Recuperação II - Algebra Linear

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1º Questão: Determinar valor de $\boxed{a}$ tal que

autovalores de $T(x,y,z) = (3ax - y + z, -x + 5ay - z, x - y + 3az)$

$\lambda$ sejam $1, 2, 6 = \lambda_1, \lambda_2, \lambda_3$

$\boxed{(A - \lambda I)v = 0}$

matriz $A = \begin{bmatrix} 3a & -1 & 1 \\ -1 & 5a & -1 \\ 1 & -1 & 3a \end{bmatrix}$

PARA $\lambda_1 = 1$

$$\begin{vmatrix} 3a-1 & -1 & 1 \\ -1 & 5a-1 & -1 \\ 1 & -1 & 3a-1 \end{vmatrix} \begin{matrix} 3a-1 & -1 \\ -1 & 5a-1 \\ 1 & -1 \end{matrix}$$

$$(5a-1).(3a-1)^2 + 1 + 1 - [\underbrace{(5a-1) + (3a-1) + (3a-1)}_{=11a}] = 0$$

$$-1 + (-1) + (1) = -3$$

$$(5a-1).(9a^2 - 6a + 1) + \underbrace{2 - 11a + 3}_{=5} = 0$$

$$45a^3 - 30a^2 + 5a - 9a^2 + 6a - 1 + 5 - 11a = 0$$

$$11a - 11a$$

$$\frac{45a^3}{3} - \frac{39a^2}{3} + \frac{4}{2} = 0$$

divisores de 3 divisor de 2

$\rightarrow 3a - 2$

$3a - 2 = 0$

$3a = 2$

$\boxed{a = \frac{2}{3}}$ raiz 1

$$\begin{array}{l|l} 45a^3 - 39a^2 + 4 & 3a-2 \\ \cancel{-45a^3} + 30a^2 & 15a^2 - 3a - 2 \\ \quad \cancel{-9a^2} + 4 & \\ \quad + 9a^2 - 6a & \\ \quad \cancel{-6a + 4} & \\ \quad \cancel{+6a - 4} & \end{array}$$

$(3a-2).(15a^2 - 3a - 2)$

$\Delta = 9 - 4.15.-2$

$\Delta = 9 + 120$

$\Delta = 129$

$a_1 = \frac{3 + \sqrt{129}}{30}$ 2

$a_2 = \frac{3 - \sqrt{129}}{30}$ 3

PARA $\lambda_2 = 2$

$$\begin{vmatrix} 3a-2 & -1 & 1 \\ -1 & 5a-2 & -1 \\ 1 & -1 & 3a-2 \end{vmatrix} \begin{matrix} 3a-2 & -1 \\ -1 & 5a-2 \\ +1 & -1 \end{matrix}$$

$$(5a-2).(3a-2)^2 + 2 - \left[(5a-2) + (3a-2) + (3a-2)\right] = 0$$

$$(5a-2).(9a^2 - 12a + 2) + 2 - 11a + 6 = 0$$

$$45a^3 - 60a^2 + 10a - 18a^2 + 24a - 4 \quad + 8 - 11a$$

$$45a^3 - 78a^2 + 23a + 4 = 0$$

{ não é possível fatorar com números racionais

PARA $\lambda_3 = 6$

$$\begin{vmatrix} 3a-6 & -1 & 1 \\ -1 & 5a-6 & -1 \\ 1 & -1 & 3a-6 \end{vmatrix} \begin{matrix} 3a-6 & -1 \\ -1 & 5a-6 \\ +1 & -1 \end{matrix}$$

$$(5a-6).(3a-6)^2 + 2 - \left[(5a-6) + (3a-6) + (3a-6)\right]$$

$$(5a-6).(9a^2 - 36a + 36) + 2 - 11a + 18$$

$$\cancel{45a^3} - \cancel{180a^2} + \cancel{180a} - \cancel{54a^2} + \cancel{216a} - \cancel{216} + \cancel{20} - \cancel{11a}$$

$$45a^3 - 234a^2 + 385a - 196 = 0 \qquad \div (3a-7)$$

$3 \quad 3 \quad 7 \quad 1$

$$3a = 7$$

$$a_1 = \frac{7}{3}$$

$$\begin{array}{l|l} 45a^3 - 234a^2 + 385a - 196 & 3a-7 \\ \hline -45a^3 + 105a^2 & (15a^2 - 43a + 28) \\ -\cancel{129a^2} + 385a - 196 & \\ +\cancel{129a^2} - 301a & \\ \quad \cancel{84a} - \cancel{196} & \\ - \cancel{84a} + \cancel{196} & \end{array}$$

$$(3a-7).(15a^2 - 43a + 28)$$

$$\Delta = 1849 - 4.15.28$$

$$\Delta = 169$$

$$a_2 = \frac{43+13}{30} = \frac{56}{30} \boxed{\frac{28}{15}}$$

$$a_3 = \frac{43-13}{30} = 1$$

Valores de a tal que autovalores sejam 1, 2, 6:

$$a = \frac{2}{3}, \quad \frac{3+\sqrt{129}}{30}, \quad \frac{3-\sqrt{129}}{30}, \quad \frac{28}{15}, \quad 1$$

QUESTÃO 2 · Seja $T: \mathbb{R}^3 \to \mathbb{R}^3$, cuja matriz em base C.

é
$$[T] = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

a) determine $T(x,y,z)$:

$T(x,y,z) = (-x-y-z, -x+z, y+z)$

b) Matriz T em relação a base $\beta' = \{(1,1,0); (1,1,1); (0,1,1)\}$

I) $T(1,1,0) = (-2,-1,1)$

II) $T(1,1,1) = (-3,0,2)$

III) $T(0,1,1) = (-2,1,2)$

I) $a_1(1,1,0) + b_1(1,1,1) + c_1(0,1,1)$

$$\begin{cases} a_1 + b_1 = -2 \longrightarrow a_1 = -2 - b_1 \\ a_1 + b_1 + c_1 = -1 \\ b_1 + c_1 = 1 \longrightarrow c_1 = 1 - b_1 \end{cases}$$

$-2 \cancel{-b_1} \cancel{+b_1} + 1 - b_1 = -1$

$b_1 = 0$

$0 + c_1 = 1$

$a_1 = -2$

$$(T(1,1,0))_{\beta'} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

(II) $a_2(1,1,0)+b_2(1,1,1)+c_2(0,1,1)$

$$\begin{cases} a_2+b_2=-3 \longrightarrow a_2=-3-b_2 \\ a_2+b_2+c_2=0 \\ b_2+c_2=2 \longrightarrow c_2=2-b_2 \end{cases}$$

$$-3\cancel{-b_2}\cancel{+b_2}+2-b_2=0$$

$$\underline{b_2=-1}$$

$a_2-1=-3$

$\underline{a_2=-2}$

$-1+c_2=2$

$\underline{c_2=3}$

$$(T(1,1,1))_{\beta'}=\begin{bmatrix}-2\\-1\\3\end{bmatrix}$$

(III) $a_3(1,1,0)+b_3(1,1,1)+c_3(0,1,1)$

$$\begin{cases} a_3+b_3=-2 \longrightarrow a_3=-2-b_3 \\ a_3+b_3+c_3=-1 \\ b_3+c_3=2 \longrightarrow c_3=2-b_3 \end{cases}$$

$$\cancel{-2}\cancel{-b_3}\cancel{+b_3}\cancel{+2}-b_3=1$$

$$\underline{b_3=-1}$$

$$(T(0,1,1))_{\beta'}=\begin{bmatrix}-1\\-1\\3\end{bmatrix}$$

$a_3-1=-2$

$\underline{a_3=-1}$

$-1+c_3=2$

$\underline{c_3=3}$

matriz $T \rightarrow$

$$(T(x,y,z))_{\beta'}=\begin{bmatrix}-2&-2&-1\\0&-1&-1\\1&3&3\end{bmatrix}$$

QUESTÃO 3

Calcular autovalores e autovetores unitários do operador $T$

$P_2(\mathbb{R}) \to P_2(\mathbb{R})$ dado por

$$T(a+bx+cx^2) = (3a-b+c) + (-a+5b-c)x + (a-b+3c)x^2$$

$$M = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \qquad \left.\begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix}\right\} \text{canônica}$$

$$\det(A - \lambda I) = 0$$

$$\left|\begin{array}{ccc|cc} 3-\lambda & -1 & 1 & 3-\lambda & -1 \\ -1 & 5-\lambda & -1 & -1 & 5-\lambda \\ 1 & -1 & 3-\lambda & 1 & -1 \end{array}\right|$$

$$(5-\lambda).(3-\lambda)^2 + 1 + 1 - \left[(5-\lambda) + (3-\lambda) + (3-\lambda)\right] = 0$$

$$(5-\lambda).(9 - 6\lambda + \lambda^2) + 2 - 11 + 3\lambda = 0$$

$$45 - 30\lambda + 5\lambda^2 - 9\lambda + 6\lambda^2 - \lambda^3 - 9 + 3\lambda = 0$$

$$-\lambda^3 + 11\lambda^2 - 36\lambda + 36 = 0 \quad \rightsquigarrow \quad \lambda - 2 \ \text{raiz}$$

$$\lambda_1 = 2$$

$$\begin{array}{l|l} -\lambda^3 + 11\lambda^2 - 36\lambda + 36 & \lambda - 2 \\ +\lambda^3 - 2\lambda^2 & (-\lambda^2 + 9\lambda - 18).(\lambda - 2) \\ \quad 9\lambda^2 - 36\lambda + 36 & \\ \quad -9\lambda^2 + 18\lambda & \\ \quad\quad -18\lambda + 36 & \\ \quad\quad +18\lambda - 36 & \\ \quad\quad\quad 0 & \end{array}$$

$$\Delta = 81 - 4.(-1)(-18)$$

$$\Delta = 81 - 72$$

$$\Delta = 9$$

$$\lambda_2 = \frac{-9+3}{-2} = 3$$

$$\lambda_3 = \frac{-9-3}{-2} = 6$$

autovalores $= \lambda_1 = 2$

$\lambda_2 = 3$

$\lambda_3 = 6$

Para $\lambda_1 = 2$ . $(a - 2I) \cdot v$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} a - b + c = 0 \\ -a + 3b - c = 0 \end{cases}$$

$$\begin{cases} b = a + c \\ 3b = a + c \Rightarrow b = 0 \end{cases}$$

$c = -a$

$$\boxed{\begin{gathered} C_{\lambda_1} = \{(ax^2 - a)\}_{a \in \mathbb{R}} \\ C_{\lambda_1} = [(1,0,-1)] \\ v_1 = (1;0;-1) \end{gathered}}$$

Para $\lambda_2 = 3$ $\quad (a - 3I) v$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -b + c = 0 \\ -a + 2b - c = 0 \\ a - b = 0 \end{cases}$$

$$\begin{cases} c = b \\ A = b \end{cases} \quad c = b = a$$

$$\boxed{\begin{gathered} C_{\lambda_2} = \{(ax^2 + ax + a)\}\, a \in \mathbb{R} \\ v_2 = (1,1,1) \end{gathered}}$$

Para $\lambda_3 = 6$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -3a - b + c = 0 \\ -a - b - c = 0 \\ -a - b - 3c = 0 \end{cases} +$$

$-2b - 4c = 0$

$b = -2c$

$\boxed{b = -2a}$

$-3a + 2b + c = 0$

$-3a + 3c = 0$

$c = a$

$$\boxed{\begin{gathered} C_{\lambda_3} = \{(ax^2 - 2ax + a)\, a \in \mathbb{R}^*\} \\ v_3 = (1,-2,1) \end{gathered}}$$

QUESTÃO 4

Determinar uma matriz P que diagonaliza a matriz

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{bmatrix}$$

Encontrar autovalores: $\det[A - \lambda I]$

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ 0 & 3-\lambda & 1 \\ 0 & 5 & -1-\lambda \end{vmatrix} \begin{matrix} 1-\lambda & 2 \\ 0 & 3-\lambda \\ 0 & 5 \end{matrix}$$

$(1-\lambda).(3-\lambda).(-1-\lambda) - [5.(1-\lambda)]$

$(3-\lambda-3\lambda+\lambda^2).(-1-\lambda) - (5-5\lambda)$

$(\lambda^2-4\lambda+3).(-1-\lambda) - (5-5\lambda)$

$-\lambda^2+4\lambda-3-\lambda^3+4\lambda^2-3\lambda - (5-5\lambda)$

$-\lambda^3+3\lambda^2+\lambda-\cancel{3} - (\cancel{5}-\cancel{5\lambda})$

$$\begin{array}{l|l} -\cancel{\lambda^3}+\cancel{3\lambda^2}-4\lambda-8 & (-\lambda-1) \\ +\cancel{\lambda^3}+\cancel{\lambda^2} & (\lambda^2-4\lambda+8) \\ \cancel{4\lambda^2}-\cancel{4\lambda}-8 & \\ -\cancel{4\lambda^2}-\cancel{4\lambda} & \\ -\cancel{8\lambda}-\cancel{8} & \\ +\cancel{8\lambda}+\cancel{8} & \\ \hline 0 & \end{array}$$

$(-\lambda-1).(\lambda^2-4\lambda+8)$

$\lambda_1 = \quad -\lambda-1=0$

$\boxed{\lambda_1 = -1}$

$\lambda_2 = \lambda^2-4\lambda+8$

$\Delta = (-4)^2 - 4.1.8$

$\Delta = 16-32$

$\Delta = -16$ !! ⌢

Não há número real

Para $\lambda_1 = -1$

$(A - \lambda_1 I)v = 0$

$\left( \quad \frac{z_1}{4} \quad \right)$

$$\begin{bmatrix} 2 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & 5 & 0 \end{bmatrix} . \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 2x_1 + 2y_1 + z_1 = 0 \\ 4y_1 + z_1 = 0 \\ 5y_1 = 0 \end{cases}$$

$4y_1 = -z_1$

$y_1 = \frac{-z_1}{4}$

$\to 2x_1 =$

$$\boxed{\left(\frac{-z_1}{4}, \frac{-z_1}{4}, z_1\right)}$$

$$2x_1 + \frac{2}{1}\left(\frac{-z_1}{4}\right) + z_1 = 0$$

$$2x_1 - \frac{2z_1}{4} + \frac{z_1}{1} = 0$$

$$2x_1 - \frac{2z_1 + 4z_1}{4} = 0$$

$$2x_1 + \frac{2z_1}{4} = 0$$

$$2x_1 = \frac{-2z_1}{4}$$

$$x_1 = \frac{-2z_1}{4} \cdot \frac{1}{2}$$

$$x_1 = \frac{-2z_1}{8}$$

$$\boxed{x_1 = \frac{-z_1}{4}}$$

Se $z_1 = 4$

$$v_1 = \left(\frac{-4}{4}, \frac{-4}{4}, 4\right)$$

$$v_1 = (-1, -1, 4)$$

$$P = \begin{bmatrix} -1 \\ -1 \\ 4 \end{bmatrix}$$ Matriz que diagonaliza A.