lista 4 – Geometria Analitica

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1. (a) $2\vec{u} \cdot (-\vec{v})$ → $2\vec{u} = (4,-6,-2)$
$-\vec{v} = (-1,1,-4$

$(4,-6,-2) \cdot (-1,1,-4)$

$4.(-1) + (-6.1) + (-2.-4)$

$-4 + (-6) + 8$

$-10 + 8$

Produto Escalar → $\boxed{-2}$

(b) $(\vec{u}+\vec{v}) \cdot (\vec{u}-\vec{v})$

$\vec{u}+\vec{v} = (2,-3,-1) + (1,-1,4)$
$(3,-4,3)$

$\vec{u}-\vec{v} = (2,-3,-1) + (-1,1,-4)$
$(1,-2,-5)$

$(3.1) + (-4.-2) + (3.-5)$

$3 + 8 + (-15)$

$\boxed{-4}$

2. $\vec{u} = (2,-1,3)$ → paralelo logo é multiplo

$\vec{v} = x.\vec{u}$

$\vec{v} = (2x, -x, 3x)$

$(2,-1,3) * (2x,-x,3x) = -42$

$4x + x + 9x = -42$

$14x = -42$

$x = \frac{-42}{14}$

$x = -3$

$\vec{v} = (-6, 3, -9)$

3.

a) $(\vec{U} - 3\vec{V}) \cdot \vec{U} \rightarrow \vec{U} \cdot \vec{U} - 3\vec{V} \cdot \vec{U}$

$= |\vec{U}|^2 - 3 \cdot -1$

$= 2^2 - 3 \cdot -1$

$= 4 + 3$

$= 7$

b) $(2\vec{V} - \vec{U}) \cdot (2\vec{V})$

$2 \cdot 2 \cdot \vec{V} \cdot \vec{V} - 2 \cdot \vec{V} \cdot \vec{U}$

$4|\vec{V}|^2 - 2 \cdot (-1)$

$4 \cdot 3^2 + 2$

$4 \cdot 9 + 2 = 38$

c) $(\vec{U} + \vec{V}) \cdot (\vec{V} - 4\vec{U})$

$\vec{U} \cdot \vec{V} - 4 \cdot \vec{U} \cdot \vec{U} + \vec{V} \cdot \vec{V} - 4\vec{U} \cdot \vec{V}$

$= -1 - 4 \cdot |\vec{U}|^2 + |\vec{V}|^2 - 4 \cdot (-1)$

$= -1 - 4 \cdot 4 + 9 + 4$

$-1 - 16 + 9 + 4$

$-17 + 13$

$= \boxed{-4}$

$(\alpha = x) \rightarrow$ para facilitar

4. $\vec{V_1} = \alpha\vec{i} + 2\vec{j} - 4\vec{k}$

$\vec{V_2} = 2\vec{i} + (1 - 2\alpha)\vec{j} + 3\vec{k}$

$\boxed{\alpha = -5}$

• $\vec{V_1} \perp \vec{V_2} = 0$

$(x + 2 - 4)(2 + (1 - 2x) + 3) = 0$

$(2x + 2(1 - 2x) - 4 \cdot 3) = 0$

$2x + 2 - 4x - 12 = 0$

$-2x - 10 = 0$

$-2x = 10$

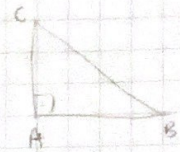
$x = \frac{10}{-2}$

$x = -5$

5. $A(m, 1, 0)$ $B(m-1, 2m, 2)$ $C(1, 3, -1)$

$\vec{AB} = B - A$

$\vec{AC} = C - A$



$\vec{AB} = (m-1)-m,\ 2m-1,\ 2-0 = (-1, 2m-1, 2)$

$\vec{AC} = 1-m,\ 3-1,\ -1-0 \rightarrow (1-m, 2, -1)$

$\vec{AB}.\vec{AC} = 0 \rightarrow \vec{AB} \perp \vec{AC}$

$(-1, 2m-1, 2).(1-m, 2, -1) = 0$

$(-1)(1-m) + (2m-1).2 - 2 = 0$

$-1 + m + 4m - 2 - 2 = 0$

$5m - 5 = 0$

$5m = 5$

$m = \frac{5}{5} \rightarrow m = 1$

6. a) $\vec{AC} \cdot \vec{BD} =$ eles são ortogonais, logo $\vec{AC} \cdot \vec{BD} = 0$

b) $\vec{AB} \cdot \vec{AD} \rightarrow$ $\boxed{\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cdot \cos\theta \qquad \cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}}$ $\rightarrow$ fórmula

$\cos 60° = \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}| \cdot |\vec{AD}|} \rightarrow \frac{1}{2} = \frac{\vec{AB} \cdot \vec{AD}}{2 \cdot 2}$

$\vec{AB} \cdot \vec{AD} = \frac{4}{2} = \boxed{2}$

c) $\vec{BA} \cdot \vec{BC} \rightarrow$ $\cos 120 = \frac{\vec{BA} \cdot \vec{BC}}{2 \cdot 2}$

$-\frac{1}{2} = \frac{\vec{BA} \cdot \vec{BC}}{2 \cdot 2} \rightarrow \vec{BA} \cdot \vec{BC} = -\frac{4}{2} = \boxed{-2}$

d. $\vec{AB} \cdot \vec{BC}$ $\rightarrow$ $\cos 120° = \dfrac{\vec{AB} \cdot \vec{BC}}{|\vec{AB}| \cdot |\vec{BC}|}$ $\rightarrow$ $\dfrac{-1}{2} = \dfrac{-(\vec{BA}) \cdot \vec{BC}}{2 \quad 2}$

$(-\vec{BA}) \cdot \vec{BC} = \dfrac{-4}{2} = -2 \Rightarrow \vec{AB} \cdot \vec{BC} = \boxed{2}$

e. $\vec{AB} \cdot \vec{DC}$ $\rightarrow$ são paralelos $\rightarrow$ $\cos 0° = \dfrac{\vec{AB} \cdot \vec{DC}}{|\vec{AB}| \cdot |\vec{DC}|}$

$=$

$1 = \dfrac{\vec{AB} \cdot \vec{DC}}{2 \cdot 2}$ $\quad \vec{AB} \cdot \vec{DC} = 4$

F. $\vec{BC} \cdot \vec{DA}$ $\rightarrow$

$\vec{BC} \parallel \vec{DA}$ $\quad \theta = 180°$ $\qquad \cos 180° = \dfrac{\vec{BC} \cdot \vec{DA}}{|\vec{BC}| \cdot |\vec{DA}|}$

mas sentido e direção $\neq$ $\qquad -1 = \dfrac{\vec{BC} \cdot \vec{DA}}{2 \cdot 2} = \boxed{-4}$

7. $\vec{u} \perp \vec{v}$ $\quad |\vec{u}| = 6$ $\quad |\vec{u} + \vec{v}| = 10$

$\Downarrow$ $\quad |\vec{v}| = 8$

$\vec{u} \cdot \vec{v} = 0$

$|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + 2\vec{u}\vec{v} + |\vec{v}|^2$

$|\vec{u} + \vec{v}|^2 = 6^2 + 2 \cdot 0 + 8^2$

$|\vec{u} + \vec{v}|^2 = 100$

$|\vec{u} + \vec{v}| = \sqrt{100}$

$|\vec{u} + \vec{v}| = 10$

$|\vec{u} - \vec{v}| =$

$|\vec{u} - \vec{v}|^2 = |\vec{u}|^2 - 2\vec{u}\vec{v} + |\vec{v}|^2$

$|\vec{u} - \vec{v}|^2 = 6^2 - 2 \cdot 0 + 8^2$

$|\vec{u} - \vec{v}|^2 = 100$

$|\vec{u} - \vec{v}| = \sqrt{100}$

$|\vec{u} - \vec{v}| = 10$

8. a) $\vec{u} = (2,-1,-1)$ $\vec{v} = (-1,-1,2)$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}$

$\cos\theta = \dfrac{(2,-1,-1)(-1,-1,2)}{\sqrt{2^2+(-1)^2+(-1)^2}\,\sqrt{(-1)^2+(-1)^2+2^2}}$

$\Downarrow$

$\dfrac{-2+1-2}{\sqrt{4+1+1}\,\sqrt{1+1+4}}$

$\sqrt{6}\,\sqrt{6} = 6$

$\downarrow$

$\dfrac{-3}{6} \rightarrow \boxed{\dfrac{-1}{2}}$

$\cos 120° = \dfrac{-1}{2}$

b) $\vec{u} = (1,-2,1)$ $\vec{v} = (-1,1,0)$

$\cos\theta = \dfrac{\vec{u}\,\vec{v}}{|\vec{u}|\,|\vec{v}|}$

$\cos\theta = \dfrac{(1,-2,1)(-1,1,0)}{\sqrt{1^2+(-2)^2+1^2}\,\sqrt{(-1)^2+1^2+0^2}}$

$\cos\theta = \dfrac{-1-2}{\sqrt{1+4+1}\,\sqrt{1+1+0}}$

$\cos\theta = \dfrac{-3}{\sqrt{6}\,\sqrt{2}} \cdot \dfrac{-3}{\sqrt{12}}$

$\dfrac{-3}{\sqrt{12}} \cdot \dfrac{\sqrt{12}}{\sqrt{12}} \rightarrow \dfrac{-3\sqrt{12}}{12}$

$\dfrac{-\sqrt{3}}{2}$ $\quad \dfrac{-3\cdot 2\sqrt{3}}{12} \rightarrow \dfrac{-6\sqrt{3}}{12}$

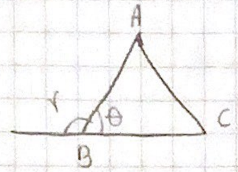
$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array}$$

$\cos\theta = 150°$

9. $A(3,4,4)$

$B(2,-3,4)$

$C(6,0,4)$



$$\cos\theta = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\,|\vec{BC}|}$$

$\vec{BA} = A - B = (3,4,4) + (-2,3,-4)$
$(1,7,0)$

$\vec{BC} = C - B = (6,0,4) + (-2,+3,-4)$
$(4,3,0)$

$|\vec{BA}| = \sqrt{1+49+0} = \sqrt{50}$

$|\vec{BC}| = \sqrt{16+9+0} = 5$

$\vec{BA}\cdot\vec{BC} = 4+21+0 = 25$

$$\cos\theta = \frac{25}{\sqrt{50}\cdot 5} = \frac{5}{\sqrt{50}} \Rightarrow \theta = \arccos\left(\frac{5}{\sqrt{50}}\right) = \boxed{45^\circ}$$

θ = ângulo interno ao vértice $B = \boxed{45^\circ}$

δ = ângulo externo ao vértice $B = 180^\circ - 45^\circ = \boxed{135^\circ}$

10. $|\vec{U}+\vec{V}| = \sqrt{37}$ $\qquad |\vec{U}| = 4 \qquad \vec{U}$ e $\vec{V} = 60^\circ$

$|\vec{U}-\vec{V}| = \sqrt{13}$ $\qquad |\vec{V}| = 3$

$(\vec{U}+\vec{V})\cdot(\vec{U}-\vec{V}) = 7$

$$\cos 60^\circ = \frac{\vec{U}\cdot\vec{V}}{4\cdot 3} \Rightarrow \frac{1}{2} = \frac{\vec{U}\cdot\vec{V}}{6}$$

$\vec{U}\cdot\vec{V} = 6$

$|\vec{U}+\vec{V}|^2 = |\vec{U}| + 2\vec{U}\cdot\vec{V} + |\vec{V}|^2$

$|\vec{U}+\vec{V}|^2 = 4^2 + 2\cdot 6 + 3^2$

$|\vec{U}+\vec{V}|^2 = 37$

$|\vec{U}+\vec{V}| = \sqrt{37}$

$|\vec{U}-\vec{V}|^2 = 4^2 - 2\cdot 6 + 3^2$

$|\vec{U}-\vec{V}| = \sqrt{13}$

$(\vec{U}+\vec{V})\cdot(\vec{U}-\vec{V}) = |\vec{U}|^2 - |\vec{V}|^2 = 4^2 - 3^2 = \boxed{7}$

11. $\vec{u} = ?$ ângulo entre $\vec{u}$ e $\vec{v} = (1,-1,0)$ é $45°$

$\vec{u}$ ortogonal a $\vec{w} = (1,1,0)$

$|\vec{u}| = 2$

$\vec{u} = (x,y,z) \rightarrow x^2 + y^2 + z^2 = 2^2$

$x^2 + y^2 + z^2 = 4$

$\langle u,v \rangle = x - y$

$|\vec{v}|^2 = 1^2 + (-1)^2 + 0^2$

$|\vec{v}|^2 = 1 + 1$

$|\vec{v}|^2 = 2$

$|v| = \sqrt{2}$

$x - y = 2\sqrt{2} \cdot \cos 45$

$x - y = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2}$

$x - y = 2$

$x + y = 0$

$x = -y$

y

$-y - y = 2$

$-2y = 2$

$y = -1$

x

$x = -(-1)$

$x = 1$

z

$1^2 + (-1)^2 + z^2 = 4$

$1 + 1 + z^2 = 4$

$2 + z^2 = 4$

$z^2 = 2$

$z = \sqrt{2}$ ou $z = -\sqrt{2}$ → logo

Vetor $\vec{u} = (1,-1,\sqrt{2})$ ou $(1,-1,-\sqrt{2})$