Avaliação 2 - Algebra Linear - Andrew Gabriel Gomes

(1) Encontrar base no $\mathbb{R}^3$ na qual $T(x,y,z)$ tem uma matriz triangular superior.

$T(x,y,z) = (x+2y+3z,\ 4y+6z,\ -y-z)$ e (3)

a matriz canônica de T é

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix}$$

a equação característica de t é:

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$$\det(a - \lambda I) = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 6 \\ 0 & -1 & -1-\lambda \end{vmatrix} = 0$$

Matriz triangular superior

$$(1-\lambda)\cdot(4-\lambda)\cdot(-1-\lambda) = 0$$

$$4-\lambda-4\lambda+\lambda^2\cdot(-1-\lambda) = 0$$

$$(\lambda^2-5\lambda+4)\cdot(-1-\lambda)$$

$$-\lambda^2-\lambda^3+5\lambda+5\lambda^2-4-4\lambda$$

$$-\lambda^3+4\lambda^2+\lambda-4 = 0$$

se $\lambda = 4$ $\quad \cancel{-4^3} + 4\cdot 4^2 \cancel{+4-4} = 0$ (4^3) $\quad 0 = 0$

$$-\lambda^3+4\lambda^2+\lambda-4 \quad \lfloor\ \lambda-4$$

$$+\lambda^3-4\lambda^2 \qquad \lambda^2-1$$

$$\cancel{\lambda-4}$$

$$\cancel{-\lambda}+4$$

$\lambda_1 = 4$
$\neq$
$\lambda_2 = 1$
$\neq$
$\lambda_3 = -1$

→ autovalores (3)

→ logo, formam uma base de $\mathbb{R}^3$

$$\lambda^2 - 1 = 0$$
$$\lambda^2 = +1$$
$$\lambda = \sqrt{1}$$
$$\lambda = \pm 1$$

Para $\lambda_1 = 4$

$(A - \lambda_1 I) \cdot v = 0$

$$\begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & -1 & -5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -3x_1 + 2y_1 + 3z_1 = 0 \\ 6z_1 = 0 \\ -y_1 - 5z_1 = 0 \end{cases}$$

$\Downarrow$

$5z_1 = -y_1$

$z_1 = \dfrac{-y_1}{5}$

$-3x_1 + 2.(-5z_1) + 3z_1$

$3x_1 - 10z_1 + 3z_1$

$3x_1 - 7z_1 = 0$

$3x_1 = 7z_1$

$x_1 = \dfrac{7z_1}{3}$ $\quad y_1 = -5z_1$

$v = \left(\dfrac{7z_1}{3}, -5z_1, z_1\right)$

se $z_1 = 3$

$v = (7, -15, 3)$

Para $\lambda_2 = 1$

$(A - \lambda_2 I) . v = 0$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 6 \\ 0 & -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 2y_2 + 3z_2 \\ 3y_2 + 6z_2 \\ -y_2 - 2z_2 \end{cases}$$

$3y_2 = -6z_2$

$y_2 = -3z_2$

$v = (0, -3z_2, z_2)$

se $z_2 = 1$

$v = (0, -3, 1)$

Se $\lambda_3 = -1$

$(A - \lambda_3 I) \cdot v = 0$

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} 2x_3 + 2y_3 + 3z_3 = 0 \\ 5y_3 + 6z_3 = 0 \\ -y_3 = 0 \end{cases}$$

$5y_3 = -6z_3$

$y_3 = \dfrac{-6z_3}{5}$

$2x_3 + 2\left(\dfrac{-6z_3}{5}\right) + 3z_3 = 0$

$2x_3 = \dfrac{12z_3}{5} + 3z_3$

$2x_3 \ \dfrac{-12z_3 + 15z_3}{5}$

$2x_3 + \dfrac{3z_3}{5} = 0$

$2x_3 = \dfrac{-3z_3}{5}$

$x_3 = \dfrac{\frac{-3z_3}{5}}{\frac{2}{1}}$

$\Rightarrow x_3 = \dfrac{-3z_3}{5} \cdot \dfrac{1}{2}$

$x_3 = \dfrac{-3z_3}{10}$

$v = \left(\dfrac{-3z_3}{10}, \dfrac{-6z_3}{5}, z_3\right)$

se $z_3 = 10$

$v = (-3, -12, 10)$

Logo, o conjunto

$\beta = \{\underbrace{(7,-15,3), (0,-3,1), (-3,-12,10)}_{\text{auto vetores}}\}$ é uma base de $\mathbb{R}^2$

Base = $\{(7,-15,3);(0,-3,1);(-3,-12,10)\}$

Matriz: $t(x,y,z)=(x+2y+3z,\ 4y+6z,\ -y-z)$

$t(7,-15,3)=(7-30+9,\ -60+18,\ 15-3)=(-14,-42,12)$

$t(0,-3,1)=(0-6+3,\ -12+6,\ 2)=(-3,-6,2)$

$t(-3,-12,10)=(-3-24+30,\ -48+60\ \ 12-10)=(3,12,2)$

$$\begin{bmatrix} -14 & -3 & 3 \\ -42 & -6 & 12 \\ 12 & 2 & 2 \end{bmatrix}$$

(2) $T: \mathbb{R}^3 \to \mathbb{R}^3$ $\qquad \beta = \{(1,0,0);(0,1,0);(0,0,1)\}$

$$[T] = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

(a) determinar $T(x,y,z)$

Pela definição de matriz de uma t.L

$$T(1,0,0) = 1(1,0,0) - 1(0,1,0) + 0(0,0,1) = (1,-1,0)$$

$$T(0,1,0) = 1(1,0,0) + 0(0,1,0) - 1(0,0,1) = (1,0,-1)$$

$$T(0,0,1) = 0(1,0,0) + 1(0,1,0) - 1(0,0,1) = (0,1,-1)$$

$(x,y,z) \in \mathbb{R}^3$

$$(x,y,z) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$$

$$\begin{cases} a_1 = x \\ a_2 = y \\ a_3 = z \end{cases}$$

$$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$$

$$T(x,y,z) = x.T(1,0,0) + y.T(0,1,0) + z.T(0,0,1)$$

$$= x.(1,-1,0) + y.(1,0,-1) + z.(0,1,-1)$$

$$= (x+y, -x+z, -y-z)$$

$$\boxed{T(x,y,z) = (x+y, -x+z, -y-z)}$$

(b) Matriz T em relação a base

$\beta' = \{(-1, 1, 0) ; (1, -1, 1) ; (0, 1, -1)\}$

$T(x, y, z) = (x+y, -x+z, -y-z)$

$* \quad T(-1, 1, 0) = \underline{(0, 1, -1)}$

$** \quad T(1, -1, 1) = \underline{(0, 0, 0)}$

$*** \quad T(0, 1, -1) = \underline{(1, -1, 0)}$

$* \quad a_1(-1, 1, 0) + b_1(1, -1, 1) + c_1(0, 1, -1)$

$$\begin{cases} -a_1 + b_1 = 0 \implies \underline{b_1 = a_1} \\ a_1 - b_1 + c_1 = 1 \\ b_1 - c_1 = -1 \end{cases}$$

$\cancel{a_1} - \cancel{a_1} + \underline{c_1 = 1}$

$b_1 - 1 = -1$

$b_1 = -1 + 1 = \underline{0}$

$a_1 = 0$
$b_1 = 0$
$c_1 = 1$

$$(T(-1, 1, 0))_{\beta'} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$** \quad a_2(-1, 1, 0) + b_2(1, -1, 1) + c_2(0, 1, -1)$

$$\begin{cases} -a_2 + b_2 = 0 & = \quad b_2 = a_2 \\ a_2 - b_2 + c_2 = 0 & \longrightarrow \cancel{a_2} - \cancel{a_2} + a_2 \implies \underline{a_2 = 0} \\ b_2 - c_2 = 0 & = \quad \underset{=0}{b_2} = \underset{=0}{c_2} \end{cases}$$

$$(T(1, -1, 1))_{\beta'} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

*** $a_3(-1,1,0)+b_3(1,-1,1)+c_3(0,1,-1)$

$$\begin{cases} -a_3+b_3=1 \\ a_3-b_3+c_3=-1 \\ b_3-c_3=0 \end{cases}$$

$a_3 \cancel{-b_3+b_3} = -1$

$b_3=c_3=0$

$1+b_3=1$

$b_3=0$

$$(T(0,1,-1))_{\beta'}=\begin{bmatrix}-1\\0\\0\end{bmatrix}$$

Matriz T $\rightarrow$ $(T(x,y,z))_{\beta'}=\begin{bmatrix}0&0&-1\\0&0&0\\1&0&0\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}$.

3) Encontre os autovalores e autovetores do operador $T(x,y,z)=(x+2y+3z,\ 4y+6z,\ -y-z)$