PROVA 2 – CÁLCULO 1

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Questão 1. $g(x) = x^4 - 2x^2 + 2$ | $g'(x) = 4x^3 - 4x$

ⓐ pontos críticos:

$g'(x) = 4x^3 - 4x$

$4x^3 - 4x = 0$

$x(4x^2 - 4) = 0$

$x = 0$

$4x^2 - 4 = 0$

$4x^2 = 4$

$x^2 = \frac{4}{4}$

$x^2 = 1$

$x = \sqrt{1}$

$x = \pm 1$

Para $x = 0$:
$0^4 - 2.0^2 + 2$
$\boxed{y = 2}$

Para $x = 1$
$1^4 - 2.1^2 + 2$
$1 \cancel{-2} \cancel{+2}$
$\boxed{y = 1}$

Para $x = -1$
$-1^4 - 2.(-1)^2 + 2$
$+1 \cancel{-2.1+2}$
$y = 1$

Pontos críticos:

$(0,2)$, $(1,1)$, $(-1,1)$

ⓑ onde a função é crescente e decrescente.

g crescente $\rightarrow g' > 0$

g decrescente $\rightarrow g' < 0$

$\rightarrow 4x^3 - 4x$

$4x(x^2 - 1)$

$\rightarrow f(x) = 4x$

$f(x) > 0$, $x > 0$

$f(x) < 0$, $x < 0$

$\boxed{h(x)} = x = \pm 1$

$\rightarrow a = 2$

$a > 0$

concavidade para cima

logo:

$h(x) > 0$ p/ $x < -1$, $x > 1$

$h(x) < 0$ p/ $-1 < x < 1$

$f(x)$ — — + +
0

$h(x)$ + — — +
-1 0 1

$g'(x)$ — + — +
-1 0 1

g crescente em $-1 < x < 0$ e $x > 1 \rightarrow (-1,0), (1,\infty)$
g decrescente em $x < -1$ e $0 < x < 1 \rightarrow (-\infty,-1), (0,1)$

(c) pontos e valores de máximo e mínimo

(onde a 1º derivada $= 0$) *

Candidatos, extremos onde g está definida

$(0,2)$
$(1,1)$
$(-1,1)$

→ com os dados onde a função é crescente e decrescente, tem-se:

→ Ponto máximo local em $x=0$. $(0,2)$
→ ponto mínimo local em $x=-1$ $(-1,1)$
→ ponto mínimo local em $x=1$ $(1,1)$

Valores:

(d) pontos de inflexão (onde a 2º derivada $=0$)

$g'(x) = 4x^3 - 4x \rightarrow g''(x) = 3.4x^2 - 4$
$\boxed{12x^2 - 4}$ → Substitui

$12x^2 - 4 = 0$
$12x^2 = 4$
$x^2 = \frac{4}{12}$
$\rightarrow x^2 = \frac{1}{3}$
$x = \sqrt{\frac{1}{3}}$
$x = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \boxed{\pm \frac{\sqrt{3}}{3}}$ →

$x^4 - 2x^2 + 2$ $\qquad x = \pm\frac{\sqrt{3}}{3}$

+ $\left(\frac{\sqrt{3}}{3}\right)^4 - 2\cdot\left(\frac{\sqrt{3}}{3}\right)^2 + 2 \Rightarrow$

$\frac{(\sqrt{3})^4}{3^4} - 2\cdot\frac{(\sqrt{3})^2}{3^2} + 2$

$\frac{9}{81} - \frac{2}{1}\cdot\frac{3}{9} + 2$

$\frac{9}{81} - \frac{6}{9} + \frac{2}{1} \Rightarrow \frac{9 - 54 + 162}{81}$

$\frac{117}{81} \Rightarrow \frac{39}{27} \Rightarrow \boxed{\frac{13}{9}}$

− $\left(-\frac{\sqrt{3}}{3}\right)^4 - 2\cdot\left(\frac{-\sqrt{3}}{3}\right)^2 + 2$

* o expoente troca o sinal e fica igual

$\boxed{\frac{13}{9}}$

Logo, pontos de inflexão: $\left(-\frac{\sqrt{3}}{3}, \frac{13}{9}\right), \left(\frac{\sqrt{3}}{3}, \frac{13}{9}\right)$

~~(−0,57, 1,44) (0,57, 1,44)~~

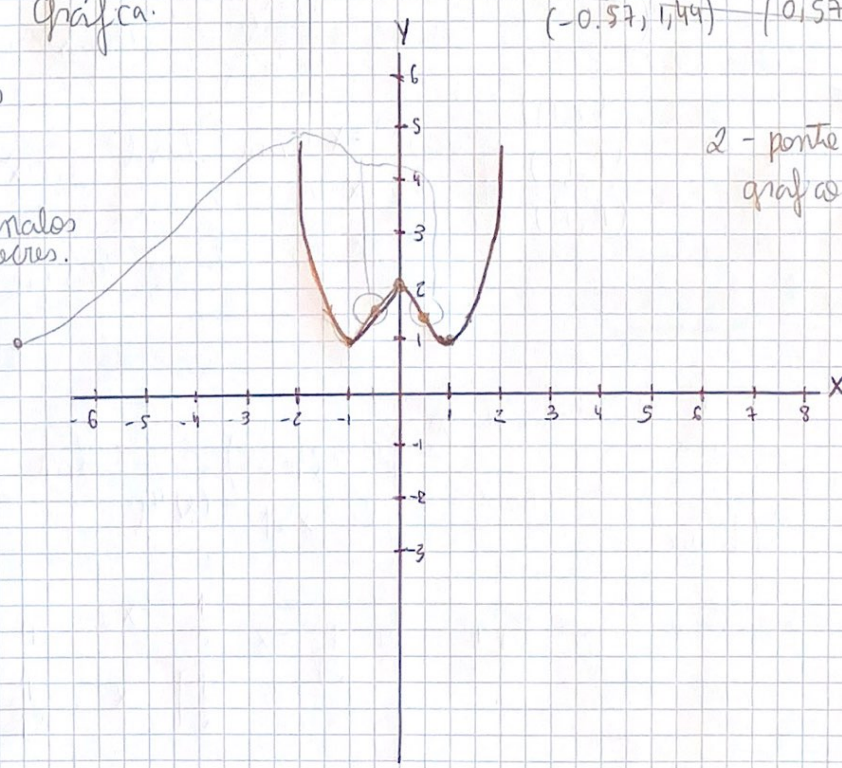
Representação gráfica.

1º: pontos críticos
(0,2) (1,1) (−1,1)

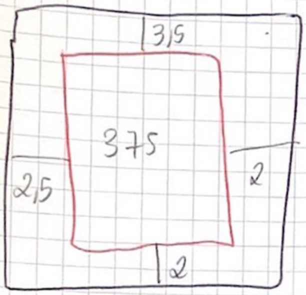
2º: usa os intervalos de cres. e decres.

3º: Pontos de inflexão

2 - ponto onde gráfico toca eixo Y



Questão 2.



→ dimensões da folha para que haja máxima economia de papel.

altura · largura

$AF \cdot LF = 375$

$AF = \dfrac{375}{LF}$ } folha

$Areat = AT \cdot LT$ → total

$At = AF + 2 + 3{,}5 \rightarrow AF + 5{,}5$

$LT = LF + 2{,}5 + 2 \rightarrow LF + 4{,}5$

$Areat = (AF + 5{,}5) \cdot (LF + 4{,}5)$

$Areat = \left(\dfrac{375}{LF} + 5{,}5\right) \cdot (LF + 4{,}5)$

$Areat = 375 + \dfrac{1687{,}5}{LF} + 5{,}5LF + 24{,}75$

$Areat = 5{,}5LF + \dfrac{1687{,}5}{LF^{1}} + 399{,}75$

→ deriva $\quad -1\left(\dfrac{LF^{-1-1=(-2)}}{1687{,}5}\right)$

$Areat' = 5{,}5 - \dfrac{1687{,}5}{LF^2}$

→ encontra quando $Areat' = 0$

$5{,}5 - \dfrac{1687{,}5}{LF^2} = 0 \rightarrow 5{,}5LF^2 - 1687{,}5 = 0$

$5{,}5LF^2 = \dfrac{1687{,}5}{5{,}5} \rightarrow \sqrt{306{,}81} \rightarrow \cong \pm 17{,}52$

* Para definir o sinal precisa ver se é ponto de máximo ou mínimo.

→ $Areat'' = \left(\cancel{5{,}5} - \dfrac{1687{,}5}{LF^2}\right)$

$(-2) \cdot -\dfrac{LF^{-2-1}}{1687{,}5} \rightarrow 2 \cdot \dfrac{1687{,}5}{LF^3}$

$= \dfrac{3375}{LF^3}$ → LF tem que ser > 0 (ponto de mínimo)

logo $LF = 17{,}52$

$AF = \dfrac{375}{17{,}52} \rightarrow AF = 21{,}40$

→ Substitui p/ achar as dimensões.

$AT = AF + 5{,}5$

$AT = 21{,}40 + 5{,}5$

$AT = 26{,}90$ (altura total)

$LT = LF + 4{,}5$

$LT = 17{,}52 + 4{,}5$

$LT = 22{,}02$ (largura total)

Questão 3. $f(x) = 5 - x^2$

$x = 0$
$x = 2$
$y = 0$

X	Y
0	5
1	4
2	1
3	−4
4	−11

Para 2 intervalos

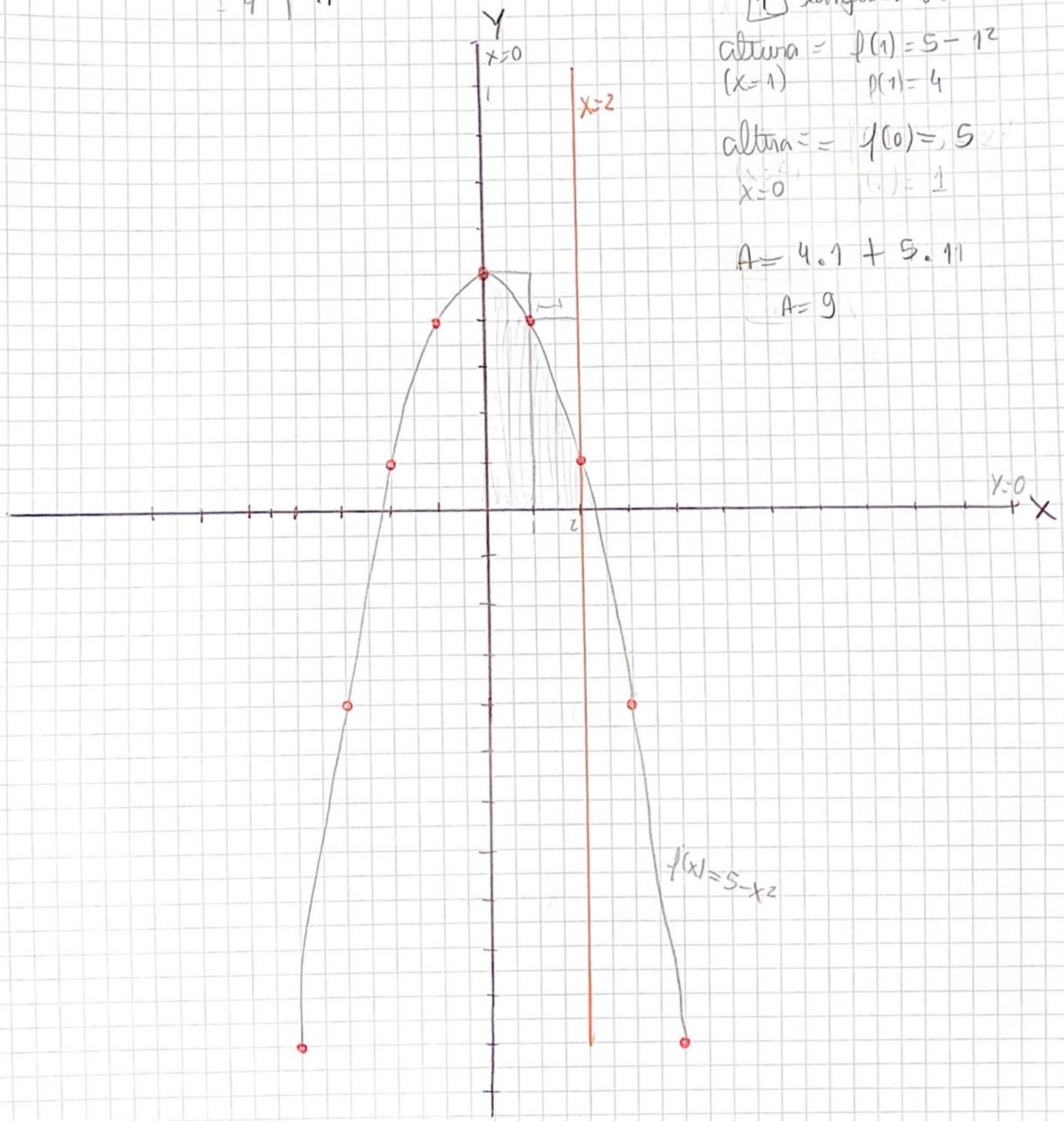
$\boxed{1}$ largura do intervalo

altura = $f(1) = 5 - 1^2$
$(x=1)$ $f(1) = 4$

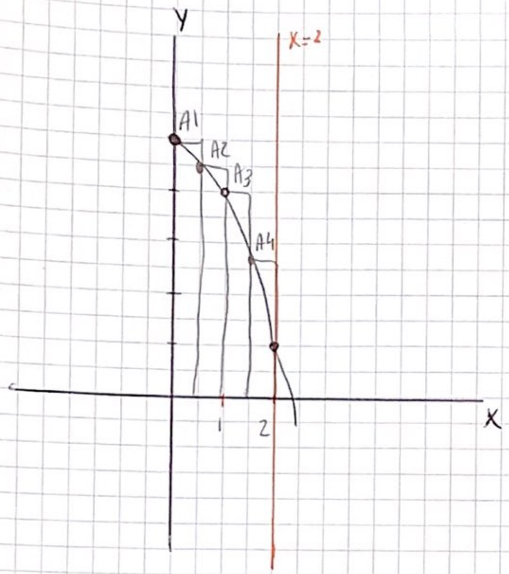
altura = $f(0) = 5$
$x = 0$

$A = 4 \cdot 1 + 5 \cdot 1$

$A = 9$



$f(x) = 5 - x^2$



Para 4

largura = $\frac{1}{2}$

alturas:

$\boxed{A2}$ $x = \frac{1}{2} \rightarrow 5 - \left(\frac{1}{2}\right)^2$

$f\left(\frac{1}{2}\right) \rightarrow \frac{5}{1} - \frac{1}{4} \rightarrow \frac{20-1}{4} \rightarrow \frac{19}{4}$

$A2 = \frac{19}{4}$

$\boxed{A3}$ $x = 1 \rightarrow f(1) = 4$

A4 $x = \frac{3}{2} \rightarrow f\left(\frac{3}{2}\right) = 5 - \left(\frac{3}{2}\right)^2$

$\frac{5}{1} - \frac{9}{4}$

$\frac{20-9}{4} \rightarrow \frac{11}{4}$

$\boxed{A1} = \frac{1}{2} \cdot 5 = \frac{5}{2} \cong 2,5$

$\boxed{A2} = \frac{1}{2} \cdot \frac{19}{4} = \frac{19}{8} \cong 2,375$

A3 $= \frac{1}{2} \cdot 4 = \frac{4}{2} \cong 2$

A4 $= \frac{1}{2} \cdot \frac{11}{4} = \frac{11}{8} \cong 1,375$

$AT \cong 8,25$

Para 8 ↓

$A1 = \frac{1}{4} \cdot 5 = 1,25$

$A2 = \frac{1}{4} \cdot \frac{79}{16} = 1,2343$

$A3 = \frac{1}{4} \cdot \frac{19}{4} = 1,1875$

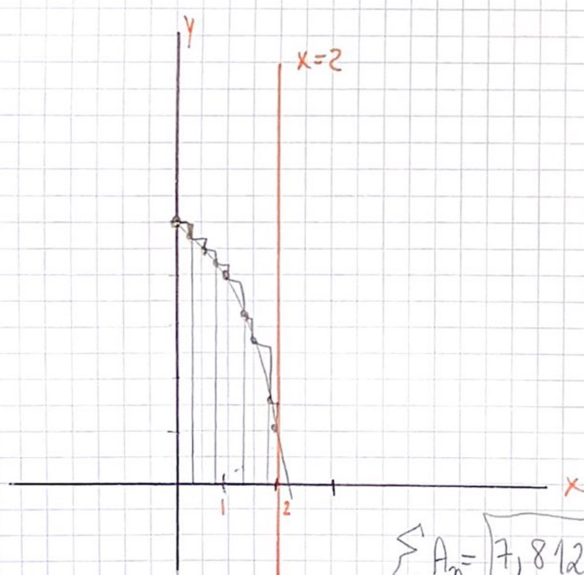
$A4 = \frac{1}{4} \cdot \frac{71}{16} = 1,1093$

$A5 = \frac{1}{4} \cdot 4 = 1$

$A6 = \frac{1}{4} \cdot \frac{55}{16} = 0,8593$

$A7 = \frac{1}{4} \cdot \frac{11}{4} = 0,6875$

$A8 = \frac{1}{4} \cdot \frac{31}{16} = 0,484375$



$\sum A_n = \boxed{7,8122}$

largura = $\frac{1}{4}$

$A2 \to f\left(\frac{1}{4}\right) = 5 - \left(\frac{1}{4}\right)^2 \quad \frac{5}{1} - \frac{1}{16} = \frac{80-1}{16} = \frac{79}{16}$

$A3 = f\left(\frac{2}{4}\right) = \frac{19}{4}$

$A4 = f\left(\frac{3}{4}\right) = 5 - \left(\frac{3}{4}\right)^2 \to \frac{5}{1} - \frac{9}{16} = \frac{80-9}{16} = \frac{71}{16}$

$A5 = f(1) = 4$

$A6 = f\left(\frac{5}{4}\right) = 5 - \left(\frac{5}{4}\right)^2 = \frac{5}{1} - \frac{25}{16} = \frac{80-25}{16} = \frac{55}{16}$

$A7 = f\left(\frac{6}{4}\right) = f\left(\frac{3}{2}\right) = \frac{11}{4}$

$A8 = f\left(\frac{7}{4}\right) = 5 - \frac{49}{16} = \frac{80-49}{16} = \frac{31}{16}$

Questão 4. determinar integrais:

① $\int \sin^4(2x)\cos(2x)\,dx$

$u = \sin 2x$

$\frac{du}{dx} = 2\cos 2x$

$dx = \frac{du}{2\cos 2x}$

$\to \int u^4 \cdot \cancel{\cos 2x} \cdot \frac{du}{2 \cdot \cancel{\cos 2x}}$

$\int \frac{u^4}{2} \cdot du$

$\frac{u^5}{10} + C$

$\frac{\sin^5 2x}{10} + C \to \boxed{\frac{1}{10} \cdot \sin^5(2x) + C}$

II $\int 4x^2 \, e^x \, dx$

$$\begin{cases} u = 4x^2 \\ du = 8x \\ dv = e^x dx \\ v = e^x \end{cases}$$

$\int u \, dv = u \cdot v - \int v \cdot du$

$= 4x^2 \cdot e^x - \int e^x \cdot 8x$

$= 4x^2 \cdot e^x - 8 \cdot \int \boxed{e^x \cdot x} \, dx$

$$\begin{cases} u = x \\ du = 1 \\ dv = e^x \\ v = e^x \end{cases}$$

$x \cdot e^x - \int e^x \cdot 1$

$= 4x^2 \cdot e^x - 8\left(x \cdot e^x - \int e^x \cdot 1 \cdot dx\right)$

$= 4x^2 \cdot e^x - 8x e^x + 8e^x$

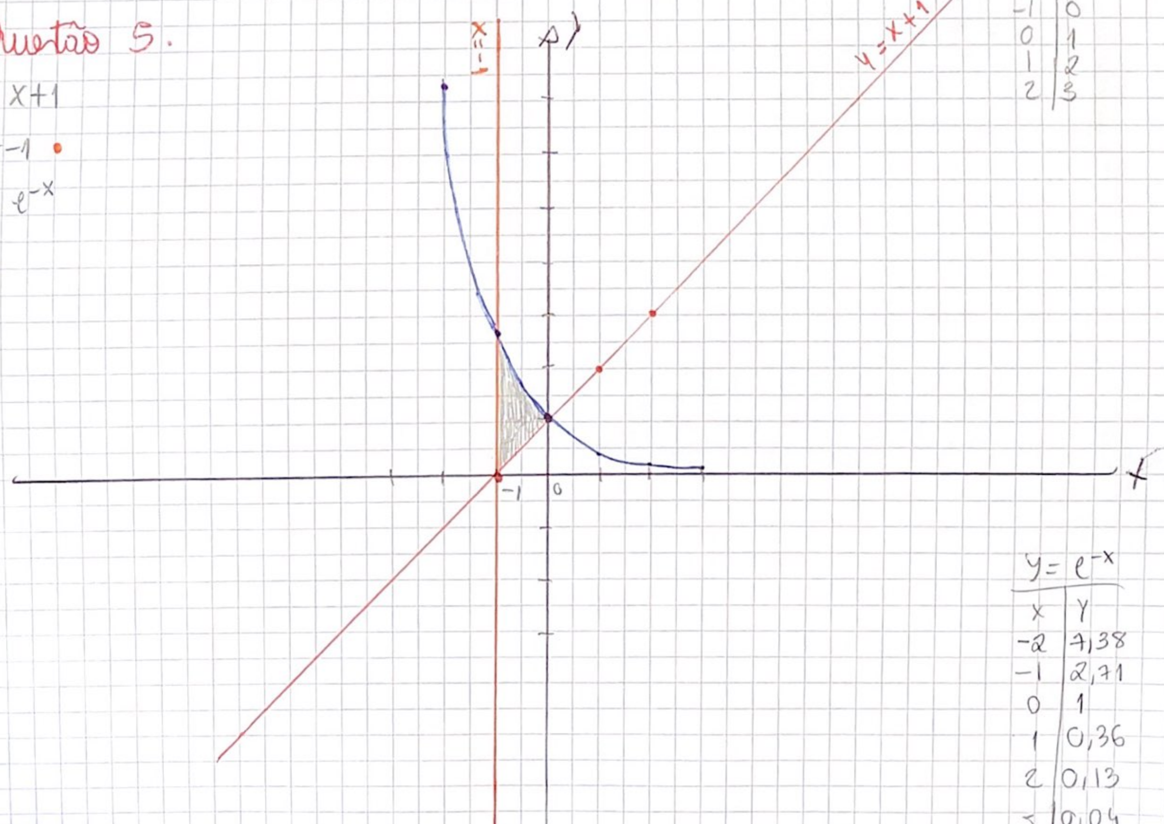
$= \boxed{4\left(x^2 e^x - 2x e^x + 2e^x\right) + C}$

Questão 5.

$y = x + 1$

$x = -1$

$y = e^{-x}$



$y = x + 1$

x	y
-1	0
0	1
1	2
2	3

$y = e^{-x}$

x	y
-2	7,38
-1	2,71
0	1
1	0,36
2	0,13
3	0,04

$$\text{Area} = \int_{-1}^{0} \left(e^{-x} - x + 1\right) dx = \left(-e^{-x} - \frac{x^2}{2} + x\right)\Big|_{-1}^{0}$$

$$0 \rightarrow \left(-e^{-0} - \frac{0^2}{2} + 0\right) - \left(-e^{1} - \frac{(-1)^2}{2} - 1\right)$$

$$(-1) - \left(-2{,}7183 - \frac{1}{2} - \frac{1}{1}\right)$$

$$-\frac{1}{2} - \frac{1}{1}$$

$$\frac{-1-2}{2} \quad \frac{-3}{2}$$

$$-1 - \left(-2{,}7183 - \frac{3}{2}\right)$$

$$-1 + 4{,}21828$$

$$\text{Area} \cong 3{,}21828$$