CSC265 Fall 2020 Homework Assignment 9
due Tuesday, December 1, 2020

Suppose that $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ are two deterministic finite automata with disjoint state sets. Let $Q = Q_1 \cup Q_2$, let $F = F_1 \cup F_2$, and let $\delta : Q \times \Sigma \to Q$ denote the function

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & \text{if } q \in Q_1 \\ \delta_2(q, a) & \text{if } q \in Q_2. \end{cases}$$

Two states $q, q' \in Q$ are *equivalent* if, for all $x \in \Sigma^*$,

$$\delta(q, x) \in F \text{ if and only if } \delta(q', x) \in F.$$

Lemma q and q' are equivalent states if and only if

- for all $a \in \Sigma$, the states $\delta(q, a)$ and $\delta(q', a)$ are equivalent, and

- either $q, q' \in F$ or $q, q' \in Q - F$.

You may use this lemma, without proof. It can be proved by induction, but that is not part of the assignment.

1. Explain how to use the DISJOINT SETS abstract data type to test whether two deterministic finite automata accept exactly the same set of strings. Justify the correctness of your algorithm.

2. What data structure should you use to implement the DISJOINT SETS abstract data type for this application? Justify your answer.