CSC265 Fall 20120 Homework Assignment 1

due Tuesday, September 22, 2020

A *rotated list* is a list that can be sorted by cyclically rotating the elements of the list. For example, 1 3 5 7 8 and 7 8 1 3 5 are rotated lists, but 1 7 3 5 8 is not.

The data structure *list of rotated lists* can be used to implement a dictionary of distinct keys. It consists of an array $A[1..s]$, and two integers n and g. The maximum size of the dictionary is s. The $n \leq s$ elements currently stored in the array are partitioned into g groups. Each group is maintained as a rotated list. The first group is stored in the first array element, $A[1]$. The second group is stored in the next two array elements, $A[2..3]$. The third group is stored in the next three array elements, $A[4..6]$. In general, for $1 \leq k < g$, the k^{th} group contains k consecutive elements of the array. However, the last group may contain fewer elements. Finally, the elements of each group are all less than the elements of the next group.

1. For $1 \leq k \leq g$, what is the location, $f(k)$, of the first element in group k? Briefly justify your answer.

2. Give an algorithm (in pseudocode) for performing SEARCH(A, x) in this data structure. It should return the location i of x in $A[1..n]$ or 0, if x is not in $A[1..n]$. If it improves clarity, you can break up your algorithm into subprogams. Briefly explain how your algorithm works. Give the high level idea, NOT a line by line description of the pseudocode.

3. Prove that the worst case time complexity of your algorithm is $\Theta(\log n)$.

4. Prove that your SEARCH algorithm is correct. Note that you may need to state and prove some additional lemmas.

5. Give an algorithm (in pseudocode) for performing INSERT in this data structure in $O(\sqrt{n} \log n)$ time. Give precise specifications for your algorithm. Briefly explain how your algorithm works, why it is correct, and why it runs in $O(\sqrt{n} \log n)$ time.

6. Give an algorithm (in pseudocode) for performing DELETE in this data structure in $O(\sqrt{n} \log n)$ time. Give precise specifications for your algorithm. Briefly explain how your algorithm works, why it is correct, and why it runs in $O(\sqrt{n} \log n)$ time.

7. What are the advantages and disadvantages of using this data structure to implement a DICTIONARY as compared to using a sorted array and an unsorted array?