Lista 6- Geometria Analítica
Andrew Gabriel Gomes

(na diagonal secundária: *Troca o sinal do resultado)

1. $\vec{u} = (3,-1,1)$, $\vec{v} = (1,2,2)$, $\vec{w} = (2,0,-3)$

a) Calcular $(\vec{u},\vec{v},\vec{w}) \rightarrow$

$$\begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 0 & -3 & 2 & 0 \end{array}$$

$(-18)+(-4)+0-(4+0+3)$

$-22-4-3$

$\boxed{-29}$

b) Calcular $(\vec{v},\vec{u},\vec{w}) \rightarrow$

$$\begin{array}{ccc|cc} 1 & 2 & 2 & 1 & 2 \\ 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -3 & 2 & 0 \end{array}$$

$3+4+0-(-4+0+(-18))$

$7+4+18$

$\boxed{29}$

2. $(\vec{u},\vec{v},\vec{w}) = -5$ | Calcular:

a) $(\vec{w},\vec{v},\vec{u}) = \boxed{5}$ (Trocou a posição de $\vec{w}$ e $\vec{u}$)

b) $(\vec{v},\vec{u},\vec{w}) = \boxed{5}$ ('' '' $\vec{u}$ e $\vec{v}$)

c) $(\vec{w},\vec{u},\vec{v}) = \boxed{-5}$ (duas permutações)

d) $\vec{v}\cdot(\vec{w}\times\vec{u}) = \boxed{-5}$ (porque $(w \times u)\cdot\vec{v}$ é a mesma coisa)

3. Verificar se $(\vec{u},\vec{v},\vec{w})$ são coplanares, ou seja, $(\vec{u},\vec{v},\vec{w}) = 0$

$\vec{u} = (1,-1,2)$, $\vec{v} = (2,2,1)$, $\vec{w} = (-2,0,-4)$

$$\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & -1 \\ 2 & 2 & 1 & 2 & 2 \\ -2 & 0 & -4 & -2 & 0 \end{array} = -6$$

, logo, não são coplanares!

$-8+2+0-(-8+0+8)$

$-6 \quad \cancel{+8-8}$.

4. $K = ?$ para que $(\vec{u}, \vec{v}, \vec{w})$ sejam coplanares.

$\vec{u} = (2, -1, K)$

$\vec{v} = (1, 0, 2)$

$\vec{w} = (K, 3, K)$

$$\begin{array}{ccccc} 2 & -1 & K & 2 & -1 \\ 1 & 0 & 2 & 1 & 0 \\ K & 3 & K & K & 3 \end{array}$$

$0 + (-2K) + 3K - (0 + 12 + (-K))$

$K - 12 + K$

$2K - 12 \longrightarrow 2K - 12 = 0$

$2K = 12 \rightarrow \boxed{K = 6}$

$K = 6$ para que $(\vec{u}, \vec{v}, \vec{w})$ sejam coplanares.

5. Verificar se os pontos são coplanares

$A(1,1,0)$ $\quad \vec{AB} = B - A = (-2, 1, -6) + (-1, -1, 0) = (-3, 0, -6)$

$B(-2,1,-6)$ $\quad \vec{AC} = C - A = (-1, 2, -1) + (-1, -1, 0) = (-2, 1, -1)$

$C(-1,2,-1)$ $\quad \vec{AD} = D - A = (2, -1, -4) + (-1, -1, 0) = (1, -2, -4)$

$D(2,-1,-4)$ $\quad (\vec{AB}, \vec{AC}, \vec{AD}) = 0$, para ser coplanar

$$\begin{array}{ccccc} -3 & 0 & -6 & -3 & 0 \\ -2 & 1 & -1 & -2 & 1 \\ 1 & -2 & -4 & 1 & -2 \end{array}$$

$12 + 0 + (-24) - (\underbrace{-6 + (-6) + 0}_{6+6})$

$\underbrace{-12 + 12}_{0}$ $\longrightarrow$ os pontos são coplanares!

6. $m = ?$, para que o Volume $= \underline{33}$. Altura $= ?$ base por $\vec{a}$ e $\vec{b}$

$\vec{a} = (0, -1, 2)$

$\vec{b} = (-4, 2, -1)$

$\vec{c} = (3, m, -2)$

Volume $= |(\vec{a}, \vec{b}, \vec{c})| = 33$

$$\begin{array}{ccccc} 0 & -1 & 2 & 0 & -1 \\ -4 & 2 & -1 & -4 & 2 \\ 3 & m & -2 & 3 & m \end{array}$$

$0 + 3 + (-8m) - (12 + 0 + (-8))$

$-8m + 3 - 12 + 8$

$-8m + 1 = 33 \quad -1$

$8m = 32$ $\qquad 8m = 34$

$\boxed{m = 4}$ ou $\quad m = -\frac{34}{8} \quad m = \boxed{-\frac{17}{4}}$

Volume $=$ área da base $\cdot h$

$h = \frac{V}{\text{área b.}}$

$\rightarrow$ área da base $= |\vec{a} \times \vec{b}|$

Cálculo área da base

$$\vec{a} \times \vec{b} = \begin{array}{ccccc} 0 & -1 & 2 & 0 & -1 \\ -4 & 2 & -1 & -4 & 2 \end{array}$$

$(1-4, -8-0, 0-4)$

$\vec{a} \times \vec{b} = (-3, -8, -4)$

$A_{base} = |\vec{a} \times \vec{b}| = \sqrt{(-3)^2 + (-8)^2 + (-4)^2}$

$|\vec{a} \times \vec{b}| = \sqrt{9 + 64 + 16}$

$\sqrt{89}$

$A_{base} = \sqrt{89}$ $\qquad$ Altura $= \frac{\text{Volume}}{\text{A. base}} \longrightarrow \boxed{\text{Altura} = \frac{33}{\sqrt{89}}}$

7. determinar ponto D para que o volume = 25 u.v

A(2,1,1) , B(-1,0,1) , C(3,2,-2) , D(0,0,x)
ele pertence a eixo Oz logo x, y = 0

$\vec{AB} = B-A \rightarrow (-1,0,1) + (-2,-1,-1) = (-3,-1,0)$
$\vec{AC} = C-A \rightarrow (3,2,-2) + (-2,-1,-1) = (1,1,-3)$
$\vec{AD} = D-A \rightarrow (0,0,x) + (-2,-1,-1) = (-2,-1,x-1)$

volume = $|(\vec{AB},\vec{AC},\vec{AD})| = 25$

volume
$$\begin{array}{ccc|cc} -3 & -1 & 0 & -3 & -1 \\ 1 & 1 & -3 & 1 & 1 \\ -2 & -1 & x-1 & -2 & -1 \end{array}$$

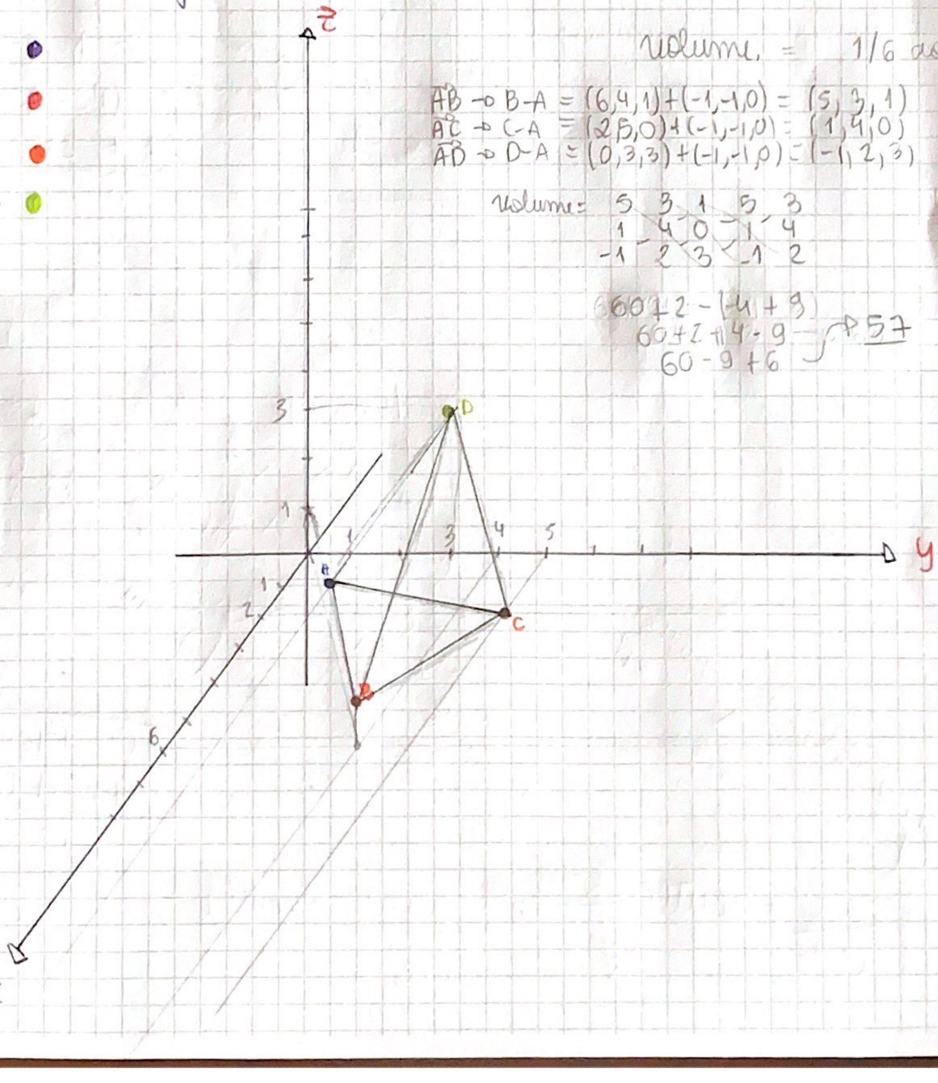
$(-3x+3) + (-6) - (-9 + x + 1)$
$= -6 - 3x + 3 + x - 1 + 9$
$-2x + 5 = 25$ ou $2x - 5 = 25$
$-2x = 20$; $2x = 30$
$x = \frac{20}{-2}$; $x = 15$
$\underline{x = -10}$

ponto $\underline{D(0,0,-10)}$ ou

D(0,0,15)

8. Representar graficamente o tetraedro $\underline{ABCD}$ e calcular seu volume

A(1,1,0) —
B(6,4,1) —
C(2,5,0) —
D(0,3,3) —



volume = 1/6 do paralelepípedo

$\vec{AB} \rightarrow B-A = (6,4,1) + (-1,-1,0) = (5,3,1)$
$\vec{AC} \rightarrow C-A = (2,5,0) + (-1,-1,0) = (1,4,0)$
$\vec{AD} \rightarrow D-A = (0,3,3) + (-1,-1,0) = (-1,2,3)$

volume =
$$\begin{array}{ccc|cc} 5 & 3 & 1 & 5 & 3 \\ 1 & 4 & 0 & 1 & 4 \\ -1 & 2 & 3 & -1 & 2 \end{array}$$

$60 + 2 - (-4 + 9)$
$60 + 2 + 4 - 9$
$60 - 9 + 6 \rightarrow \underline{57}$

$\frac{1}{6} \cdot 57$

$\boxed{\frac{19}{2}}$ u.v

9. ponto vertice D?

tetraedro / $V=6$

$A(-2,4,-1)$
$B(-3,2,3)$
$C(1,-2,-1)$

$D \in Oy$ / logo x e $z = 0$

$D(0,y,0)$

$\vec{AB} \to B-A = (-3,2,3)+(2,-4,1) = (-1,-2,4)$
$\vec{AC} \to C-A = (1,-2,-1)+(2,-4,1) = (3,-6,0)$
$\vec{AD} \to D-A = (0,y,0)+(2,-4,1) = (2,y-4,1)$

$$\begin{matrix} -1 & -2 & 4 & -1 & -2 \\ 3 & -6 & 0 & 3 & -6 \\ 2 & y-4 & 1 & 2 & y-4 \end{matrix}$$

$6+12y-48-(-48-6)$
$6+12y-\cancel{48}+\cancel{48}+6$

$1/6 \cdot \frac{12y}{6}+\frac{12}{6}$

$2y+2=6$
$2y=6-2$
$2y=4$ → $y=\frac{4}{2}$ = $\boxed{y=2}$

$-2y-2=6$
$-2y=8$ → $\boxed{y=-4}$

$D(0,2,0)$ ou $D(0,-4,0)$

10. $|\vec{u}|=3$, $|\vec{v}|=4$ (120° entre $\vec{u}$ e $\vec{v}$)

a) $|\vec{u}+\vec{v}| = |\vec{u}+\vec{v}|^2 = u^2+v^2+2.u.v.\cos(120)$

$= 3^2+4^2+2.3.4.\left(-\frac{1}{2}\right)$

$= 9+16+24 \cdot -\frac{1}{2}$

$25 + \frac{-24}{2}$

$25-12$

$|\vec{u}+\vec{v}| = \sqrt{13}$?

b) $|\vec{u} \times (\vec{v}-\vec{u})|$

$= |\vec{u}|.|\vec{v}|.\sin\theta$

$= 3.4.\sin\theta$

$= 12.\frac{\sqrt{3}}{2}$

$= 6\sqrt{3}$

c) → $V = ||(\vec{u}\times\vec{v}).(\vec{u}\times\vec{v})|| \to ||\vec{u}\times\vec{v}||^2 = 6\sqrt{3}^2$

$36.\sqrt{9}$
36.3 → $108\,u.v$