Row Reduction and Inverse Calculator

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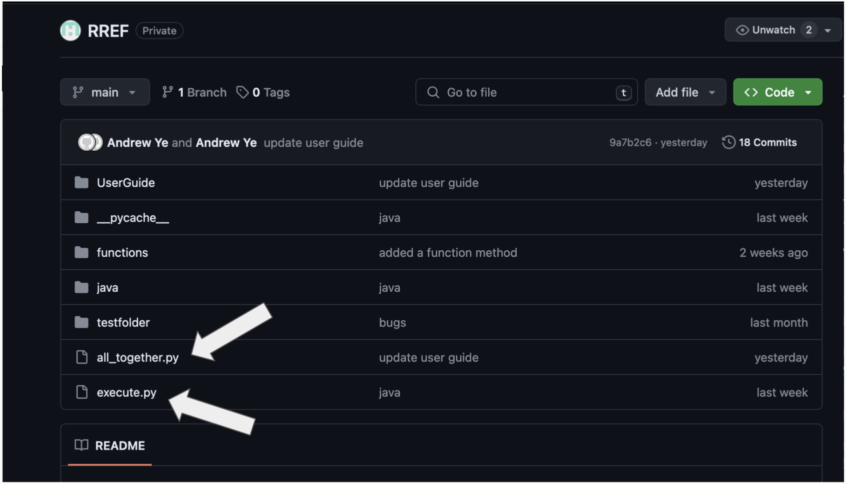
Objective

Our objective for this project was to create a program that would allow the user to perform matrix operations found in a linear algebra class.

Accessing the Calculator

Github

Go to this link `https://github.com/AndrewYe12/RREF`.
Download the *alltogether.py* and *execute.py* files.



Using the Calculator

Creating Your Matrix

To encode your matrix in a way the program can act upon you must format it as a list of lists of integers, with each list of numbers representing a row.

```
x = matrix([[1,2],[2,1]])
```

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Matrix Operations as Methods

Our Methods

We wanted to program an algorithm to row reduce a matrix, find the inverse of a matrix, add/multiply two matrices, and find the determinant of a matrix.

Intro to Matrix Operation Methods

$.rref()$: Reduces the matrix to reduced echelon form

$.inverse()$: Finds and returns the inverse of the matrix

$+$: Adds two matrices together

$*$: Multiplies two matrices together

$.fraction()$: Returns the matrix as a string of fractions

$.det()$: Calculates the determinant of a matrix.

Row Reduction

Transforms a matrix into row reduced echelon form.

```
from all_together import matrix
x = matrix([[-3,3,4,5], [2,2,3,3],[2,2,3,22]])
print(x.rref().matrix)
```

$$\begin{bmatrix} -3 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 22 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0.08333333333333326 & 0 \\ 0 & 1 & 1.4166666666666665 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse

Outputs the inverse of a matrix.

```
from all_together import matrix
x = matrix([[-3,3,4,5], [2,2,3,3],[2,2,3,22]])
print(x.inverse().matrix)
```

$A = \begin{bmatrix} -3 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 22 \end{bmatrix}$. This code finds the matrix $P = \begin{bmatrix} -0.166666666 & 0.24561403 & 0.00438596491 \\ 0.16666666666 & 0.333333333 & -0.08333333 \\ 0 & -0.0526315789 & 0.0526315789 \end{bmatrix}$ so that

$$PA = rref(A)$$

Addition

Inputs two matrices and returns one with the added terms.

```
from all_together import matrix
x = matrix([[-3,3,4,5], [2,2,3,3],[2,2,3,22]])
y = matrix([[-3,3,4,10], [2,22,3,3],[2,11,3,22]])
print((x + y).matrix)
```

$$\begin{bmatrix} -3 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 22 \end{bmatrix} + \begin{bmatrix} -3 & 3 & 4 & 10 \\ 2 & 22 & 3 & 3 \\ 2 & 11 & 3 & 22 \end{bmatrix} = \begin{bmatrix} -6 & 6 & 8 & 15 \\ 4 & 24 & 6 & 6 \\ 4 & 13 & 6 & 44 \end{bmatrix}$$

Multiplication

Inputs two matrices and outputs the matrix resulting from performing matrix multiplication.

```
from all_together import matrix
x = matrix([[-3,3,4,5], [2,2,3,3],[2,2,3,22]])
y = matrix([[-3,3,4,10,9], [2,22,3,3,8],[2,11,3,22,7],[4,2,1,34,5]])
print((x * y).matrix)
```

$$\begin{bmatrix} -3 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 22 \end{bmatrix} \begin{bmatrix} -3 & 3 & 4 & 10 & 9 \\ 2 & 22 & 3 & 3 & 8 \\ 2 & 11 & 3 & 22 & 7 \\ 4 & 2 & 1 & 34 & 5 \end{bmatrix} = \begin{bmatrix} 43 & 111 & 14 & 237 & 50 \\ 16 & 89 & 26 & 194 & 70 \\ 92 & 127 & 45 & 840 & 165 \end{bmatrix}$$

Fraction

Inputs a matrix and returns a copy of it with the entries as fractions.

```
from all_together import matrix
x = matrix([[-3,3,4,5], [2,2,3,3],[2,2,3,22]])
print(x.fraction().matrix)
```

$$\begin{bmatrix} -3 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 \\ 2 & 2 & 3 & 22 \end{bmatrix} \to \begin{bmatrix} `\frac{-3}{1}' & `\frac{3}{1}' & `\frac{4}{1}' & `\frac{5}{1}' \\ `\frac{2}{1}' & `\frac{2}{1}' & `\frac{3}{1}' & `\frac{3}{1}' \\ `\frac{2}{1}' & `\frac{2}{1}' & `\frac{3}{1}' & `\frac{22}{1}' \end{bmatrix}$$

Determinant

Inputs a matrix and returns the determinant.

```
from all_together import matrix
x = [[1,0,0],[0,1,0],[0,0,1]]
print(x.det())
```

$$det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1$$

Reflection

What Went Well

The matrix multiplication and addition methods were relatively easy to implement because they had an explicit formula. For example, if A is a $m \times n$ matrix and B is a $n \times p$ matrix, then

$$(AB)_{ij} = \sum_{r=1}^{n} A_{ir} * B_{rj} \tag{1}$$

Problems We Faced

The inverse and row reduction methods were difficult to implement. The hardest part was swapping the rows. The rows only have a position relative to each other.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Results

This program has already helped us in our linear algebra class by allowing us to compute these steps in an efficient way.

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Are there any questions?