Effective Field Method at Zero Temperature with Field Along Various Directions

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1 Introduction

The effective field method was used to 3000 iterations (except where noted) to determine the 0 temperature states of the 12x12x12 3D FCC kagome lattice while being subjected to a changing magnetic field along various field directions.

There are several types of runs which differ in either the starting state, the field direction, the maximum field to which the field was increased, and whether the field was then decreased after being saturated. When starting from a ground state for any of the simulations included in this pdf, the ground state had characteristic angles theta = 0.206275 and phi = 3.11867.

The purpose of these simulations was to observe the behaviour of the lattice's properties when subject to fields of various conditions. Analyses that was performed on the resulting data included the following:

- Plots of magnetization versus field
- Plots of energy versus field
- Composite plots of energy for both increasing and decreasing fields
- Composite plots of magnetization for both increasing and decreasing fields
- Snap shots of the characteristic 6 spins for each lattice at significant points of each simulation
- Determination of the number of "unique" spins that populate the lattice
- Determination of the components of the unique spins
- Dot products of each of the 6 spins with their respective "neighbors".

Preliminary results providing insight on the causes of particular spin configurations due to application of field are also contained within the PDF.

Note: In all 6-spin snapshots, the spins are as such: A-Red, B-Green, C-Blue, D-Pink, E-Brown, F-Purple. Determining the number of unique spins that populate a particular lattice was not done for all simulations, but so far it seems that the lattice always maintains a 6 spin system, except during unstable periods in which the lattice undergoes a transition. It's likely that if the number of iterations used for EFM were increased and reran for transition phases the number of unique spins would probably reduce to 6. In other words, I think the system is changing too quickly for EFM with 3000 iterations to properly find its ground state during transitions.

2 (001) Increasing Field, Ground State

The 6 spins begin to transition to the planar state at H=0.0105 and achieves the planar state at H=0.0121. The pink and brown spins swap positions, as do the blue and purple spins, as the field is increased beyond the planar state. The spins gradually align with the 001 field direction, until approximately at 0.14 the lattice becomes saturated and the red and green and spins become parallel to the field direction.

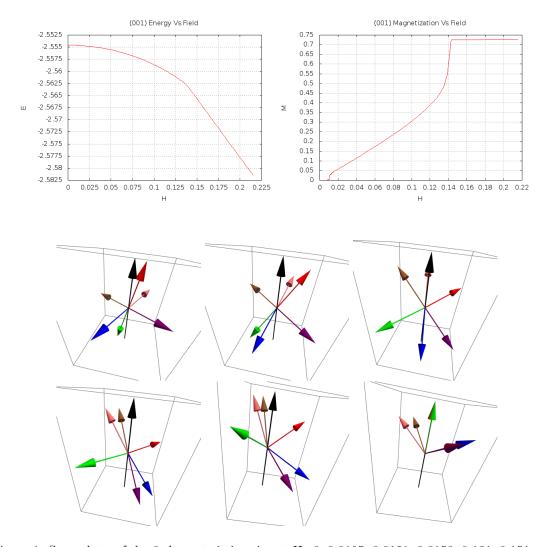


Figure 1: Snap shots of the 6 characteristic spins at H=0, 0.0105, 0.0121, 0.0150, 0.131, 0.151. The black arrow indicates the direction of the field. In the dot product graph, AB dot product goes to zero once the lattice is saturated. This agrees with A and B (red and green) lining up in the above snapshots.

3 (001) Decreasing Field, Ground State

The lattice leaves the saturated state at a field lower than what was required to induce it while increasing the field. This transition from saturation occurs at approximately H=0.13, compared to the transition to saturation at H=0.14 when increasing the field. The spins gradually unalign and rest in a planar state at zero field, and is characterized by the groundstate angles theta=89.9 degrees and phi=44.94 degrees.

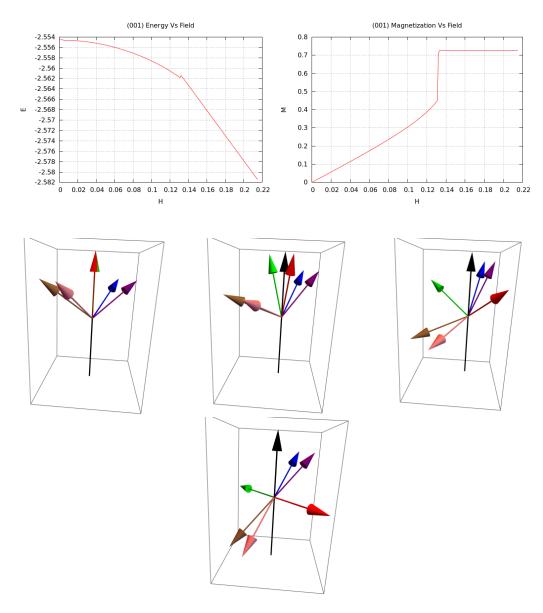


Figure 2: Snapshots at H=0.215, 0.132, 0.130, 0

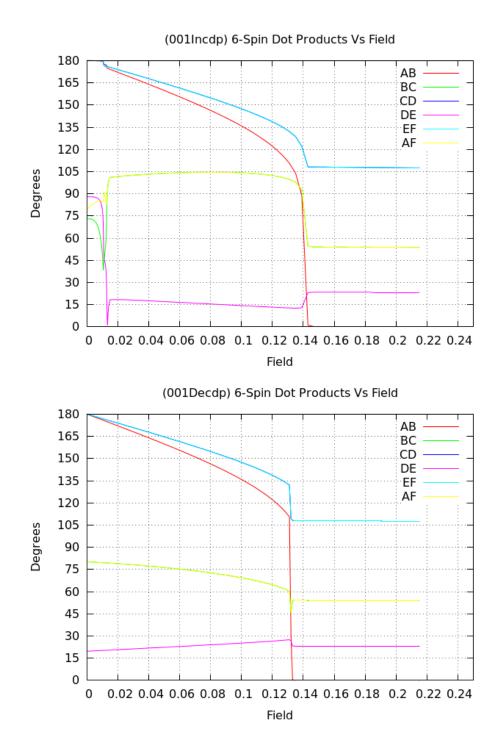


Figure 3: Dot products between the characteristic spins for both increasing and decreasing field.

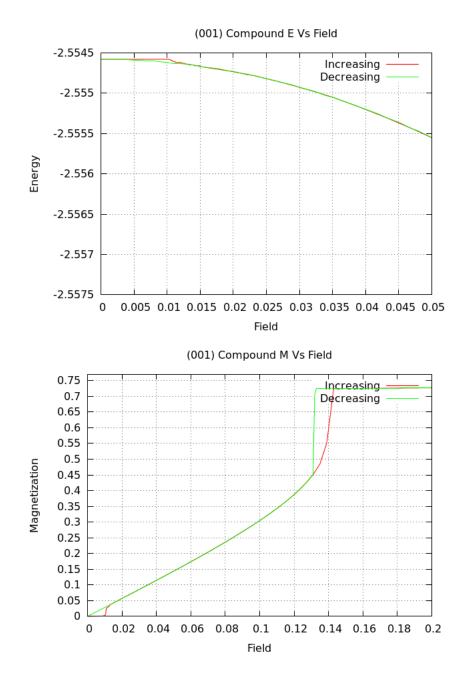


Figure 4: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude. Note the different scales for the energy and magnetization graph. This is because plotting energy vs field on a graph which has an xrange of 0.2 reduces the ability to see any difference in the energy curves. This was done for all composite graphs.

4 (010) Increasing Field, Ground State

The lattice begins to undergo a transition at 0.0110. At 0.0114 the planar state is achieved. Similar to 001, two pairs of spins begin to swap positions at around 0.015, though the pairs consist of spins different from 001. The blue and green spins and red and pink spins appear to be swapping positions, but never do. The spins gradually align with the 010 direction until at approximately H=0.14 the lattice becomes saturated and the purple and brown spins become parallel to the field direction.

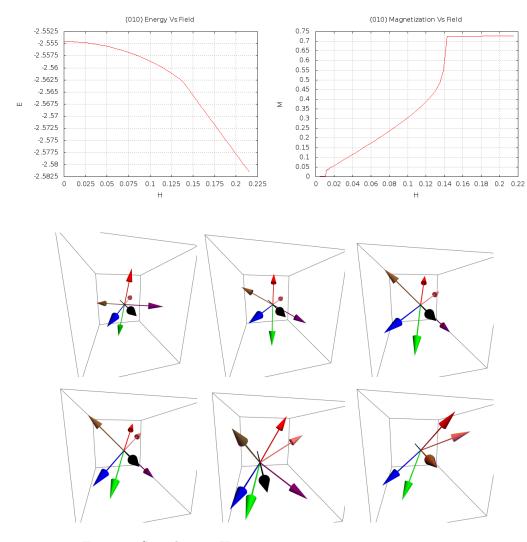


Figure 5: Snapshots at H=0, 0.0110, 0.0117, 0.0150, 0.139, 0.179

5 (010) Decreasing Field, Ground State

The lattice is released from saturation at approximately field=0.13, a field magnitude lower than the increasing field required to induce saturation. The spins gradually unalign with the decreasing field, and rest at a zero field planar state characterized by angles theta=52.7 degrees and phi=117.275 degrees.

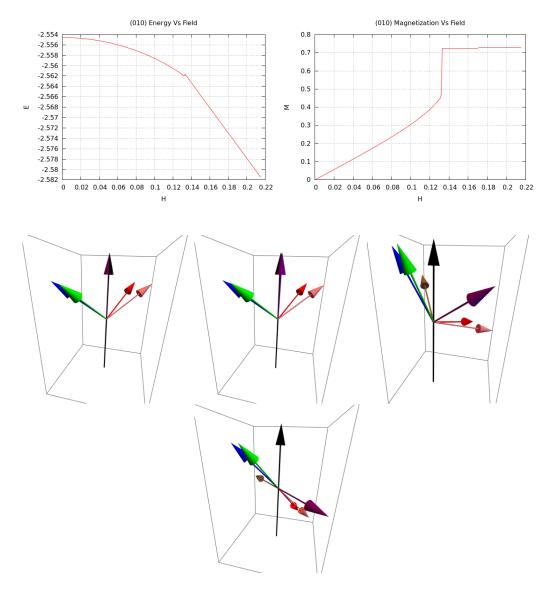


Figure 6: Snapshots of the 6 characteristic spins at H=0.215, 0.132, 0.131, 0. While the field arrow is pointing up, it is still pointing in the 010 direction.

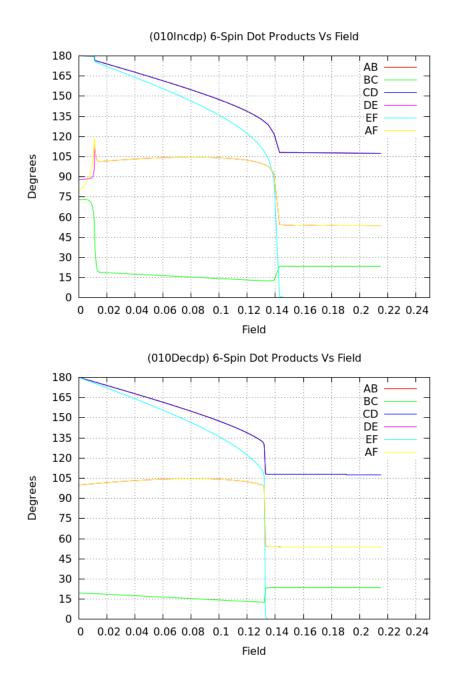


Figure 7: Dot products between the characteristic spins for both increasing and decreasing field.

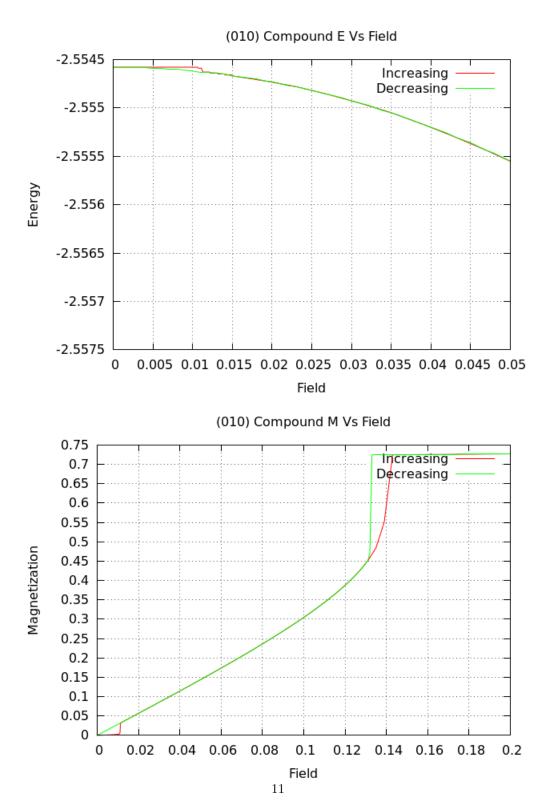


Figure 8: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude.

6 (011) Increasing Field, Ground State

The lattice begins to transition at approximately 0.007. At 0.0074 the planar state is achieved. At 0.0093, the pink and red spins swap position, and the blue and green spins swap position. As the field is increased further to 0.0115, the green and brown spins begin to swap positions. At 0.143, this is achieved. Once saturated, no spins are parallel with the field direction.

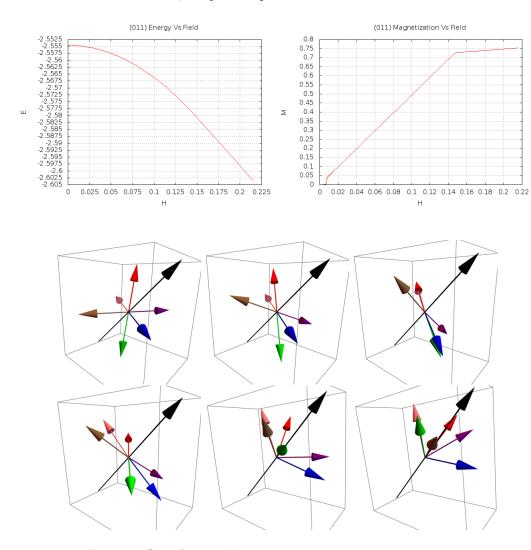


Figure 9: Snapshots at H=0, 0.0066, 0.0082, 0.0094, 0.115, 0.167

7 (011) Decreasing Field, Ground State

As the field is decreased to 0.134, the brown and green spins swap positions again. All 6 spins gradually unalign with the field until they reach a zero field planar state characterized by angles theta=69.33 degrees and phi=-139.133 degrees.

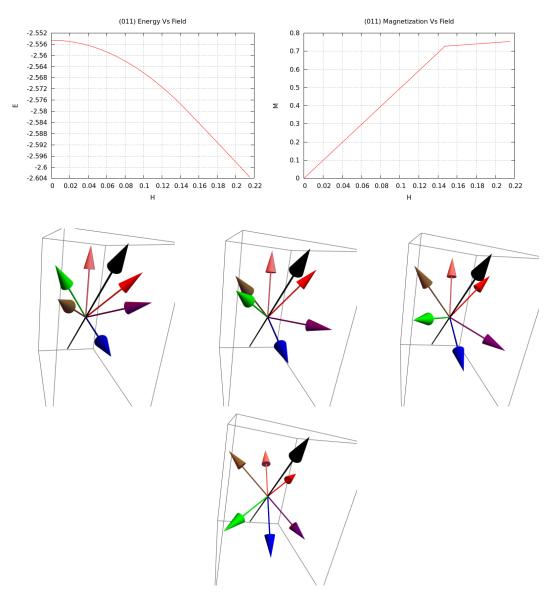


Figure 10: Snapshots at H=0.215, 0.134, 0.094, 0

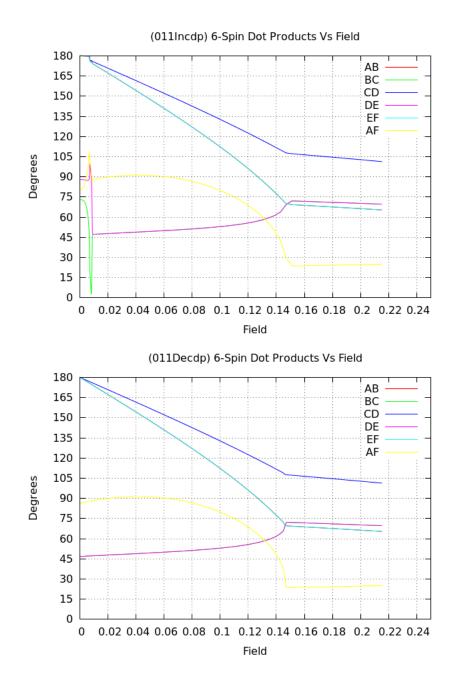


Figure 11: Dot products between the characteristic spins for both increasing and decreasing field.

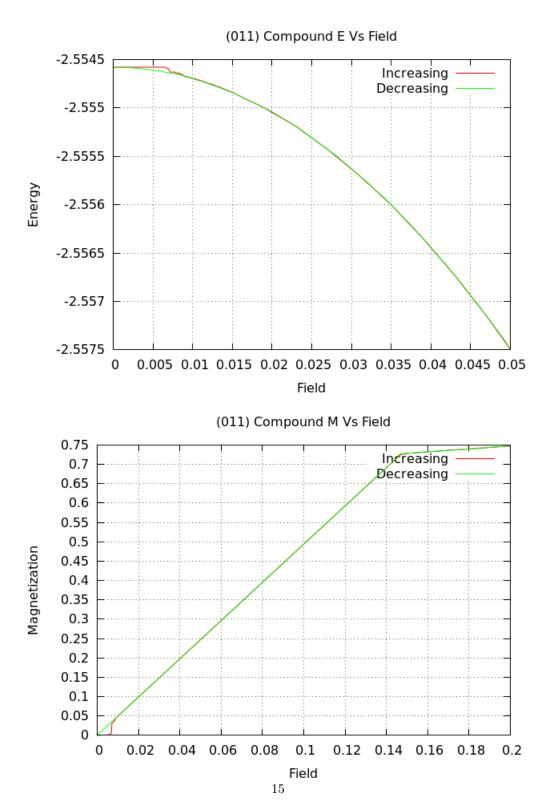


Figure 12: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude.

8 (100) Increasing Field, Ground State

Two inflection points are observed in the magnetization graph. There are two inflection points in the energy graph as well, however they are not as obvious. A planar state is achieved at 0.0041. Between 0.0041 and 0.0064, the brown and green spins become closer to one another, as do the red and purple spins. The pink and blue spins remain fixed. The spins gradually align with the field, until it suddenly snaps into its final position where the blue and pink spins point in the same direction, in addition to lying within the xy plane. The lattice is saturated beyond this point.

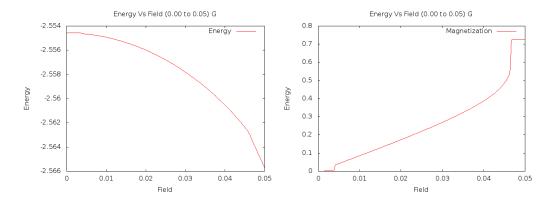


Figure 13: Energy vs increasing field and Magnetization versus increasing field

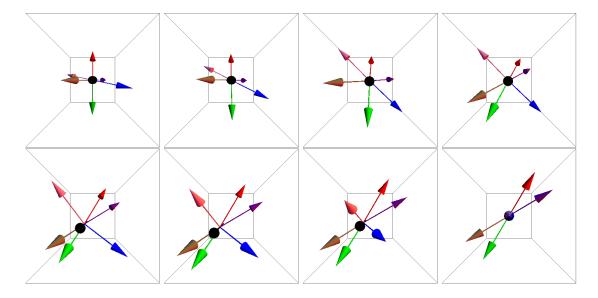


Figure 14: H=0, 0.0038, 0.0041, 0.0054, 0.0418, 0.0465, 0.0466 and 0.05

9 (100) Decreasing Field, Ground State

As the field is decreased, the spins are released from the saturated state, and return to the typically observed planar state, that gradually unaligns with the field direction as the field is decreased. Finally, the spins return to a full planar state, and not the original zero field ground state. The starting configuration of this run was not the final configuration of the (100) Increasing Field, Ground State run, but the same zero field ground state that the (100) Increasing Field, Ground state started from. The final configuration is characterized by angles theta=51.63 degrees and phi=154.414 degrees.

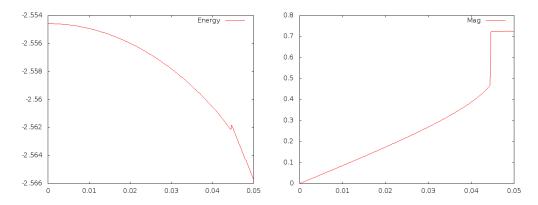


Figure 15: Energy vs decreasing field and Magnetization versus decreasing field

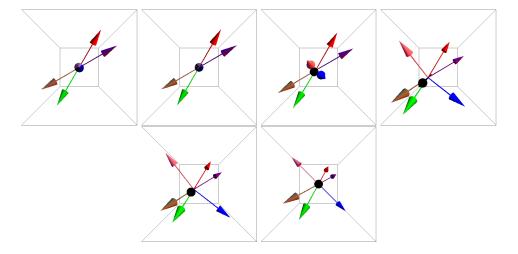


Figure 16: H=0.05, 0.0498, 0.0446, 0.0445, 0.0372, and 0. It can be seen that the pink and blue spins have not yet entirely lined up with the field vector in the first picture, but they have in the second picture. This indicates that simply using a zero field groundstate and subjecting it to a high field immediately with only 3000 steps does not yield the true state for that field.

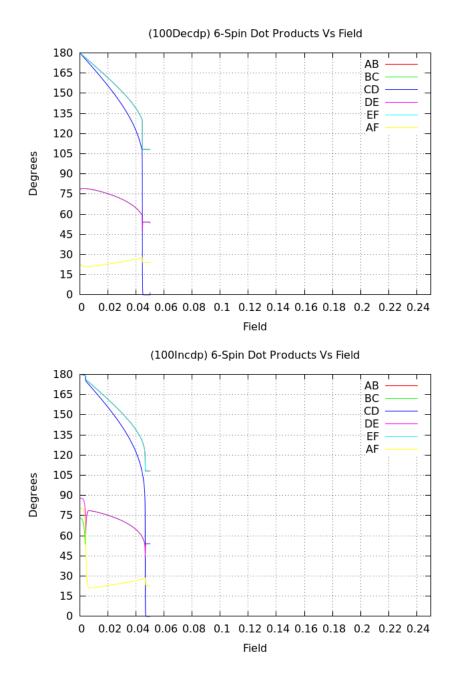


Figure 17: Dot products between the characteristic spins for both increasing and decreasing field. The data only continues to a low field relative to other simulations due to the saturation field occurring so early. A simulation continuing this run is currently being worked on.

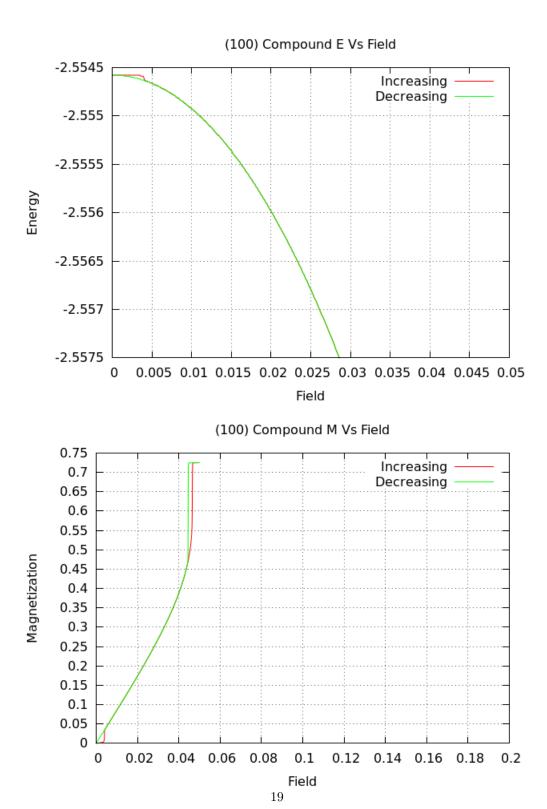


Figure 18: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude.

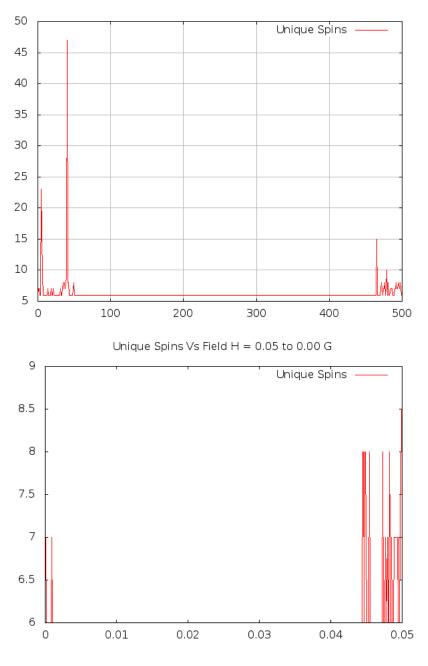


Figure 19: The number of unique spins within the lattice. The labelling is not clear, and not consistent throughout this pdf since these graphs were not completely finished at the time of making the pdf. The first graph's x axis begins at H=0, and ends at H=0.05, and depicts the number of spins within the system during an increasing field. The second is for decreasing field.

10 (100) Increasing Field, Random State

The energy drops very quickly at near zero field, which is likely because of insufficient number of iterations. Two points of inflection are observed. The transition of the spins is very similar to starting from a ground state.

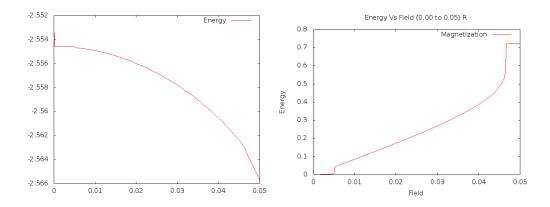


Figure 20: Energy vs increasing field and Magnetization versus increasing field

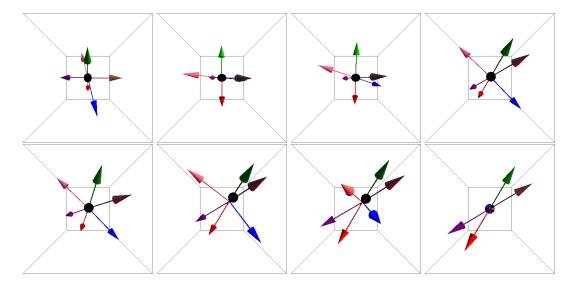


Figure 21: H = 0.00, 0.0001, 0.0047, 0.0053, 0.0054, 0.0410, 0.0466, 0.05

11 (100) Decreasing Field, Random State

The lattice starts off in a planar state. As the field is decreased, spins gradually unalign with the field. Suddenly, at 0.0455, the spins snap back into a slight realigned planar state. Another sudden readjustment is observed at 0.025. Finally, the spins return a full planar state at 0 field. The red and brown spins swap positions as the field lowers, as do the green and purple spins. This is likely a result of insufficient EFM iterations.

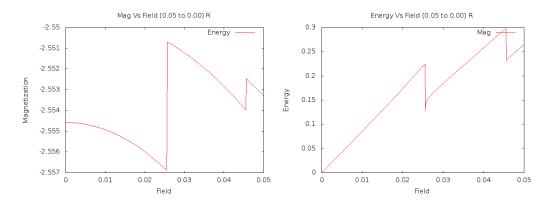


Figure 22: Energy vs decreasing field and Magnetization versus decreasing field

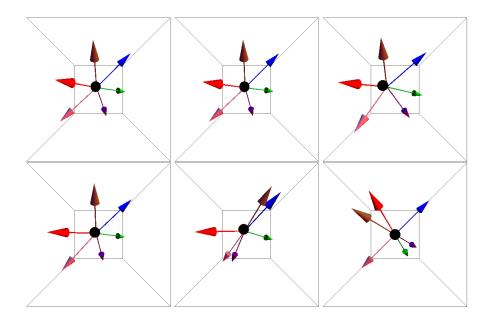


Figure 23: H = 0.05, 0.0460, 0.0455, 0.0257, 0.0256, 0.00.

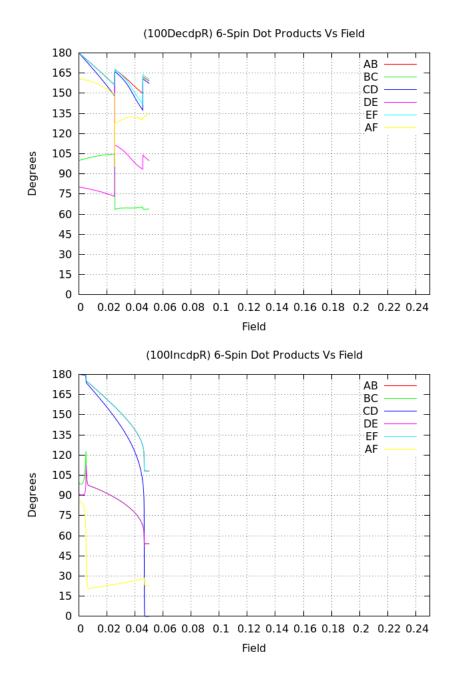
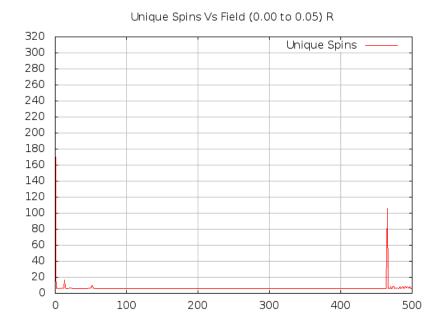


Figure 24: Dot products between the characteristic spins for both increasing and decreasing field. The data only continues to a low field relative to other simulations due to the saturation field occurring so early. A simulation continuing this run is currently being worked on.



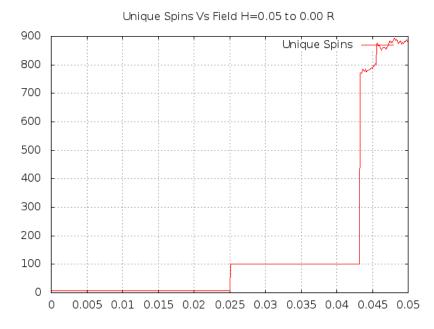


Figure 25: At high field, while decreasing it, a huge number of "unique" spins was observed. When looking through one of the configuration files, most spins only agreed with some other spins to at most 1 decimal place. The program that determines whether a spin is unique considers a spin to be unique when it doesn't agree with any previously found spin up to at least two decimal places. Hence, the large number of unique spins present. The plateau in the middle is actually just false data I put into the text file, since I cancelled the program since it was taking a very long time. I reran the program from around H = 0.0250 to 0, and found that the lattice in this field range always had 6 unique spins.

12 (101) Increasing Field, Ground State

The lattice begins to undergo a transition to a planar state at 0.0055, where it is achieved at 0.0060. At 0.0115 the purple and red spins and green and brown spins begin to swap positions, which is complete at 0.0128. At approximately 0.14, the green and pink spins swap position. Upon swapping, the lattice becomes saturated.

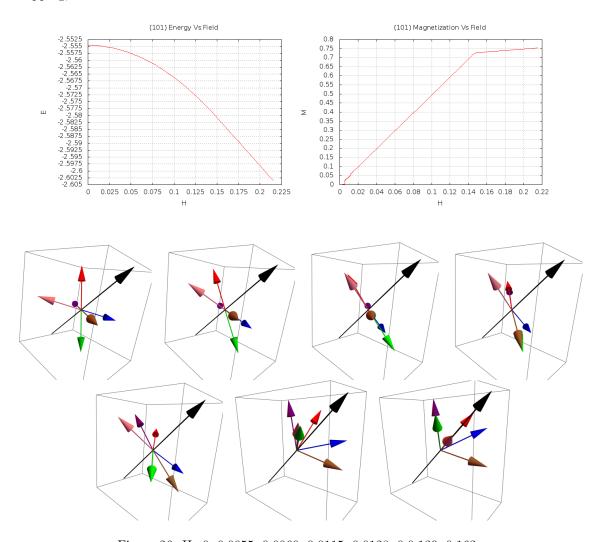


Figure 26: H=0, 0.0055, 0.0060, 0.0115, 0.0128, 0.0.139, 0.163

13 (101) Decreasing Field, Ground State

When decreasing the field, instead of the pink and green re-swapping positions the blue and red spins, the spins that mirror them, swap positions at approximately 0.137. The final configuration is characterized by angles theta=110.826 degrees and phi=49.1513 degrees.

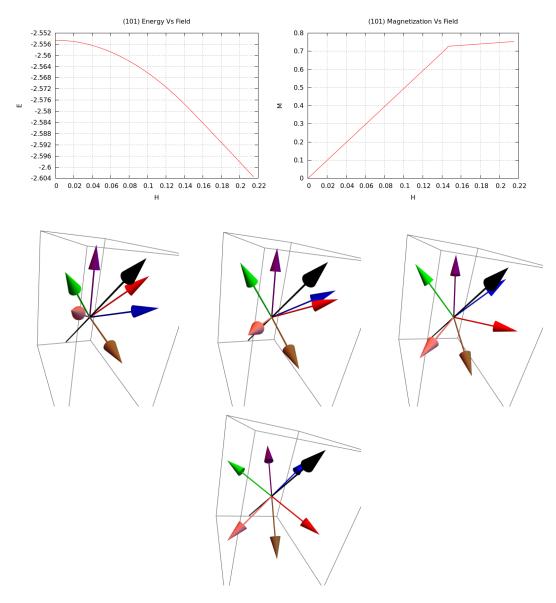


Figure 27: Snapshots at H=0.2150, 0.137, 0.09, 0

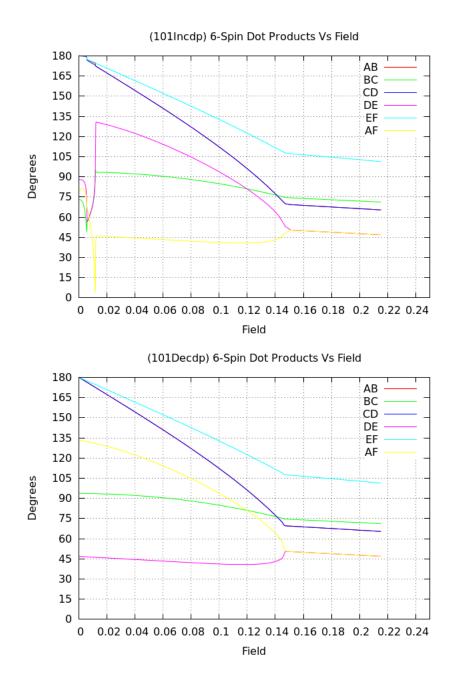


Figure 28: Dot products between the characteristic spins for both increasing and decreasing field.

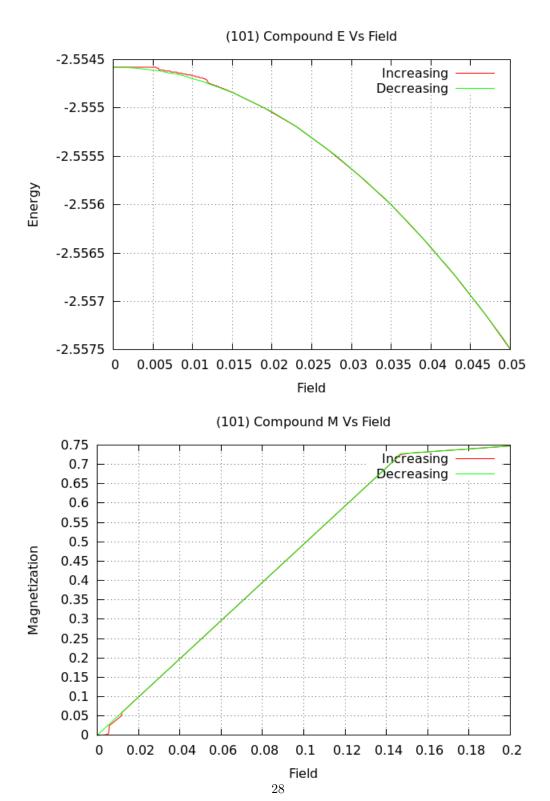


Figure 29: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude.

14 (110) Increasing Field, Ground State

Two inflection points are evident in the graphs of energy and magnetization, indicating the occurrence of a sudden change in orientation of the spins. The first inflection point occurs at H 0.005, at which the spins snap into a planar state. The second occurs at H 0.009, where another planar state forms but oriented in a different direction. This is the result of the red and purple spins swapping positions, and the brown and green spins swapping positions.

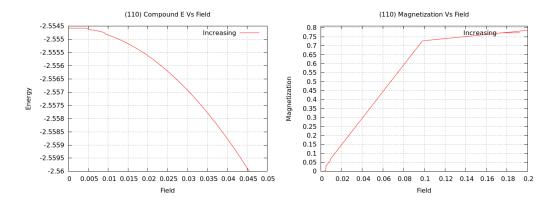


Figure 30: Energy vs increasing field and Magnetization versus increasing field

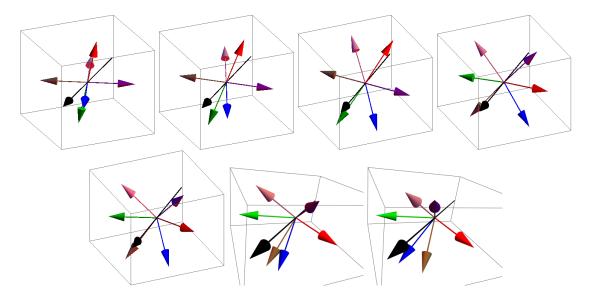


Figure 31: Snapshots of the 6 characteristic spins of the lattice at H=0, 0.0046, 0.0049, 0.0095, 0.05, 0.089, and 0.1

15 (110) Decreasing Field, Ground State

Similar to starting with a high field in the 111 direction, the spins are in a planar state that is aligned with the field. When the field is lowered, the spins are stuck in a planar state. In the first snapshot of the spins, the brown and green spins are partially aligned. This is likely due to insufficient number of iterations for EFM. The final configuration has characteristic angles theta=90.096 degrees and phi=135.034 degrees. The starting configuration of this run was not the final configuration of the (110) Increasing Field, Ground State run, but the same zero field ground state that the (110) Increasing Field, Ground state started from.

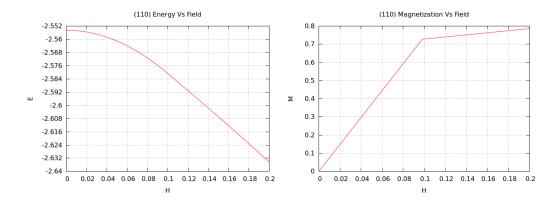


Figure 32: Energy vs decreasing field and Magnetization versus decreasing field

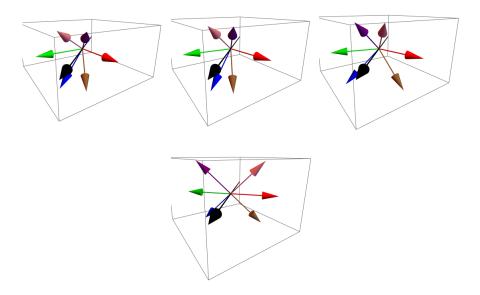


Figure 33: Snapshots of the 6 characteristic spins of the lattice at H=0.2, 0.121, 0.101, and 0

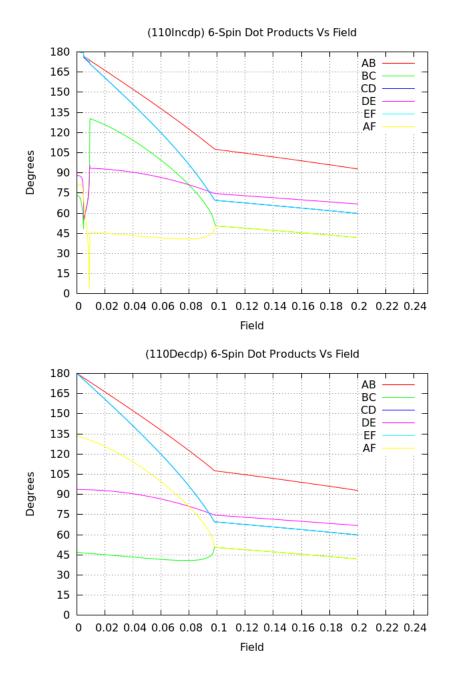


Figure 34: Dot products between the 6 characteristic spins of the lattice.

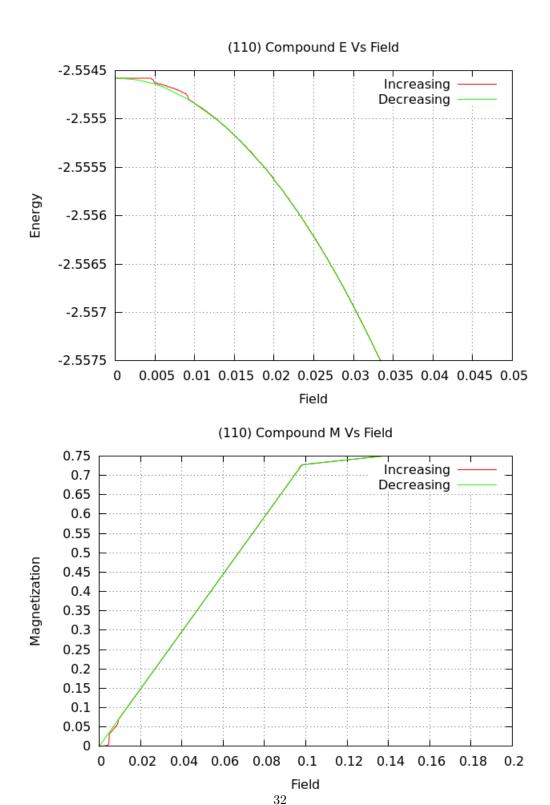


Figure 35: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude.

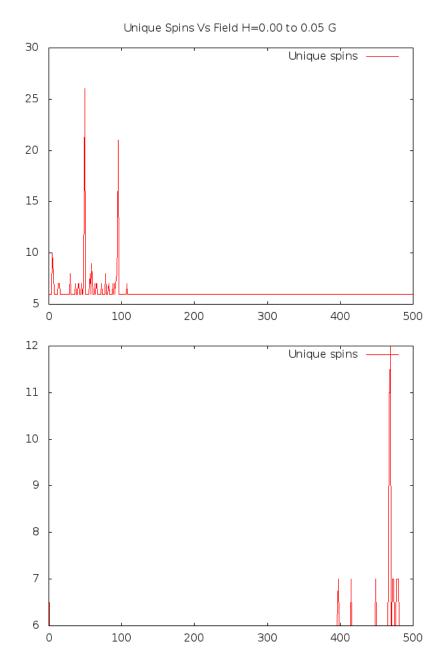


Figure 36: As mentioned before, these graphs aren't finished. The top graph's x-axis begins at H=0, and ends at H=0.05. The bottom graph's x-axis begins at H=0.05 and ends at H=0.

16 (110) Increasing Field, Random State

Very similar to beginning with the ground state, in that there are 2 transitions at approximately 0.0038 and 0.0075. The transformation of the 6 spins is similar as well, with the spins beginning in the groundstate, transition to a planar state, 2 pairs of spins rotate and switch places, and the plane the spins lie in reorients itself. The spins then gradually align with the field.

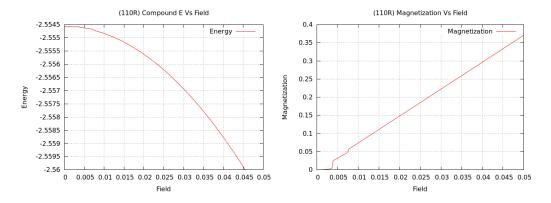


Figure 37: Energy vs increasing field and Magnetization versus increasing field

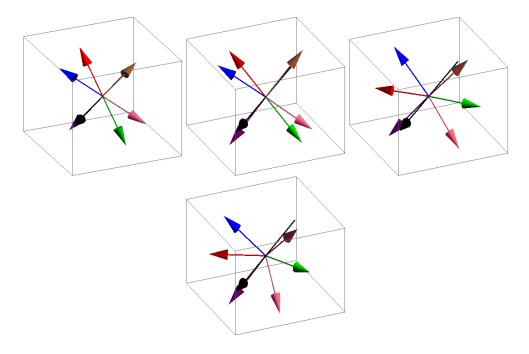


Figure 38: Snapshots of the 6 spins at $H=0.00,\,0.0068,\,0.0081,\,\mathrm{and}\,\,0.05$

17 (110) Decreasing Field, Random State

A sudden transition occurs at approximately 0.022. The green and pink spins swap places with another, and the blue and red spins also follow this swap. It's possible this transition only occurs because there is insufficient steps being used for EFM, as was the case with applying the field in the 111 direction. Any transitions disappeared when increasing the number of steps from 2000 to 3000 steps when decreasing the field with a random initial configuration.

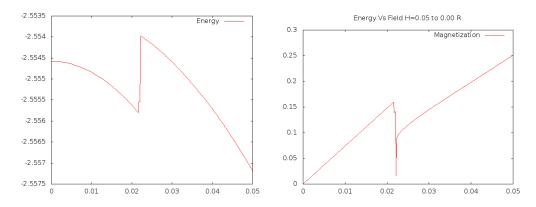


Figure 39: Energy vs decreasing field and Magnetization versus decreasing field

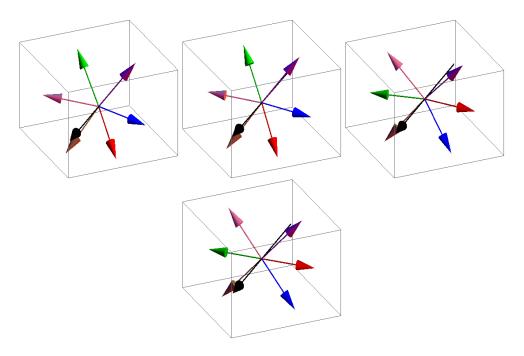


Figure 40: Snapshots at H=0.05, H=0.0260, 0.0201, and 0.

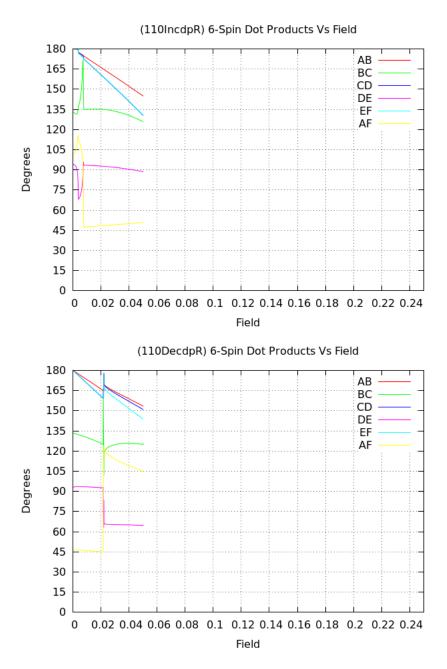


Figure 41: The curves from starting with a random initial state is different than that of starting with the ground state. The curves follow a similar trend, but are markably different. This indicates whatever states it falls into is dependent on starting state, and that there are multiple planar states the spin configuration after transitioning. This simulation was also only run to 0.05, passed the transition point.

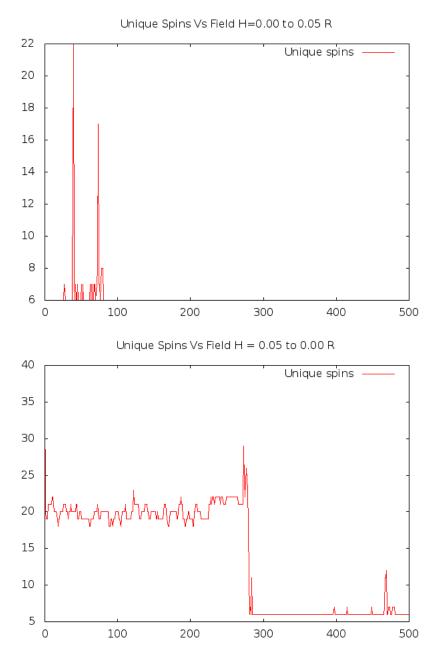


Figure 42: When decreasing the field, the lattice seems to be having trouble finding a stable 6 spin configuration. A sudden transition is observed at approximately 0.028 field, which can be observed in all graphs from this simulation.

18 2K (111) Increasing Field, Ground State

Steps persist in the energy graphs. This can probably be fixed by increasing precision. A sudden drop in energy occurs at field 0.006. This corresponds to the spin configuration snapping into a planar state, where the applied field vector intersects the plane. The angle of intersection is not perpendicular, but looks close to it. Once in the planar state, the spins gradually align with the field, and nothing else interesting happens.

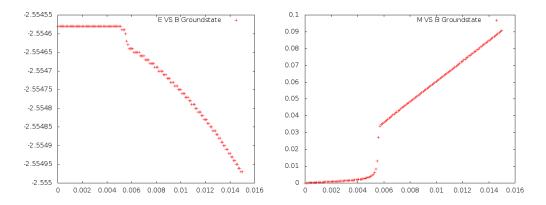


Figure 43: Energy vs increasing field and Magnetization versus increasing field

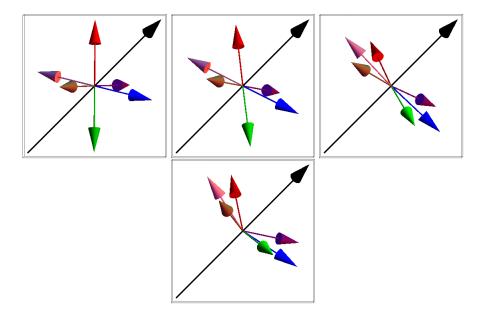


Figure 44: Snapshots of the 6 characteristic spins of the lattice at B=0, B=0.0052, B=0.0077, and B=0.05

0.0049	0.0048	0.0047	0.0046	0.0049	0.0044	0.0043	0.0042	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0024	0.0023	0.0022	0.0021	0.0020	0.0019	0.0018	0.0017	0.0016	0.0015	0.0014	0.0013	0.0012	0.0011	0.0010	0.0009	0.0008	0.0007	0.0006	0.0005	0.0004	0.0003	0.0002	0.000	0.0000
o o	o	0	o	-7	-7	ø	o	o	0	σ	0	σ	σ	0	o	o	o	0	6	o	o	o	o	o	o	o	o	<u>о</u>	0	o	o o	ი ი	о О	о. О	o,	o o	o o	6	o	00	72	9	o,	о. О	0	o	o	<u>о</u>	σ
0.0099	0.0098	0.0097	0.0096	0.0095	0.0094	0.0093	0.0092	0.0091	0.0090	6800.0	0.0088	0.0087	0.0086	0.0085	0.0084	0.0083	0.0082	0.0081	0.0080	0.0079	0.0078	0.0077	0.0076	0.0075	0.0074	0.0073	0.0072	0.0071	0.0070	0.0069	0.0068	0.0067	0.0066	0.0065	0.0064	0.0063	0.0062	0.0061	0.0060	0.0059	0.0058	0.0057	0.0056	0.0055	0.0054	0.0053	0.0052	0.0051	0.0050
o	o	o	o	o	o	o	o	o	o	σ	. თ	0	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	-7	o	o	-7	o	o	o	o	7	o	o	o	28	o	o	o	o	o	σ
0.0149	0.0148	0.0147	0.0146	0.0145	0.0144	0.0143	0.0142	0.0141	0.0140	0.0139	0.0138	0.0137	0.0136	0.0135	0.0134	0.0133	0.0132	0.0131	0.0130	0.0129	0.0128	0.0127	0.0126	0.0125	0.0124	0.0123	0.0122	0.0121	0.0120	0.0119	0.0118	0.0117	0.0116	0.0115	0.0114	0.0113	0.0112	0.0111	0.0110	0.0109	0.0108	0.0107	0.0106	0.0105	0.0104	0.0103	0.0102	0.0101	0.0100
o	o	o	o	o	o	o	o	o	o	σ	. თ	0	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ
0.0199	0.0198	0.0197	0.0196	0.0195	0.0194	0.0193	0.0192	0.0191	0.0190	0.0189	0.0188	0.0187	0.0186	0.0185	0.0184	0.0183	0.0182	0.0181	0.0180	0.0179	0.0178	0.0177	0.0176	0.0175	0.0174	0.0173	0.0172	0.0171	0.0170	0.0169	0.0168	0.0167	0.0166	0.0165	0.0164	0.0163	0.0162	0.0161	0.0160	0.0159	0.0158	0.0157	0.0156	0.0155	0.0154	0.0153	0.0152	0.0151	0.010.0
o	o	o	o	o	o	o	o	o	o	σ	0	0	σ	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ
0.0249	0.0248	0.0247	0.0246	0.0245	0.0244	0.0243	0.0242	0.0241	0.0240	0.0239	0.0238	0.0237	0.0236	0.0235	0.0234	0.0233	0.0232	0.0231	0.0230	0.0229	0.0228	0.0227	0.0226	0.0225	0.0224	0.0223	0.0222	0.0221	0.0220	0.0219	0.0218	0.0217	0.0216	0.0215	0.0214	0.0213	0.0212	0.0211	0.0210	0.0209	0.0208	0.0207	0.0206	0.0205	0.0204	0.0203	0.0202	0.0201	0.0200
o	o	o	o	o	o	o	o	o	o	σ	0	0.	σ.	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ
0.0299	0.0298	0.0297	0.0296	0.0295	0.0294	0.0293	0.0292	0.0291	0.0290	0.0289	0.0288	0.0287	0.0286	0.0285	0.0284	0.0283	0.0282	0.0281	0.0280	0.0279	0.0278	0.0277	0.0276	0.0275	0.0274	0.0273	0.0272	0.0271	0.0270	0.0269	0.0268	0.0267	0.0266	0.0265	0.0264	0.0263	0.0262	0.0261	0.0260	0.0259	0.0258	0.0257	0.0256	0.0255	0.0254	0.0253	0.0252	0.0251	0.0250
o	o	o	o	o	o	o	o	o	o	σ	. თ	0	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ
0.0349	0.0348	0.0347	0.0346	0.0345	0.0344	0.0343	0.0342	0.0341	0.0340	0.0339	0.0338	0.0337	0.0336	0.0335	0.0334	0.0333	0.0332	0.0331	0.0330	0.0329	0.0328	0.0327	0.0326	0.0325	0.0324	0.0323	0.0322	0.0321	0.0320	0.0319	0.0318	0.0317	0.0316	0.0315	0.0314	0.0313	0.0312	0.0311	0.0310	0.0309	0.0308	0.0307	0.0306	0.0305	0.0304	0.0303	0.0302	0.0301	0.0500
o	o	o	o	o	o	o	o	o	o	σ	0	0.00	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	ø	o	o	o	o	o
0.0399	0.0398	0.0397	0.0396	0.0395	0.0394	0.0393	0.0392	0.0391	0.0390	0.0389	0.0388	0.0387	0.0386	0.0385	0.0384	0.0383	0.0382	0.0381	0.0380	0.0379	0.0378	0.0377	0.0376	0.0375	0.0374	0.0373	0.0372	0.0371	0.0370	0.0369	0.0368	0.0367	0.0366	0.0365	0.0364	0.0363	0.0362	0.0361	0.0360	0.0359	0.0358	0.0357	0.0356	0.0355	0.0354	0.0353	0.0352	0.0351	0.0350
o	o	o	o	o	o	o	o	o	o	σ	0	0.00	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ
0.0449	0.0448	0.0447	0.0446	0.0445	0.0444	0.0443	0.0442	0.0441	0.0440	0.0439	0.0438	0.0437	0.0436	0.0435	0.0434	0.0433	0.0432	0.0431	0.0430	0.0429	0.0428	0.0427	0.0426	0.0425	0.0424	0.0423	0.0422	0.0421	0.0420	0.0419	0.0418	0.0417	0.0416	0.0415	0.0414	0.0413	0.0412	0.0411	0.0410	0.0409	0.0408	0.0407	0.0406	0.0405	0.0404	0.0403	0.0402	0.0401	0.0400
o	o	o	o	o	ø	o	ø	o	o	σ	0	0	σ.	0	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	ø	o	o	o	o	σ
0.0499	0.0498	0.0497	0.0496	0.0495	0.0494	0.0493	0.0492	0.0491	0.0490	0.0489	0.0488	0.0487	0.0486	0.0485	0.0484	0.0483	0.0482	0.0481	0.0480	0.0479	0.0478	0.0477	0.0476	0.0475	0.0474	0.0473	0.0472	0.0471	0.0470	0.0469	0.0468	0.0467	0.0466	0.0465	0.0464	0.0463	0.0462	0.0461	0.0460	0.0459	0.0458	0.0457	0.0456	0.0455	0.0454	0.0453	0.0452	0.0451	0.0450
o	o	o	o	o	o	o	o	o	o	σ	0	0	σ	0	o	o	o	σ	o	o	o	o	o	o	0	σ	o	o	o	თ	o	თ	o	თ	o	o	o	o	o	o	o	o	o	o	o	o	o	o	σ

Figure 45: A chart outlining the number of unique spins within the lattice for various field magnitudes in the 111 direction. The simulation began with the groundstate configuration, and used 2000 iterations.

19 2K (111) Decreasing Field, Ground State

Steps persist in the energy graphs. Increasing precision can probably fix this. Unlike the case where the field was increased, a sudden transition does not occur within the spin configuration when decreasing the field. The field is initially at its highest, and the spins are partially aligned with the field. As the field is lowered, the spins gradually relax to a planar state, and do not return to the ground state that is typically observed at near zero or zero field.

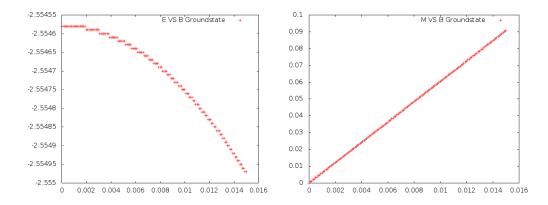


Figure 46: Energy vs decreasing field and Magnetization versus decreasing field

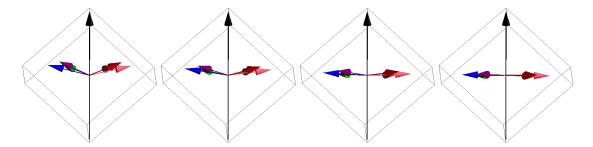


Figure 47: Snapshots of the 6 characteristic spins of the lattice at B=0.05, B=0.0309, B=0.01, and B=0.00

æ	0.001	o	0.0051	o	0.0101	o	0.0151	o	6 0.0201	_	0.0251	,	0.0301	n	00351	n	2000	n	000
6	0.0002	o	0.0052	o	0.0102	o	0.0152	o	6 0.0202	_	0.0252	0	0.0302	o	0.0352	o	0.0402	თ	0.0452
1	0.0003	o	0.0053	o	0.0103	o	0.0153	o	6 0.0203	o	0.0253	o	0.0303	o	0.0353	o	0.0403	o	0.0453
е.	0.0004	o	0.0054	o	0.0104	ø	0.0154	o	6 0.0204	o	0.0254	6	0.0304	o	0.0354	o	0.0404	o	0.0454
е.	0.0005	o	0.0055	o	0.0105	ø	0.0155	o	6 0.0205	o	0.0255	6	0.0305	o	0.0355	o	0.0405	o	0.0455
е.	0.0006	o	0.0056	o	0.0106	ø	0.0156	o	6 0.0206	o	0.0256	6	0.0306	o	0.0356	o	0.0406	o	0.0456
e.	0.0007	o	0.0057	0	0.0107	o	0.0157	o	6 0.0207	_	0.0257	0	0.0307	o	0.0357	o	0.0407	o	0.0457
e.	0.0008	o	0.0058	0	0.0108	o	0.0158	o	6 0.0208		0.0258	0	0.0308	o	0.0358	o	0.0408	o	0.0458
m	0.0009	0	0.0059	o	0.0109	ø	0.0159	o	6 0.0209	o	0.0259	6	0.0309	o	0.0359	o	0.0409	o	0.0459
	0.0010	o	0.0060	o	0.0110	o	0.0160	o	6 0.0210		0.0260	0	0.0310	o	0.0360	o	0.0410	o	0.0460
_	0.0011	o	0.0061	o	0.0111	o	0.0161	o	6 0.0211	0	0.0261	o	0.0311	o	0.0361	o	0.0411	o	0.0461
	0.0012	o	0.0062	o	0.0112	o	0.0162	o	6 0.0212	o	0.0262	o	0.0312	o	0.0362	o	0.0412	o	0.0462
	0.0013	o	0.0063	o	0.0113	o	0.0163	o	6 0.0213	o	0.0263	0	0.0313	o	0.0363	o	0.0413	o	0.0463
	0.0014	o	0.0064	o	0.0114	o	0.0164	o	6 0.0214	o	0.0264	0	0.0314	o	0.0364	o	0.0414	o	0.0464
_	0.0015	o	0.0065	o	0.015	o	0.0165	o	6 0.0215	o	0.0265	0	0.0315	o	0.0365	o	0.0415	o	0.0465
_	0.0016	o	0.0066	o	0.0116	o	0.0166	o	6 0.0216	o	0.0266	o	0.0316	o	0.0366	o	0.0416	o	0.0466
_	0.0017	o	0.0067	o	0.0117	o	0.0167	o	6 0.0217	o	0.0267	0	0.0317	o	0.0367	o	0.0417	o	0.0467
_	0.0018	o	0.0068	o	0.0118	o	0.0168	o	6 0.0218	o	0.0268	o	0.0318	o	0.0368	o	0.0418	o	0.0468
_	0.0019	o	0.0069	o	0.0119	o	0.0169	o	6 0.0219	o	0.0269	o	0.0319	o	0.0369	o	0.0419	o	0.0469
_	0.0020	o	0.0070	o	0.0120	o	0.0170	o	6 0.0220	_ _	0.0270	6	0.0320	o	0.0370	o	0.0420	o	0.0470
_	0.0021	o	0.0071	o	0.0121	o	0.0171	o	6 0.0221	- О	0.0271	o	0.0321	o	0.0371	o	0.0421	o	0.0471
_	0.0022	o	0.0072	o	0.0122	o	0.0172	o	6 0.0222	o	0.0272	o	0.0322		0.0372	o	0.0422	o	0.0472
	0.0023	o	0.0073	o	0.0123	o	0.0173	o			0.0273	o	0.0323		0.0373	o	0.0423	o	0.0473
	0.0024	o	0.0074	o	0.0124	o	0.0174	o			0.0274	o	0.0324		0.0374	o	0.0424	o	0.0474
	0.0025	o	0.0075	o	0.0125	o	0.0175	o			0.0275	o	0.0325		0.0375	o	0.0425	o	0.0475
	0.0026	o	0.0076	o	0.0126	o	0.0176	o			0.0276	0	0.0326		0.0376	o	0.0426	o	0.0476
	0.0027	o	0.0077	o	0.0127	o	0.0177	o			0.0277	0	0.0327		0.0377	o	0.0427	o	0.0477
	0.0028	o	0.0078	o	0.0128	o	0.0178	o			0.0278	0	0.0328	o	0.0378	o	0.0428	o	0.0478
	0.0029	σ.	0.0079	σ.	0.0129	σ.	0.0179	σ.			0.0279	σ.	0.0329	σ.	0.0379	σ,	0.0429	σ.	0.0479
	0.0030	on .	0.0080	on .	0.0130	on .	0.0180	on .			0.0280		0.0330	on .	0.0380	on .	0.0430	on .	0.0480
	0.003	o	0.00	o	0.0131	o	0.00	o			0.0281	o .	0.03		0.0381	o	0.0431	o	0.0481
	0.0032	o	0.0082	o	0.0132	o	0.0182	o			0.0282	0	0.0332		0.0382	o	0.0432	o	0.0482
	0.0033	o	0.0083	o	0.0133	o	0.0183	o			0.0283	o	0.033		0.0383	o	0.0433	o	0.0483
	0.0034	o	0.0084	o	0.0134	o	0.0184	o			0.0284	o	0.0334		0.0384	o	0.0434	o	0.0484
	0.0035	0	0.0085	o	0.0135	o	0.0185	o			0.0285	0	0.0335	o	0.0385	o	0.0435	o	0.0485
	0.0036	o	0.008	o	0.0136	o	0.0186	o			0.0286	0	0.0336		0.0386	o	0.0436	o	0.0486
	0.0037	o	0.0087	o	0.0137	o	0.0187	o			0.0287	0	0.0337		0.0387	o	0.0437	o	0.0487
	0.0038	o	0.008	o	0.0138	o	0.0188	o			0.0288	0	0.0338		0.0388	o	0.0438	o	0.0488
	0.0039	on.	0.0089	on	0.0139	m	0.0189	m			0.0289	o o	0.0339		0.0389	o	0.0439	on	0.0489
	0.0040	o	0.0090	o	0.0140	o	0.0190	o			0.0290	0	0.0340	o	0.0390	o	0.0440	o	0.0490
	0.0041	o	0.0091	o	0.0141	o	0.0191	o			0.0291	o	0.0341	o	0.0391	o	0.0441	o	0.0491
_	0.0042	o	0.0092	0	0.0142	o	0.0192	o			0.0292	0	0.0342	o	0.0392	o	0.0442	o	0.0492
	0.0043	o	0.0093	0	0.0143	o	0.0193	o			0.0293	0	0.0343	o	0.0393	o	0.0443	o	0.0493
_	0.0044	o	0.0094	0	0.0144	o	0.0194	o			0.0294	0	0.0344	o	0.0394	o	0.0444	o	0.0494
_	0.0045	o	0.0095	o	0.0145	o	0.0195	o			0.0295	0	0.0345		0.0395	o	0.0445	o	0.0495
	0.0046	o	0.0096	o	0.0146	o	0.0196	o	6 0.0246	o	0.0296	o	0.0346		0.0396	o	0.0446	o	0.0496
_	0.0047	6	0.0097	6	0.0147	o	0.0197	o			0.0297	6	0.0347	o	0.0397	o	0.0447	o	0.0497
_	0.0048	6	0.0098	6	0.0148	o	0.0198	o	6 0.0248	0	0.0298	6	0.0348	o	0.0398	o	0.0448	o	0.0498
_	0.0049	o	0.0099	o	0.0149	o	0.0199	o	6 0.0249	_	0.0299	o	0.0349	o	0.0399	o	0.0449	o	0.0499
	0.0000	1		ſ	0.000	•	0.000	•											

Figure 48: A chart outlining the number of unique spins within the lattice for various field magnitudes in the 111 direction for a decreasing field. The spin configuration was initially the groundstate, and used 2000 iterations.

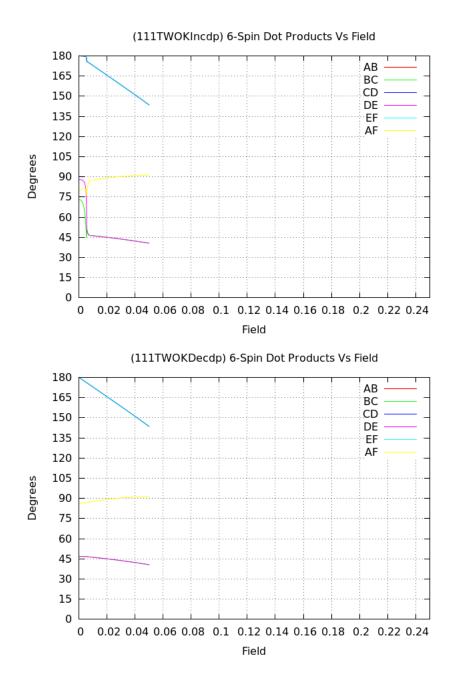


Figure 49: Dot products of the 6 characteristic spins for both increasing and decreasing fields.

20 2K (111) Increasing Field, Random State

Very similar to run 1, where the lattice starts off at a ground state configuration and snaps into a planar configuration, followed by gradual alignment with the field. Note: the first data point in the energy and magnetization plots were removed since it was much higher than any other points on the graph, which caused the plot to become flattened in order to fit the entire range of data onto the same plot. This is likely due to the fact that 2000 iterations is insufficient for the energy to be minimized, and so starting from a random initial configuration causes the energy of the lattice at the first field value (B=0) to be much higher than the ground state since it has yet to become a ground state.

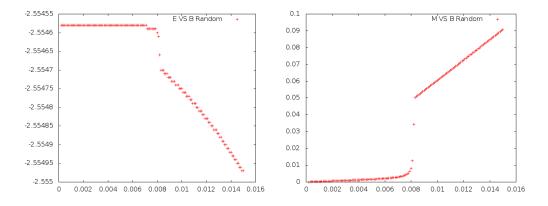


Figure 50: Energy vs increasing field and Magnetization versus increasing field

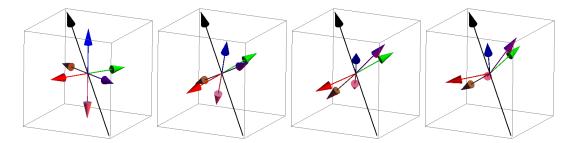


Figure 51: Snapshots of the 6 characteristic spins of the lattice. Fields not recorded, but snapshot 1 is the groundstate at zero field, the second is prior to the point of inflection, the third is immediately after the point of inflection, and the fourth is likely at 0.05, the maximum field for this simulation.

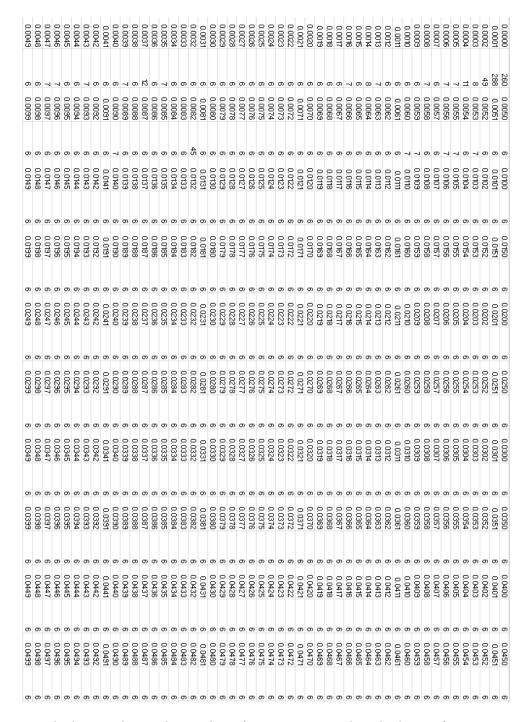


Figure 52: A chart outlining the number of unique spins within the lattice for increasing field magnitude in the 111 direction. The lattice was initially random. EFM used 2000 iterations.

21 2K (111) Decreasing Field, Random State

Between the first two images of the 6 spins, there is little difference even though the field had changed by about 0.03. At 0.21, the 6 spins undergo sudden and rapid changes in orientation. Eventually, the 6 spins rest in a near planar state, and gradually relax to a full planar state at B=0. When referring to the spin chart, it's clear that trying to visualize the entire lattice by choosing 6 spins won't work, since there are far more than 6 spins for the majority of fields. An alternate approach was used, which involved looking at a small, manageable section of the entire lattice.

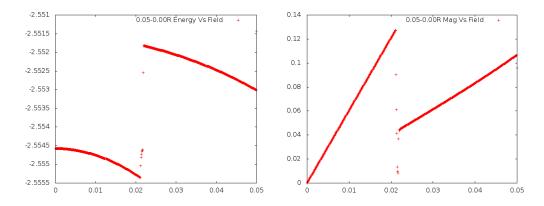


Figure 53: Energy vs decreasing field and Magnetization versus decreasing field

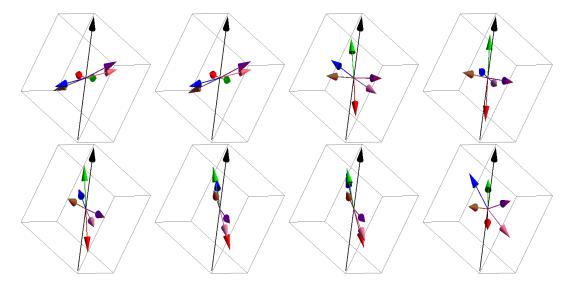


Figure 54: Snapshots of the 6 characteristic spins of the lattice over the course of increasing field

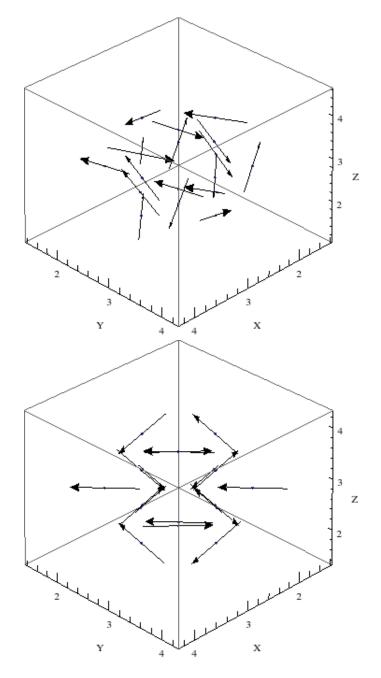


Figure 55: Visualization of a small section of the entire lattice used in the 2K iteration 111 decreasing field simulation. The spins are initially highly disordered, until they snap into a final planar configuration. The field at which this transition occurred can be deduced from the spin chart for this simulation. Ie, at H=0.02 when the number of unique spins equal 6.

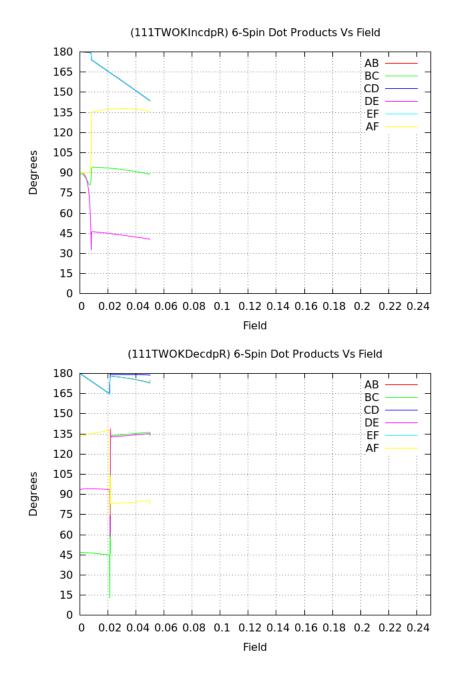


Figure 56: Dot products of the characteristic spins for increasing and decreasing field. Since this was a preliminary simulation, the field was only increased to 0.05.

88 0.03 88 0.028 88 0.028 88 0.0298 88 0.0298 88 0.0294 88 0.0294 88 0.0293 84 0.0293 84 0.0289 84 0.0288 84 0.0288 86 0.0288 87 0.0288 88 0.0288 88 0.0288 88 0.0288 88 0.0277 88 0.0277 88 0.0277 88 0.0277 89 0.0277 80 0.0277 80 0.0277 80 0.0277 80 0.0277 80 0.0277 80 0.0268 80 0.0277 80 0.0268 80 0.0277 80 0.0268 80 0.0277 80 0.0268 80 0.0278 80 0.0277 80 0.0268	o	0.0001	o	0.0051	б	0.0101	o	0.0151	o	0.0201	78	0.0251	82	0.0301	8	0.0351	20	0.0401	8	0.0451
Big Dig	o	0.0002	o	0.0052	o	0.0102	o	0.0152	o	0.0202	82	0.0252	82	0.0302	92	0.0352	20	0.0402	8	0.0452
Bolith Color Boli	o	0.0003	o	0.0053	o	0.0103	o	0.0153	o	0.0203	8	0.0253	82	0.0303	8	0.0353	20	0.0403	8	0.0453
Bolith B	o	0.0004	o	0.0054	o	0.0104	o	0.0154	o	0.0204	8	0.0254		0.0304	8	0.0354	20	0.0404	8	0.0454
Bolitical Colores Boli	o	0.0005	6	0.0055	o	0.0105	o	0.0155	o	0.0205	8	0.0255	22	0.0305	9	0.0355	20	0.0405	88	0.0455
Be Dicas Color	o	0.0006	o	0.0056	o	0.0106	o	0.0156	o	0.0206	8	0.0256		0.0306	9	0.0356	84	0.0406	88	0.0456
Beauty Colore C	o	0.0007	o	0.0057	-	0.0107	o	0.0157	o	0.0207	8	0.0257		0.0307		0.0357	8	0.0407	22	0.0457
Beauty	o	0.0008	6	0.0058	0	0.0108	6	0.0158	o	0.0208	82	0.0258		0.0308	9	0.0358	8	0.0408	22	0.0458
Beauty Colore C	o	0.0009	o	0.0059	o	0.0109	0	0.0159	o	0.0209	82	0.0259	8	0.0309	9	0.0359	84	0.0409	8	0.0459
Box Doug	o	0.001	o	0.006	o	0.011	o	0.016	o	0.021	82	0.026	8	0.031	9	0.036	82	0.041	8	0.0460
Box Document Doc	o	0.0011	o	0.0061	o	0.0111	o	0.0161	00	0.0211	82	0.0261	8	0.0311	9	0.0361	82	0.0411	22	0.0461
Box Doug	o	0.0012	o	0.0062	o	0.0112	o	0.0162	ಕ	0.0212	œ	0.0262		0.0312	8	0.0362	82	0.0412	22	0.0462
Box Dig	o	0.0013	o	0.0063	o	0.0113	o	0.0163	9	0.0213	8	0.0263	8	0.0313	8	0.0363	20	0.0413	2	0.0463
Mathematical Colores	o	0.0014	o	0.0064	o	0.0114	o	0.0164	9	0.0214	20	0.0264		0.0314	8	0.0364	88	0.0414	8	0.0464
Box Dig	o	0.0015	o	0.0065		0.0115	o	0.0165	ⅎ	0.0215	8	0.0265	8	0.0315	92	0.0365	8	0.0415	8	0.0465
Box Dig Box	o	0.0016	o	0.0066	o	0.0116	o	0.0166	9	0.0216	88	0.0266	8	0.0316	92	0.0366	88	0.0416	8	0.0466
Box Dig Box	o	0.0017	o	0.0067	-	0.0117	o	0.0167		0.0217	88	0.0267	88	0.0317	94	0.0367	88	0.0417	22	0.0467
Box Doug Box	o	0.0018	o	0.0068		0.0118	o	0.0168	8	0.0218	8	0.0268	88	0.0318	92	0.0368	88	0.0418	2	0.0468
Box Doug Ray Doug Ray Doug Ray Doug Ray Doug Ray Doug Ray	o	0.0019	o	0.0069	o	0.0119	o	0.0169	큠	0.0219	8	0.0269	8	0.0319	92	0.0369	88	0.0419	82	0.0469
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0143 6 0.0049 88 0.0239 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0239 82 0.0247 78 0.0198 6 0.0048 6 0.0048 88 0.0239 82 0.0244 78 0.0193 6 0.0144 6 0.0043 6 0.0044 88 0.0234 80 0.0244 78 0.0193 6 0.0143 6 0.0043 6 0.0043 88 0.0233 80 0.0244 78 0.0193 6 0.0143 6 0.0043 6 0.0043 88 0.0231 80 0.0232 80 0.0183 6 0.0133 6 0.0033 6 0.00	o	0.002	o	0.007	o	0.012	o	0.017	98	0.022	9	0.027	8	0.032	94	0.037	88	0.042	8	0.0470
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0143 6 0.0043 88 0.0239 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0239 82 0.0247 78 0.0198 6 0.0048 6 0.0047 88 0.0234 80 0.0244 80 0.0194 6 0.0144 6 0.0043 6 0.0044 88 0.0234 80 0.0244 80 0.0193 6 0.0143 6 0.0043 88 0.0233 80 0.0244 78 0.0193 6 0.0143 6 0.0042 80 0.0231 80 0.0244 78 0.0193 6 0.0144 6 0.0043 81 0.0233 81 0.0233 81	o	0.0021	o	0.0071	o	0.0121	o	0.0171	œ	0.0221	9	0.0271		0.0321		0.0371	88	0.0421	8	0.0471
88 0.03 83 0.025 78 0.02 6 0.015 6 0.015 6 0.005 88 0.0239 82 0.0249 78 0.0198 6 0.0143 6 0.0049 88 0.0239 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0237 82 0.0244 78 0.0193 6 0.0145 6 0.0047 88 0.0236 82 0.0244 78 0.0193 6 0.0145 6 0.0044 88 0.0234 80 0.0244 80 0.0193 6 0.0143 6 0.0044 88 0.0233 80 0.0241 78 0.0193 6 0.0141 6 0.0042 80 0.0234 78 0.0234 78 0.0189 6 0.0141 6 0.0043 81 0.0237 80 0.0237 80 </td <td>o</td> <td>0.0022</td> <td>o</td> <td>0.0072</td> <td>-</td> <td>0.0122</td> <td>o</td> <td>0.0172</td> <td>87</td> <td>0.0222</td> <td>20</td> <td>0.0272</td> <td>8</td> <td>0.0322</td> <td>æ</td> <td>0.0372</td> <td>8</td> <td>0.0422</td> <td>8</td> <td>0.0472</td>	o	0.0022	o	0.0072	-	0.0122	o	0.0172	87	0.0222	20	0.0272	8	0.0322	æ	0.0372	8	0.0422	8	0.0472
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0143 6 0.0049 88 0.0239 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0237 82 0.0247 78 0.0196 6 0.0047 6 0.0047 88 0.0236 82 0.0244 78 0.0195 6 0.0146 6 0.0046 6 0.0047 88 0.0236 82 0.0244 78 0.0193 6 0.0143 6 0.0044 6 0.0044 88 0.0233 80 0.0241 78 0.0193 6 0.0141 6 0.0042 88 0.0233 81 0.0234 78 0.0189 6 0.0141 6 0.0033 6 0.0042 80 </td <td>o</td> <td>0.0023</td> <td>o</td> <td>0.0073</td> <td>-</td> <td>0.0123</td> <td>o</td> <td>0.0173</td> <td></td> <td>0.0223</td> <td>8</td> <td>0.0273</td> <td></td> <td>0.0323</td> <td>8</td> <td>0.0373</td> <td>8</td> <td>0.0423</td> <td>8</td> <td>0.0473</td>	o	0.0023	o	0.0073	-	0.0123	o	0.0173		0.0223	8	0.0273		0.0323	8	0.0373	8	0.0423	8	0.0473
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0149 6 0.0049 88 0.0238 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0237 82 0.0247 78 0.0198 6 0.0148 6 0.0048 88 0.0237 82 0.0244 78 0.0195 6 0.0144 6 0.0045 6 0.0044 88 0.0234 80 0.0244 80 0.0193 6 0.0144 6 0.0043 6 0.0043 88 0.0233 80 0.0244 78 0.0193 6 0.0143 6 0.0043 88 0.0233 80 0.0244 78 0.0193 6 0.0143 6 0.0042 80 0.0233 81<	o	0.0024	o	0.0074		0.0124	o	0.0174	20	0.0224	82	0.0274		0.0324	8	0.0374	8	0.0424	8	0.0474
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0149 6 0.0049 88 0.0238 82 0.0244 78 0.0198 6 0.0143 6 0.0048 88 0.0238 82 0.0247 78 0.0198 6 0.0143 6 0.0048 88 0.0237 82 0.0247 78 0.0198 6 0.0144 6 0.0049 6 0.0044 88 0.0234 80 0.0244 80 0.0193 6 0.0144 6 0.0043 6 0.0043 88 0.0233 80 0.0241 78 0.0193 6 0.0143 6 0.0042 88 0.0233 80 0.0241 78 0.0193 6 0.0141 6 0.0043 80 0.0233 81<	o	0.0025	o	0.0075		0.0125	o	0.0175		0.0225	88	0.0275		0.0325		0.0375	20	0.0425	8	0.0475
88 0.03 83 0.025 78 0.012 6 0.014 6 0.004 89 0.0239 82 0.0249 78 0.0193 6 0.0143 6 0.0043 88 0.0238 82 0.0244 78 0.0198 6 0.0048 6 0.0049 88 0.0236 82 0.0247 78 0.0198 6 0.0143 6 0.0047 6 0.0047 88 0.0237 82 0.0244 78 0.0195 6 0.0144 6 0.0045 6 0.0044 88 0.0234 80 0.0244 80 0.0193 6 0.0143 6 0.0044 88 0.0233 80 0.0242 78 0.0193 6 0.0143 6 0.0043 80 0.0231 80 0.0244 78 0.0193 6 0.0144 6 0.0034 6 0.0034 84 </td <td>o</td> <td>0.0026</td> <td>o</td> <td>0.0076</td> <td>-</td> <td>0.0126</td> <td>o</td> <td>0.0176</td> <td></td> <td>0.0226</td> <td>88</td> <td>0.0276</td> <td></td> <td>0.0326</td> <td></td> <td>0.0376</td> <td>82</td> <td>0.0426</td> <td>8</td> <td>0.0476</td>	o	0.0026	o	0.0076	-	0.0126	o	0.0176		0.0226	88	0.0276		0.0326		0.0376	82	0.0426	8	0.0476
88 0.03 83 0.025 78 0.02 6 0.015 6 0.004 88 0.0239 82 0.0249 78 0.0198 6 0.0149 6 0.0049 88 0.0239 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0237 82 0.0247 78 0.0193 6 0.0147 6 0.0047 88 0.0237 82 0.0244 78 0.0195 6 0.0145 6 0.0044 88 0.0234 80 0.0244 78 0.0193 6 0.0144 6 0.0044 88 0.0233 80 0.0241 78 0.0193 6 0.0141 6 0.0042 80 0.0231 80 0.0241 78 0.0193 6 0.0141 6 0.0042 81 0.0233 80 0.0189 6 0.0138 6 </td <td>o</td> <td>0.0027</td> <td>o</td> <td>0.0077</td> <td></td> <td>0.0127</td> <td>o</td> <td>0.0177</td> <td>8</td> <td>0.0227</td> <td>78</td> <td>0.0277</td> <td></td> <td>0.0327</td> <td>99</td> <td>0.0377</td> <td>82</td> <td>0.0427</td> <td>22</td> <td>0.0477</td>	o	0.0027	o	0.0077		0.0127	o	0.0177	8	0.0227	78	0.0277		0.0327	99	0.0377	82	0.0427	22	0.0477
88 0.023 83 0.025 78 0.025 6 0.014 6 0.005 82 0.0239 82 0.0249 78 0.018 6 0.0149 6 0.0049 82 0.0239 82 0.0244 78 0.018 6 0.0143 6 0.0039 6 0.0049 83 0.0239 82 0.0247 78 0.0198 6 0.0143 6 0.0039 6 0.0044 83 0.0236 82 0.0247 78 0.0198 6 0.0144 6 0.0037 6 0.0047 83 0.0237 82 0.0246 78 0.0195 6 0.0145 6 0.0037 6 0.0047 83 0.0236 82 0.0244 80 0.0195 6 0.0145 6 0.0035 6 0.0044 83 0.0234 80 0.0244 80 0.0193 6 0.0143 6 0.0033 6 0.0043 83 0.0237 82 0.0241 80 0.0193 6 0.0143 6 0.0033 6 0.0043 83 0.0237 82 0.0241 78 0.0193 6 0.0141 6 0.0031 6 0.0042 83 0.0231 80 0.0241 78 0.0193 6 0.0141 6 0.0031 6 0.0044 84 0.0238 81 0.0239 81 0.0239 81 0.0239 81 0.0239 81 0.0239 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0236 81 0.0336 81 0.0336 81 0.0336 81 0.0336 81 0.0336 81 0.0337 82 0.0185 81 0.0336 81 0	o	0.0028	o	0.0078		0.0128	თ	0.0178		0.0228	78	0.0278		0.0328		0.0378	8	0.0428	22	0.0478
88 0.03 83 0.025 78 0.025 6 0.014 6 0.004 89 0.0239 82 0.0249 78 0.019 6 0.0149 6 0.0049 88 0.0236 82 0.0244 78 0.0198 6 0.0048 6 0.0048 88 0.0236 82 0.0244 78 0.0198 6 0.0144 6 0.0048 88 0.0237 82 0.0244 78 0.0196 6 0.0144 6 0.0048 88 0.0234 80 0.0244 78 0.0193 6 0.0143 6 0.0043 88 0.0233 80 0.0244 80 0.0193 6 0.0143 6 0.0043 80 0.0234 80 0.0244 78 0.0193 6 0.0143 6 0.0043 81 0.0233 81 0.0234 81 0.0133 6<	o	0.0029	o	0.0079		0.0129	o	0.0179		0.0229	8	0.0279		0.0329		0.0379	82	0.0429	8	0.0479
88 0.023 83 0.025 78 0.029 6 0.014 6 0.005 8 0.005 8 0.0239 82 0.0249 77 0.0199 6 0.0149 6 0.0099 6 0.0049 8 0.0239 82 0.0249 77 0.0198 6 0.0149 6 0.0099 6 0.0049 8 0.0239 82 0.0247 78 0.0198 6 0.0148 6 0.0098 6 0.0047 8 0.0239 82 0.0247 78 0.0198 6 0.0147 6 0.0099 6 0.0047 8 0.0047 8 0.0239 82 0.0244 78 0.0195 6 0.0144 6 0.0099 6 0.0044 8 0.0239 80 0.0245 80 0.0195 6 0.0145 6 0.0094 6 0.0044 8 0.0239 80 0.0244 80 0.0193 6 0.0143 6 0.0093 6 0.0044 8 0.0239 80 0.0242 78 0.0193 6 0.0142 6 0.0093 6 0.0044 8 0.0239 80 0.0241 78 0.0193 6 0.0142 6 0.0093 6 0.0044 8 0.0239 80 0.0241 78 0.0193 6 0.0143 6 0.0093 6 0.0044 8 0.0239 80 0.0241 78 0.0193 6 0.0143 6 0.0093 6 0.0043 8 0.0239 80 0.0234 80 0.0189 6 0.0133 6 0.0039 6 0.0033 8 0.0234 80 0.0239 80 0.0239 80 0.033 6 0.0133 6 0.0093 6 0.0033 8 0.0234 80 0.0237 80 0.0185 6 0.0133 6 0.0089 6 0.0033 8 0.0234 80 0.0235 80 0.0135 6 0.0035 6 0.0035 8 0.0037 8 0.0234 80 0.0185 6 0.0135 6 0.0085 6 0.0034 8 0.0234 80 0.0235 8 0.0135 6 0.0085 6 0.0034 8 0.0033 8 0.0234 8 0.0234 8 0.0033 6 0.0035 6 0.0034 6 0.0035 8 0.0034 8 0.0234 8 0.0235 8 0.0033 6 0.0035 6 0.0034 6 0.0035 8 0.0034 8 0.0033 8 0.0234 8 0.0234 8 0.0033 6 0.0035 6 0.0034 6 0.0034 8 0.0033 8 0.0234 8 0.0234 8 0.0033 6 0.0035 6 0.0034 6 0.0035 8 0.0033 8 0.0234 8 0.0234 8 0.0033 6 0.0035 6 0.0034 6 0.0035 8 0.0033 8 0.0234 8 0.0234 8 0.0033 6 0.0035 6 0.0034 6 0.0035 8 0.0033 8 0.0234 8 0.0033 8 0.0035 6 0.0035 6 0.0034 6 0.0035 8	o	0.003	o	0.008	o	0.013	o	0.018	20	0.023	8	0.028		0.033		0.038	82	0.043	8	0.0480
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 8 0.023 82 0.0249 78 0.018 6 0.0149 6 0.0049 8 0.0239 82 0.0249 78 0.018 6 0.0143 6 0.0049 8 0.0239 82 0.0247 78 0.018 6 0.0143 6 0.0038 6 0.0048 8 0.0237 82 0.0247 78 0.0198 6 0.0147 6 0.0037 6 0.0047 8 0.0237 82 0.0246 78 0.0198 6 0.0147 6 0.0037 6 0.0047 8 0.0237 82 0.0245 78 0.0185 6 0.0145 6 0.0035 6 0.0045 8 0.0234 80 0.0234 80 0.0245 80 0.0185 6 0.0145 6 0.0035 6 0.0044 8 0.0232 82 0.0244 80 0.0183 6 0.0143 6 0.0033 6 0.0043 8 0.0234 80 0.0244 78 0.0193 6 0.0142 6 0.0033 6 0.0044 8 0.0232 82 0.0241 78 0.0193 6 0.0141 6 0.0031 6 0.0041 8 0.0231 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 8 0.0231 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 8 0.0231 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 8 0.0231 80 0.0231 80 0.0181 6 0.0131 6 0.0031 6 0.0031 8 0.0033 8	o	0.0031	o	0.0081		0.0131	o	0.0181		0.0231	8	0.0281		0.0331		0.0381	82	0.0431	8	0.0481
88 0.023 83 0.025 78 0.025 6 0.014 6 0.005 88 0.0239 82 0.0249 78 0.018 6 0.0143 6 0.0049 88 0.0239 82 0.0249 78 0.018 6 0.0143 6 0.0039 88 0.0239 82 0.0247 78 0.0180 6 0.0143 6 0.0039 6 0.0044 88 0.0237 82 0.0247 78 0.0193 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0246 78 0.0195 6 0.0145 6 0.0037 6 0.0047 88 0.0236 82 0.0245 78 0.0195 6 0.0145 6 0.0036 6 0.0045 88 0.0237 80 0.0245 80 0.0184 6 0.0145 6 0.0039 6 0.0044 88 0.0238 80 0.0244 80 0.0193 6 0.0143 6 0.0033 6 0.0043 88 0.0239 80 0.0241 78 0.0193 6 0.0141 6 0.0031 6 0.0042 82 0.0231 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 84 0.0238 81 0.0234 78 0.0181 6 0.0141 6 0.0039 6 0.0041 84 0.0238 81 0.0238 81 0.0183 6 0.0138 6 0.0038 6 0.0038 84 0.0238 81 0.0238 81 0.0183 6 0.0138 6 0.0038 6 0.0037 84 0.0236 79 0.0236 82 0.0186 6 0.0136 6 0.0086 6 0.0037 84 0.0286 79 0.0236 82 0.0186 6 0.0136 6 0.0086 6 0.0037 85 0.0284 78 0.0234 82 0.0186 6 0.0135 6 0.0086 6 0.0035 86 0.0283 78 0.0234 82 0.0184 6 0.0134 6 0.0086 6 0.0035 86 0.0283 78 0.0234 82 0.0184 6 0.0135 6 0.0084 6 0.0035	o	0.0032	o	0.0082		0.0132	o	0.0182		0.0232	78	0.0282		0.0332		0.0382	82	0.0432	2	0.0482
88 0.023 83 0.025 78 0.029 6 0.014 6 0.005 88 0.0239 82 0.0249 78 0.019 6 0.0149 6 0.0049 88 0.0239 82 0.0249 78 0.0198 6 0.0149 6 0.0049 88 0.0239 82 0.0247 78 0.0198 6 0.0148 6 0.0039 6 0.0049 88 0.0239 82 0.0247 78 0.0198 6 0.0147 6 0.0039 6 0.0047 88 0.0239 82 0.0246 78 0.0197 6 0.0147 6 0.0039 6 0.0047 88 0.0239 80 0.0245 78 0.0196 6 0.0145 6 0.0039 6 0.0045 88 0.0239 80 0.0245 80 0.0194 6 0.0143 6 0.0039 6 0.0043 88 0.0239 80 0.0241 78 0.0193 6 0.0143 6 0.0033 6 0.0042 88 0.0239 80 0.0241 78 0.0193 6 0.0141 6 0.0039 6 0.0041 88 0.0239 80 0.0241 78 0.013 6 0.0141 6 0.003 6 0.0041 88 0.0239 80 0.0241 78 0.013 6 0.0141 6 0.003 6 0.0041 88 0.0239 80 0.0241 78 0.013 6 0.0141 6 0.003 6 0.0041 88 0.0239 80 0.0241 78 0.013 6 0.013 6 0.003 6 0.0041 88 0.0238 80 0.0237 80 0.0189 6 0.013 6 0.0089 6 0.003 88 0.0238 80 0.0237 80 0.0189 6 0.013 6 0.0089 6 0.003 89 0.0238 81 0.0238 81 0.0189 6 0.013 6 0.0089 6 0.003 80 0.0238 80 0.0237 80 0.0189 6 0.013 6 0.0089 6 0.003 80 0.0238 80 0.0237 80 0.0189 6 0.013 6 0.0089 6 0.003 80 0.0238 80 0.0238 81 0.0189 6 0.013 6 0.0089 6 0.003 80 0.0239 80 0.0236 80 0.0189 6 0.013 6 0.0089 6 0.003 80 0.0239 80 0.0236 80 0.0189 6 0.013 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.013 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.013 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.013 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.013 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0089 6 0.0033 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0189 6 0.0139 6 0.0039 80 0.0239 80 0.0239 80 0.0239 80	o	0.0033	o	0.0083		0.0133	o	0.0183	æ	0.0233	78	0.0283	8	0.0333	8	0.0383	20	0.0433	22	0.0483
88 0.03 83 0.025 78 0.022 6 0.015 6 0.005 89 0.0239 82 0.0249 78 0.0193 6 0.0149 6 0.0049 89 0.0236 82 0.0244 78 0.0198 6 0.0149 6 0.0049 88 0.0236 82 0.0244 78 0.0198 6 0.0147 6 0.0047 88 0.0237 82 0.0244 78 0.0198 6 0.0144 6 0.0049 6 0.0047 88 0.0236 82 0.0244 80 0.0194 6 0.0144 6 0.0045 6 0.0045 88 0.0234 80 0.0245 80 0.0193 6 0.0143 6 0.0043 80 0.0231 80 0.0241 78 0.0193 6 0.0143 6 0.0042 81 0.0233 81<	o	0.0034	o	0.0084		0.0134	o	0.0184	8	0.0234	78	0.0284		0.0334		0.0384	8	0.0434	22	0.0484
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 88 0.0239 82 0.0249 78 0.0198 6 0.0143 6 0.0098 6 0.0049 88 0.0239 82 0.0248 78 0.0198 6 0.0143 6 0.0098 6 0.0043 88 0.0237 82 0.0247 78 0.0198 6 0.0147 6 0.0039 6 0.0044 88 0.0237 82 0.0247 78 0.0197 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0245 78 0.0198 6 0.0148 6 0.0096 6 0.0048 88 0.0236 80 0.0245 80 0.0198 6 0.0145 6 0.0095 6 0.0048 88 0.0237 80 0.0244 80 0.0198 6 0.0144 6 0.0095 6 0.0044 88 0.0238 80 0.0244 80 0.0193 6 0.0144 6 0.0093 6 0.0044 88 0.0239 80 0.0244 78 0.0193 6 0.0143 6 0.0092 6 0.0044 88 0.0231 80 0.0241 78 0.0191 6 0.0143 6 0.0093 6 0.0041 84 0.0280 80 0.0241 78 0.0193 6 0.0143 6 0.0093 6 0.0041 84 0.0280 80 0.0241 78 0.0193 6 0.013 6 0.0093 6 0.0033 84 0.0280 80 0.0239 81 0.0188 6 0.0133 6 0.0087 6 0.0033 84 0.0286 79 0.0236 82 0.0186 6 0.0136 6 0.0087 6 0.0037 84 0.0286 79 0.0236 82 0.0186 6 0.0136 6 0.0087 6 0.0037	o	0.0035	o	0.0085		0.0135	o	0.0185		0.0235	78	0.0285		0.0335	88	0.0385	8	0.0435	22	0.0485
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 88 0.0239 82 0.0249 78 0.018 6 0.0143 6 0.0049 88 0.0239 82 0.0248 78 0.018 6 0.0143 6 0.0049 88 0.0237 82 0.0248 78 0.0188 6 0.0143 6 0.0038 6 0.0048 88 0.0237 82 0.0247 78 0.0198 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0246 78 0.0187 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0245 78 0.0186 6 0.0145 6 0.0036 6 0.0048 88 0.0236 80 0.0245 80 0.0185 6 0.0145 6 0.0035 6 0.0044 88 0.0237 80 0.0244 80 0.0194 6 0.0045 6 0.0045 88 0.0233 80 0.0244 80 0.0193 6 0.0144 6 0.0033 6 0.0043 88 0.0232 82 0.0242 78 0.0183 6 0.0142 6 0.0033 6 0.0043 88 0.0232 80 0.0241 78 0.0193 6 0.0142 6 0.0031 6 0.0041 89 0.0230 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 80 0.0230 80 0.0241 78 0.0191 6 0.0141 6 0.0031 6 0.0041 80 0.0230 80 0.0241 78 0.0193 6 0.0143 6 0.0031 6 0.0041 81 0.0230 80 0.0241 78 0.0193 6 0.0143 6 0.0031 6 0.0041 82 0.0230 80 0.0241 78 0.0193 6 0.0143 6 0.0031 6 0.0041 83 0.0230 80 0.0231 80 0.0193 6 0.0133 6 0.0033 84 0.0238 80 0.0238 81 0.0239 80 0.0138 6 0.0038 6 0.0033	o	0.0036	o	0.0086		0.0136	o	0.0186		0.0236	79	0.0286	2	0.0336		0.0386	8	0.0436	22	0.0486
88 0.023 83 0.025 78 0.02 6 0.014 6 0.016 6 0.004 8 0.023 82 0.0249 78 0.029 6 0.0149 6 0.0049 6 0.0049 8 0.0239 82 0.0249 78 0.018 6 0.0149 6 0.0049 6 0.0049 8 0.0239 82 0.0248 78 0.018 6 0.0148 6 0.0038 6 0.0048 8 0.0236 82 0.0247 78 0.018 6 0.0147 6 0.0037 6 0.0047 8 0.0247 78 0.018 6 0.0147 6 0.0037 6 0.0047 8 0.0235 80 0.0245 78 0.018 6 0.0145 6 0.0036 6 0.0045 8 0.0235 80 0.0245 80 0.018 6 0.0145 6 0.0044 6 0.0033 6 0.0044 8 0.0235 80 0.0244 80 0.018 6 0.0143 6 0.0033 6 0.0043 8 0.0234 80 0.0243 80 0.018 6 0.0143 6 0.0033 6 0.0043 8 0.023 82 0.0241 78 0.018 6 0.0141 6 0.0031 6 0.0041 8 0.023 8 0.0242 78 0.018 6 0.0141 6 0.0031 6 0.0041 8 0.003 8 0.0044 8 0.023 8 0.023 8 0.0241 78 0.018 6 0.014 6 0.003 6 0.0043 8 0.0043 8 0.023 8 0.023 8 0.023 8 0.018 6 0.018 6 0.003 6 0.0043 8 0.003 8 0.023 8 0.023 8 0.023 8 0.023 8 0.018 6 0.018 6 0.003 6 0.003 8 0.003 8 0.023 8 0.023 8 0.023 8 0.018 6 0.018 6 0.003 6 0.003 8 0.00	o	0.0037	o	0.0087		0.0137	o	0.0187		0.0237	78	0.0287		0.0337		0.0387	8	0.0437	88	0.0487
88 0.03 83 0.025 78 0.02 6 0.075 6 0.075 6 0.005 89 0.0259 82 0.0249 78 0.0199 6 0.0049 6 0.0049 80 0.0259 82 0.0249 78 0.0199 6 0.0049 6 0.0049 80 0.0258 82 0.0248 78 0.0198 6 0.0143 6 0.0098 6 0.0048 80 0.0257 82 0.0247 78 0.0198 6 0.0147 6 0.0057 6 0.0047 80 0.0258 82 0.0246 78 0.0197 6 0.0146 6 0.0059 6 0.0047 80 0.0259 80 0.0245 80 0.0198 6 0.0145 6 0.0095 6 0.0045 80 0.0259 80 0.0245 80 0.0194 6 0.0145 6 0.0095 6 0.0043 80 0.0259 80 0.0244 80 0.0194 6 0.0143 6 0.0093 6 0.0043 80 0.0259 80 0.0241 78 0.0193 6 0.0142 6 0.0093 6 0.0042 81 0.0259 80 0.0241 78 0.0192 6 0.0141 6 0.0093 6 0.0041 82 0.0259 80 0.0241 78 0.0193 6 0.0141 6 0.0093 6 0.0041 83 0.0259 80 0.0241 78 0.0193 6 0.0141 6 0.0093 6 0.0041 84 0.0259 80 0.0241 78 0.0193 6 0.0141 6 0.0093 6 0.0041 85 0.0259 80 0.0241 78 0.0193 6 0.0141 6 0.0093 6 0.0041 86 0.0259 80 0.0241 78 0.0193 6 0.0141 6 0.0093 6 0.0041	o	0.0038	o	0.0088		0.0138	o	0.0188		0.0238	8	0.0288		0.0338		0.0388	82	0.0438	88	0.0488
88 0.03 83 0.025 78 0.02 6 0.005 6 0.00 6 0.005 80 0.0259 82 0.0249 78 0.018 6 0.0149 6 0.0049 80 0.0259 82 0.0248 78 0.0189 6 0.0149 6 0.0059 80 0.0259 82 0.0248 78 0.0189 6 0.0143 6 0.0059 6 0.0048 80 0.0257 82 0.0247 78 0.0187 6 0.0147 6 0.0057 6 0.0047 80 0.0256 82 0.0246 78 0.0187 6 0.0147 6 0.0057 6 0.0048 80 0.0258 80 0.0245 80 0.0185 6 0.0146 6 0.0056 6 0.0045 80 0.0259 80 0.0245 80 0.0184 6 0.0145 6 0.0059 6 0.0044 80 0.0259 80 0.0245 80 0.0184 6 0.0144 6 0.0059 6 0.0043 80 0.0259 80 0.0243 80 0.0184 6 0.0143 6 0.0059 6 0.0043 81 0.0259 80 0.0241 78 0.0182 6 0.0141 6 0.0051 6 0.0041 82 0.0259 80 0.0241 78 0.0181 6 0.0141 6 0.0051 6 0.0041 83 0.0259 80 0.0241 78 0.0181 6 0.0141 6 0.0051 6 0.0041	o	0.0039	o	0.0089		0.0139	o	0.0189	8	0.0239	으	0.0289	20	0.0339		0.0389	20	0.0439	8	0.0489
88 0.03 83 0.025 78 0.02 6 0.075 6 0.07 6 0.005 80 0.0259 82 0.0249 78 0.078 6 0.049 6 0.0049 80 0.0239 82 0.0248 78 0.078 6 0.0143 6 0.0098 6 0.0049 80 0.0239 82 0.0248 78 0.0188 6 0.0143 6 0.0098 6 0.0048 80 0.0237 82 0.0247 78 0.0197 6 0.0147 6 0.0097 6 0.0047 80 0.0236 82 0.0246 78 0.0195 6 0.0146 6 0.0096 6 0.0046 80 0.0236 80 0.0245 80 0.0195 6 0.0145 6 0.0095 6 0.0045 80 0.0239 80 0.0244 80 0.0193 6 0.0144 6 0.0093 6 0.0044 80 0.0232 82 0.0243 80 0.0193 6 0.0142 6 0.0093 6 0.0043 80 0.0232 80 0.0244 78 0.0193 6 0.0142 6 0.0093 6 0.0043 80 0.0231 80 0.0241 78 0.0191 6 0.0141 6 0.0091 6 0.0041	o	0.004	o	0.009	o	0.014	o	0.019	78	0.024	8	0.029	2	0.034	8	0.039	8	0.044	88	0.0490
88 0.03 83 0.025 78 0.02 6 0.075 6 0.07 6 0.005 89 0.0259 82 0.0249 78 0.0199 6 0.049 6 0.0049 88 0.0239 82 0.0248 78 0.0198 6 0.0143 6 0.0039 6 0.0049 88 0.0239 82 0.0248 78 0.0198 6 0.0143 6 0.0039 6 0.0047 88 0.0237 82 0.0247 78 0.0197 6 0.0147 6 0.0037 6 0.0047 88 0.0238 80 0.0245 78 0.0195 6 0.0145 6 0.0095 6 0.0045 88 0.0239 80 0.0245 80 0.0195 6 0.0145 6 0.0095 6 0.0045 88 0.0239 80 0.0244 80 0.0194 6 0.0144 6 0.0095 6 0.0044 88 0.0233 80 0.0243 80 0.0194 6 0.0143 6 0.0093 6 0.0043 80 0.0239 82 0.0242 78 0.0193 6 0.0142 6 0.0092 6 0.0042	~	0.0041	o	0.0091	o	0.0141	o	0.0191	78	0.0241	8	0.0291		0.0341		0.0391	82	0.0441	8	0.0491
88 0.023 83 0.025 78 0.02 6 0.075 6 0.07 6 0.005 89 0.0239 82 0.0249 78 0.0199 6 0.0049 6 0.0049 88 0.0238 82 0.0248 78 0.0198 6 0.0048 6 0.0049 88 0.0238 82 0.0247 78 0.0198 6 0.0148 6 0.0038 6 0.0048 88 0.0236 82 0.0247 78 0.0197 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0245 78 0.0196 6 0.0146 6 0.0095 6 0.0047 88 0.0236 80 0.0245 80 0.0196 6 0.0145 6 0.0095 6 0.0045 88 0.0234 80 0.0244 80 0.0194 6 0.0143 6 0.0093 6 0.0044 88 0.0233 80 0.0243 80 0.0193 6 0.0143 6 0.0093 6 0.0043	o	0.0042	6	0.0092		0.0142	6	0.0192		0.0242	82	0.0292	83	0.0342	84	0.0392	82	0.0442	88	0.0492
88 0.03 83 0.025 78 0.02 6 0.005 6 0.01 6 0.005 89 0.0239 82 0.0249 78 0.0199 6 0.0149 6 0.0049 80 0.0239 82 0.0248 78 0.0199 6 0.0149 6 0.0098 6 0.0049 80 0.0236 82 0.0247 78 0.0197 6 0.0147 6 0.0037 6 0.0047 80 0.0236 82 0.0247 78 0.0197 6 0.0146 6 0.0096 6 0.0047 80 0.0236 82 0.0246 78 0.0196 6 0.0146 6 0.0036 6 0.0048 80 0.0236 80 0.0246 80 0.0195 6 0.0145 6 0.0035 6 0.0045 80 0.0254 80 0.0244 80 0.0194 6 0.0144 6 0.0094 6 0.0044	o	0.0043	6	0.0093	0	0.0143	6	0.0193		0.0243	8	0.0293	8	0.0343	9	0.0393	20	0.0443	88	0.0493
88 0.03 83 0.025 78 0.02 6 0.005 6 0.01 6 0.005 80 0.0259 82 0.0249 78 0.0198 6 0.0149 6 0.0099 80 0.0259 82 0.0248 78 0.0198 6 0.0143 6 0.0098 6 0.0049 80 0.0259 82 0.0247 78 0.0198 6 0.0147 6 0.0097 6 0.0047 80 0.0256 82 0.0247 78 0.0197 6 0.0147 6 0.0097 6 0.0047 80 0.0256 82 0.0246 78 0.0196 6 0.0146 6 0.0096 6 0.0046 80 0.0255 80 0.0245 80 0.0195 6 0.0145 6 0.0095 6 0.0045	o	0.0044	o	0.0094	o	0.0144	o	0.0194	8	0.0244	8	0.0294	88	0.0344	9	0.0394	8	0.0444	88	0.0494
88 0.023 83 0.025 78 0.02 6 0.005 6 0.01 6 0.005 83 0.0239 82 0.0243 78 0.0198 6 0.0143 6 0.0039 6 0.0049 88 0.0239 82 0.0248 78 0.0198 6 0.0143 6 0.0039 6 0.0043 88 0.0237 82 0.0247 78 0.0197 6 0.0147 6 0.0037 6 0.0047 88 0.0236 82 0.0246 78 0.0196 6 0.0146 6 0.0096 6 0.0046	o	0.0045	o	0.0095	o	0.0145	o	0.0195	8	0.0245	8	0.0295	88	0.0345		0.0395	8	0.0445	8	0.0495
88 0.023 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 89 0.0259 82 0.0249 78 0.0159 6 0.0143 6 0.0059 6 0.0049 88 0.0238 82 0.0248 78 0.0158 6 0.0148 6 0.0038 6 0.0048 88 0.0237 82 0.0247 78 0.0197 6 0.0147 6 0.0097 6 0.0047	6	0.0046		0.0096	-	0.0146	o	0.0196		0.0246	8	0.0296	8	0.0346		0.0396	8	0.0446	8	0.0496
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 89 0.0239 82 0.0249 78 0.0139 6 0.0149 6 0.0039 6 0.0049 88 0.0238 82 0.0248 78 0.0188 6 0.0148 6 0.0038 6 0.0048	o	0.0047		0.0097		0.0147	o	0.0197		0.0247	82	0.0297	88	0.0347		0.0397	82	0.0447	88	0.0497
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005 89 0.0299 82 0.0249 78 0.0189 6 0.0149 6 0.0099 6 0.0049	თ	0.0048		0.0098		0.0148	o	0.0198		0.0248	82	0.0298	8	0.0348	88	0.0398	8	0.0448	8	0.0498
88 0.03 83 0.025 78 0.02 6 0.015 6 0.01 6 0.005	o	0.0049	o	0.0099		0.0149	თ	0.0199	78	0.0249	82	0.0299	89	0.0349		0.0399	8	0.0449	92	0.0499
	o	0.005	o	0.01	6	0.015	6	0.02	78	0.025	83	0.03	88	0.035	82	0.04	84	0.045	102	0.0500

Figure 57: The number of unique spins within the lattice for this simulation was extremely high. This is likely a result of insufficient EFM iterations

22 3K (111) Increasing Field, Ground State

Steps persist in the energy graphs when plotting on a smaller scale. This can probably be fixed by increasing precision. A sudden drop in energy occurs at field 0.0049. There is little difference between the 2000 and 3000 step simulations for increasing field up to H=0.05. The 2000 and 3000 iterations can't be compared for fields beyond that, since the 2000 step simulation was only run to H=0.05.

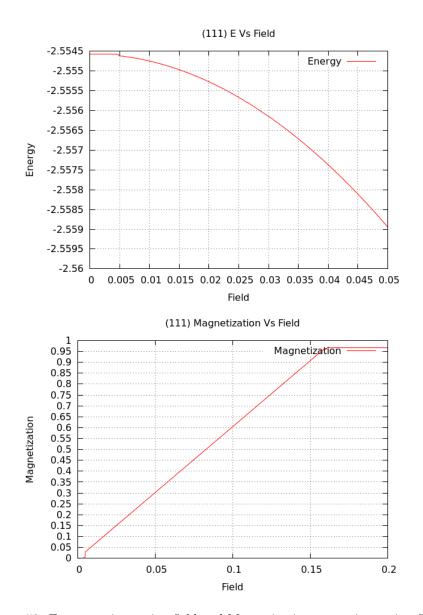


Figure 58: Energy vs increasing field and Magnetization versus increasing field

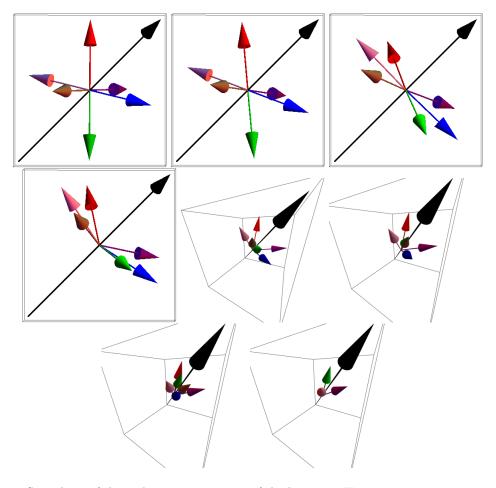


Figure 59: Snapshots of the 6 characteristic spins of the lattice at $H=0,\,0.0040,\,0.0055,\,0.05,\,0.140,\,0.152,\,0.158,\,0.172$. As the spins close up, the brown and green spins become parallel, then unalign. Finally, the 6 spin system appears to reduce to a 3-spin system at H=0.17.

0.0000	6	0.005	6	0.01	6	0.015	6	0.02	6	0.025	6	0.03	6	0.035	6	0.04	6	0.045	6
0.0001	6	0.0051	6	0.0101	6	0.0151	6	0.0201	6	0.0251	6	0.0301	6	0.0351	6	0.0401	6	0.0451	6
0.0002		0.0052	7	0.0102	6	0.0152	6	0.0202	6	0.0252	6	0.0302	6	0.0352	6	0.0402	6	0.0452	6
0.0003	6	0.0053	6	0.0103	6	0.0153	6	0.0203	6	0.0253	6	0.0303	6	0.0353	6	0.0403	6	0.0453	6
0.0004		0.0054	6	0.0104	6	0.0154	6	0.0204	6	0.0254	6	0.0304	6	0.0354	6	0.0404	6	0.0454	6
0.0005		0.0055	6	0.0105	6	0.0155	6	0.0205	6	0.0255	6	0.0305	6	0.0355	6	0.0405	6	0.0455	6
0.0006	7	0.0056	6	0.0106	6	0.0156	6	0.0206	6	0.0256	6	0.0306	6	0.0356	6	0.0406	6	0.0456	6
0.0007	6	0.0057	6	0.0107	6	0.0157	6	0.0207	6	0.0257	6	0.0307	6	0.0357	6	0.0407	6	0.0457	6
0.0008	6	0.0058	6	0.0101	6	0.0158	6	0.0208	6	0.0258	6	0.0308	6	0.0358	6	0.0401	6	0.0458	6
0.0009	6	0.0059	6	0.0100	6	0.0159	6	0.0209	6	0.0259	6	0.0309	6	0.0359	6	0.0400	6	0.0459	6
0.0003	7	0.0055	6	0.0103	6	0.0155	6	0.0203	6	0.0255	6	0.0303	6	0.0333	6	0.0403	6	0.0455	6
0.0010		0.0061	6	0.0111	6	0.0161	6	0.021	6	0.0261	6	0.031	6	0.0361	6	0.041	6	0.0461	6
															6				
0.0012		0.0062	6	0.0112	6 6	0.0162	6 6	0.0212	6 6	0.0262	6	0.0312	6	0.0362		0.0412	6 6	0.0462	6
0.0013	6	0.0063	6	0.0113	-	0.0163	-	0.0213	-	0.0263	6	0.0313	6	0.0363	6	0.0413	-	0.0463	6
0.0014	6	0.0064	6	0.0114	6	0.0164	6	0.0214	6	0.0264	6	0.0314	6	0.0364	6	0.0414	6	0.0464	6
0.0015	6	0.0065	6	0.0115	6	0.0165	6	0.0215	6	0.0265	6	0.0315	6	0.0365	6	0.0415	6	0.0465	6
0.0016	6	0.0066	6	0.0116	6	0.0166	6	0.0216	6	0.0266	6	0.0316	6	0.0366	6	0.0416	6	0.0466	6
0.0017	6	0.0067	6	0.0117	6	0.0167	6	0.0217	6	0.0267	6	0.0317	6	0.0367	6	0.0417	6	0.0467	6
0.0018	6	0.0068	6	0.0118	6	0.0168	6	0.0218	6	0.0268	6	0.0318	6	0.0368	6	0.0418	6	0.0468	6
0.0019	7	0.0069	6	0.0119	6	0.0169	6	0.0219	6	0.0269	6	0.0319	6	0.0369	6	0.0419	6	0.0469	6
0.0020	6	0.007	6	0.012	6	0.017	6	0.022	6	0.027	6	0.032	6	0.037	6	0.042	6	0.047	6
0.0021		0.0071	6	0.0121	6	0.0171	6	0.0221	6	0.0271	6	0.0321	6	0.0371	6	0.0421	6	0.0471	6
0.0022	6	0.0072	6	0.0122	6	0.0172	6	0.0222	6	0.0272	6	0.0322	6	0.0372	6	0.0422	6	0.0472	6
0.0023	6	0.0073	6	0.0123	6	0.0173	6	0.0223	6	0.0273	6	0.0323	6	0.0373	6	0.0423	6	0.0473	6
0.0024	6	0.0074	6	0.0124	6	0.0174	6	0.0224	6	0.0274	6	0.0324	6	0.0374	6	0.0424	6	0.0474	6
0.0025	6	0.0075	6	0.0125	6	0.0175	6	0.0225	6	0.0275	6	0.0325	6	0.0375	6	0.0425	6	0.0475	6
0.0026	6	0.0076	6	0.0126	6	0.0176	6	0.0226	6	0.0276	6	0.0326	6	0.0376	6	0.0426	6	0.0476	6
0.0027	6	0.0077	6	0.0127	6	0.0177	6	0.0227	6	0.0277	6	0.0327	6	0.0377	6	0.0427	6	0.0477	6
0.0028	6	0.0078	6	0.0128	6	0.0178	6	0.0228	6	0.0278	6	0.0328	6	0.0378	6	0.0428	6	0.0478	6
0.0029	6	0.0079	6	0.0129	6	0.0179	6	0.0229	6	0.0279	6	0.0329	6	0.0379	6	0.0429	6	0.0479	6
0.0030	6	0.008	6	0.013	6	0.018	6	0.023	6	0.028	6	0.033	6	0.038	6	0.043	6	0.048	6
0.0031	6	0.0081	6	0.0131	6	0.0181	6	0.0231	6	0.0281	6	0.0331	6	0.0381	6	0.0431	6	0.0481	6
0.0032	6	0.0082	6	0.0132	6	0.0182	6	0.0232	6	0.0282	6	0.0332	6	0.0382	6	0.0432	6	0.0482	6
0.0033	6	0.0083	6	0.0133	6	0.0183	6	0.0233	6	0.0283	6	0.0333	6	0.0383	6	0.0433	6	0.0483	6
0.0034	6	0.0084	6	0.0134	6	0.0184	6	0.0234	6	0.0284	6	0.0334	6	0.0384	6	0.0434	6	0.0484	6
0.0035	6	0.0085	6	0.0135	6	0.0185	6	0.0235	6	0.0285	6	0.0335	6	0.0385	6	0.0435	6	0.0485	6
0.0036	6	0.0086	6	0.0136	6	0.0186	6	0.0236	6	0.0286	6	0.0336	6	0.0386	6	0.0436	6	0.0486	6
0.0037	6	0.0087	6	0.0137	6	0.0187	6	0.0237	6	0.0287	6	0.0337	6	0.0387	6	0.0437	6	0.0487	6
0.0038	6	0.0088	6	0.0138	6	0.0188	6	0.0238	6	0.0288	6	0.0338	6	0.0388	6	0.0438	6	0.0488	6
0.0039	6	0.0089	6	0.0139	6	0.0189	6	0.0239	6	0.0289	6	0.0339	6	0.0389	6	0.0439	6	0.0489	6
0.0040	6	0.009	6	0.014	6	0.019	6	0.024	6	0.029	6	0.034	6	0.039	6	0.044	6	0.049	6
0.0041		0.0091	6	0.0141	6	0.0191	6	0.0241	6	0.0291	6	0.0341	6	0.0391	6	0.0441	6	0.0491	6
0.0042		0.0092	6	0.0142	6	0.0192	6	0.0242	6	0.0292	6	0.0342	6	0.0392	6	0.0442	6	0.0492	6
0.0043	6	0.0093	6	0.0143	6	0.0193	6	0.0243	6	0.0293	6	0.0343	6	0.0393	6	0.0443	6	0.0493	6
0.0044	6	0.0094	6	0.0144	6	0.0194	6	0.0244	6	0.0294	6	0.0344	6	0.0394	6	0.0444	6	0.0494	6
0.0045	6	0.0095	6	0.0145	6	0.0195	6	0.0245	6	0.0295	6	0.0345	6	0.0395	6	0.0445	6	0.0495	6
0.0046	6	0.0036	6	0.0146	6	0.0196	6	0.0246	6	0.0236	6	0.0346	6	0.0396	6	0.0446	6	0.0436	6
0.0047	6	0.0030	6	0.0147	6	0.0197	6	0.0247	6	0.0230	6	0.0347	6	0.0330	6	0.0447	6	0.0430	6
0.0041	6	0.0031	6	0.0141	6	0.0198	6	0.0241	6	0.0231	6	0.0341	6	0.0398	6	0.0448	6	0.0431	6
0.0049		0.0030	6	0.0149	6	0.0130	6	0.0240	6	0.0230	6	0.0349	6	0.0330	6	0.0449	6	0.0430	6
0.0043	J	3.0033		3.0143	0	0.0100		3.0243	0	0.0200	0	3.0343		0.0000	0	3.0443	0	0.0400	0

Figure 60: A noticeable change in the number of unique spins occurs at 0.0049, coinciding with the transition in the energy and magnetization graphs. This chart does not include data on the lattice at fields beyond 0.05.

23 3K (111) Decreasing Field, Ground State

No transition is observed at low field in this scenario, similar to the 2000 step case. A transition does occur at high field when the lattice is released from a saturated state. Once released from the saturated state, the system expands from the saturated 3-spin system into the typically observed 6-spin system. Finally, the spin configuration becomes planar at zero field. The 111 field direction is the only direction that creates a 3-spin system out of all directions tested. The final configuration is characterized by theta=52.1404 degrees and phi=-116.39 degrees.

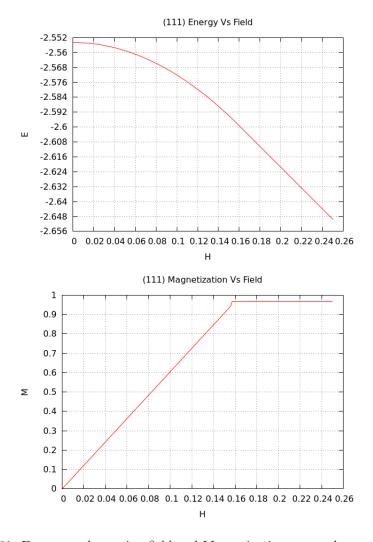


Figure 61: Energy vs decreasing field and Magnetization versus decreasing field

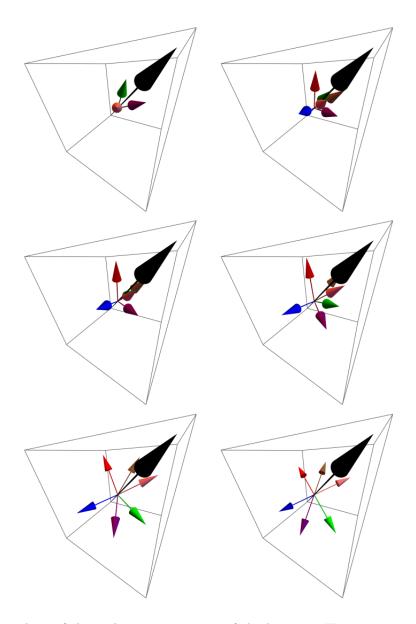


Figure 62: Snapshots of the 6 characteristic spins of the lattice at H=0.25, 0.156, 0.152, 0.136, 0.081, and 0

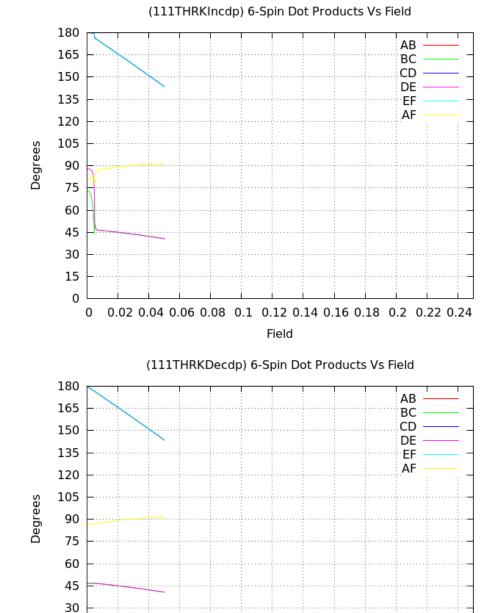


Figure 63: Dot products between the various characteristic spins for both increasing and decreasing fields.

0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2 0.22 0.24 Field

15 0

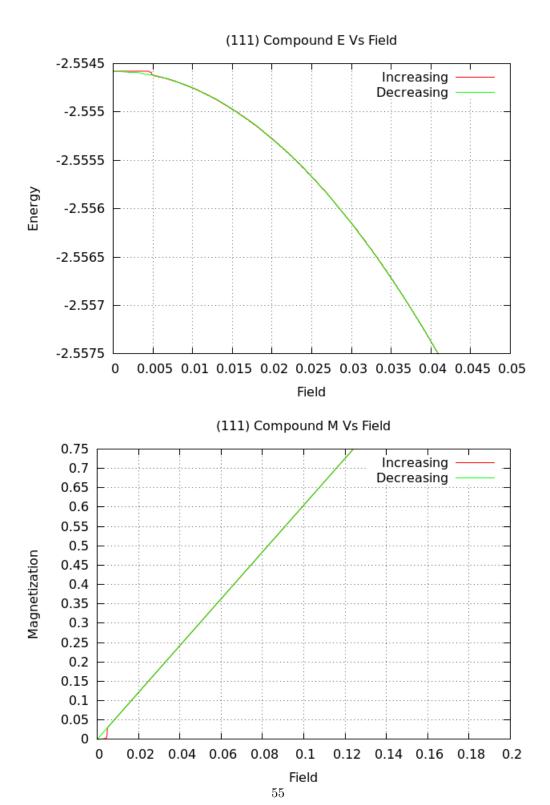


Figure 64: Composite graphs of energy and magnetization for both decreasing and increasing field magnitude. The magnetization actually plateaus at M=0.81, which is not shown here.

0.0500	6	0.045	6	0.04	6	0.035	6	0.03	6	0.025	6	0.02	6	0.015	6	0.01	6	0.005	6
0.0499	6	0.0449	6	0.0399	6	0.0349	6	0.0299	6	0.0249	6	0.0199	6	0.0149	6	0.0099	6	0.0049	6
0.0498	6	0.0448	6	0.0398	6	0.0348	6	0.0298	6	0.0248	6	0.0198	6	0.0148	6	0.0098	6	0.0048	6
0.0497	6	0.0447	6	0.0397	6	0.0347	6	0.0297	6	0.0247	6	0.0197	6	0.0147	6	0.0097	6	0.0047	6
0.0496	6	0.0446	6	0.0396	6	0.0346	6	0.0296	6	0.0246	6	0.0196	6	0.0146	6	0.0096	6	0.0046	6
0.0495	6	0.0445	6	0.0395	6	0.0345	6	0.0295	6	0.0245	6	0.0195	6	0.0145	6	0.0095	6	0.0045	6
0.0494	6	0.0444	6	0.0394	6	0.0344	6	0.0294	6	0.0244	6	0.0194	6	0.0144	6	0.0094	6	0.0044	6
0.0493	6	0.0443	6	0.0393	6	0.0343	6	0.0293	6	0.0243	6	0.0193	6	0.0143	6	0.0093	6	0.0043	6
0.0492	6	0.0442	6	0.0392	6	0.0342	6	0.0292	6	0.0242	6	0.0192	6	0.0142	6	0.0092	6	0.0042	6
0.0491	6	0.0441	6	0.0391	6	0.0341	6	0.0291	6	0.0241	6	0.0191	6	0.0141	6	0.0091	6	0.0041	6
0.0490	6	0.044	6	0.039	6	0.034	6	0.029	6	0.024	6	0.019	6	0.014	6	0.009	6	0.004	6
0.0489	6	0.0439	6	0.0389	6	0.0339	6	0.0289	6	0.0239	6	0.0189	6	0.0139	6	0.0089	6	0.0039	6
0.0488	6	0.0438	6	0.0388	6	0.0338	6	0.0288	6	0.0238	6	0.0188	6	0.0138	6	0.0088	6	0.0038	6
0.0487	6	0.0437	6	0.0387	6	0.0337	6	0.0287	6	0.0237	6	0.0187	6	0.0137	6	0.0087	6	0.0037	6
0.0486	6	0.0436	6	0.0386	6	0.0336	6	0.0286	6	0.0236	6	0.0186	6	0.0136	6	0.0086	6	0.0036	6
0.0485	6	0.0435	6	0.0385	6	0.0335	6	0.0285	6	0.0235	6	0.0185	6	0.0135	6	0.0085	6	0.0035	6
0.0484	6	0.0434	6	0.0384	6	0.0334	6	0.0284	6	0.0234	6	0.0184	6	0.0134	6	0.0084	6	0.0034	6
0.0483	6	0.0433	6	0.0383	6	0.0333	6	0.0283	6	0.0233	6	0.0183	6	0.0133	6	0.0083	6	0.0033	6
0.0482	6	0.0432	6	0.0382	6	0.0332	6	0.0282	6	0.0232	6	0.0182	6	0.0132	6	0.0082	6	0.0032	6
0.0481	6	0.0431	6	0.0381	6	0.0331	6	0.0281	6	0.0231	6	0.0181	6	0.0131	6	0.0081	6	0.0031	6
0.0480	6	0.043	6	0.038	6	0.033	6	0.028	6	0.023	6	0.018	6	0.013	6	0.008	6	0.003	6
0.0479	6	0.0429	6	0.0379	6	0.0329	6	0.0279	6	0.0229	6	0.0179	6	0.0129	6	0.0079	6	0.0029	6
0.0478	6	0.0428	6	0.0378	6	0.0328	6	0.0278	6	0.0228	6	0.0178	6	0.0128	6	0.0078	6	0.0028	6
0.0477	6	0.0427	6	0.0377	6	0.0327	6	0.0277	6	0.0227	6	0.0177	6	0.0127	6	0.0077	6	0.0027	6
0.0476	6	0.0426	6	0.0376	6	0.0326	6	0.0276	6	0.0226	6	0.0176	6	0.0126	6	0.0076	6	0.0026	6
0.0475	6	0.0425	6	0.0375	6	0.0325	6	0.0275	6	0.0225	6	0.0175	6	0.0125	6	0.0075	6	0.0025	6
0.0474	6	0.0424	6	0.0374	6	0.0324	6	0.0274	6	0.0224	6	0.0174	6	0.0124	6	0.0074	6	0.0024	6
0.0473	6	0.0423	6	0.0373	6	0.0323	6	0.0273	6	0.0223	6	0.0173	6	0.0123	6	0.0073	6	0.0023	6
0.0472	6	0.0422	6	0.0372	6	0.0322	6	0.0272	6	0.0222	6	0.0172	6	0.0122	6	0.0072	6	0.0022	6
0.0471	6	0.0421	6	0.0371	6	0.0321	6	0.0271	6	0.0221	6	0.0171	6	0.0121	6	0.0071	6	0.0021	6
0.0470	6	0.042	6	0.037	6	0.032	6	0.027	6	0.022	6	0.017	6	0.012	6	0.007	6	0.002	6
0.0469	6	0.0419	6	0.0369	6	0.0319	6	0.0269	6	0.0219	6	0.0169	6	0.0119	6	0.0069	6	0.0019	6
0.0468	6	0.0418	6	0.0368	6	0.0318	6	0.0268	6	0.0218	6	0.0168	6	0.0118	6	0.0068	6	0.0018	6
0.0467	6	0.0417	6	0.0367	6	0.0317	6	0.0267	6	0.0217	6	0.0167	6	0.0117	6	0.0067	6	0.0017	6
0.0466	6	0.0416	6	0.0366	6	0.0316	6	0.0266	6	0.0216	6	0.0166	6	0.0116	6	0.0066	6	0.0016	6
0.0465	6	0.0415	6	0.0365	6	0.0315	6	0.0265	6	0.0215	6	0.0165	6	0.0115	6	0.0065	6	0.0015	6
0.0464	6	0.0414	6	0.0364	6	0.0314	6	0.0264	6	0.0214	6	0.0164	6	0.0114	6	0.0064	6	0.0014	6
0.0463	6	0.0413	6	0.0363	6	0.0313	6	0.0263	6	0.0213	6	0.0163	6	0.0113	6	0.0063	6	0.0013	6
0.0462	6	0.0412	6	0.0362	6	0.0312	6	0.0262	6	0.0212	6	0.0162	6	0.0112	6	0.0062	6	0.0012	6
0.0461	6	0.0411	6	0.0361	6	0.0311	6	0.0261	6	0.0211	6	0.0161	6	0.0111	6	0.0061	6	0.0011	6
0.0460	6	0.041	6	0.036	6	0.031	6	0.026	6	0.021	6	0.016	6	0.011	6	0.006	6	0.001	6
0.0459	6	0.0409	6	0.0359	6	0.0309	6	0.0259	6	0.0209	6	0.0159	6	0.0109	6	0.0059	6	0.0009	6
0.0458	6	0.0408	6	0.0358	6	0.0308	6	0.0258	6	0.0208	6	0.0158	6	0.0108	6	0.0058	6	0.0008	7
0.0457	6	0.0407	6	0.0357	6	0.0307	6	0.0257	6	0.0207	6	0.0157	6	0.0107	6	0.0057	6	0.0007	6
0.0456	6	0.0406	6	0.0356	6	0.0306	6	0.0256	6	0.0206	6	0.0156	6	0.0106	6	0.0056	6	0.0006	6
0.0455	6	0.0405	6	0.0355	6	0.0305	6	0.0255	6	0.0205	6	0.0155	6	0.0105	6	0.0055	6	0.0005	6
0.0454	6	0.0404	6	0.0354	6	0.0304	6	0.0254	6	0.0204	6	0.0154	6	0.0104	6	0.0054	6	0.0004	6
0.0453	6	0.0403	6	0.0353	6	0.0303	6	0.0253	6	0.0203	6	0.0153	6	0.0103	6	0.0053	6	0.0003	6
0.0452	6	0.0402	6	0.0352	6	0.0302	6	0.0252	6	0.0202	6	0.0152	6	0.0102	6	0.0052	6	0.0002	7
0.0451	6	0.0401	6	0.0351	6	0.0301	6	0.0251	6	0.0201	6	0.0151	6	0.0101	6	0.0051	6	0.0001	6
																		0	6

Figure 65: Chart indicating the number of unique spins for decreasing the field in the 111 direction, when starting at a groundstate. No notable change in number of spins can be observed, except for the low field section when the number increases to 7 for two field values.

24 3K (111) Increasing Field, Random State

Similar to run 1, a transition to a planar state is observed at around 0.0037. This contrasts the transition field of 0.49 in the run starting from a ground state. This could be due to the fact the ground state that is initially generated at H=0 is slightly different than that of the one used in run 1 and run 2. Maybe there is a relationship that tells us what field the transition will occur for a given theta and phi?

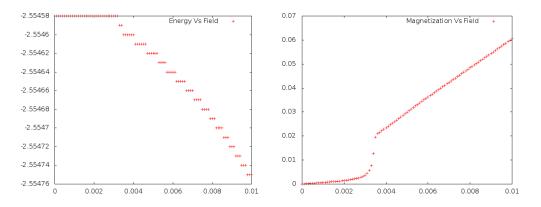


Figure 66: Energy vs increasing field and Magnetization versus increasing field

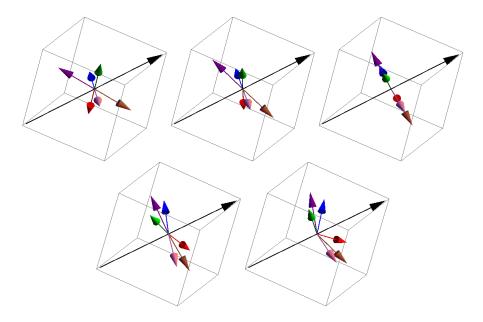


Figure 67: Snapshots of the 6 characteristic spins of the lattice at H=0.00, 0.0033, 0.0041, 0.0069, and 0.0500

0.0000 8	3	0.005	6	0.01	6	0.015	6	0.02	6	0.025	6	0.03	6	0.035	6	0.04	6	0.045	6
0.0001 6	3	0.0051	8	0.0101	6	0.0151	6	0.0201	6	0.0251	6	0.0301	6	0.0351	6	0.0401	6	0.0451	6
0.0002 6	3	0.0052	6	0.0102	6	0.0152	6	0.0202	6	0.0252	6	0.0302	6	0.0352	6	0.0402	6	0.0452	6
0.0003 6	3	0.0053	6	0.0103	6	0.0153	6	0.0203	6	0.0253	6	0.0303	6	0.0353	6	0.0403	6	0.0453	6
0.0004 6		0.0054	6	0.0104	6	0.0154	6	0.0204	6	0.0254	6	0.0304	6	0.0354	6	0.0404	6	0.0454	6
0.0005 6		0.0055	6	0.0105	6	0.0155	6	0.0205	6	0.0255	6	0.0305	6	0.0355	6	0.0405	6	0.0455	6
0.0006 6	_	0.0056	6	0.0106	6	0.0156	6	0.0206	6	0.0256	6	0.0306	6	0.0356	6	0.0406	6	0.0456	6
0.0007 6		0.0057	6	0.0107	6	0.0157	6	0.0207	6	0.0257	6	0.0307	6	0.0357	6	0.0407	6	0.0457	6
0.0008 6		0.0058	6	0.0108	6	0.0158	6	0.0208	6	0.0258	6	0.0308	6	0.0358	6	0.0408	6	0.0458	6
0.0000 6		0.0059	6	0.0109	6	0.0159	6	0.0200	6	0.0259	6	0.0309	6	0.0359	6	0.0409	6	0.0459	6
																	6		
0.0010 6		0.006	6	0.011	6	0.016	6	0.021	6	0.026	6	0.031	6 6	0.036	6	0.041	-	0.046	6
0.0011 6		0.0061	6	0.0111	6	0.0161	6	0.0211	6	0.0261	6	0.0311	_	0.0361	6	0.0411	6	0.0461	6
0.0012 6		0.0062	6	0.0112	6	0.0162	6	0.0212	6	0.0262	6	0.0312	6	0.0362	6	0.0412	6	0.0462	6
0.0013 6		0.0063	6	0.0113	6	0.0163	6	0.0213	6	0.0263	6	0.0313	6	0.0363	6	0.0413	6	0.0463	6
0.0014 6		0.0064	6	0.0114	6	0.0164	6	0.0214	6	0.0264	6	0.0314	6	0.0364	6	0.0414	6	0.0464	6
0.0015 6	-	0.0065	6	0.0115	6	0.0165	6	0.0215	6	0.0265	6	0.0315	6	0.0365	6	0.0415	6	0.0465	6
0.0016 6	3	0.0066	6	0.0116	6	0.0166	6	0.0216	6	0.0266	6	0.0316	6	0.0366	6	0.0416	6	0.0466	6
0.0017 6	3	0.0067	6	0.0117	6	0.0167	6	0.0217	6	0.0267	6	0.0317	6	0.0367	6	0.0417	6	0.0467	6
0.0018 6	3	0.0068	6	0.0118	6	0.0168	6	0.0218	6	0.0268	6	0.0318	6	0.0368	6	0.0418	6	0.0468	6
0.0019 6	3	0.0069	6	0.0119	6	0.0169	6	0.0219	6	0.0269	6	0.0319	6	0.0369	6	0.0419	6	0.0469	6
0.0020	7	0.007	6	0.012	6	0.017	6	0.022	6	0.027	6	0.032	6	0.037	6	0.042	6	0.047	6
0.0021 8	3	0.0071	6	0.0121	6	0.0171	6	0.0221	6	0.0271	6	0.0321	6	0.0371	6	0.0421	6	0.0471	6
0.0022 6	3	0.0072	6	0.0122	6	0.0172	6	0.0222	6	0.0272	6	0.0322	6	0.0372	6	0.0422	6	0.0472	6
0.0023 6	3	0.0073	6	0.0123	6	0.0173	6	0.0223	6	0.0273	6	0.0323	6	0.0373	6	0.0423	6	0.0473	6
0.0024 6		0.0074	6	0.0124	6	0.0174	6	0.0224	6	0.0274	6	0.0324	6	0.0374	6	0.0424	6	0.0474	6
0.0025 6		0.0075	6	0.0125	6	0.0175	6	0.0225	6	0.0275	6	0.0325	6	0.0375	6	0.0425	6	0.0475	6
0.0026 6		0.0076	6	0.0126	6	0.0176	6	0.0226	6	0.0276	6	0.0326	6	0.0376	6	0.0426	6	0.0476	6
0.0027 7		0.0077	6	0.0127	6	0.0177	6	0.0227	6	0.0277	6	0.0327	6	0.0377	6	0.0427	6	0.0477	6
0.0028		0.0078	6	0.0128	6	0.0178	6	0.0228	6	0.0278	6	0.0328	6	0.0378	6	0.0428	6	0.0478	6
0.0029 6		0.0079	6	0.0129	6	0.0179	6	0.0229	6	0.0279	6	0.0329	6	0.0379	6	0.0429	6	0.0479	6
0.0023 6		0.0013	6	0.0123	6	0.0113	6	0.0223	6	0.0213	6	0.0323	6	0.0313	6	0.0423	6	0.0413	6
0.0030 6		0.0081	6	0.0131	6	0.0181	6	0.0231	6	0.0281	6	0.0331	6	0.0381	6	0.0431	6	0.0481	6
																	-		
0.0032 6		0.0082	6	0.0132	6	0.0182	6	0.0232	6	0.0282	6	0.0332	6	0.0382	6	0.0432	6	0.0482	6
0.0033		0.0083	6	0.0133	6	0.0183	6	0.0233	6	0.0283	6	0.0333	6	0.0383	6	0.0433	6	0.0483	6
0.0034 6		0.0084	6	0.0134	6	0.0184	6	0.0234	6	0.0284	6	0.0334	6	0.0384	6	0.0434	6	0.0484	6
0.0035 8		0.0085	6	0.0135	6	0.0185	6	0.0235	6	0.0285	6	0.0335	6	0.0385	6	0.0435	6	0.0485	6
0.0036 6		0.0086	6	0.0136	6	0.0186	6	0.0236	6	0.0286	6	0.0336	6	0.0386	6	0.0436	6	0.0486	6
0.0037 10		0.0087	6	0.0137	6	0.0187	6	0.0237	6	0.0287	6	0.0337	6	0.0387	6	0.0437	6	0.0487	6
0.0038 10)	0.0088	6	0.0138	6	0.0188	6	0.0238	6	0.0288	6	0.0338	6	0.0388	6	0.0438	6	0.0488	6
0.0039 8	3	0.0089	6	0.0139	6	0.0189	6	0.0239	6	0.0289	6	0.0339	6	0.0389	6	0.0439	6	0.0489	6
0.0040 6	3	0.009	6	0.014	6	0.019	6	0.024	6	0.029	6	0.034	6	0.039	6	0.044	6	0.049	6
0.0041 6	3	0.0091	6	0.0141	6	0.0191	6	0.0241	6	0.0291	6	0.0341	6	0.0391	6	0.0441	6	0.0491	6
0.0042 6	3	0.0092	6	0.0142	6	0.0192	6	0.0242	6	0.0292	6	0.0342	6	0.0392	6	0.0442	6	0.0492	6
0.0043 6	3	0.0093	6	0.0143	6	0.0193	6	0.0243	6	0.0293	6	0.0343	6	0.0393	6	0.0443	6	0.0493	6
0.0044 8	3	0.0094	6	0.0144	6	0.0194	6	0.0244	6	0.0294	6	0.0344	6	0.0394	6	0.0444	6	0.0494	6
0.0045 6	3	0.0095	6	0.0145	6	0.0195	6	0.0245	6	0.0295	6	0.0345	6	0.0395	6	0.0445	6	0.0495	6
0.0046 6	3	0.0096	6	0.0146	6	0.0196	6	0.0246	6	0.0296	6	0.0346	6	0.0396	6	0.0446	6	0.0496	6
0.0047 6		0.0097	6	0.0147	6	0.0197	6	0.0247	6	0.0297	6	0.0347	6	0.0397	6	0.0447	6	0.0497	6
0.0048		0.0098	6	0.0148	6	0.0198	6	0.0248	6	0.0298	6	0.0348	6	0.0398	6	0.0448	6	0.0498	6
0.0049 6		0.0099	6	0.0149	6	0.0199	6	0.0249	6	0.0299	6	0.0349	6	0.0399	6	0.0449	6	0.0499	6
5.00.0	+			0.0.10		2.0,00	-	5.02 70	-	5.0250		3.00 70	-	3.0000	-	5.0.70	-	0.05	6
	+																	0.00	J

Figure 68: Number of unique spins within the lattice while increasing the field in the 111 direction after starting with a random initial configuration. The number of unique spins increases to 10 at H=0.0037.

25 3K (111) Decreasing Field, Random State

Very similar to decreasing the field in the 111 direction after starting with a ground state. However, it is different than decreasing the field in the 111 direction after starting with a random intial configuration with only 2000 steps. Here, 3000 steps were used and the resulting difference between this and the 2000 step case is the lack of a transition. It behaves exactly the same way if you were to start from a ground state.

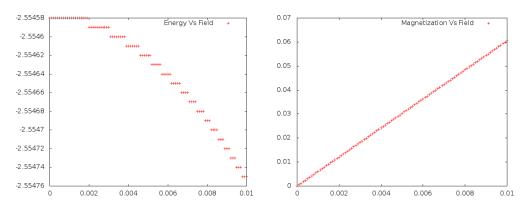


Figure 69: Energy vs decreasing field and Magnetization versus decreasing field

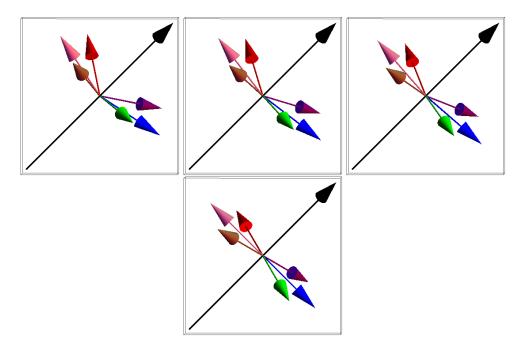
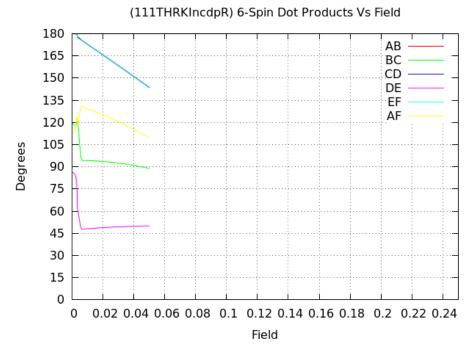


Figure 70: Snapshots of the 6 characteristic spins at H=0.05, 0.0329, 0.0176, and 0



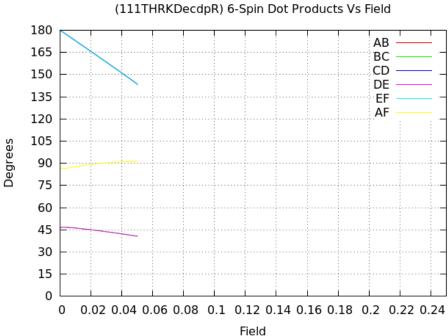


Figure 71: Dot products between the 6 characteristic spins.

0.0500	61	0.045		6	0.04	61	0.035	6	0.03		6	0.025	6		0.02	6	0.015	f	31	0.01	61	0.005	6
0.0499	6	0.0449			0.0399	6	0.0349	6	0.0299		6	0.0249	6	0.	.0199	6	0.0149		3	0.0099	6	0.0049	6
0.0498	6	0.0448			0.0398	6	0.0348	6	0.0298		6	0.0248	6		.0198	6	0.0148			0.0098	6	0.0048	6
0.0497	6	0.0447	-		0.0397	6	0.0347	6	0.0297		6	0.0247	6		.0197	6	0.0147	6		0.0097	6	0.0047	6
0.0496	6	0.0446	-		0.0396	6	0.0346	6	0.0296		6	0.0246	6		.0196	6	0.0146	6		0.0096	6	0.0046	6
0.0495	6	0.0445			0.0395	6	0.0345	6	0.0295		6	0.0245	6		.0195	6	0.0145			0.0095	6	0.0045	6
0.0494	6	0.0444	-	6 (0.0394	6	0.0344	6	0.0294		6	0.0244	6	0.	.0194	6	0.0144	6	3	0.0094	6	0.0044	6
0.0493	6	0.0443			0.0393	6	0.0343	6	0.0293		6	0.0243	6		.0193	6	0.0143			0.0093	6	0.0043	6
0.0492	6	0.0442	-		0.0392	6	0.0342	6	0.0292		6	0.0242	6		.0192	6	0.0142	6		0.0092	6	0.0042	6
0.0491	6	0.0441		6	0.0391	6	0.0341	6	0.0291		6	0.0241	6	0	0.0191	6	0.0141	6	3	0.0091	6	0.0041	6
0.0490	6	0.044	-	6	0.039	6	0.034	6	0.029		6	0.024	6	- 1	0.019	6	0.014	6	3	0.009	6	0.004	6
0.0489	6	0.0439	-	6 (0.0389	6	0.0339	6	0.0289		6	0.0239	6	0.	.0189	6	0.0139	6	3	0.0089	6	0.0039	6
0.0488	6	0.0438			0.0388	6	0.0338	6	0.0288		6	0.0238	6		.0188	6	0.0138	6		0.0088	6	0.0038	6
0.0487	6	0.0437	-	6 (0.0387	6	0.0337	6	0.0287		6	0.0237	6	0.	.0187	6	0.0137	6	3	0.0087	6	0.0037	6
0.0486	6	0.0436	-	6 (0.0386	6	0.0336	6	0.0286		6	0.0236	6	0.	.0186	- 6	0.0136	6	3	0.0086	6	0.0036	6
0.0485	6	0.0435	-	6 (0.0385	6	0.0335	6	0.0285		6	0.0235	6	0.	.0185	6	0.0135		3	0.0085	6	0.0035	6
0.0484	6	0.0434	-	6 (0.0384	6	0.0334	6	0.0284		6	0.0234	6	0.	.0184	6	0.0134	6	3	0.0084	6	0.0034	6
0.0483	6	0.0433			0.0383	6	0.0333	6	0.0283		6	0.0233	6		.0183	6	0.0133	6		0.0083	6	0.0033	6
0.0482	6	0.0432	-		0.0382	6	0.0332	6	0.0282		6	0.0232	6		.0182	- 6	0.0132			0.0082	6	0.0032	6
0.0481	6	0.0431		6	0.0381	6	0.0331	6	0.0281		6	0.0231	6	0	0.0181	6	0.0131	6	3	0.0081	6	0.0031	6
0.0480	6	0.043	- 1	6	0.038	6	0.033	6	0.028		6	0.023	6	- 1	0.018	- 6	0.013	6	3	0.008	6	0.003	6
0.0479	6	0.0429	-	6 (0.0379	6	0.0329	6	0.0279		6	0.0229	6	0.	.0179	- 6	0.0129	6	3	0.0079	6	0.0029	6
0.0478	6	0.0428	- 1	6 (0.0378	6	0.0328	6	0.0278		6	0.0228	6	0.	.0178	6	0.0128	6	3	0.0078	6	0.0028	6
0.0477	6	0.0427	- 1	6 (0.0377	6	0.0327	6	0.0277		6	0.0227	6	0.	.0177	6	0.0127	6	3	0.0077	6	0.0027	6
0.0476	6	0.0426		6 (0.0376	6	0.0326	6	0.0276		6	0.0226	6	0.	.0176	6	0.0126	6	3	0.0076	6	0.0026	6
0.0475	6	0.0425		6 (0.0375	6	0.0325	6	0.0275		6	0.0225	6	0.	.0175	6	0.0125	6	3	0.0075	6	0.0025	6
0.0474	6	0.0424		6 (0.0374	6	0.0324	6	0.0274		6	0.0224	6	0.	.0174	6	0.0124	6	3	0.0074	6	0.0024	6
0.0473	6	0.0423		6 (0.0373	6	0.0323	6	0.0273		6	0.0223	6	0.	.0173	6	0.0123	6	3	0.0073	6	0.0023	6
0.0472	6	0.0422		6 (0.0372	6	0.0322	6	0.0272		6	0.0222	6	0.	.0172	- 6	0.0122	6	3	0.0072	6	0.0022	6
0.0471	6	0.0421		6	0.0371	6	0.0321	6	0.0271		6	0.0221	6	0	0.0171	6	0.0121	6	3	0.0071	6	0.0021	6
0.0470	6	0.042		6	0.037	6	0.032	6	0.027		6	0.022	6	- (0.017	6	0.012	6	3	0.007	6	0.002	6
0.0469	6	0.0419	- 1	6 (0.0369	6	0.0319	6	0.0269		6	0.0219	6	0.	.0169	6	0.0119	6	3	0.0069	6	0.0019	6
0.0468	6	0.0418		6 (0.0368	6	0.0318	6	0.0268		6	0.0218	6	0.	.0168	6	0.0118	6	3	0.0068	6	0.0018	6
0.0467	6	0.0417		6 (0.0367	6	0.0317	6	0.0267		6	0.0217	6	0.	.0167	6	0.0117	6	3	0.0067	6	0.0017	6
0.0466	6	0.0416		6 (0.0366	6	0.0316	6	0.0266		6	0.0216	6	0.	.0166	6	0.0116	6	3	0.0066	6	0.0016	6
0.0465	6	0.0415		6 (0.0365	6	0.0315	6	0.0265		6	0.0215	6	0.	.0165	6	0.0115	6	3	0.0065	6	0.0015	6
0.0464	6	0.0414	- 1	6 (0.0364	6	0.0314	6	0.0264		6	0.0214	6	0.	.0164	6	0.0114	6	3	0.0064	6	0.0014	6
0.0463	6	0.0413		6 (0.0363	6	0.0313	6	0.0263		6	0.0213	6	0.	.0163	6	0.0113	6	3	0.0063	6	0.0013	6
0.0462	6	0.0412	- 1	6 (0.0362	6	0.0312	6	0.0262		6	0.0212	6	0.	.0162	6	0.0112	6	3	0.0062	6	0.0012	6
0.0461	6	0.0411	- 1	6	0.0361	6	0.0311	6	0.0261		6	0.0211	6	0	0.0161	6	0.0111	6	3	0.0061	6	0.0011	6
0.0460	6	0.041		6	0.036	6	0.031	6	0.026		6	0.021	6		0.016	6	0.011	6	_	0.006	6	0.001	6
0.0459	6	0.0409	- 1	6 (0.0359	6	0.0309	6	0.0259		6	0.0209	6	0.	.0159	6	0.0109	6	3	0.0059	6	0.0009	6
0.0458	6	0.0408	- 1		0.0358	6	0.0308	6	0.0258		6	0.0208	6	0.	.0158	6	0.0108	6	3	0.0058	6	0.0008	7
0.0457	6	0.0407			0.0357	6	0.0307	6	0.0257		6	0.0207	6		.0157	6	0.0107	6		0.0057	6	0.0007	6
0.0456	6	0.0406			0.0356	6	0.0306	6	0.0256		6	0.0206	6		.0156	6	0.0106			0.0056	6	0.0006	6
0.0455	6	0.0405		6 (0.0355	6	0.0305	6	0.0255		6	0.0205	6	0.	.0155	6	0.0105	6	3	0.0055	6	0.0005	6
0.0454	6	0.0404	- 1	6 (0.0354	6	0.0304	6	0.0254		6	0.0204	6	0.	.0154	6	0.0104	6	3	0.0054	6	0.0004	6
0.0453	6	0.0403			0.0353	6	0.0303	6	0.0253		6	0.0203	6		.0153	6	0.0103			0.0053	6	0.0003	6
0.0452	6	0.0402			0.0352	6	0.0302	6	0.0252		6	0.0202	6		.0152	6	0.0102	(_	0.0052	6	0.0002	7
0.0451	6	0.0401	- 1	6	0.0351	6	0.0301	6	0.0251	ı	6	0.0201	6	0	0.0151	6	0.0101	- 6	3	0.0051	6	0.0001	6

Figure 72: Number of unique spins within the lattice while decreasing the field in the 111 direction after starting from an initially random configuration.

26 Saturation of the Lattice

Using the same groundstate as used in all simulations, 7 simulations were run with differing field directions. The field was increased up until saturating the lattice. 001 and 010 both have identical magnetization curves, as do 011 and 101. However, 100 differs from 001 and 010, and 110 differs from 011 and 101, which is unexpected. When using the 111 field direction, saturation occurs at a field that is higher than the saturation fields of any other simulations.

Magnetization Vs. Increasing Field In Various Directions 1 001 0.9 010 011 8.0 100 101 0.7 110 111 0.6 Σ 0.5 0.4 0.3 0.2 0.1 0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2 0.22 0.24 0.26 Н

Figure 73: Magnetization curves starting with the same ground state and subjected to fields of various directions

27 Effect of Starting State on Switching Field (111)

To determine the effect the starting state has on when the lattice transitions to a planar state, 97 pairs of theta and phi were generated. These pairs were not randomly generated, but were generated by incrementing theta and phi through a for loop. While the contour plot gives the impression that pairs of theta and phi were generated from the forbidden zone and underwent a transition, this is simply the result of interpolation by Mathematica. The field used was along the 111 axis.

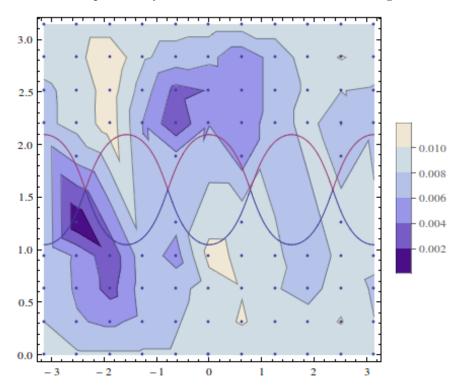


Figure 74: Contour plot indicating what pairs of theta and phi generate groundstates that transition sooner than others in the presence of the 111 field direction. The values in the legend are the magnitudes of the fields at which a point of inflection occurred and resulted in a transition. The reason why some states transition sooner than others could just be from the fact that certain states are already closer to the planar state at zero field. So there's less change in the spin components required to transition to the planar state. For example, at the darkest region there was a pair of theta and phi that gave rise to a starting state that I think was already planar. So in a way it had already transitioned at zero field. The region surrounding this point also transitions fairly quickly in comparison to most of the plane. I think there might be something special about the ground state that had already transitioned at zero field; that is, its theta and phi might describe 1 or more post-transition planar states.

28 Effect of Field Direction on Switching Field

To observe the effect of field direction on the switching field for a particular ground state (the same groundstate used in all groundstate simulations), the z-component of the applied magnetic field was varied for 20 different simulations. The result is that the switching field seems to change with a linear relationship with respect to the changing z-component. The switching field was approximated by finding the point of inflection for each of the magnetization curves.

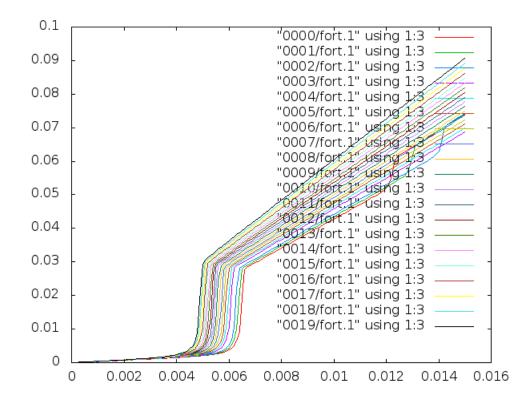


Figure 75: Magnetization curves for each simulation. The z-component was varied by increments of 5 percent for each simulation

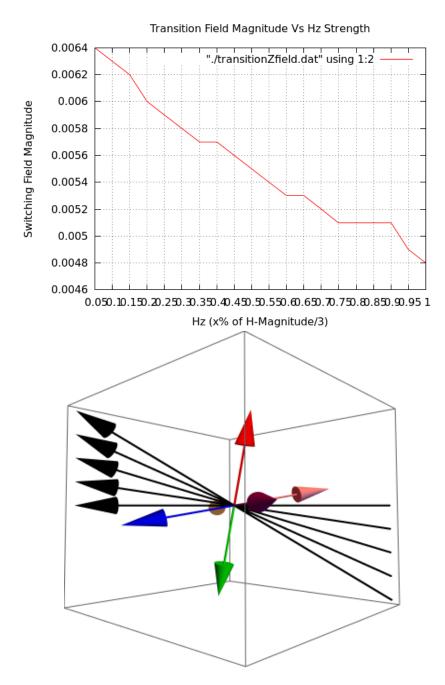


Figure 76: Switching field magnitude as a function of z-component strength. A visualization depicting how the field was changed for each independent simulation is also shown.

29 Reduction of Degeneracy By Application of Field

To gain insight on the possibility of being able to reduce the degree of the degeneracy by application of a field, 21 initially random spin configurations were created and subjected to an increasing magnetic field in the 111 direction. Once they surpassed the transition point, the field was then decreased to zero. What was observed was that each state fell into 1 of 6 possible zero field configurations, illustrated below:

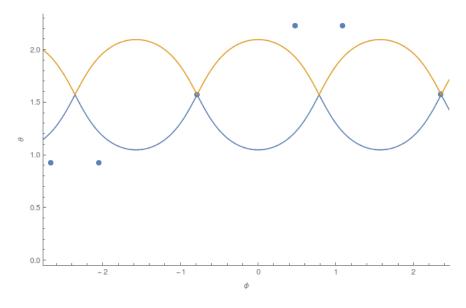


Figure 77: Depiction of what pairs of theta and phi characterize the groundstates after decreasing the field to zero for the 111 field.

Theta	Phi	Percent
127.26	27.3352	28.57
52.7912	-117.385	14.29
52.8063	-152.599	19.05
127.249	62.6494	14.29
90.02	-45.016	14.29
90.1097	135	9.52

Table 1: The 6 pairs of characteristic ground state angles describing the 6 possible states that result after decreasing the 111 field.

It can also be noted that when decreasing the field, after transitioning, for both the 010 and 111 field directions, after starting from the same ground state, the characteristic angles were the same: 52.7815, -117.275 (010), 52.1404 -116.39 (111).

30 Concluding Remarks

There are several things that can be noted through this pdf, and I'll try to summarize them here:

- 1. The field magnitude at which the lattice undergoes a transition is dependent on the starting spin configuration
- 2. The field magnitude at which the lattice undergoes a transition is dependent on the field direction
- 3. The continuous degeneracy collapses to a collection of likely 6 possible groundstates, regardless of initial configuration.

Some starting states result in a transition sooner than others, which can be seen in the contour plot. This makes sense since some starting configurations are probably "closer" to the planar state (or one of 6 planar states) and require less field to bring them to their pending planar state.

By changing the field direction, transitions can be made to occur earlier for a particular ground state configuration. What has yet to be determined, however, is whether the change in field will have any effect on the types of planar states that the spin configuration turns into once the field is decreased to zero.

To gain some more insight on how the ground state degeneracy changes as a result of being subject to a field, I'm planning as of now to do the following:

Analyze recently completed simulations that involved generating 20 initially random configurations for each of the field directions used in this pdf. By increasing the field, then decreasing the field for each simulation, I will determine what effect changing the field direction has on collapsing the continuous degeneracy. Will it change the possible ground state combinations? Is the possible ground state combinations the same regardless of field direction?

Finally, there are several long term goals that should be addressed at some point in the future, and are as follows:

- 1. Analytical description of how spins change before transitioning, or how the Byron-Andrew relations change with respect to changing field prior to transitioning.
- 2. An analytical description of when the lattice will undergo a transition for a given starting configuration and field direction
- 3. A description of how the spins change after transitioning, but prior to the lattice becoming saturated. This should be fairly straightforward since there is a linear relationship between the spin component and field magnitude for a field of some direction.
- 4. A description that predicts when saturation will occur for a particular field direction.
- 5. Determining how the spins will change after becoming saturated. This should also be straight forward, since the change is linear.
- 6. Why aren't 100, 010, 001 magnetization curves all the same? Furthermore, why aren't 011, 101, and 110 magnetization curves all the same?