Worth: 15%

1. [20 marks]
   Let A be an arbitrary language.

   (a) Show that A is semi-decidable if and only if $A \leq_m A_{TM}$. (This, combined with the fact that A_{TM} is semi-decidable, shows that A_{TM} is **complete** for the class of semi-decidable problems.)

   (b) Show that if A is semi-decidable and $A \leq_m A^c$, then A is decidable.

2. [20 marks]
   Let A be an arbitrary language.

   (a) Show that there exists a language B such that $B \not\leq_m A$.

   (b) Show that there exists a language C such that $A \leq_m C$ and $C \not\leq_m A$.

3. [20 marks]
   Show that every infinite semi-decidable language has an infinite decidable subset.

4. [20 marks]
   A Turing machine M has a useless state if there is some state q that is never entered on M's computation beginning on any string w. Let $U = \{\langle M \rangle \mid M \text{ is a Turing machine with a useless state}\}$. Show that $U \in coSD \setminus D$.