Prova 1 - Cálculo I

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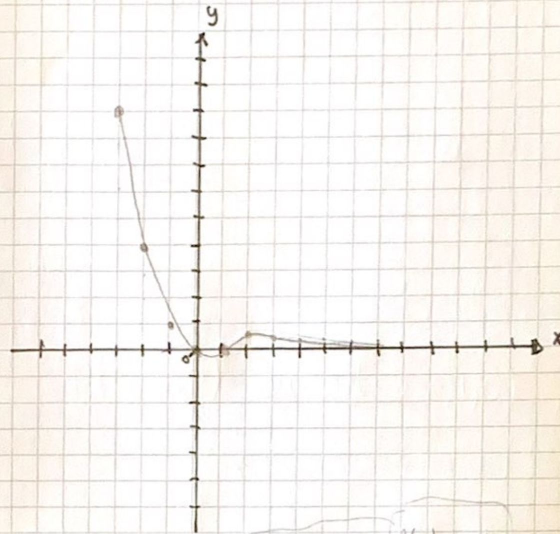
$f(x) = 0, \; x = 1$



1. $f(x) = \begin{cases} x^2 & \text{se } x < 1 \\ 0 & \text{se } x = 1 \\ \frac{1}{x} & \text{se } x > 1 \end{cases}$

$f(-1) = 1$
$f(-2) = 4$
$f(-3) = 9$
$f(-4) = 16$
$\to x < 1$

$f(2) = \frac{1}{2} \quad x > 1$
$f(3) \; \frac{1}{3}$
$f(4) \; \frac{1}{4}$

$f(0) = 0^2 = 0$

(a) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2$

$\lim_{x \to 1^-} (1)^2$

$\lim_{x \to 1^-} = \boxed{1}$

(b) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} \to \lim_{x \to 1^+} \frac{1}{1} = \boxed{1}$

(c) $\lim_{x \to 1} f(x) = \lim_{x \to 1} = \boxed{0}$

(d) $\lim_{x \to -1} f(x) = \lim_{x \to -1} x^2 \to \lim_{x \to -1} (-1)^2 \to \lim_{x \to -1} = \boxed{1}$

(e) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 1/x = \boxed{0}$ (ela aproxima de 0 o tanto quanto quisermos)

2. Usando propriedades, determinar os seguintes limites:

(i) $\lim_{x \to 4} \frac{x^2 - 4x}{x^3 - 4x^2 + x - 4} = \frac{\lim_{x \to 4} x^2 - 4x}{\lim_{x \to 4} x^3 - 4x^2 + x - 4} \to \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x^2+1)}$

$\lim_{x \to 4} \frac{x}{x^2 + 1} \to \frac{4}{4^2 + 1} = \frac{4}{16 + 1} = \boxed{\frac{4}{17}}$

(ii) $\lim_{x\to 2} \dfrac{\sqrt{4x+1}-3}{x-2} = \dfrac{\lim_{x\to 2} \sqrt{4x+1}-3}{\lim_{x\to 2} x-2}$

$$\frac{\sqrt{4x+1}-3}{x-2} \cdot \frac{(\sqrt{4x+1}+3)}{(\sqrt{4x+1}+3)} = \frac{(\sqrt{4x+1})^2 + \cancel{3\sqrt{4x+1}} - \cancel{3\sqrt{4x+1}} - 9}{(x-2)(\sqrt{4x+1}+3)}$$

$$\to \frac{4x+1-9}{(x-2)(\sqrt{4x+1}+3)} \to \frac{4x-8}{(x-2)(\sqrt{4x+1}+3)} \to \frac{4\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{4x+1}+3)}$$

$$\frac{4}{\sqrt{4x+1}+3} \xrightarrow{\text{subs.}} \frac{4}{\sqrt{4.(2)+1}+3} \to \frac{4}{\sqrt{9}+3} = \frac{4}{6}$$

$$\lim_{x\to 2} \frac{\sqrt{4x+1}-3}{x-2} \boxed{=\frac{2}{3}}$$

(iii) $\lim_{x\to -\infty} \dfrac{3x^4-6x^2+1}{6x-x^3-2x^4} = \dfrac{\lim_{x\to-\infty} 3x^4-6x^2+1}{\lim_{x\to-\infty} 6x-x^3-2x^4}$

$$\frac{3x^4-6x^2+1}{6x-x^3-2x^4} \to \frac{\frac{3x^4}{x^4}-\frac{6x^2}{x^4}+\frac{1}{x^4}}{\frac{6x}{x^4}-\frac{x^3}{x^4}-\frac{2x^4}{x^4}} \to \frac{3-\cancel{\frac{6}{x^2}}+\cancel{\frac{1}{x^4}}}{\cancel{\frac{6}{x^3}}-\cancel{\frac{1}{x}}-2} \boxed{=\frac{3}{-2}}$$

$-\infty$ $-\infty$

$-\infty$ $-\infty$

(iv) $\lim_{x \to +\infty} \left(1 + \frac{4}{x}\right)^{x+2}$

3. Assíntotas verticais e horizontais da função → $f(x) \; \frac{2x^2}{x^2-4}$

vertical → $\lim_{x \to 0^+} f(x) \; \frac{2x^2}{x^2-4} = \boxed{\begin{matrix} 2 \\ \text{e} \\ -2 \end{matrix}}$ pois é onde a expressão está indefinida

horizontal → $\lim_{x \to +\infty} f(x) \; \frac{2x^2}{x^2-4} \to \boxed{2}$

como as potências são iguais, a assíntota horizontal é a reta $\boxed{y = \frac{a}{b}}$

$f(x) = \frac{ax^n}{bx^m}$

$n = m = 2$

$a = 2$

$b = 1$

$y = \frac{2}{1} = 2$

4. $x^2 - e^x + 2 = 0$

$f(x) = x^2 - e^x + 2$

$f(-1) = (-1)^2 - e^{-1} + 2$

$= 1 - 0{,}3679 + 2$

$\cong 2{,}6321 > 0$

$f(0) = 0 - 1 + 2$

$f(0) = 1 > 0$

$f(1) = 1 - 2{,}71828 + 2$

$\cong 0{,}28172 > 0$

$f(2) = 4 - 7{,}3890 + 2$

$\cong -1{,}389 < 0$

$f\left(\frac{3}{2}\right) = 2{,}25 - 4{,}4816 + 2$

$= -0{,}2316 < 0$

intervalo $\leq 0{,}5$

$\left[1, \frac{3}{2}\right]$

→ intervalo $\leq 0{,}5$ e conforme o TVI tem solução (raiz)

5 $f(x) = x^2 - 4$ → achar derivada utilizando a definição!

$$f'(1) = \lim_{h\to 0} \frac{((1+h)^2 - 4) - (1^2 - 4)}{h}$$

$$(1^2 + 2h + h^2 - 4) - (-3)$$

$$(\cancel{-3} + 2h + h^2) \cancel{-(-3)}$$

→ $f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$

$f'(x) = 2x$

$f(x) = x^2 - 4$

$$\frac{2h + h^2}{h} \to \frac{\cancel{h}(2+h)}{\cancel{h}} = 2 + \cancel{h} \to 0$$

$$\lim_{h\to 0} = \boxed{2}$$

$f'(1) = 2$

$f(1) = 1^2 - 4$

$f(1) = -3$

$f(2) = 0$

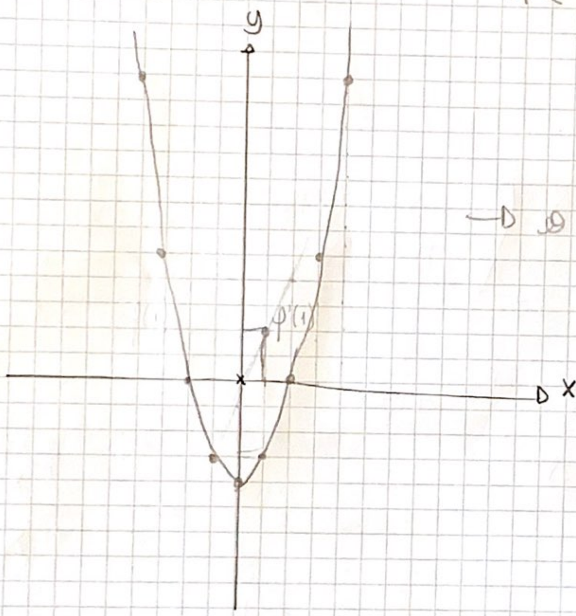
$f(3) = 5$

$f(4) = 12$

$f(5) = 21$

$f(0) = -4$

$f(-1) =$



→ o $f'(1)$ pode ser interpretado como a inclinação da reta tangente ao gráfico de f!

6. determinar as derivadas

(i)

$$y = \frac{(x^2-1)}{(x^3-\operatorname{sen}(x))} \rightarrow y' = \frac{(x^2-1)'.(x^3-\operatorname{sen}(x)) - (x^2-1).(x^3-\operatorname{sen}(x))}{(x^3-\operatorname{sen}(x))^2}$$

$$\frac{2x.(x^3-\operatorname{sen}(x)) - (x^2-1).\ 3x^2-\cos(x)}{(x^3-\operatorname{sen}(x))^2}$$

(ii) $y = (x-\cos x^2).\ln(3x^4-2)$

$$y' = (x-\cos x^2)'.\ln(3x^4-2) + (x-\cos x^2).[\ln(3x^4-2)]'$$

$$(2x\operatorname{sen}(x^2)+1).\ln(3x^4-2) + (x-\cos x^2).\left(\frac{12x^3}{3x^4-2}\right)$$

(iii) $y = tg(x^3-7x) - 3 + cotg(5x)$

$$y' = (tg(x^3-7x))' - [3+cotg(5x)]'$$

$$y' = sec^2(x^3-7x)(3x^2-7) - (-5\operatorname{cossec}^2(5x))$$

(iv) $y = 3(4x^3-5)^5\left(-\frac{3}{x^6}\right)' + e^{x-x^2}$

$g(u) = 3.(u)^5 =' 15.(u)^4$

$u(x) = 4x^3-5 = 12.x^2$

$15.(u)^4.12x^2$

$15(4x^3-5).12x^2$

$y' \rightarrow 15(4x^3-5).12x^2 - \frac{18}{x^7} + e^{x-x^2}(1-2x)$

(V) $y = \left(\sec(3-2x^3)\right)' + \left(\csc(2x+4)\right)' + \left(\sqrt{1+x^2}\right)'$

$y' = -4x \sec(3-2x^2)\tan(3-2x^2) - 2\cot(2x+4)\csc(2x+4)$

$+ \dfrac{x}{(x^2+1)^{1/2}}$