Theoretical Change in State based on Angular Velocity

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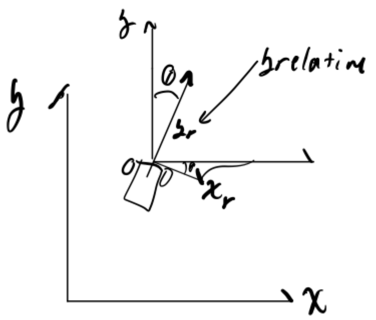
January 24, 2019

1 Defining the Axes.

We shall define a collection of axis. The first is the arbitrary y and x axis with an arbitrary origin representing a static position in space and two static orthogonal directions. (Perhaps based on the corner of the room the robot is placed in.) We will also define a relative axis y_r and x_r whose origin is represented by midpoint between the contacts of the wheels. x_r is parallel to axis defined by connecting the two wheels, and y_r is perpendicular to it and also represents the direction the vehicle would go when the angular momentum of the two wheels are equal and positive.

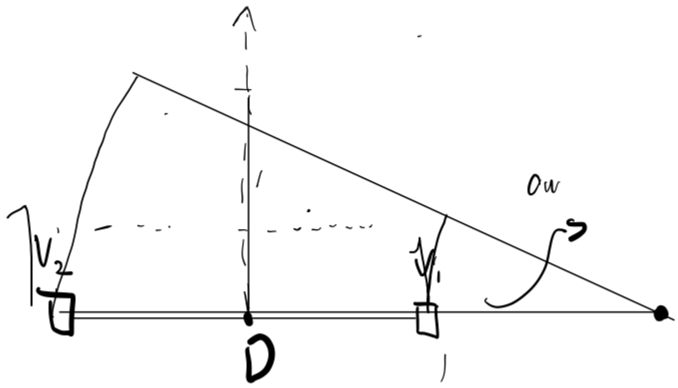


The value θ shall be defined as the angle between the absolute axis y and the relative axis y_r. Thus if directional velocity is represented by $\frac{dy}{dt}$ and $\frac{dx}{dt}$ we can say that:

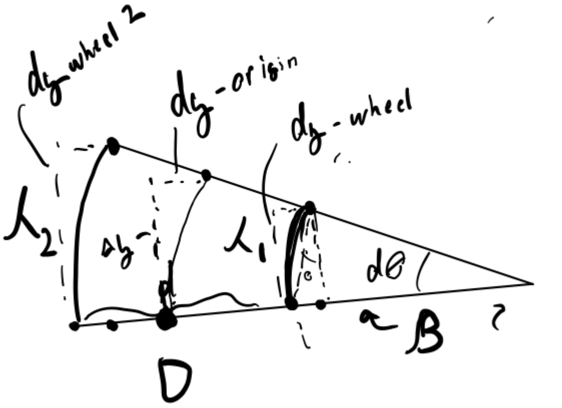
$$dy = dy_r cos(\theta) - dx_r sin(\theta)$$

$$dx = dy_r sin(\theta) + dx_r cos(\theta)$$

1.1 Defining movement based on rotational velocity.



Should we imagine that the angular velocities of the wheels ω_1 and ω_2 are not equal the car would follow an arched path rotating along an arbitrary axis.



We can then define a differential distances $\lambda_1 = \omega_1 r dt$ and $\lambda_2 = \omega_1 r dt$ (where r is the radius of the wheel) representing the differential arched distance each wheel travels and $d\theta$ representing the differential change in angle of the device. D represents the width of the vehicle and β represents the distance between the right wheel and the temporary axis of rotation. We thus we can say:

$d\theta * \beta = \lambda_1$
$d\theta * (\beta + D) = \lambda_2$
$d\theta = \frac{\lambda_2 - \lambda_1}{D}$

Thus:

$\boldsymbol{\frac{d\theta}{dt} = \frac{\omega_2 r - \omega_1 r}{D}}$

Now dy_r is the average of the change in distances of the wheels dy_1 and dy_2. Since this forms an arbitrary unit circle, we can easily say:$dy_1 = \beta sin(d\theta) = \frac{\lambda_1}{d\theta} sin(d\theta)$ and $dy_2 = \frac{\lambda_2}{d\theta} sin(d\theta)$ thus $dy_r = \frac{\lambda_2+\lambda_1}{2d\theta} sin(d\theta)$. We can then substitute in $d\theta$ as defined above, and replace λ with ωr to yield the resulting final equations:

$$dy_r = \frac{D}{2}\frac{\omega_2+\omega_1}{\omega_2-\omega_1} sin(\frac{\omega_1-\omega_2}{D} r dt)$$

$$dx_r = \frac{D}{2}\frac{\omega_2+\omega_1}{\omega_2-\omega_1} (1 - cos(\frac{\omega_1-\omega_2}{D} r dt))$$

If $\omega_2 - \omega_1 \neq 0$. If $\omega_2 - \omega_1 = 0$ we will end up dividing 0 by 0, thus we must take the limit as this difference approaches 0 which yields:

$dy_r = r\omega_1 = r\omega_2$
$dx_r = 0$

If $\omega_2 - \omega_1 = 0$. This intuitively makes sense. Also if $\omega_1 = \omega_2$ then both distances would be 0, which also checks out since this would be the condition where the cart is simply spinning in a circle. To obtain change in absolute position one must simply plug in these relative equations into the aforementioned relative equation. So once again:

$$dy_r = \begin{cases} \frac{D}{2}\frac{\omega_2+\omega_1}{\omega_2-\omega_1} sin(\frac{\omega_1-\omega_2}{D} r dt), & \text{if } \omega_2 - \omega_1 \neq 0 \\ r\omega_1 = r\omega_2, & \text{otherwise} \end{cases}$$

$$dx_r = \begin{cases} \frac{D}{2}\frac{\omega_2+\omega_1}{\omega_2-\omega_1} (1 - cos(\frac{\omega_1-\omega_2}{D} r dt)), & \text{if } \omega_2 - \omega_1 \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

$dy = dy_r cos(\theta) - dx_r sin(\theta)$
$dx = dy_r sin(\theta) + dx_r cos(\theta)$

$d\theta = \frac{\omega_2 r - \omega_1 r}{D} dt$