CSC265 Fall 2020 Homework Assignment 3

due Tuesday, October 6, 2020

1. Construct a data structure based on a LOBS that represents a sequence of positive integers S of length n and supports the following operations in $O(\log n)$ time:

   SUBSEQMAX(S, i, j): returns the maximum value among the i'th through j'th elements of the sequence S, where $1 \leq i \leq j \leq n$.

   APPEND(S, v): appends the integer v to the end of the sequence S.

   INSERT(S, i, v): inserts the integer v immediately before the i'th element in the sequence S, where $1 \leq i \leq n$.

   For example, if $S = 12, 3, 8, 8$, then SUBSEQMAX$(S, 2, 3)$ returns 8 and INSERT$(S, 2, 2)$ changes S to $12, 2, 3, 8, 8$.

   (a) Clearly explain how your data structure represents a sequence of positive integers.

   (b) Give an algorithm (in pseudocode) for performing SUBSEQMAX(S, i, j) in this data structure. Briefly explain how your algorithm works. Give the high level idea, NOT a line by line description of the pseudocode.

   (c) Prove that your algorithm for SUBSEQMAX is correct.

   (d) Prove that the worst case time complexity of your algorithm is $\Theta(\log n)$.

   (e) Clearly explain in English how to perform APPEND and INSERT in $O(\log n)$ time. Do not use pseudocode. Briefly explain why your data structure is correct and satisfies the required time complexity bound.

   You may use algorithms and results in the solutions to Homework Assignment 2.

2. Consider the abstract datatype CACHE. An object of this abstract data type is a subset C of $\{1, \ldots, m\}$ of size at most k.

   Initially, $C = \emptyset$. The only operation is ACCESS(p), which adds the number $p \in \{1, \ldots, m\}$ to the set C and, if this causes the size of C to become bigger than k, removes the element from C that was least recently the parameter of an access operation.

   Give a data structure for this abstract data type that uses $O(k \log m)$ space (measured in bits) and has worst case time complexity $O(\log k)$.
   Draw a diagram of your data structure when $k = 2$ and the sequence

   $$\text{ACCESS}(3),\ \text{ACCESS}(1),\ \text{ACCESS}(3),\ \text{ACCESS}(2),\ \text{ACCESS}(2)$$

   is performed starting with $C = \emptyset$.
   Clearly describe the representation of your data structure, how to perform ACCESS, why it is correct, and why it satisfies the required complexity bounds.
   Do not use pseudocode.