where m_A is the meson mass. We can try to estimate it by looking at the form factors slope at $\Delta^2 = 0$. in eqs. (7), (10) the terms with the pion pole are enhanced in taking derivatives, masking the smoother behaviour of the other terms. From eqs. (8), (9) assuming $F_1^V(\Delta^2) = F_2^V(\Delta^2)/3.71 = m_\rho^2/(\Delta^2 + m_\rho^2)$, we obtain respectively $m_A = 1.07$ GeV and $m_A = 1.18$ GeV.

In a previous work [4], from the commutators

$$[\bar{I}^{(3)}, V_\mu^{(3)}] = 0 , \qquad [\bar{I}^{(-)}, V_\mu^{(3)}] = A_\mu^{(-)}$$

and in the same approximation as above, we derived

$$G(\Delta^2) = F_1^V(\Delta^2) + \frac{\Delta^2}{4m^2} F_2^V(\Delta^2) .$$

From the slope at $\Delta^2 = 0$, we obtained $m_A = 1.24$ GeV (in a more refined calculation of $G(\Delta^2)$ the result was 1.1 GeV). The agreement among the various estimates seems to be rather encouraging.

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ON THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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An exact calculation of the anomalous magnetic moment of the muon to second order in the fine structure constant is given. The uncertainties which limit the possible precision in testing quantum electrodynamics with this effect are also discussed.

In recent years much interest has been devoted to testing of quantum electrodynamics. Up to now the most severe limit is given by the anomalous magnetic moment of the muon [1]. The second order corrections of this quantity in the fine structure constant α have already been calculated by several authors [2], where, however, terms of the order m/M and higher terms are neglected, m and M being the electron and the muon masses respectively. Thus there is an uncertainty of about 5×10^{-7} in the theoretical value of the g-factor of the muon. On the other hand a new measurement of $(g-2)$ of the muon is to be made at CERN in the near future, the experimental error of which is hoped to be of the order of 2×10^{-7} or smaller. These facts stimulated an exact calculation of the second order corrections, which will be reported on here. Moreover, we shall discuss the other uncertainties which limit the possible precision in testing quantum electrodynamics with this effect.

Up to second order in α the anomalous magnetic moment of the muon is represented by the same Feynman diagrams as for the electron except for vacuum polarization of a virtual electron pair by the muon [2]. The contribution of this diagram is given in the standard way

$$\delta_2 = \left(\frac{\alpha}{\pi}\right)^2 \int_0^1 du \int_0^1 dv \, \frac{u^2(1-u)v^2(1-\frac{1}{3}v^2)}{u^2(1-v^2)+\lambda(1-u)}, \qquad \lambda = 4\left(\frac{m}{M}\right)^2 . \tag{1}$$

After a somewhat tedious calculation we obtain the following exact expression

$$\begin{aligned}\delta_2 = \left(\frac{\alpha}{\pi}\right)^2 \Big\{ & -\tfrac{25}{36} + \tfrac{1}{3}\ln\frac{2}{\sqrt{\lambda}} + \frac{\sqrt{\lambda}}{6} + \lambda[\tfrac{11}{12} - \tfrac{3}{4}\ln 2 + \tfrac{3}{8}\ln\lambda] + \\ & + \left(\frac{\sqrt{\lambda}}{4} - \tfrac{5}{16}\lambda^{\frac{3}{2}}\right)\left[\frac{\pi^2}{4} + \tfrac{1}{2}\ln\frac{\lambda}{4}\ln\frac{2+\sqrt{\lambda}}{2-\sqrt{\lambda}} + f\left(\frac{4}{2-\sqrt{\lambda}}\right) + \right. \\ & \left. - f\left(\frac{2\sqrt{\lambda}}{\sqrt{\lambda}-2}\right) - 2f(\tfrac{1}{2}\sqrt{\lambda}) + 2f(-\tfrac{1}{2}\sqrt{\lambda})\right] + \frac{\lambda^2}{8}\left[\frac{\pi^2}{6} - \ln^2 2 + \right. \\ & \left. + \tfrac{1}{4}\ln^2\lambda + \tfrac{1}{2}\ln\frac{4}{\lambda}\ln(4-\lambda) - \tfrac{1}{2}f(\tfrac{1}{4}\lambda)\right]\Big\} = \\ = \left(\frac{\alpha}{\pi}\right)^2 \Big\{ & -\tfrac{25}{36} + \tfrac{1}{3}\ln\frac{M}{m} + \left(\tfrac{1}{3} + \frac{\pi^2}{4}\right)\frac{m}{M} + \\ & + 4\left(\tfrac{2}{3} - \ln\frac{M}{m}\right)\left(\frac{m}{M}\right)^2 + O\left(\frac{m^3}{M^3}\right)\Big\} \approx 1.09586\left(\frac{\alpha}{\pi}\right)^2 .\end{aligned} \tag{2}$$

Here $f(x)$ denotes the Spence function [3]. The two leading terms of (2) are just those which have been given previously [2]. The contribution of the other second order diagrams can be taken from the calculation of the anomalous magnetic moment of the electron [4]

$$\delta_1 = \left(\frac{\alpha}{\pi}\right)^2 \left\{\tfrac{197}{144} + \frac{\pi^2}{12} + \tfrac{3}{4}\zeta(3) - \tfrac{1}{2}\pi^2 \ln 2\right\} \approx -0.32848\left(\frac{\alpha}{\pi}\right)^2 . \tag{3}$$

From (2) and (3) the following numerical value for the total radiative corrections of the g-factor of the muon up to the second order in α is obtained

$$\begin{aligned}(g_\mu - 2) &= 2\left[\frac{\alpha}{2\pi} + \delta_1 + \delta_2 + O(\alpha^3)\right] = \\ &= 2\left[\frac{\alpha}{2\pi} + 0.7674\left(\frac{\alpha}{\pi}\right)^2 + O(\alpha^3)\right] = \\ &= 2 \times 0.001\,165\,526 .\end{aligned} \tag{4}$$

In refs. 2-4 the following numerical data have been used [5, 6], where the error of α is subject to some discussion [5]

$$\alpha^{-1} = 137.0388 \pm 0.0006, \qquad M = (206.767 \pm 0.005)\, m . \tag{5}$$

This result includes also the diagram with vacuum polarization of a $\mu^{\pm}$-pair.

There are several uncertainties involved in the result (4). First of all we have to consider the influence of the errors in α and λ. From (2) we obtain for the absolute error of g_μ

$$\Delta g_\mu = 0.32\ \Delta\alpha + 0.019\ \Delta\lambda . \tag{6}$$

According to (5) the error of α causes an uncertainty of 1×10^{-8} for g_μ, whereas the influence of the error of λ is less important: 9×10^{-11}. Moreover, the third order corrections in α are of course not included in (4), since they are unknown at present. But they can reasonably be assumed to be at most of the order $(\alpha/\pi)^3 = 1.25 \times 10^{-8}$. Thus, the value of g_μ given in (4) is the prediction of pure quantum electrodynamics within about one unit of 10^{-8}.

Deviations from pure quantum electrodynamics have to be expected first of all by weak and strong interactions. The influence of weak interactions has been studied by several authors. Their results differ considerably, but in general they are of the order of 10^{-9} or smaller [7]. The situation is different with strong interaction. Here it has been shown [8] that the existence of the ρ- and ω-resonances results in a considerable pionic contribution to the photon propagator associated with the radiative corrections to the g-factor of the muon. If we use the data of Rosenfeld et al. [6], this contribution is estimated to about 5×10^{-8}. The influence of the strongly interacting particles thus exceeds the contribution of the third order terms as well as the inaccuracy of g_μ caused by the errors of α and λ. Therefore, third order terms can probably not be tested without a more detailed knowledge of the influence of strong interactions. Nevertheless to test quantum electrodynamics as far as possible with this effect it would be very worthwhile to improve the accuracy of the planned experiment at least half an order of magnitude further.

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THE SATURATION PROBLEM IN THE NON-RELATIVISTIC QUARK MODEL OF "ELEMENTARY" PARTICLES

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It is shown how a situation in which a three quark system is strongly bound to form a proton, while a four or more quark system is unbound or has a binding energy much smaller than expected at first sight may produce itself in the quark model of "elementary" particles.

Among the questions which we have listed as needing clarification in the non-relativistic quark model of elementary particles [1], that of "saturation" is a prominent one; in view of the many succesfull tests of the model [2-4] it now appears appropriate to propose a possible qualitative solution to this question. This is the purpose of this note.

In what follows we concentrate our attention on the baryons although what we are going to say may clearly extend to the mesons; we may think of quarks having a mass $M \approx 10$ GeV*, but our remarks will be independent on the precise value of the mass of the quark. We need only assume:

1) that the mass M of the quark is much larger then the mass M_B of the baryons:

$$M \gg M_B$$

2) that

$$M \gg \mu$$

where μ^{-1} is the range of the interaction between two quarks.

If the force between quarks is dominantly transmitted by vector mesons $\mu^{-1} \approx 5 m_{\pi}$. In the following we may think, for concreteness, of the potential between two quarks as a potential well with a radius having the above order of magnitude. Note that 1) and 2) are the basic assumptions of the non-relativistic quark model.

The question of saturation can in its essentials be formulated as follows: why systems with 4 or 5 quarks are not much more bound (or, in other words, why they are not much lighter) than a system of three quarks, say, to be definite, a proton? And also: why a system of 6 quarks is a deuteron and not a collapsed aggregate **

To clarify this question we begin by recalling that, assuming we deal only with two body forces, the average potential energy of a pair of quarks in the proton is:

$$|\overline{V}| \approx M - \tfrac{1}{3} M_P \qquad (1)$$

In fact, in the non relativistic quark model the kinetic energy is negligible, with respect to

* Compare ref.5, where a crude estimate of 10 GeV is given for the mass M of the 5 quark based on the difference between the average mass of the odd parity and even parity mesons.

** Note that the "saturation" question is masked, but nevertheless exists also in all the more abstract approaches; it can be stated in this way: why are the representations $\underline{8}$ for the mesons and $\underline{8}$, $\underline{10}$ for the baryons the lowest ones and in fact the only ones so far observed