→ Avaliação 1 – Álgebra Linear

Andrew Gabriel Gomes - Mátricula: 201110015

1°

$S = 2+1+1+1+1+5$

$\boxed{S = 11}$ $\boxed{S^2 = 121}$

$a = 121 \% 8$

$$a = \begin{array}{r|l} 121 & 8x \\ -120 & 15 \\ \hline \boxed{1} & \end{array}$$

$\boxed{a = 1}$

informações

→ determinar uma T.L $T: \mathbb{R}^3 \to \mathbb{R}^3$

onde o $N(T) = [(1,0,2)]$

→ determinar a base para a imagem de T.

---

1) base β para $D_T = \mathbb{R}^3$ onde $(1,0,2) \in \beta$

$\beta = \{(1,0,2); (1,1,0); (0,0,1)\} \subset \mathbb{R}^3$

β' para $\text{Im}\,T = \mathbb{R}^3$

$\beta' = \{(1,0,0); (0,1,0); (0,0,1)\}$ base canônica

$$\begin{cases} T(1,0,2) = (0,0,0) \\ T(1,1,0) = (1,0,0) \\ T(0,0,1) = (0,1,0) \end{cases}$$

$$(x,y,z) = a(1,0,2) + b(1,1,0) + c(0,0,1)$$

$$(x,y,z) = (a+b,\ b+c,\ 2a+c)$$

$$\begin{cases} a+b = x \to \underline{a = x-y} \\ \underline{b = y} \\ 2a+c = z \to c = z-2(x-y) \end{cases}$$

$$\underline{c = z-2x-2y}$$

$$(x,y,z) = (x-y)(1,0,2) + y(1,1,0) + (z-2x-2y)(0,0,1)$$

$$T(x,y,z) = T\big((x-y)(1,0,2)\big) + T\big(y(1,1,0)\big) + T\big((z-2x-2y)(0,0,1)\big)$$

$$T(x,y,z) = (x-y)\,T(1,0,2) + y \cdot T(1,1,0) + (z-2x-2y)\,T(0,0,1)$$

$$T(x,y,z) = (x-y)(0,0,0) + y(1,0,0) + (z-2x-2y)(0,1,0)$$

$$T(x,y,z) = (y,\ z-2x-2y,\ 0)$$

$T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T(x,y,z) = (y,\ z-2x-2y,\ 0)$$

base para $Im(T)$:

$T: \mathbb{R}^3 \to \mathbb{R}^3$

$T(x,y,z) = (y, z-2x-2y, 0)$

$Im(T) = \{ u \in \mathbb{R}^3, T(x,y,z) = u \}$

$= \{ (a,b,c) \in \mathbb{R}^3 ; (y, z-2x-2y, 0) = (a,b,c)$

$\{ (a,b,c) \in \mathbb{R}^3 ; \underline{y = a, z-2x-2y = b, 0 = c} \}$

$Im(T) = \{ (a,b,c) \in \mathbb{R}^4 \mid c = 0 \}$

$\{ (a,b,0) ; a e b \in \mathbb{R} \}$

$\{ a(1,0,0) + b(0,1,0), a,b \in \mathbb{R} \}$

$Im(T) = [(1,0,0);(0,1,0)]$

2^a questão $\underline{a=1}$

→ determinar uma t.l $\mathbb{R}^4 \to \mathbb{R}^3$ tal que

$Im(T) = [(1,0,2);(1,-1,1)]$

① Base β p/ $D_T = \mathbb{R}^4$

$\beta = \{(1,0,0,0);(0,1,0,0);(0,0,1,0);(0,0,0,1)\}$ (canônica)

$T(1,0,0,0) = (1,0,2)$
$T(0,1,0,0) = (1,-1,1)$
$T(0,0,1,0) = (0,0,0)$
$T(0,0,0,1) = (0,0,0)$

$(x,y,z,w) = x(1,0,0,0) + y(0,1,0,0) + z(0,0,1,0) + w(0,0,0,1)$

$T(x,y,z,w)\ x.T(1,0,0,0) + y.T(0,1,0,0) + z.T(0,0,1,0) + w.T(0,0,0,1)$

$T(x,y,z,w) = x.(1,0,2) + y.(1,-1,1) + z(0,0,0) + w(0,0,0)$

$T(x,y,z,w) = (x+y, -y, 2x+y)$

$T: \mathbb{R}^4 \to \mathbb{R}^3$

② base p/ núcleo de T

$T: \mathbb{R}^4 \to \mathbb{R}^3$ — $T(x,y,z,w) = (x+y, -y, 2x+y)$

$N(T) = \{(x,y,z,w) \in \mathbb{R}^4 \mid T(x,y,z,w) = 0\}$

$$\begin{cases} x + y = 0 \\ -y = 0 \\ 2x + y = 0 \end{cases} \rightarrow \begin{matrix} x = 0 \\ y = 0 \end{matrix}$$

$$\left\{ (0, 0, z, w) \mid z \text{ e } w \in \mathbb{R} \right\}$$

$$\left\{ z(0,0,1,0) + w(0,0,0,1) \right\}$$

$$\hookrightarrow \left[(0,0,1,0) ; (0,0,0,1) \right] \quad //$$

Questão 3) $\qquad a = 1)$

$\rightarrow$ Determinar T.L $\quad P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$

$T(1+x) = 2 \quad$ e $\quad T(x^3 + x^2) = x^2 + 2x$

1) base β p $D_T = P_3(\mathbb{R}) = \{ a_0 + a_1 t + a_2 t^2 + a_3 t^3 \mid a_{0,1,2,3} \in \mathbb{R} \}$

$\beta = \{ 1 + x, x^3 + x^2, x^2, x^3 \}$

β' p $\operatorname{Im} T \subset P_2(\mathbb{R}) = \{ a_0 + a_1 t + a_2 t^2 \mid a_{0,1,2} \in \mathbb{R} \}$

$$\beta' = \{2,\ x^2+2x,\ x,\ x^2\}$$

$$\begin{cases} T(1+x) = 2 \\ T(x^3+x^2) = x^2+2x \\ T(x^2) = 0 \\ T(x^3) = 0 \end{cases}$$

$$(t + yx + zx^2 + wx^3) = a(1+x) + b(x^3+x^2) + c(x^2) + d(x^3)$$

$$\begin{cases} a + b^3 = t \\ a + b^2 = y \\ c = z \\ d = w \end{cases} \rightarrow \underline{b = t - y}$$

$$a + (t-y)^2 = y$$
$$\underline{a = y - (t-y)^2}$$

$$(t + yx + zx^2 + wx^3) = (y-(t-y)^2)(1+x) + (t-y)(x^3+x^2) + z(x^2) + w(x^3)$$

$$T(t + yx + zx^2 + wx^3) = (y-(t-y)^2)\,T(1+x) + (t-y)\cdot T(x^3+x^2) + \cancel{3.T(x^2) + wT(x^3)}$$

$$T(t + yx + zx^2 + wx^3) = (y-(t-y)^2)\cdot 2 + (t-y)(x^2+2x)$$

$$\boxed{T(t + yx + zx^2 + wx^3) = 2\cdot(y-(t-y)^2) + (t-y)\cdot(x^2+2x)}$$

Questão 4

Determinar T^{-1} (inversa) de

$T: \mathbb{R}^3 \to \mathbb{R}^3 \quad T(x,y,z) = (x,-y,z)$

$T(1,0,0) = (1,0,0)$
$T(0,1,0) = (0,-1,0)$
$T(0,0,1) = (0,0,1)$

$\beta' = \{(1,0,0);(0,-1,0);(0,0,1)\}$

$T^{-1}(1,0,0) = (1,0,0)$
$T^{-1}(0,-1,0) = (0,1,0)$
$T^{-1}(0,0,1) = (0,0,1)$

$(x,y,z) = a(1,0,0) + b(0,-1,0) + c(0,0,1)$

$(x,y,z) = a, -b, c$

$$\begin{cases} a = x \\ -b = y \quad (-1) \\ c = z \end{cases}$$

$(x,y,z) = x(1,0,0) - y(0,-1,0) + z(0,0,1)$

$T^{-1}(x,y,z) = x.T^{-1}(1,0,0) - y.T^{-1}(0,-1,0) + z.T^{-1}(0,0,1)$

$T^{-1}(x,y,z) = x.(1,0,0) - y.(0,1,0) + z(0,0,1)$

$\boxed{T^{-1}(x,y,z) = (x,-y,z)}$

$T(1,2,3) = (1,-2,3)$
$T^{-1}(1,-2,3) = (1,2,3)$ ✓

$T(4,-5,6) = (4,5,6)$
$T^{-1}(4,5,6) = (4,-5,6)$ ✓

→ Como a Imagem de T é $\mathbb{R}^3$

Concluímos então que T é sobrejetora,

logo T também é bijetora,

consequentemente $\exists T^{-1}$ e como

calculado acima:

$$T^{-1}(x,y,z) = (x,-y,z)$$

//