Avaliação Matemática Discreta

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Matrícula: 2011100015

1) $n = 2011100015$

Calcular R (resto da divisão de n por 4)

$$
\begin{array}{l|l}
2011100015 & 4 \\
\hline
-200 & 502775003 \\
\quad 00111 & \\
\quad -108 & \\
\qquad 3000 & \\
\qquad -3000 & \\
\qquad\quad 015 & \\
\qquad\quad -012 & \\
\qquad\qquad 3 = R &
\end{array}
$$

$R = 3$

$\boxed{R=3}$

(2) Encontrar o 83º termo da Sequência dada por:

$a_0 = -1$ $\quad a_n = -k \cdot a_{n-1} + k^2 \cdot a_{n-2} + k^3 \cdot a_{n-3} + (n^2+2n+1) \cdot k^n$

$a_1 = 1$

$a_2 = 0$ $\quad$ onde $\boxed{k=2}$

~~Sequência~~: $\underbrace{-2 \cdot a_{n-1} + 2^2 \cdot a_{n-2} + 2^3 \cdot a_{n-3} + (n^2+2n+1) \cdot 2^n}$

83º termo $\rightsquigarrow a_{82}$ $\quad$ Relação de recorrência

$a_n = -2\, a_{n-1} + 4 \cdot a_{n-2} + 8 \cdot a_{n-3} + (n^2+2n+1) \cdot 2^n$

Encontrando a solução geral da relação a_n

grau = 3

$r^3 - (-2) \cdot r^2 - (4) \cdot r^1 - (8) \cdot r^0$

$\underbrace{r^3 + 2r^2 - 4r - 8} \rightarrow \boxed{r=2}$ $\quad 2^3 + 2 \cdot 2^2 - 4r - 8$

$8 + 8 - 8 - 8$

0

2 é raiz.

$$\begin{array}{l|l} r^3 + 2r^2 - 4r - 8 & r+2 \\ \hline -\cancel{r^3} + \cancel{2r^2} & r^2 - 4 \\ \quad +4r + 8 & \\ \quad\quad 0 & \end{array}$$

$(r+2) \cdot (r^2 - 4)$ $\quad$ $r^2 - 4 = 0$

$r^2 = \sqrt{4}$

$r = \pm 2$

$=$

$r^3 - 4r + 2r^2 - 8$

$r^3 + 2r^2 - 4r - 8$

Raízes = $\boxed{2 \text{ e } -2}$

$-2^3 + 2 \cdot (-2)^2 - 4 \cdot (-2) - 8 = -8 + 8 + 8 - 8 = 0$

$$a_n = \alpha(2)^n + \beta(-2)^n \qquad \boxed{\alpha, \beta \in \mathbb{R}}$$

Encontrando o 83º termo

$$a_0 = \alpha \cdot 2^0 + \beta \cdot -2^0 = -1$$
$$\alpha + \beta = -1$$

$$a_1 = \alpha \cdot 2^1 + \beta \cdot -2^1 = 1$$
$$2\alpha - 2\beta = 1$$

$$a_2 = \alpha \cdot 2^2 + \beta \cdot -2^2 = 0$$
$$4\alpha + 4\beta = 0$$

$$\begin{cases} \alpha + \beta = -1 \\ 2\alpha - 2\beta = +1 \\ 4\alpha + 4\beta = 0 \end{cases}$$

$$\alpha + \beta = -1$$
$$\alpha = -1 - \beta$$

$$2(-1-\beta) - 2\beta = 1$$
$$-2 - 2\beta - 2\beta = 1$$
$$-4\beta = 3$$
$$\boxed{\beta = -\frac{3}{4}}$$

$$\alpha = -1 - \left(-\frac{3}{4}\right)$$

$$\alpha = \frac{-1}{1} + \frac{3}{4}$$

$$\alpha = \frac{-4+3}{4} = \boxed{\frac{-1}{4} = \alpha}$$

O 83º termo:

$$\boxed{a_n = -\frac{1}{4} \cdot 2^n - \frac{3}{4} \cdot (-2)^n}$$

$$a_{82} = \left(\frac{-1}{4} \cdot 2^{82}\right) - \left(\frac{3}{4} \cdot (-2)^{82}\right)$$

$$\frac{-1}{4} \cdot 2^{82} - \frac{3}{4} \cdot (-2)^{82} = \boxed{(-1) \cdot 2^{82}} \text{ ou } = \boxed{-2^{82}}$$

$$-4.835.703.278.458.520.000.000.000 \;\; ||$$

código de bolito !!

Questão 3 | Mostrar por Indução, que $\forall n \in \mathbb{N}$

$$1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{n}} > 2 \cdot \sqrt{n+1} - 2$$

Caso base: $\boxed{\forall n \in \mathbb{N}}$ $\rightarrow n = 3$

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} > 2 \cdot \sqrt{3+1} - 2$$

$$1 + \frac{1}{1{,}41} + \frac{1}{1{,}73} > 2 \cdot \sqrt{4} - 2$$

$$1 + 0{,}71 + 0{,}58 > 2 \cdot 2 - 2$$

$$2{,}29 > 4 - 2$$

$$2{,}29 > 2$$ ok!

Hipótese I Para K

$$1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{K}} > 2 \cdot \sqrt{K+1} - 2$$

Aí aqui que $\frac{1}{\sqrt{K}} > 2 \cdot \sqrt{K+1} - 2$

Tese Para K+1

$$\underbrace{1 + \frac{1}{\sqrt{2}} + \dots +}_{\frac{1}{\sqrt{K}}} \frac{1}{\sqrt{K+1}} > 2 \cdot \sqrt{K+2} - 2$$

$$\underbrace{1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{K+1}}}_{\frac{1}{\sqrt{K}}} > 2.\sqrt{K+2} - 2$$

D MDC

$$\frac{K^{1/2}}{1}, \frac{(K+1)^{\cancel{1/2}}}{(K+1)} \Bigg| \begin{matrix} K^{1/2} \\ (K+1) \end{matrix}$$

$$1$$

$$\frac{1}{\sqrt{K}} + \frac{1}{\sqrt{K+1}} > 2.\sqrt{K+2} - 2$$

$$\frac{\sqrt{K}.\sqrt{K+1} + K + 1}{\sqrt{K}.(K+1)}$$

↓ simplificando

$$\frac{K^{\frac{1}{2}}.(K+1)^{1/2} + K + 1}{K^{1/2}.(K+1)} > 2.(K+2)^{\frac{1}{2}} - 2$$

$$\overset{||}{\frown}$$

EXERCICIO 4:

$F_n = G_n$

Mostrar por indução, que $\forall n \geq 1$ onde

$F_1 = 1$
$F_2 = 2$
$F_n = F_{n-1} + F_{n+2}$ se $n \geq 3$ e

$$G_n = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$F_n = G_n$

PASSO BASE

$$\begin{cases} F_1 = 1 \\ F_2 = 1 \\ F_n = F_{n-1} + F_{n-2} \end{cases}$$

$$G_n = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$$G_1 = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right]$$

$$\frac{\sqrt{5}}{5}\left(\frac{\cancel{1}+\sqrt{5}}{2} - \frac{\cancel{1}-\sqrt{5}}{2}\right) = \frac{\cancel{\sqrt{5}}}{\cancel{5}}\left(\frac{2\cancel{\sqrt{5}}}{\cancel{2}}\right) = 1 = F_1 = G_1$$

$$G_2 = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right] \Rightarrow \frac{\sqrt{5}}{5}\left(\frac{\cancel{1}+2\sqrt{5}+\cancel{5}}{4} - \frac{\cancel{1}-2\sqrt{5}+\cancel{5}}{4}\right) =$$

$$\rightarrow \frac{\sqrt{5}}{5}\left(\frac{\cancel{2}\sqrt{5}}{\cancel{2}}\right) \rightarrow \frac{\sqrt{5}}{5} \cdot \sqrt{5} = 1 = F_2 = G_2$$

HIPOTESE DE INDUÇÃO: a formula fechada funciona para K e também para K+1

$F_K = G_K$

$F_{K-1} = G_{K-1}$
$F_{K-2} = G_{K-2}$
$\vdots$

TESE $\quad F_{k+2} = G_{k+2}$

$F_k + F_{k+1}$

$$\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k} - \left(\frac{1-\sqrt{5}}{2}\right)^{2}\right] + \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right]$$

$$\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k} + \left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right] = G_{k+2}$$

$$\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k}\cdot\left(1+\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k}\cdot\left(1+\frac{1-\sqrt{5}}{2}\right)\right] = G_{k+2}$$

$$\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k}\cdot\left(\frac{3+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^{k}\cdot\left(\frac{3-\sqrt{5}}{2}\right)\right] = G_{k+2}$$

$$\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k}\cdot\left(\frac{1+\sqrt{5}}{2}\right)^{2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k}\cdot\left(\frac{1-\sqrt{5}}{2}\right)^{2}\right] = G_{k+2}$$

$$\boxed{\frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+2}\right] = G_{k+2}} \checkmark$$

RASCUNHO

$$\left(\frac{1+\sqrt{5}}{2}\right)^{2} = \frac{1+2\sqrt{5}+5}{4}$$

$$\div 2\left(\frac{6+2\sqrt{5}}{4}\right) = \frac{3+\sqrt{5}}{2}$$

EXERCÍCIO 5:

mostre que, se $\begin{cases} \boxed{F_1 = 1} \\ F_2 = 2 \\ F_n = F_{n-1} + F_{n-2}, & \forall n \geq 3 \end{cases}$

então

$$F_n < \left(\frac{7}{4}\right)^n \quad \forall n \geq 1$$

BASE. sendo $P(n) = F_n < \left(\frac{7}{4}\right)^n \quad \forall n \geq 1$

temos $n=1$

$F_1 = 1 < \overset{\cong 1,75}{\frac{7}{4}}$, então $P(1)$ é verdadeira,

supondo que $P(1), P(2), \ldots, P(n), \forall n \geq 2$ sejam verdadeiras

HIPÓTESE DE INDUÇÃO

$$F_{n+1} < \left(\frac{7}{4}\right)^{n+1}, \text{ daí}$$

$$F_{n+1} = F_n + F_{n-1} < \left(\frac{7}{4}\right)^n + \left(\frac{7}{4}\right)^{n-1}$$

$$< \frac{7}{4}\left(\frac{7}{4}\right)^{n-1} + \left(\frac{7}{4}\right)^{n-1}$$

$$< \left(1 + \frac{7}{4}\right)\left(\frac{7}{4}\right)^{n-1}$$

$$F_{n+1} < \left(\frac{7}{4}\right)^2\left(\frac{7}{4}\right)^{n-1}$$

$$F_{n+1} < \left(\frac{7}{4}\right)^{n+1} \quad \forall n \geq 1.$$

//

RASCUNHO:

$1 + \frac{7}{4} \cong 2,75$

preciso de $\left(\frac{7}{4}\right)^2$ e que seja $> 1 + \frac{7}{4}$

$\left(\frac{7}{4}\right)^2 = \frac{49}{16} \simeq 3,0625$