– Prova 1 – Calculo II

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Questão 1. → Esboçar gráfico. | curva de nível.

$$T(x,y) = 30 - \left(x^2 + \frac{1}{4} \cdot y^2 + \frac{1}{9} \cdot z^2\right)$$

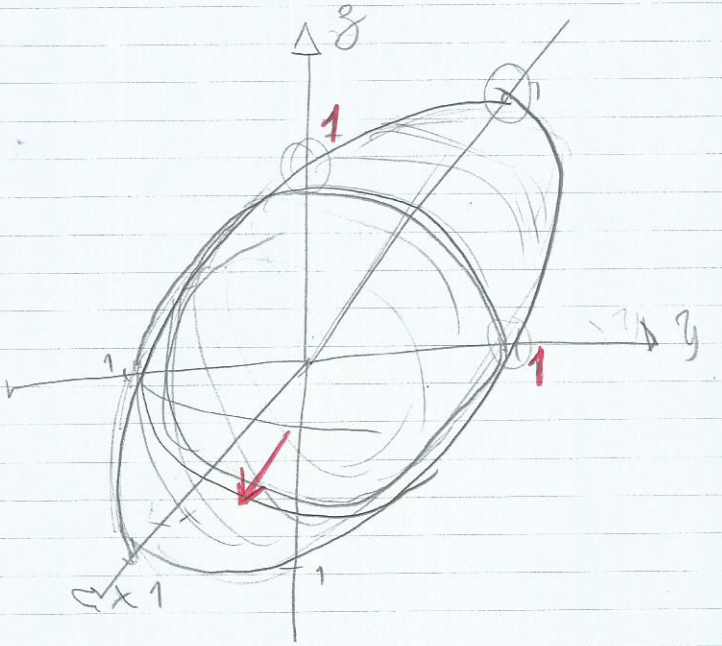
$$30 = x^2 + \frac{1}{4} y^2 + \frac{1}{9} \cdot z^2$$

(a) na origem, onde $x, y, z = 0$ e temperatura $= 30$

(b) na superfície do elipsoide, quando

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

(c) diminuição



3MM

② mostrar que $\lim_{x,y \to 0} \frac{x^2 \cdot y^3}{x^3+y}$ ñ existe

$\frac{x^2 \cdot y^3}{x^3+y}$ → usando + anulamento

$\boxed{\lim_{x,y\to 0} x^2 = 0}$ $\lim_{x,y\to 0} \frac{y^3}{x^3+y}$ limitar

⇓

$0 \leq \frac{y^3}{x^3+y} \leq x^3+y$

$\frac{0}{x^3+y} \leq \frac{y^3}{x^3+y} \leq \frac{x^3+y}{x^3+y}$ → $0 \leq \frac{y^3}{x^3+y} \leq 1$

limitada

$\times$

||

O limite existe!

Questão 3.

n̂ consegui por 2 caminhos

ⓐ $\lim\limits_{(x,y)\to 0} x.\sin\left(\frac{1}{x^2+y^2}\right)$

limitar

$$.x \quad -1 \leq \sin\left(\frac{1}{x^2+y^2}\right) \leq 1 \quad .x$$
$$.x$$

$$\lim -x \leq x\sin\left(\frac{1}{x^2+y^2}\right) \leq \lim x$$

$$\to 0$$

$$\lim_{x,y\to 0} x.\sin\left(\frac{1}{x^2+y^2}\right) = 0$$

ⓑ $\lim\limits_{x,y\to 0} \frac{x^3}{x^2+y^2}$

① $y=x$ $\quad \frac{x^3}{x^2+x^2} \to \frac{\cancel{x}^3}{2\cancel{x}^2} \to \frac{x}{2} = \frac{0}{2} = 0$ ✓

② $y=x^2$ $\quad \frac{x^3}{x^2+x^4} \to \frac{\cancel{x^3}}{\cancel{x^2}(1+x^2)} \to \frac{x}{1+x^2} \to \frac{0}{1} = 0$ ✓

$$\lim_{(x,y)\to 0} \frac{x^3}{x^2+y^2} = 0$$

c) $\lim\limits_{(x,y)\to 0} \frac{3yx^2}{x^3+y}$

1) $y=x$ $\quad \frac{3.x.x^2}{x^3+x} \rightarrow \frac{3.x^3}{x^3+x} \rightarrow \frac{3.\cancel{x^3}}{\cancel{x}(1+x^2)} = \frac{3.x^2}{1+x^2}$

$\downarrow$

2) $y=x^2$ $\quad \frac{3.x^2.x^2}{x^3+x^2} \rightarrow \frac{3\cancel{x^4}}{\cancel{x^2}(1+x)} \rightarrow \frac{3x^2}{1+x} \rightarrow \frac{3.0^2}{1+} = \frac{0}{1} = 0 \qquad \frac{3.0^2}{1+0^2} = \frac{0}{1} = 0$

$\lim\limits_{(x,y)\to 0} \frac{3yx^2}{x^3+y} = 0$

d) $\lim\limits_{(x,y)\to 0} \frac{3x^2-y^2+5}{x^2+y^2+2}$

1) $y=x$ $\quad \frac{3x^2-x^2+5}{x^2+x^2+2} \rightarrow \lim\limits_{x,y\to 0} \frac{3x^2-x^2+5}{x^2+x^2+2} = \frac{5}{2}$ ✓

2) $y=x^2$ $\quad \frac{3x^2-x^4+5}{x^2+x^4+2} \rightarrow \lim\limits_{x,y\to 0} \frac{3x^2-x^4+5}{x^2+x^4+2} = \frac{5}{2}$

Questão 4) $f(x,y) = \frac{3x}{y-x}$

a) $f(1,2) = \frac{3 \cdot 1}{2-1} = \frac{3}{1} = 3$

b) $f(3,-7) = \frac{3 \cdot 3}{-7-3} = \frac{9}{-10} = \boxed{\simeq -0{,}9}$

c) $f(1,-1) = \frac{3 \cdot 1}{-1-1} = \frac{3}{-2} \simeq -1{,}5$

d) $y - x \neq 0 \rightarrow y \neq x$ $\quad D = \{x, y \in \mathbb{R} \mid y \neq x\}$

Questão 5)

a) $f(x,y) = \sqrt{x+y-1} \rightarrow x+y-1 \geq 0$

$x + y \geq 1$

$D = \{x, y \in \mathbb{R} \mid x + y \geq 1\}$

b) $f(x,y) = \frac{1}{2x-y+1} \rightarrow 2x - y + 1 \neq 0$

$2x - y \neq -1$

$D = \{x, y \in \mathbb{R} \mid 2x - y \neq -1\}$

c) $f(x,y) = \ln(x^2 - y + 1)$

$\longrightarrow x^2 - y + 1 > 0$

$D = \{x, y \in \mathbb{R} \mid x^2 - y > -1\}$

$x^2 - y > -1$

d) $f(x,y) = \dfrac{\ln x}{x-1}$ $\longrightarrow x > 0$

$\longrightarrow x - 1 \neq 0$

$x \neq 1$

$D = \{x, y \in \mathbb{R} \mid x \neq 1 \text{ e } x > 0\}$

6) $f(x,y) = \begin{cases} \dfrac{1}{x^2+y^2}, & \text{se } (x,y) \neq (0,0) \\ 1, & \text{se } (x,y) = (0,0) \end{cases}$

→ Verificar se é contínua em (0,0)

$$\lim_{(x,y)\to(0,0)} \frac{1}{x^2+y^2} = \infty$$

O limite tendendo a 0 formam assíntotas.
e em (0,0) a função = 1.

Por 2 caminhos)

1) $y = x$ $\quad \dfrac{1}{x^2+x^2} \to \dfrac{1}{2x^2} \to \dfrac{1}{2 \cdot 0^2} = \cancel{\dfrac{1}{0}} \to \infty$ Contínua em 0,0

2) $y = x^2$ $\quad \dfrac{1}{x^2+x^4} \to \dfrac{1}{0^2+0^4} = \cancel{\dfrac{1}{0}} \to \infty$ V

Questão 7. $f(x,y) = y - x$

a) $D = \{\mathbb{R}\}$

b) $Im = \{\mathbb{R}\}$

c) $C1 = 1 \rightarrow y - x = 1$

$\boxed{y = 1 + x}$

$C3 = 3 \quad y - x = 3$

$\boxed{y = 3 + x}$

$C2 = 2 \longrightarrow y - x = 2$

$\boxed{y = 2 + x}$

$C4 = 4 \quad y - x = 4$

$y = 4 + x$

C1

X	Y
−2	−1
−1	0
0	1
1	2
2	3

C2

x	y
−2	0
−1	1
0	2
1	3
2	4

C3

x	y
−2	1
−1	2
0	3
1	4
2	5

C4

x	y
−2	2
−1	3
0	4
1	5
2	6