Lista de exercícios — Calculo I — Andrew Gabriel Gomes.

Questão 1:

a) $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \frac{\lim_{x\to 2} x^2+x-6}{\lim_{x\to 2} x-2} = \frac{2^2+2-6}{2-2} = \frac{4+2-6}{0}$

Expressão indeterminada $\to \frac{0}{0}$

$x^2+x-6 \to (x-2)(x+3)$

$x^2+3x-2x-6$

x^2+x-6

$x-2 \to \lim_{x\to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}}$

$\lim_{x\to 2} x+3 = 2+3=5$

$\lim_{x\to 2} x+3 = \boxed{5}$

b) $\lim_{x\to -4} \frac{x^2+5x+4}{x^2+3x-4} = \frac{\lim_{x\to -4} x^2+5x+4}{\lim_{x\to -4} x^2+3x-4} = \frac{(-4)^2+5(-4)+4}{(-4^2)+3.(-4)-4} = \frac{16-20+4}{16-12-4} = \frac{0}{0}$ ∄

x^2+5x+4

$(x+1)(x+4)$

x^2+3x-4

$(x-1)(x+4)$

$= \frac{\lim_{x\to -4} (x+1)\cancel{(x+4)}}{\lim_{x\to -4} (x-1)\cancel{(x+4)}}$

$\frac{x+1}{x-1} = \frac{-4+1}{-4-1} = \frac{-3}{-5}$

c) $\lim_{x\to 2} \frac{x^2-x+6}{x-2} = \frac{\lim_{x\to 2} x^2-x+6}{\lim_{x\to 2} x-2}$

(x^2-x+6) ñ tem como fatorar!

logo:

$\lim_{x\to 2} \frac{x^2-x+6}{x-2} = \nexists$

d) $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} = \frac{\lim_{x\to 4} x^2-4x}{\lim_{x\to 4} x^2-3x-4} \to \frac{(4)^2-4(4)}{4^2-3\cdot 4-4} \to \frac{16-16}{0} = \frac{0}{0}$!!

x^2-4x | x^2-3x-4

$x(x-4)$ | $(x-4)(x+1)$ $\to \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} \to \frac{x}{x+1} - \frac{4}{4+1} = \boxed{\frac{4}{5}}$

e) $\lim_{t\to -3} \frac{t^2-9}{2t^2+7t+3} \to \frac{\lim_{t\to -3} t^2-9}{\lim_{t\to -3} 2t^2+7t+3} \to \frac{0}{2(-3)^2+7(-3)+3} \to \frac{0}{0}$!!

$18-21+3$

t^2-9 | $2t^2+7t+3$

| $(2x+1)(x+3)$ $\to \frac{\cancel{(x+3)}(x-3)}{(2x+1)\cancel{(x+3)}} \quad \frac{(x-3)}{(2x+1)} \to \frac{-3-3}{2.(-3)+1}$

$\boxed{\frac{-6}{-5}}$

$\to a^2-b^2 = (a+b)(a-b) \to a = x$
$b = \sqrt{9} = 3$

$(x+3)(x-3)$

f) $\lim_{x\to -1} \frac{x^2-4x}{x^2-3x-4} \to \frac{\lim_{x\to -1} x^2-4x}{\lim_{x\to -1} x^2-3x-4} \to \frac{(-1)^2-4(-1)}{(-1)^2-3(-1)-4} \to \frac{1+4}{1+3-4} = \frac{5}{0}$!!

x^2-4x | x^2-3x-4

$x(x-4)$ | $(x-4)(x+1)$ $\to \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \frac{x}{x+1} = \frac{-1}{-1+1} \quad \frac{-1}{0}$!!

$=\infty$?

Questão 2

$\lim_{x\to 3} \frac{x^4-8x^3+18x^2-27}{x^4-10x^3+36x^2-54x+27} = \frac{\lim_{x\to 3} x^4-8x^3+18x^2-27}{\lim_{x\to 3} x^4-10x^3+36x^2-54x+27}$

$x^4-8x^3+18x^2-27$ | $x^4-10x^3+36x^2-54x+27$

factor | factor

$(x+1)(x-3)^3$ | $(x-1)(x-3)^3$

$\frac{(x+1)\cancel{(x-3)^3}}{(x-1)\cancel{(x-3)^3}} = \frac{x+1}{x-1} = \frac{3+1}{3-1} \to \frac{4}{2}$

$\boxed{2}$

$$\lim_{x\to 2}\left(\frac{x-2}{\sqrt{2x}-4}\right)\cdot\left(\frac{\sqrt{2x}+4}{\sqrt{2x}+4}\right) = \frac{(x-2)(\sqrt{2x}+4)}{(\sqrt{2x})^2-4^2} \rightarrow \frac{(x-2)(\sqrt{2x}+4)}{(2x-16)} \rightarrow \frac{(x-2)(\sqrt{2x}+4)}{2(x-8)}$$

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} \rightarrow \frac{0}{0} \longrightarrow \left(\frac{x-4}{\sqrt{x}-2}\right)\cdot\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4}\frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x})^2-2^2}$$

$(A+B)(A-B) = a^2 - B^2$

$$\boxed{\lim_{x\to 4}\sqrt{x}+2} \leftarrow \lim_{x\to 4}\frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}$$

$$\sqrt{4}+2 \rightarrow (2+2) = \boxed{4}$$

$$\lim_{x\to 0}\frac{x}{2-\sqrt{4-x}} \rightarrow \frac{0}{0} \rightarrow \frac{(x)}{(2-\sqrt{4-x})}\cdot\frac{(2+\sqrt{4-x})}{(2+\sqrt{4-x})} = \frac{(x)(2+\sqrt{4-x})}{(2)^2-(\sqrt{4-x})^2}$$

$$\frac{\cancel{(x)}(2+\sqrt{4-x})}{\cancel{4}-\cancel{4}-\cancel{x}}$$

$$\lim_{x\to 0} 2+\sqrt{4-x}$$

$$2+\sqrt{4-0}$$

$$2+\sqrt{4}$$

$$2+2 = \boxed{4}$$

Questão 3

a) $\lim_{x\to\infty}\frac{1}{2x+3} = 0$ → quanto maior o valor de x, mais próximo de 0 chegamos

(b) $\lim_{x \to \infty} \frac{3x+5}{x-4} \to \frac{\cancel{x}\left(3+\frac{5}{x}\right)}{\cancel{x}\left(1-\frac{4}{x}\right)} \to \frac{3+\cancel{\frac{5}{x}}_{\to 0}}{1-\cancel{\frac{4}{x}}_{\to 0}} \to \lim_{x \to \infty} 3$

(c) $\lim_{t \to -\infty} \frac{1-x-x^2}{2x^2-7} \to \frac{x^2\left(\frac{1}{x^2}-\frac{x}{x^2}-\frac{\cancel{x^2}}{\cancel{x^2}}\right)}{\left(\frac{2\cancel{x^2}}{\cancel{x^2}}-\frac{7}{x^2}\right)} \to \frac{\cancel{\frac{1}{x^2}}_{\to 0}-\cancel{\frac{1}{x}}-1}{2-\cancel{\frac{7}{x^2}}_{\to 0}} = \boxed{\frac{-1}{2}}$

(d) $\lim_{x \to \infty} \frac{2-3y^2}{5y^2+4y} \to \frac{\cancel{y^2}\left(\frac{2}{y^2}-\frac{3\cancel{y^2}}{\cancel{y^2}}\right)}{\cancel{y^2}\left(\frac{5\cancel{y^2}}{\cancel{y^2}}+\frac{4y}{y^2}\right)} \to \frac{\cancel{\frac{2}{y^2}}-3}{5+\cancel{\frac{4y}{y^2}}} \to \boxed{\frac{-3}{5}}$

(e) $\lim_{x \to \infty} \frac{x^3+5x}{2x^3-x^2+4} \left(\div x^3\right) \to \frac{\left(\frac{\cancel{x^3}}{\cancel{x^3}}+\frac{5x}{x^3}\right)}{\left(\frac{2\cancel{x^3}}{\cancel{x^3}}-\frac{x^2}{x^3}+\frac{4}{x^3}\right)} \to \frac{\left(1+\cancel{\frac{5}{x^2}}_{0}\right)}{\left(2-\cancel{\frac{1}{x}}_{0}+\cancel{\frac{4}{x^3}}_{0}\right)} = \boxed{\frac{1}{2}}$

(f) $\lim_{t \to -\infty} \frac{t^2+2}{t^3+t^2-1} \to \frac{\left(\frac{t^2}{t^3}+\frac{2}{t^3}\right)}{\left(\frac{t^3}{t^3}+\frac{t^2}{t^3}-\frac{1}{t^3}\right)} \to \frac{\left(\cancel{\frac{1}{t}}_{\to 0}+\cancel{\frac{2}{t^3}}_{\to 0}\right)}{\left(1+\cancel{\frac{1}{t}}_{\to 0}-\cancel{\frac{1}{t^3}}_{\to 0}\right)} \to \boxed{0}$

$\to \lim_{x \to +\infty} \left(5x^3-3x^2-2x-1\right) \to \left(\infty-\infty-\infty-1\right) \to \boxed{\text{ind.}}$

$\to x^3\left(5-\frac{3x^2}{x^3}-\frac{2x}{x^3}-1\right) \to g(x) = \left(5-\cancel{\frac{3}{x}}_{\to 0}-\cancel{\frac{2}{x^2}}_{\to 0}-1\right) \to \boxed{g(x) = 4}$

$f(x) = x^3$

$\lim_{x \to +\infty} f(x) = +\infty$ e $\lim_{x \to +\infty} g(x) = 4$ / $f(x) \cdot g(x) = +\infty$ $= \boxed{0}$

→ $\lim\limits_{x \to -\infty} (2x^5 - x^4 + 2x^2 - 1)$ → $x^5\left(\frac{2x^5}{x^5} - \frac{x^4}{x^5} + \frac{2x^2}{x^5} - \frac{1}{x^5}\right)$

$x^5\left(2 - \frac{1}{x} + \frac{2}{x^3} - \frac{1}{x^5}\right)$

$f(x) = x^5$ $\quad$ $g(x) = \left(2 - \underbrace{\frac{1}{x}}_{0} + \underbrace{\frac{2}{x^3}}_{0} - \underbrace{\frac{1}{x^5}}_{0}\right)$

$g(x) = 2$

$\lim\limits_{x \to -\infty} f(x) = -\infty$

e

$\lim\limits_{x \to -\infty} g(x) = 2$

→ $f(x) \cdot g(x) = -\infty$

→ 0

4)

a) → $\frac{6}{x-5} = \frac{6}{5+5} = \frac{6}{0^+} = +\infty$ $\left(\lim\limits_{x \to 5^+} \frac{6}{x-5}\right)$

b) → $\frac{6}{x-5} = \frac{6}{5-5} = \frac{6}{0^-} = -\infty$ $\left(\lim\limits_{x \to 5^-} \frac{6}{x-5}\right)$

c) → $\lim\limits_{x \to 3} \frac{1}{(x-3)^8}$ → ∞

1 → >0

$(x-3)^8$ → • se aproxima de 0
• >0 para x perto de 3

d) → $\lim\limits_{x \to 0} \frac{x-1}{x^2(x+2)}$ → $-\infty$

$x-1$ → <0 (negativo)

$x^2(x+2)$ → • se aproxima de 0
→ >0 para x perto de 0
em ambos os lados
a função diminui
sem limites

e) → $\lim\limits_{x \to -2^+} \frac{x-1}{x^2(x+2)}$ → $-\infty$

$x-1$ → negativo

$x^2(x+2)$ → se aproxima de 0
>0 para x próximo de -2
do lado direito

→ • A função decresce sem limite

f) $\lim_{x \to 5^+} \ln(x-5) \to$

a medida que x se aproxima de 5 a partir da direita, diminui sem limites

$$-\infty$$

5) b) $\lim_{t \to 0} \frac{\text{sen}(3t)}{2t} \xrightarrow{x=3t} \frac{\text{sen}\, x}{\frac{2x}{3}} \to \text{sen}\, x . \frac{3}{2x}$

$x = 3t$
$t = \frac{x}{3}$

$\to \frac{\text{sen}\, x}{2x} . 3 \to \frac{3}{2} . \frac{\cancel{\text{sen}\, x}}{\cancel{x}} \to 1 \to \boxed{\frac{3}{2}}$

e) $\lim_{x \to 0} \frac{\text{sen}\, 2x}{\text{sen}\, 3x} = \lim_{x \to 0} \frac{\frac{\text{sen}\, 2x}{2x} . 2x}{\frac{\text{sen}\, 3x}{3x} . 3x} \to \boxed{\frac{2}{3}}$

a) $\lim_{x \to 0} \frac{10^x - 1}{5^x - 1} \to \frac{\frac{10^x - 1}{x}}{\frac{5^x - 1}{x}}$ $\ln(10)$, $\ln(5)$ $\to \boxed{\frac{\ln 10}{\ln 5}}$

c) $\lim_{x \to 0} \left(1 + \frac{x}{3}\right)^{1/x} = e^{1/3} = \sqrt[3]{e} \to$

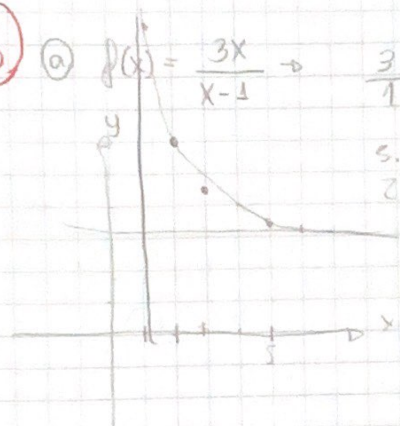
$$\left(\left(1 + \frac{1}{x}\right)^{x}\right)^{1/3}$$

d) $\lim_{x \to +\infty} \left(1 + \frac{2}{x}\right)^{x} \to e \to \boxed{e^2}$

(f) $\lim_{x\to 0}\left(1+\frac{4x}{7}\right)^{\frac{1}{x}} = e^{\frac{4}{7}}$

$\left(\left(1+\frac{4x}{7}\right)^{1/x} \to \left(\left(1+\frac{1}{x}\right)^{x}\right)^{\frac{4}{7}}$

6 (a) $f(x) = \frac{3x}{x-1} \to \frac{3\cdot 1}{1-1} \to \frac{3}{0}$ → logo



$\frac{3\cdot 2}{2-1} \to \frac{6}{1} \to 6$

$\to \frac{3\cdot 3}{3-1} \to \frac{9}{2} \to 4,5$

$\frac{3\cdot 1,5}{1,5-1} \to \frac{4,5}{0,5} = 9$

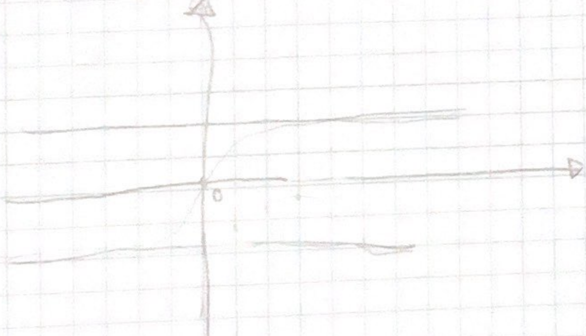
$\frac{3\cdot 5}{5-1} \to \frac{15}{4} \to \frac{3\cdot 6}{6-1} \to$

Assin. vertical → $x=1$

Assin. horizontal → $y=3$

(b) $\frac{2x}{\sqrt{x^2+4}} \to \lim_{x\to\pm\infty} \frac{2x}{\sqrt{x^2+4}} \to \frac{2\cdot 1}{\sqrt{1+4}} = \frac{2\cdot 2}{\sqrt{4+4}}$

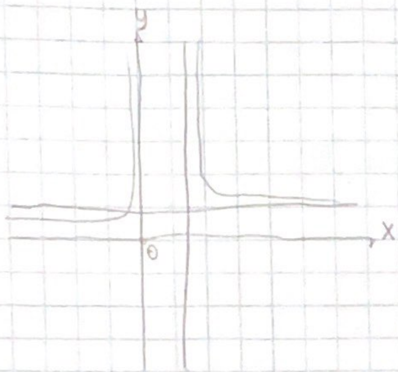
$\frac{2\cdot 3}{\sqrt{9+4}}$



horizontal assintotas = $y = 2, -2$

c) $\dfrac{2x^2+1}{2x^2-3x} \to \lim\limits_{x \to \infty} \dfrac{2x^2+1}{2x^2-3x} \to \dfrac{1}{0} - 0$

$y = 1$



d) $\dfrac{x}{\sqrt{x^2-4}} \to \dfrac{2}{\sqrt{4-4}} = \dfrac{2}{\sqrt{0}} = \dfrac{2}{0}$

determinar asimtota

e) $f(x) = \dfrac{x^3+1}{x^2+4} \to$

f)

7) Analisar a continuidade

A) $f(x) = \begin{cases} \frac{x^2-4}{x+2}, & x \neq -2 \\ 1 & x = -2 \end{cases}$ ⟶ n̂ é contínua em $x = -2$

⟶ $(-2) \in D(f)$ e $f(-2) = -4$ X

⟶ $\lim_{x \to -2} \frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{x+2}$

$\lim_{x \to -2} x-2 = -2-2 = -4$ X

b) $f(x) = x^3 - 2x + 3$ ⟶ $\lim_{x \to 0} x^3 - 2x + 3$ ⟶ $0^3 - 2.(0) + 3$

3

$= 3$

existe limite

$x_0 \in I$, $3 \in D(f)$

⟶ é contínua

c) $f(x) = \frac{x}{x^2 - 1}$ $\lim_{x \to 0} \frac{x}{x^2-1}$ ⟶ $\frac{0}{0^2-1} = \frac{0}{-1}$

↓

$\frac{x}{(x+1)(x-1)}$ ⟶ ?

n̂ tem limite

n̂ é contínua logo

8)

$$f(0) = 0$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} 1 + ax \to \underbrace{1 + a \cdot 0 \to 0}_{1}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^4 + 2a \to \cancel{0}^{4\to 0} + 2a \to 2a$$

Para f ser contínua em 0 precisamos que $\lim_{x \to 0} f(x) = f(0)$

$$\Rightarrow 2a = 0$$
$$a = 0/2 \to a = 0$$

9) a) f é contínua (função polinomial)

$f(x) = x^3 + x - 1$

$f(0) = 0^3 + 0 - 1$

$f(0) = -1$

$f(1) = 1^3 + 1 - 1$

$f(1) = 1$

0 está entre f(0) e f(1) logo existe ao menos 1 valor em [0,1]

b) f é contínua

$f(x) = x^3 + 3x - 5$

$f(1) = 1^3 + 3 \cdot 1 - 5$

$1 + 3 - 5$

$f(1) = -1$

$f(2) = 2^3 + 3 \cdot 2 - 5$

$8 + 6 - 5$

$f(2) = 9$

0 está entre f(1) e f(2)

(c) f é contínua

$\hookrightarrow f\left(\frac{1}{2}\right) = 1 + \frac{\sqrt{2}}{4} > 0$

$\hookrightarrow f\left(\frac{3}{2}\right) = 1 - \frac{3\sqrt{2}}{4} < 0$