Assignment # 1
(Due Jan 29, 3:00 pm)

Worth: 15%

1. [20 marks]
   Show that the following functions are computable:

   (a) $f(x) = 3x$

   (b) $f(x,y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise.} \end{cases}$

   You can assume the input numbers are in unary, and for part (b), the input is given as $x\$y$.

2. [20 marks]
   A *Turing machine with left reset* is a Turing machine with a singly infinite tape and the following transition function:

   $$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, RESET\}.$$

   If $\delta(q,a) = (q', b, RESET)$, then the Turing machine replaces a with b at its current tape position, moves the tape head to the leftmost position on the tape, and transitions from state q to state q'.

   Show that the set of languages recognized by the class of Turing machines with left reset is the same as the set of languages recognized by the class of standard Turing machines.

3. [20 marks]
   Prove that a language $A \subseteq \Sigma^*$ is semi-decidable if and only if there is a decidable binary relation $R \subseteq \Sigma^* \times \Sigma^*$ such that for all $x \in \Sigma^*$,

   $$x \in A \text{ if and only if there is some } y \in \Sigma^* \text{ for which } (x,y) \in R.$$

   Recall that a binary relation R is decidable if there is a Turing machine M that determines in finite time whether or not $(x,y) \in R$ for a given pair $(x,y) \in \Sigma^* \times \Sigma^*$.

4. [20 marks]
   Show that a language L is decidable if and only if there is some enumerator E that prints the strings of L in lexicographic order.