Lista 2- Andrew Gabriel Gomes

Geometria Analitica

① $\vec{u} = (2,-3)$

$\vec{v} = (1,-1)$

$\vec{w} = (-2,1)$

(a) $2\vec{u} - \vec{v} \rightarrow 2\vec{u} + (-\vec{v})$

$2(2,-3) + (-1,+1)$

$(4,-6) + (-1,1)$

$2\vec{u} - \vec{v} = (3,-5)$

(b) $\vec{v} - \vec{u} + 2\vec{w}$

$\vec{v} + (-\vec{u}) + 2\vec{w}$

$(1,1) + (-2,3) + (-4,2)$

$\vec{v} - \vec{u} + 2\vec{w} = (-5,4)$

$\vec{v} - \vec{u} + 2\vec{w} = (-5,4)$

(c) $\frac{1}{2}\vec{u} - 2\vec{v} - \vec{w}$ $(\cdot 2)$

$\vec{u} - 4\vec{v} - 2\vec{w} \longrightarrow \frac{1}{2}\vec{u} - 2\vec{v} - \vec{w} = \left(1, -\frac{1}{2}\right)$

$(2,-3) + (-4,4) + (4,-2)$

$(2,-1) \rightarrow \left(1, -\frac{1}{2}\right)$

② $\vec{u} = (3,-1)$, $\vec{v} = (-1,2)$

(a) $4(\vec{u} - \vec{v}) + \frac{1}{3}\vec{x} = 2\vec{u} - \vec{x}$

$4(\vec{u} + (-\vec{v}))$

$\vec{u} + (-\vec{v}) = (3,-1) + (1,-2)$

$\rightarrow (4,-3)$

$4(4,-3) + \frac{1}{3}\vec{x} = (6,-2) - \vec{x}$

$(16,-12) + \frac{1}{3}\vec{x} = (6,-2) - \vec{x} \quad \cdot (3)$

$(48,-36) + \vec{x} = (18,-6) - 3\vec{x}$

$\vec{x} + 3\vec{x} = (18,-6) - (48,-36)$

$(18,-6) + (-48,+36)$

$4\vec{x} = (-30,30)$

$\vec{x} = \left(\frac{-30}{4}, \frac{30}{4}\right) \longrightarrow \vec{x} = \left(\frac{-15}{2}, \frac{15}{2}\right)$

$\vec{x} = \left(\frac{-15}{2}, \frac{15}{2}\right)$

② (B) $3\vec{x} - (2\vec{v} - \vec{u}) = 2(4\vec{x} - 3\vec{u})$

$3\vec{x} - (2\vec{v} + (-\vec{u})) = 8\vec{x} + (-6\vec{u})$

$3\vec{x} - ((-2,4) + (-3,1)) = 8\vec{x} + (-18,6)$

$3\vec{x} - (-5,5) = 8\vec{x} + (-18,6)$

$3\vec{x} - 8\vec{x} = (-18,6) + (-5,5)$

$-5\vec{x} = (-23,11)$

$\vec{x} = \left(\frac{23}{5}, \frac{-11}{5}\right) \rightarrow \vec{x} = \left(\frac{23}{5}, -\frac{11}{5}\right)$

③ $\vec{w} = a_1.\vec{u} + a_2.\vec{v}$

$a_1 = -1$

$a_2 = 2$

$(-12,6) = a_1.(2,-4) + a_2(-5,1)$

$(-12,6) = -1(2,-4) + 2(-5,1)$

$(-12,6) = (-2,4) + (-10,2)$

$(-12,6) = (-12,6)$

↳ fiz por substituição

$2 \times (-1) + -5 \times 2 = -12$

$-2 + -10$

④ (a) $B = A + 2v$

$(1,3) = (-5,1) + 2(a,b)$

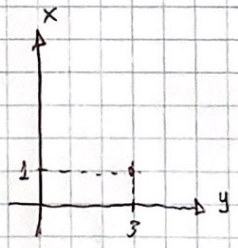
$(1,3) = (-5,1) + (2a,2b)$

$(1,3) = (-5+2a, 1+2b)$

$1 = -5 + 2a \rightarrow 2a = (1+5) \rightarrow a = 6/2 = 3$ (2)

$3 = 1 + 2b \rightarrow 2b = (3-1) \rightarrow b = 2/2 = 1$ (2)

$\vec{v} = (3,1) \rightarrow \vec{v} = (3,1)$



(b) $A = B + 3\vec{v}$

$(-5,1) = (1,3) + 3(a,b)$

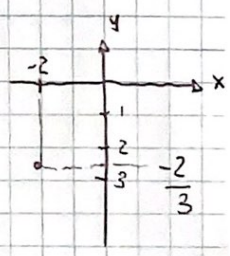
$(-5,1) = (1,3) + (3a,3b)$

$(-5,1) = (1+3a, 3+3b)$

$-5 = 1 + 3a \rightarrow 3a = \frac{-6}{3} = -2$

$1 = 3 + 3b \rightarrow 3b = \frac{-2}{3} = \frac{-2}{3}$

$\vec{v} = \left(-2, \frac{-2}{3}\right) \rightarrow \vec{v} = \left(-2, \frac{-2}{3}\right)$



⑤ Vetor → $\vec{V} = (-1, 3)$

extremidade final = B(3,1)

Ponto Inicial → A(4,-2)

→ Vetor AB → $AB = (xb - xa, yb - ya)$

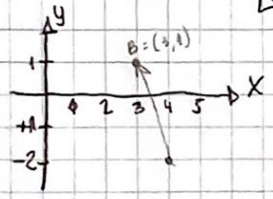
$(-1, 3) = (3 - xa, 1 - ya)$

Ponto Inicial = A(4,-2)

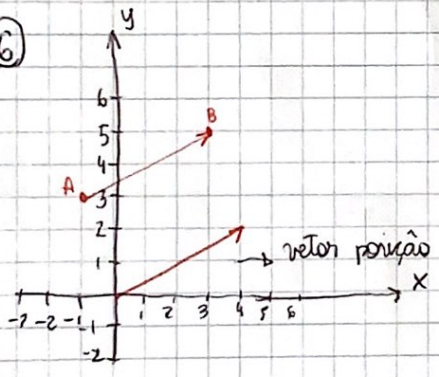
$3 - xa = -1$; $1 - ya = 3$

$xa = 4$; $ya = -2$

Representação Gráfica



⑥

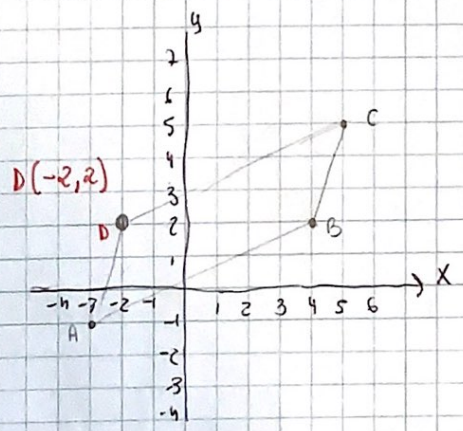


A(-1,3)

B(3,5)

$\vec{V} = \overrightarrow{AB} = (4,2)$

⑦



A(-3,-1)

B(4,2)

C(5,5)

D(-2,2)

⑧ $A(-3,2)$ $B(5,-2)$ $M=(1,0)$ $N=\left(\frac{7}{3},\frac{-2}{3}\right)$

$\vec{AM}=\frac{1}{2}\vec{AB}$ → ponto médio

$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$M=\left(\frac{-3+5}{2},\frac{2-2}{2}\right)$

$M=(1,0)$

$AN=\frac{2}{3}\vec{AB}$ → $AN=N-A$, $AB=B-A$ → $N-A=\frac{2}{3}(B-A)$

$(x,y)=\frac{2}{3}[(5,-2)-(-3,2)]$

$(x,y)=\frac{2}{3}[(5,-2)+(3,-2)]$

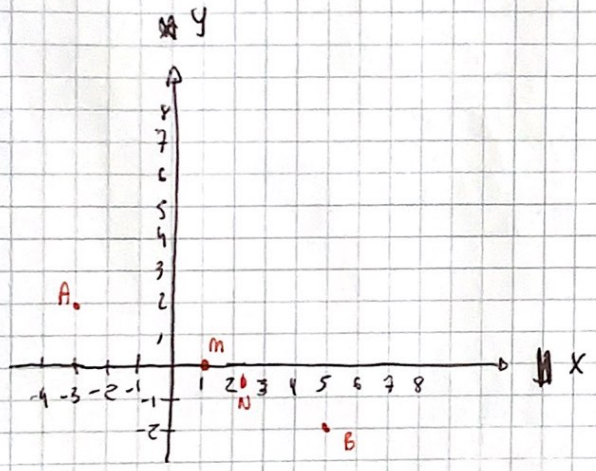
$(x,y)=\frac{2}{3}(8,-4)$

$(x,y)=\left(\frac{16}{3},\frac{-8}{3}\right)+(-3,2)$

$\frac{16}{3}-\frac{3}{1}=\frac{16-9}{3}=\frac{7}{3}$

$(x,y)=\left(\frac{7}{3},\frac{-2}{3}\right)$

$\frac{-8}{3}+\frac{2}{1}=\frac{-8+6}{3}=\frac{-2}{3}$



⑨ $\vec{u}=(1,-1)$

$\vec{v}=(-3,4)$

a) $|\vec{u}|$ → $(1,-1)$ → $\sqrt{1+1}$ → $\sqrt{2}$

b) $|\vec{v}|$ → $(-3,4)$ → $\sqrt{(-3)^2+(4)^2}$ → $\sqrt{9+16}$ → $\sqrt{25}=5$

c) $|\vec{u}+\vec{v}|$ → $\vec{u}+\vec{v}=(1,-1)+(-3,4)$, $u+v=(-2,3)$ → $\sqrt{(-2)^2+(3)^2}$ → $\sqrt{4+9}$ → $\sqrt{13}$

9) d) $\frac{\vec{v}}{|\vec{v}|}$ → vetor → $\vec{v}$

$|\vec{v}| = 5$

$\frac{(-3,4)}{5} \rightarrow \left(\frac{-3}{5}, \frac{4}{5}\right)$

e) $\left|\frac{\vec{v}}{|\vec{v}|}\right| \rightarrow \left|\frac{(-3,4)}{5}\right| \rightarrow \sqrt{\left(\frac{-3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \rightarrow \sqrt{\frac{9}{25} + \frac{16}{25}}$

$\sqrt{\frac{25}{25}}$

$\sqrt{1} = \boxed{1}$

10) $a = ?$

$\vec{v} = (a, -2)$

$|\vec{v}| = 4$

$|\vec{v}^2| = x^2 + y^2$

$4^2 = a^2 + 4$

$16 = a^2 + 4$

$a^2 = 12$

$a = \pm\sqrt{12} \rightarrow a = \pm 2\sqrt{3}$

$a = \pm 2\sqrt{3}$

11) $\vec{v} = (1, -3)$

a) $-2\vec{v} \rightarrow -2.(1,-3)$

$\boxed{(-2, 6)}$

b) $2.\frac{(1,3)}{\sqrt{(1^2 + (-3)^2}} \rightarrow \frac{2(1,3)}{\sqrt{(1+9)}}$

$\frac{(2,-6)}{\sqrt{10}} \leftarrow \frac{2.(1,3)}{\sqrt{10}}$

$\left(\frac{2}{\sqrt{10}}, \frac{-6}{\sqrt{10}}\right)$

$\left(\frac{2}{\sqrt{10}}, -\frac{6}{\sqrt{10}}\right)$

c) $-4.\frac{\vec{v}}{|\vec{v}|} \rightarrow \frac{-4.(1,-3)}{\sqrt{1^2 + (-3)^2}} \rightarrow \frac{(-4, 12)}{\sqrt{10}}$

$\left(\frac{-4}{\sqrt{10}}, \frac{12}{\sqrt{10}}\right)$

$\left(-\frac{4}{\sqrt{10}}, \frac{12}{\sqrt{10}}\right)$