.

c. Simulate 10,000 values of the random variable and estimate $E(X)$ and $V(X)$.

```r
# part c.
set.seed(89)
sims <- 10^4
X <- runif(sims, 10, 20)
EX <- mean(X)
VX <- var(X)
c(EX, VX)
```

```
[1] 14.997757  8.392661
```

d. Find $E(\bar{X})$ and $V(\bar{X})$ for $n = 8$ exactly and via simulation.

$E(\bar{X}) = \mu_{\bar{X}} = \mu_X = 15$, $V(\bar{X}) = \frac{\sigma_X^2}{n} = \frac{\frac{100}{12}}{8} = 1.0416667$

```r
set.seed(46)
sims <- 10^4
n <- 8
a <- 10
b <- 20
xbar <- numeric(sims)
for(i in 1:sims){
  X <- runif(n, a, b)
  xbar[i] <- mean(X)
}
mean(xbar)
```

```
[1] 15.01263
```

```r
var(xbar)
```

```
[1] 1.043381
```

2 Exponential

2.1 Problem

Let $X_1, X_2, \ldots, X_{20} \overset{i.i.d}{\sim} \text{Exp}(\lambda = 2)$. Let $Y = \sum_{i=1}^{20} X_i$.

a. Simulate the sampling distribution of Y in `R`.

```r
sims <- 10^4
Y <- numeric(sims)
for(i in 1:sims){
 Y[i] <- sum(rexp(20, 2))
}
EY <- mean(Y)
VY <- var(Y)
c(EY, VY)
```

```
[1] 10.017660  5.028047
```

```r
mean(Y <= 10)
```

```
[1] 0.5268
```