Chapter 4 Problem 22

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Problem 22

Let $X_1, X_2, \dots X_n \stackrel{iid}{\sim} F$ with corresponding pdf $f(x) = 3x^2, 0 \le x \le 1$.

- (a) Find the pdf for X_{\min} .
- (b) Find the pdf for X_{max} .
- (c) If n=10, find the probability that the largest value, $X_{\rm max}$, is greater than 0.92
- (d) if n = 10, find the expected value of X_{max} .



$f_{\min}(x)$

(a) The pdf for X_{\min} is:

$$f_{\min}(x) = n(1 - F(X))^{n-1} f(x)$$
 (1)

Since $F(x) = \int_0^x 3t^2 dt = x^3, 0 \le x \le 1$, it follows that

$$f_{\min}(x) = n (1 - x^3)^{n-1} 3x^2, 0 \le x \le 1$$



$f_{\max}(x)$

(b) The pdf for X_{max} is:

$$f_{\max}(x) = nF^{n-1}(x)f(x) \tag{2}$$

Since $F(x) = \int_0^x 3t^2 dt = x^3, 0 \le x \le 1$, it follows that

$$f_{\max}(x) = n(x^3)^{n-1} 3x^2, 0 \le x \le 1$$

$P(X_{\rm max} > 0.92)$

(c)
$$f_{\max}(x) = 10 (x^3)^{10-1} 3x^2 = 30x^{29}, 0 \le x \le 1$$

[1] 0.918

$$P(X_{\text{max}} > 0.92) = \int_{0.92}^{1} 30x^{29} dx = 0.918.$$



$$E[X_{\text{max}}] = \int_0^1 x_{\text{max}} f_{\text{max}}(x) = \int_0^1 30x^{30}$$

f1 <- function(x) $\{30*x^30\}$ ans <- integrate(f1, 0, 1)\$value ans

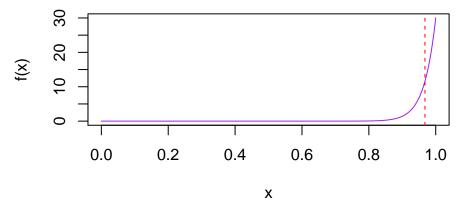
[1] 0.9677

$$E[X_{\rm max}] = 0.9677$$



What does the pdf look like?

```
curve(f, 0, 1, n = 500, col ="purple")
abline(v = ans, lty = "dashed", col = "red")
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