

# Rasterization

## **CS4620 Lecture 12**

# The graphics pipeline

- **The standard approach to object-order graphics**
- **Many versions exist**
  - software, e.g. Pixar's REYES architecture
    - many options for quality and flexibility
  - hardware, e.g. graphics cards in PCs
    - amazing performance: millions of triangles per frame
- **We'll focus on an abstract version of hardware pipeline**
- **“Pipeline” because of the many stages**
  - very parallelizable
  - leads to remarkable performance of graphics cards (many times the flops of the CPU at  $\sim 1/5$  the clock speed)

# Pipeline

you are here →

**APPLICATION**

**COMMAND STREAM**

3D transformations; shading →

**VERTEX PROCESSING**

**TRANSFORMED GEOMETRY**

conversion of primitives to pixels →

**RASTERIZATION**

**FRAGMENTS**

blending, compositing, shading →

**FRAGMENT PROCESSING**

**FRAMEBUFFER IMAGE**

user sees this →

**DISPLAY**

# Primitives

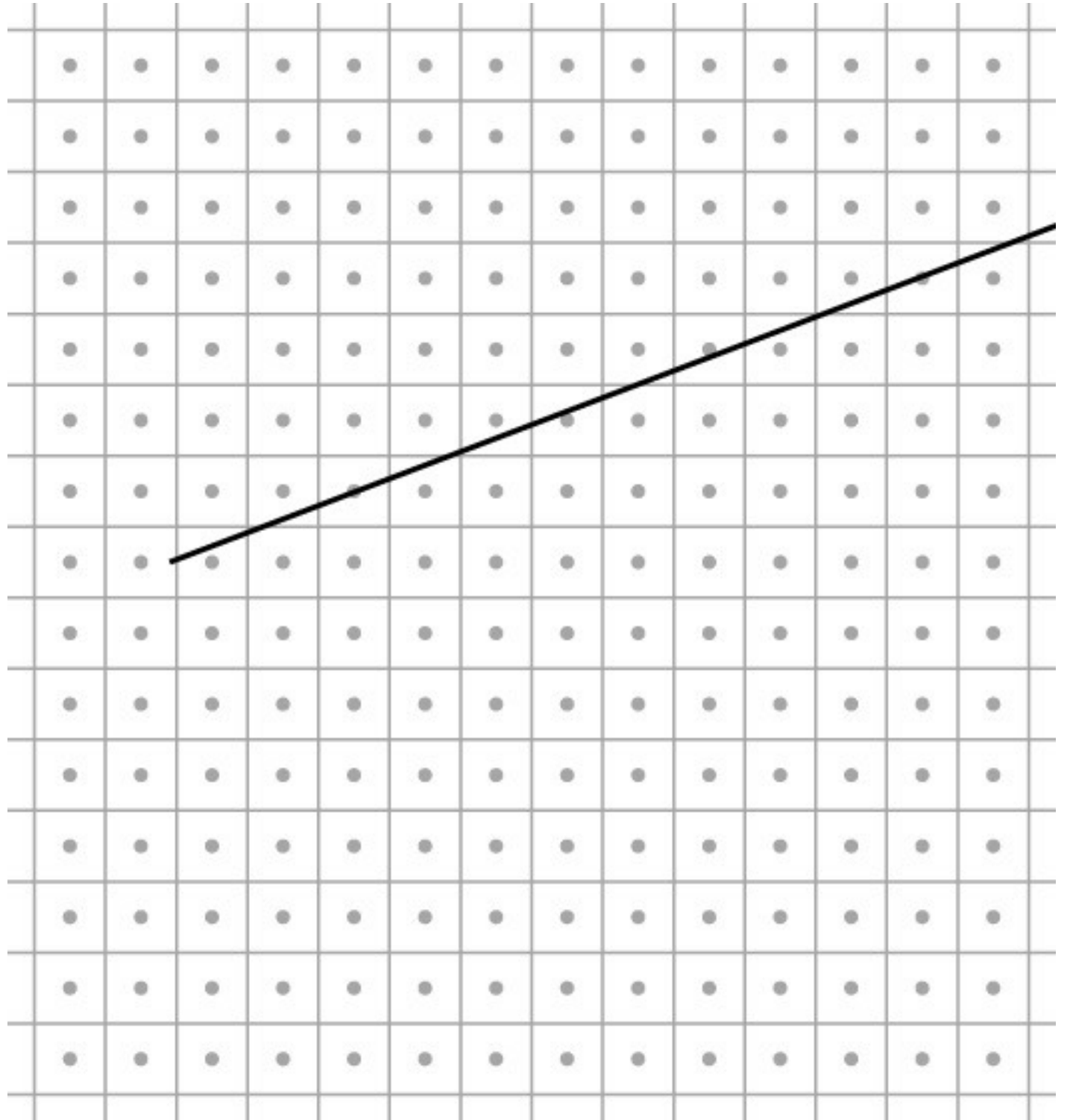
- **Points**
- **Line segments**
  - and chains of connected line segments
- **Triangles**
- **And that's all!**
  - Curves? Approximate them with chains of line segments
  - Polygons? Break them up into triangles
  - Curved surfaces? Approximate them with triangles
- **Trend over the decades: toward minimal primitives**
  - simple, uniform, repetitive: good for parallelism

# Rasterization

- **First job: enumerate the pixels covered by a primitive**
  - simple, aliased definition: pixels whose centers fall inside
- **Second job: interpolate values across the primitive**
  - e.g. colors computed at vertices
  - e.g. normals at vertices
  - e.g. texture coordinates

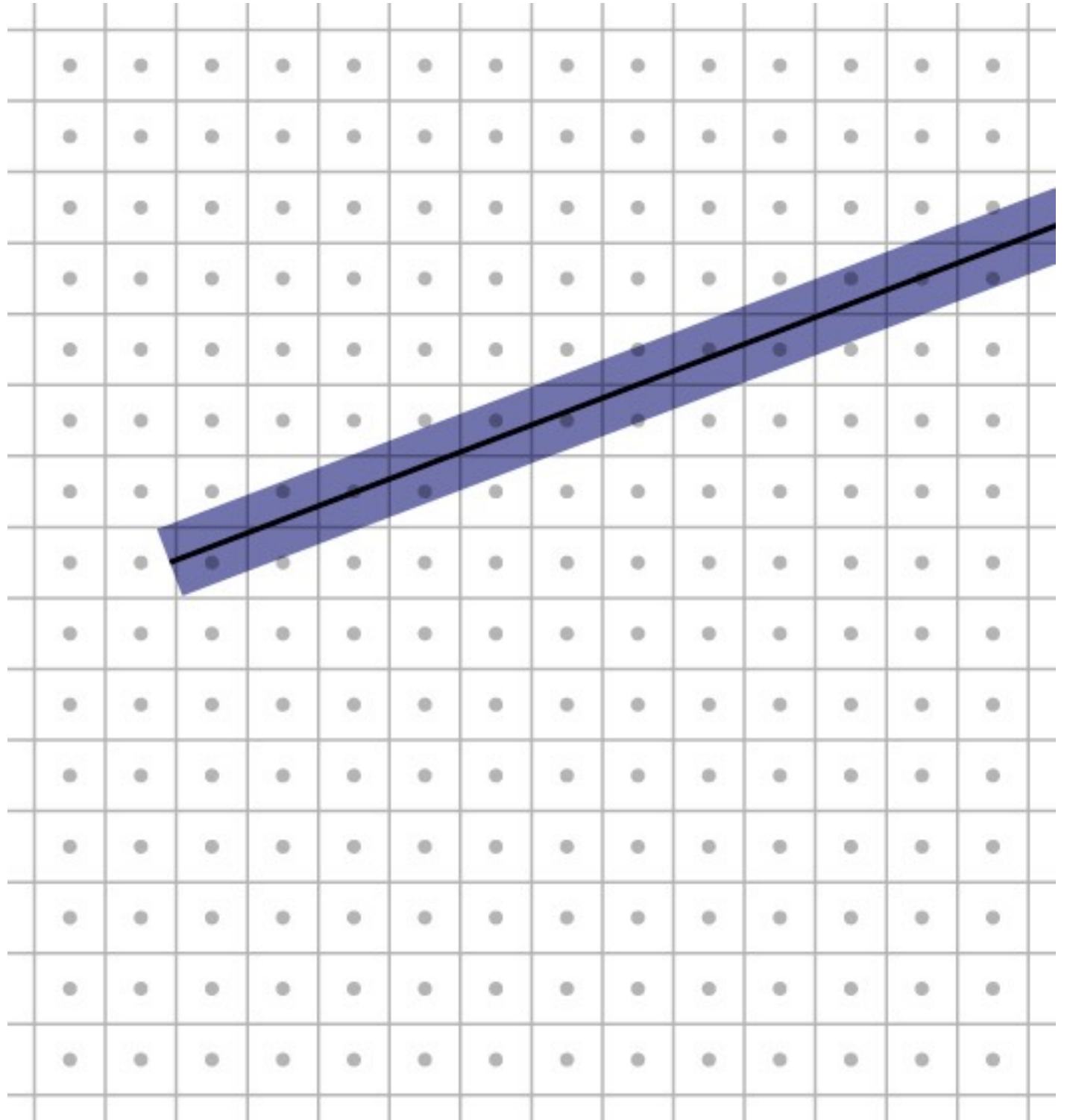
# Rasterizing lines

- **Define line as a rectangle**
- **Specify by two endpoints**
- **Ideal image: black inside, white outside**



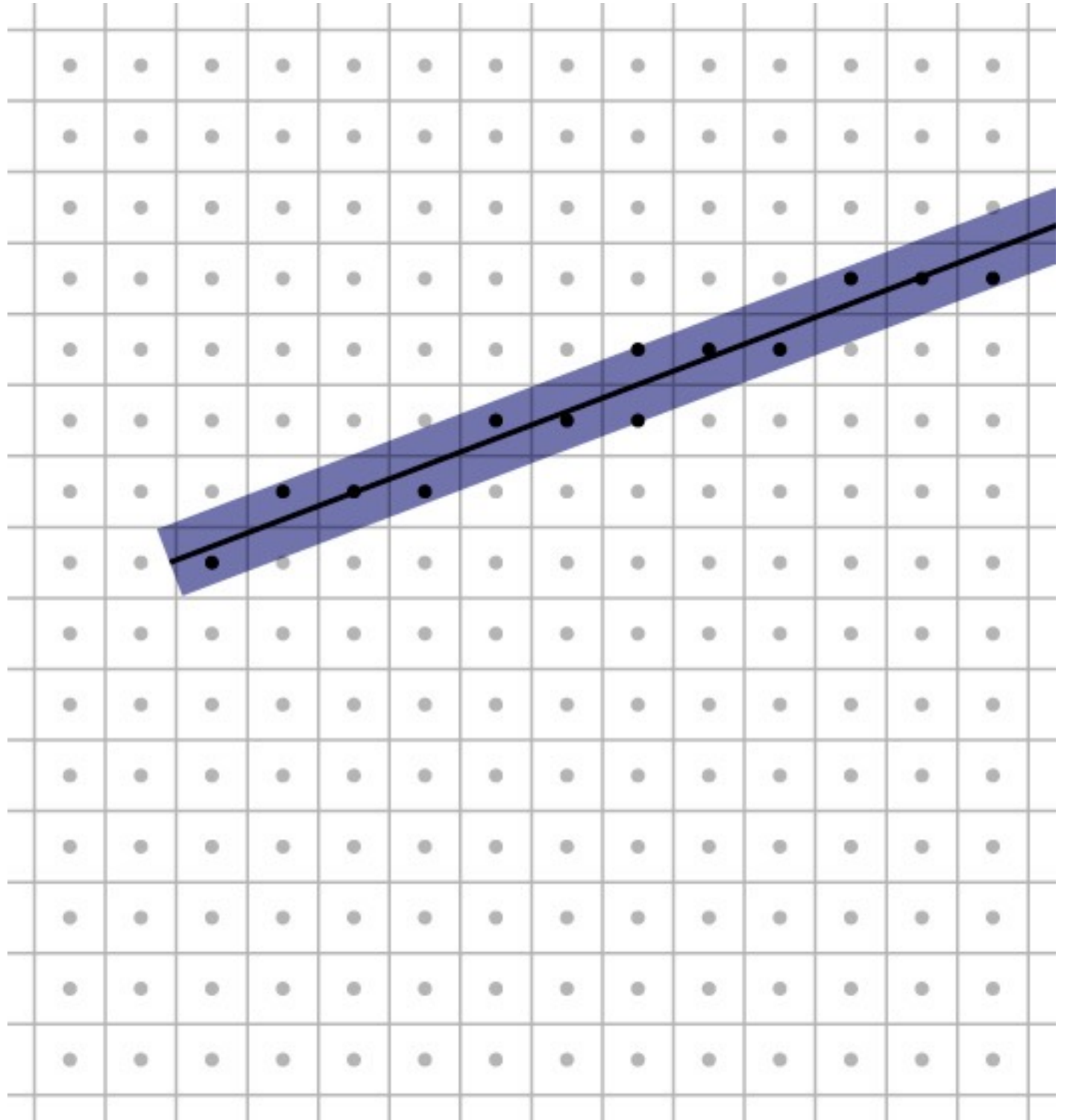
# Rasterizing lines

- **Define line as a rectangle**
- **Specify by two endpoints**
- **Ideal image: black inside, white outside**



# Point sampling

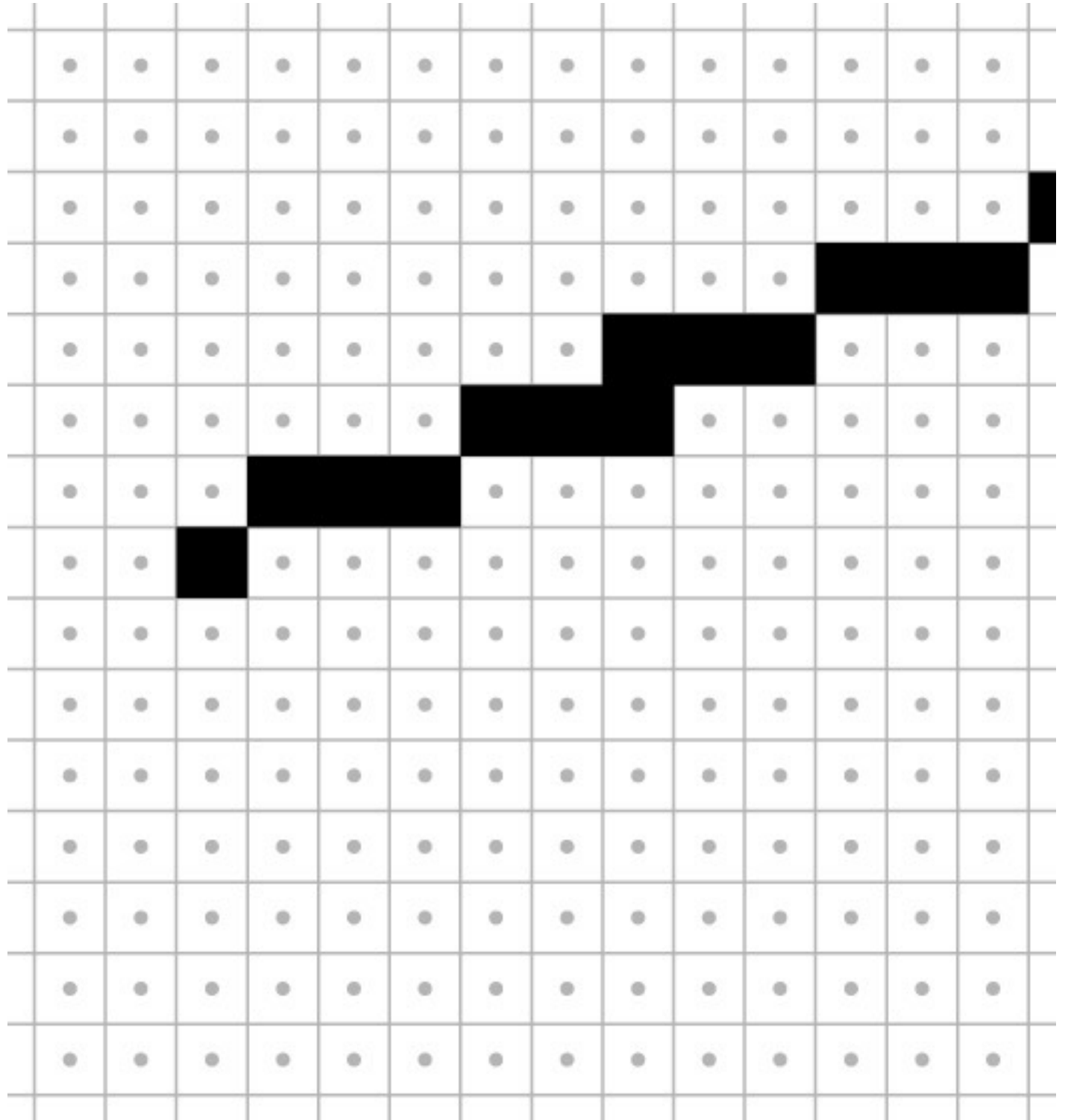
- **Approximate rectangle by drawing all pixels whose centers fall within the line**
- **Problem: sometimes turns on adjacent pixels**



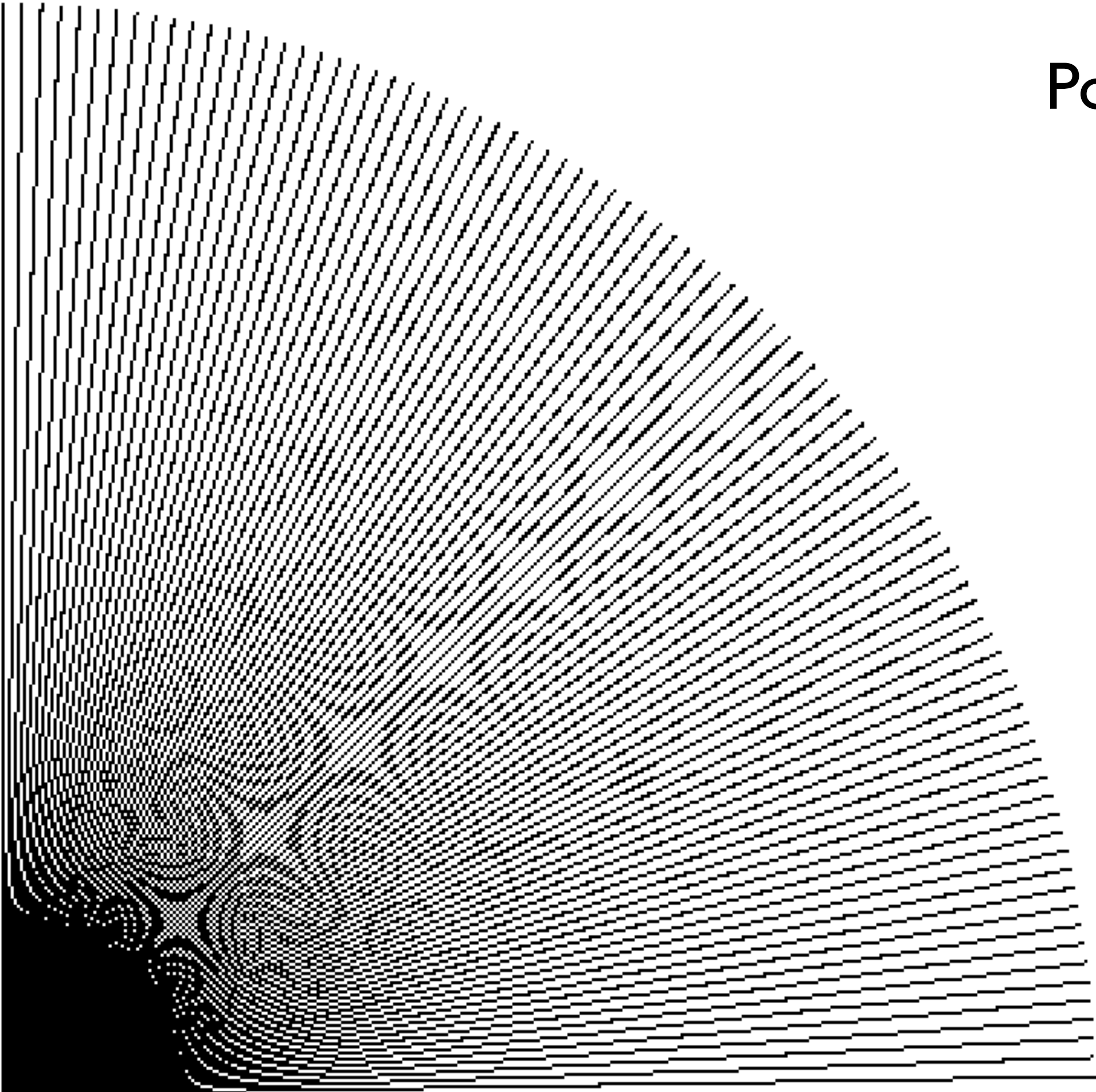


# Point sampling

- **Approximate rectangle by drawing all pixels whose centers fall within the line**
- **Problem: sometimes turns on adjacent pixels**

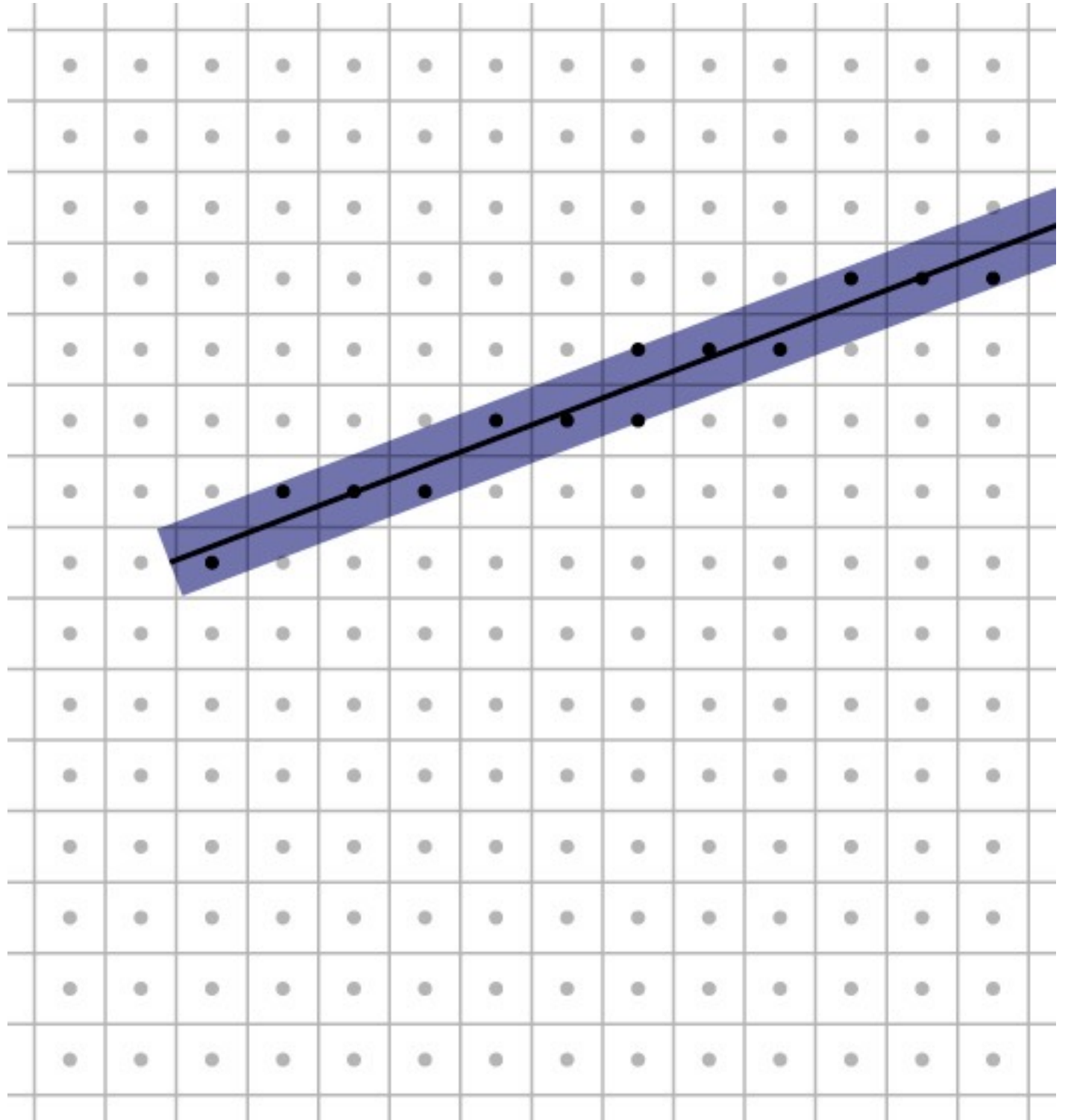


# Point sampling in action



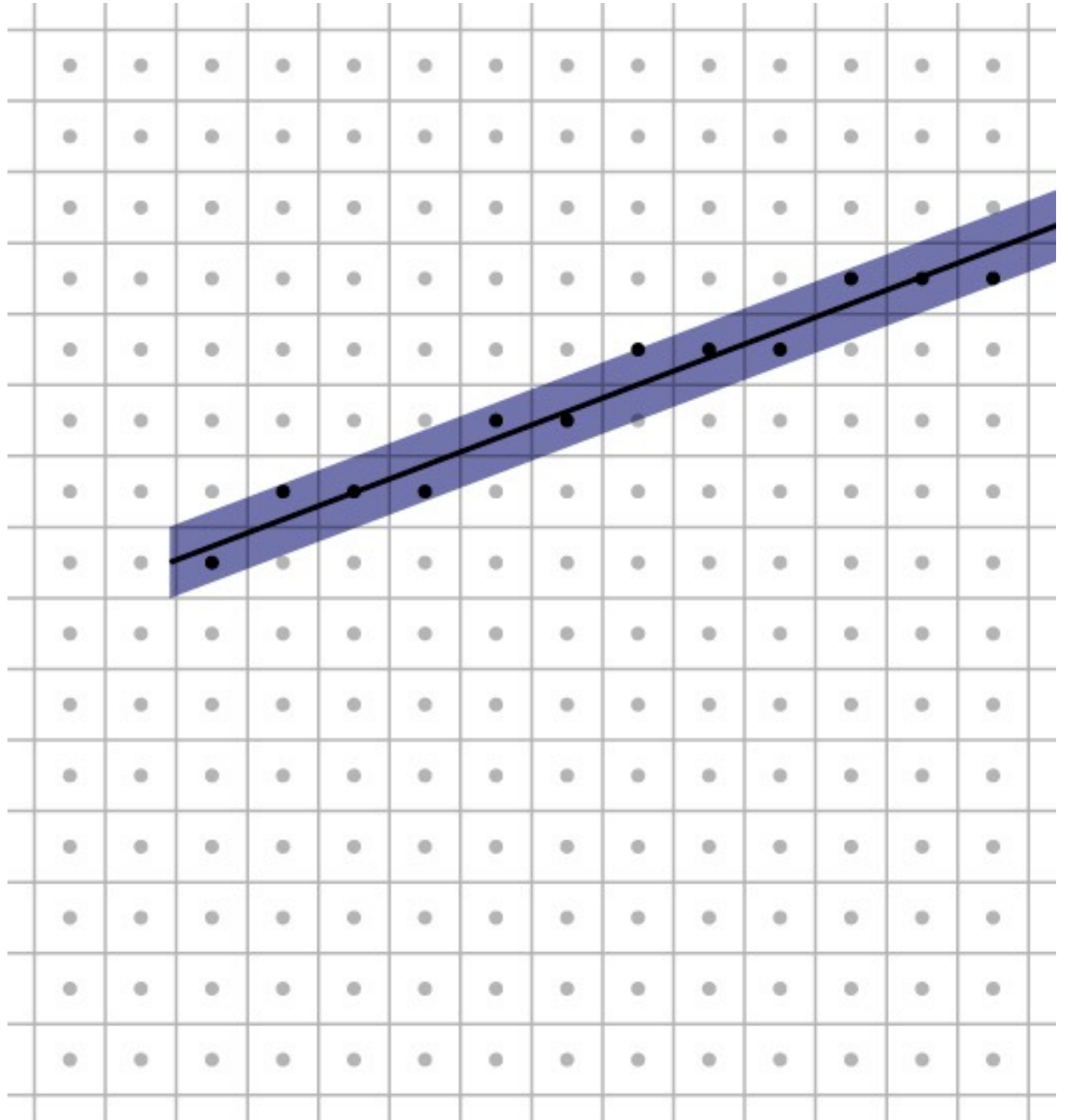
# Bresenham lines (midpoint alg.)

- **Point sampling unit width rectangle leads to uneven line width**
- **Define line width parallel to pixel grid**
- **That is, turn on the single nearest pixel in each column**
- **Note that  $45^\circ$  lines are now thinner**



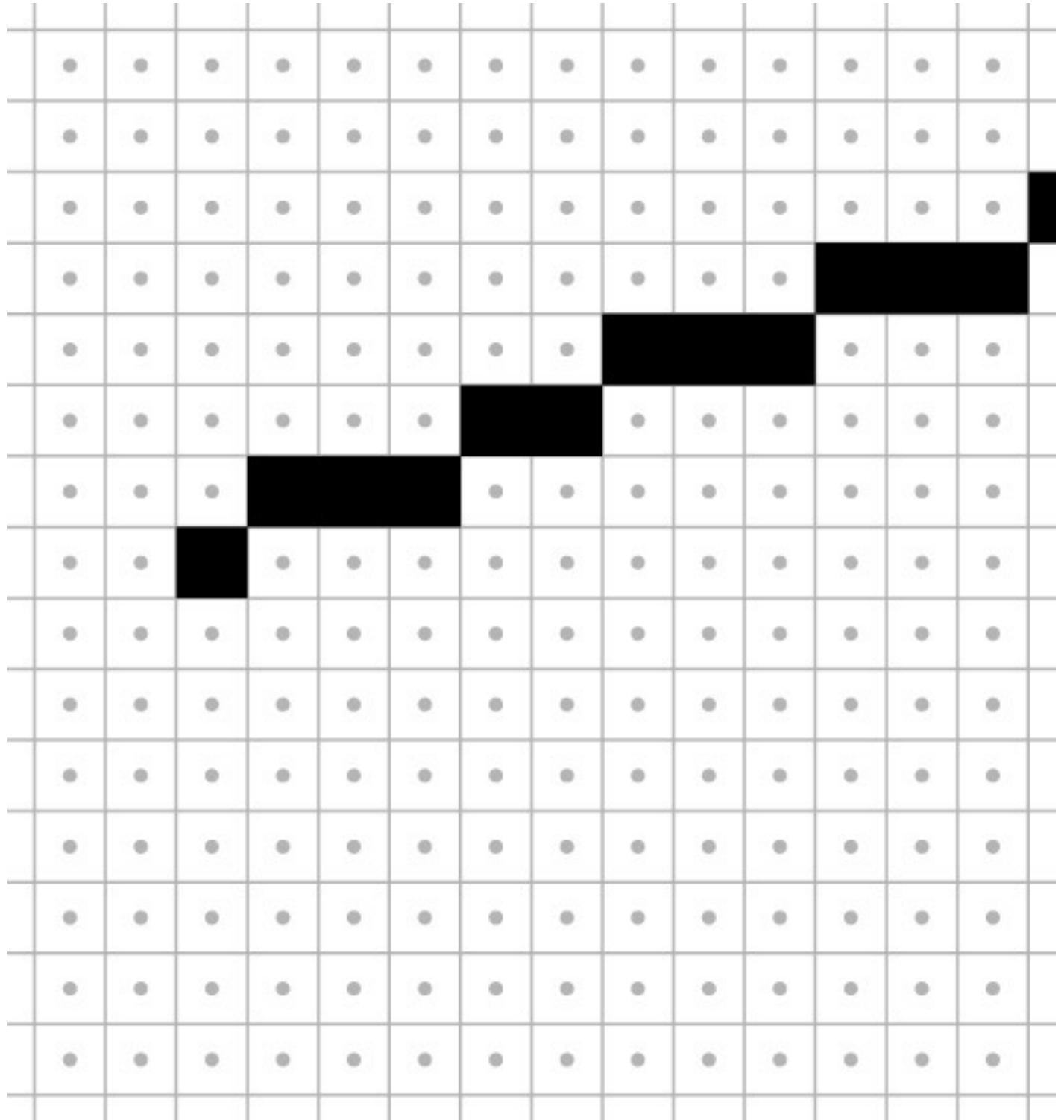
# Bresenham lines (midpoint alg.)

- **Point sampling unit width rectangle leads to uneven line width**
- **Define line width parallel to pixel grid**
- **That is, turn on the single nearest pixel in each column**
- **Note that  $45^\circ$  lines are now thinner**

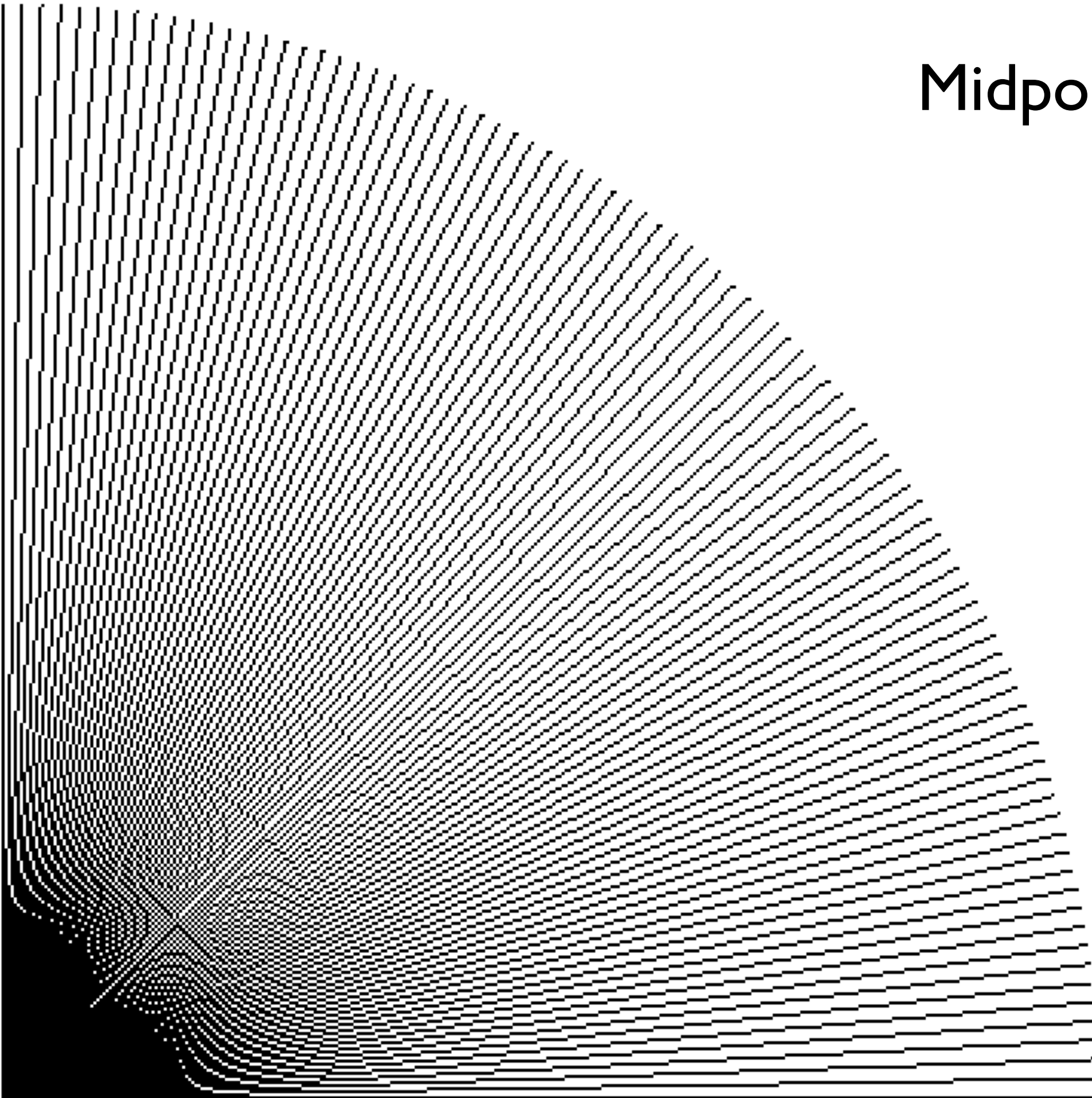


# Bresenham lines (midpoint alg.)

- **Point sampling unit width rectangle leads to uneven line width**
- **Define line width parallel to pixel grid**
- **That is, turn on the single nearest pixel in each column**
- **Note that 45° lines are now thinner**

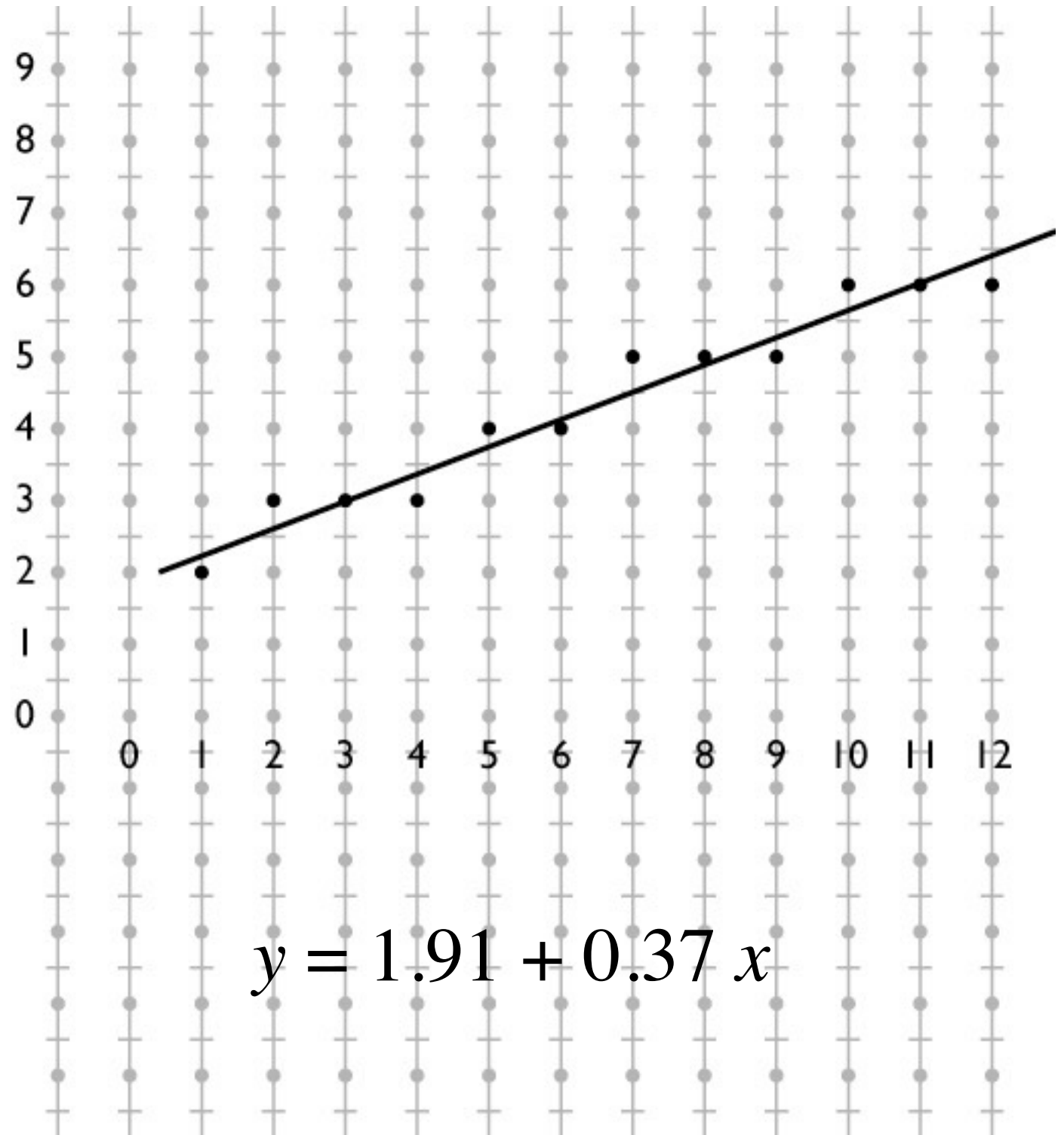


# Midpoint algorithm in action



# Algorithms for drawing lines

- **line equation:**  
 $y = b + m x$
- **Simple algorithm:**  
evaluate line equation  
per column
- **W.l.o.g.  $x_0 < x_1$ ;**  
 $0 \leq m \leq 1$

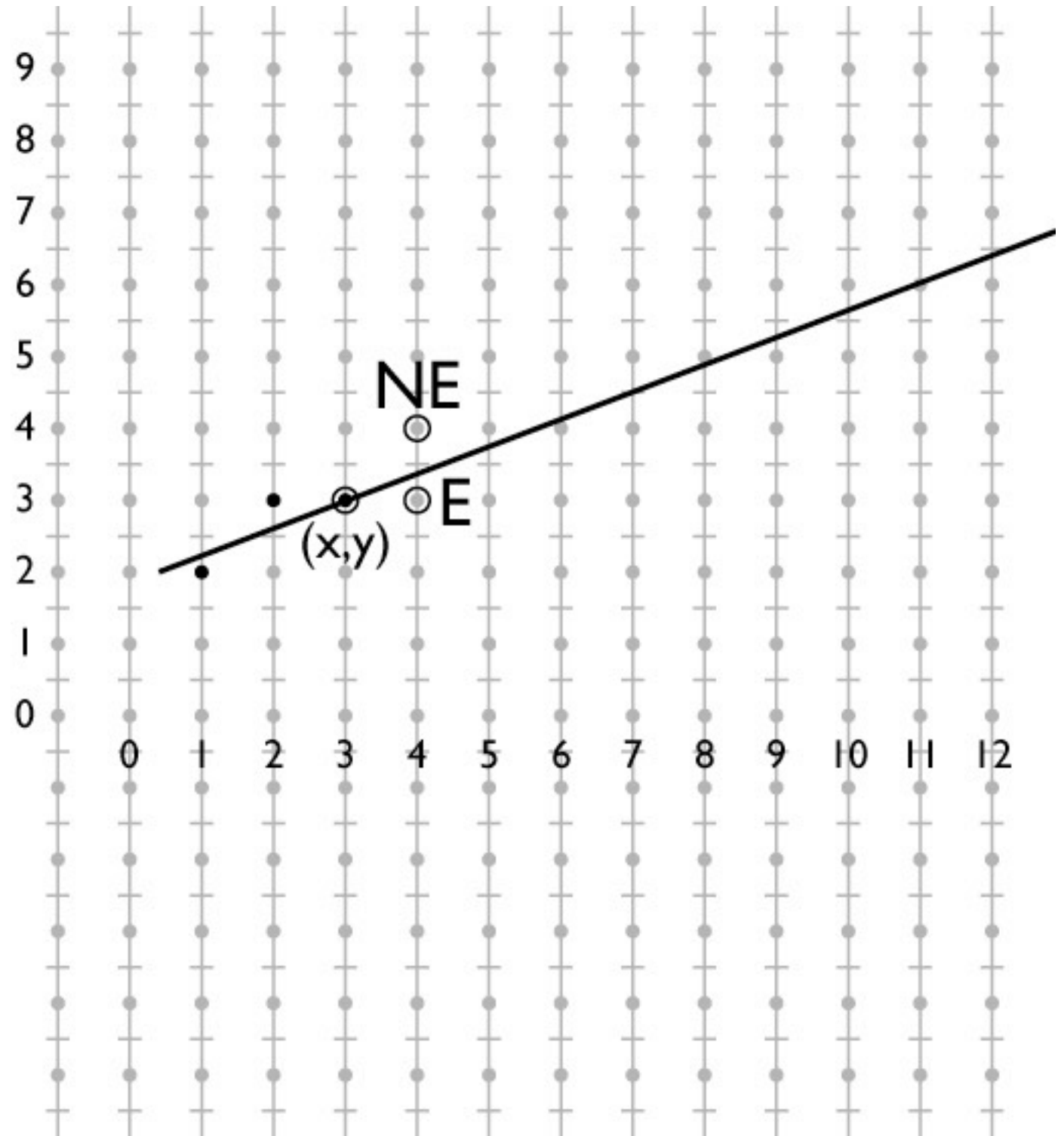


```
for x = ceil(x0) to floor(x1)
  y = b + m * x
  output(x, round(y))
```

$$y = 1.91 + 0.37 x$$

# Optimizing line drawing

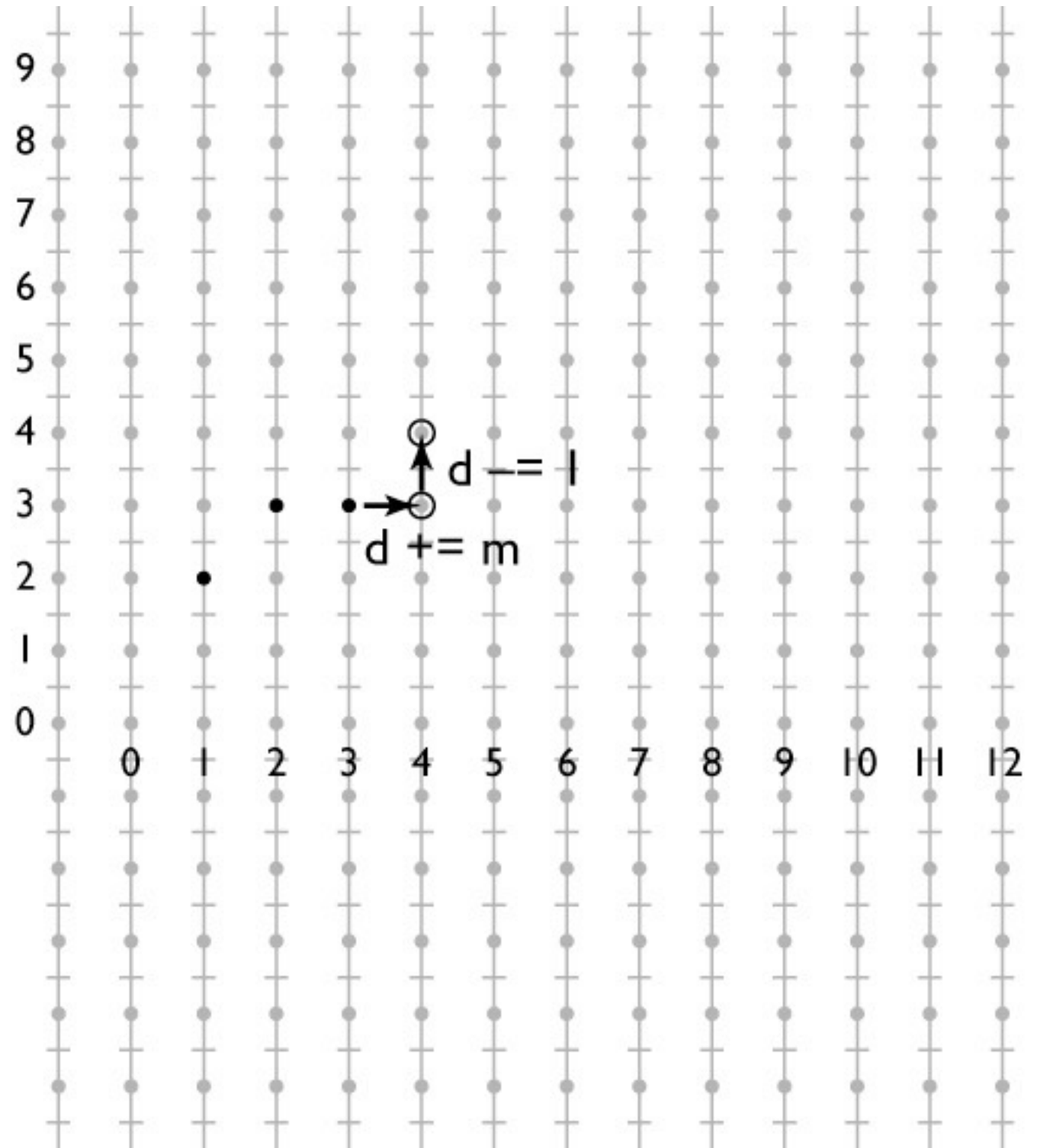
- **Multiplying and rounding is slow**
- **At each pixel the only options are E and NE**
- $d = m(x + 1) + b - y$
- $d > 0.5$  decides between E and NE





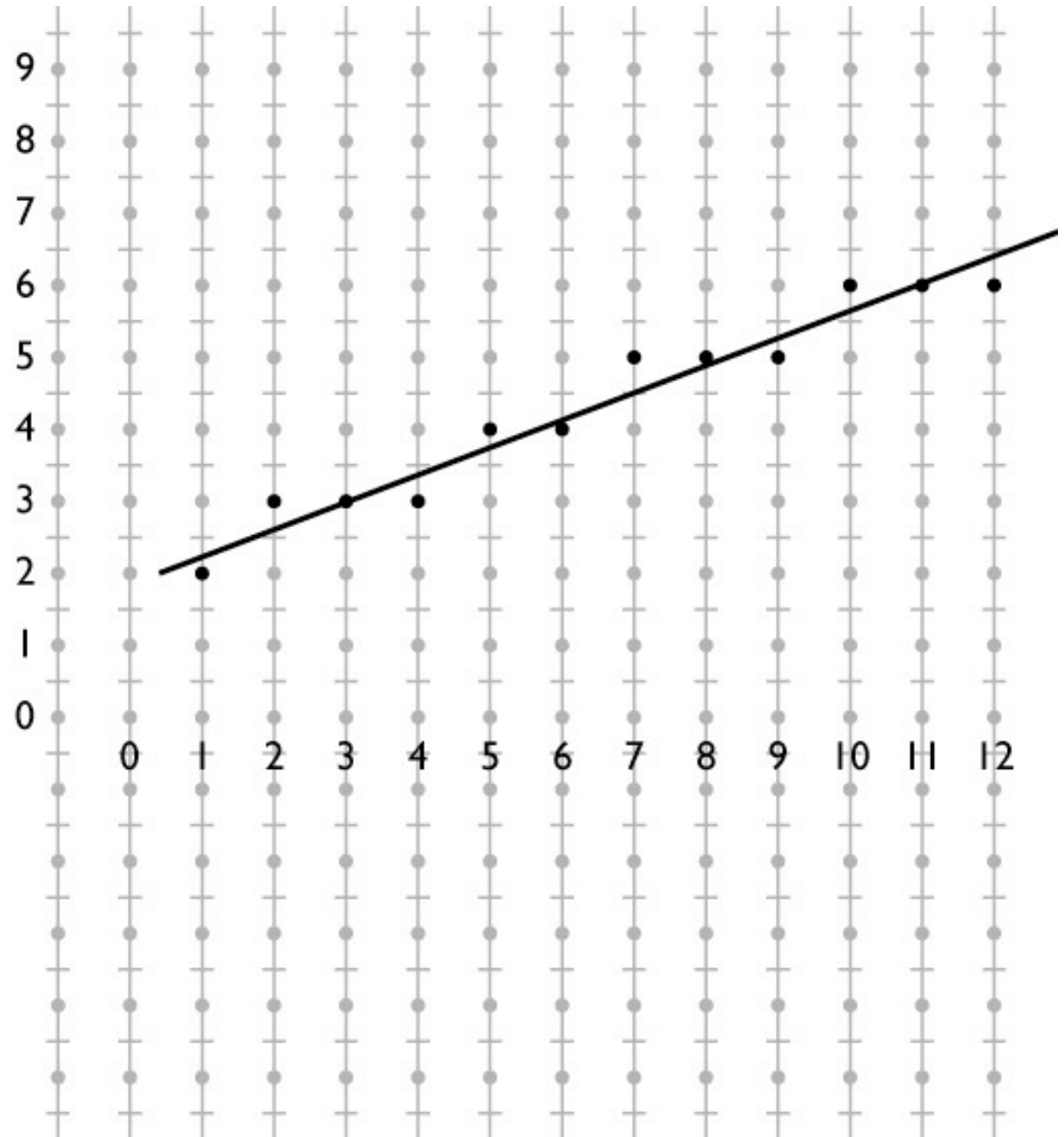
# Optimizing line drawing

- $d = m(x + 1) + b - y$
- Only need to update  $d$  for integer steps in  $x$  and  $y$
- Do that with addition
- Known as “DDA” (digital differential analyzer)



# Midpoint line algorithm

```
x = ceil(x0)
y = round(m * x + b)
d = m * (x + 1) + b - y
while x < floor(x1)
    if d > 0.5
        y += 1
        d -= 1
    x += 1
    d += m
    output(x, y)
```

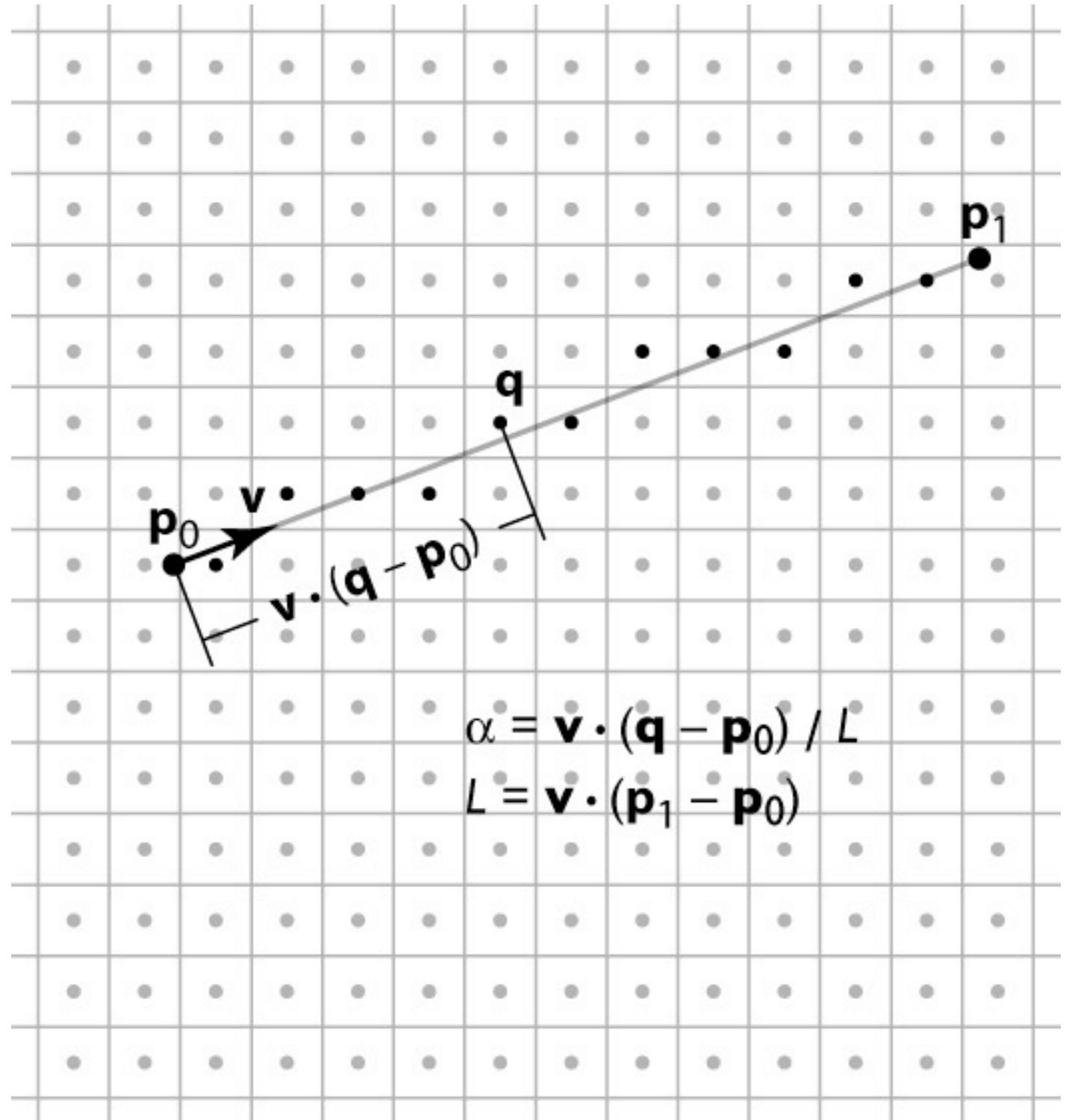


# Linear interpolation

- **We often attach attributes to vertices**
  - e.g. computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments
- **Recall basic definition**
  - 1D:  $f(x) = (1 - \alpha) y_0 + \alpha y_1$
  - where  $\alpha = (x - x_0) / (x_1 - x_0)$
- **In the 2D case of a line segment, alpha is just the fraction of the distance from  $(x_0, y_0)$  to  $(x_1, y_1)$**

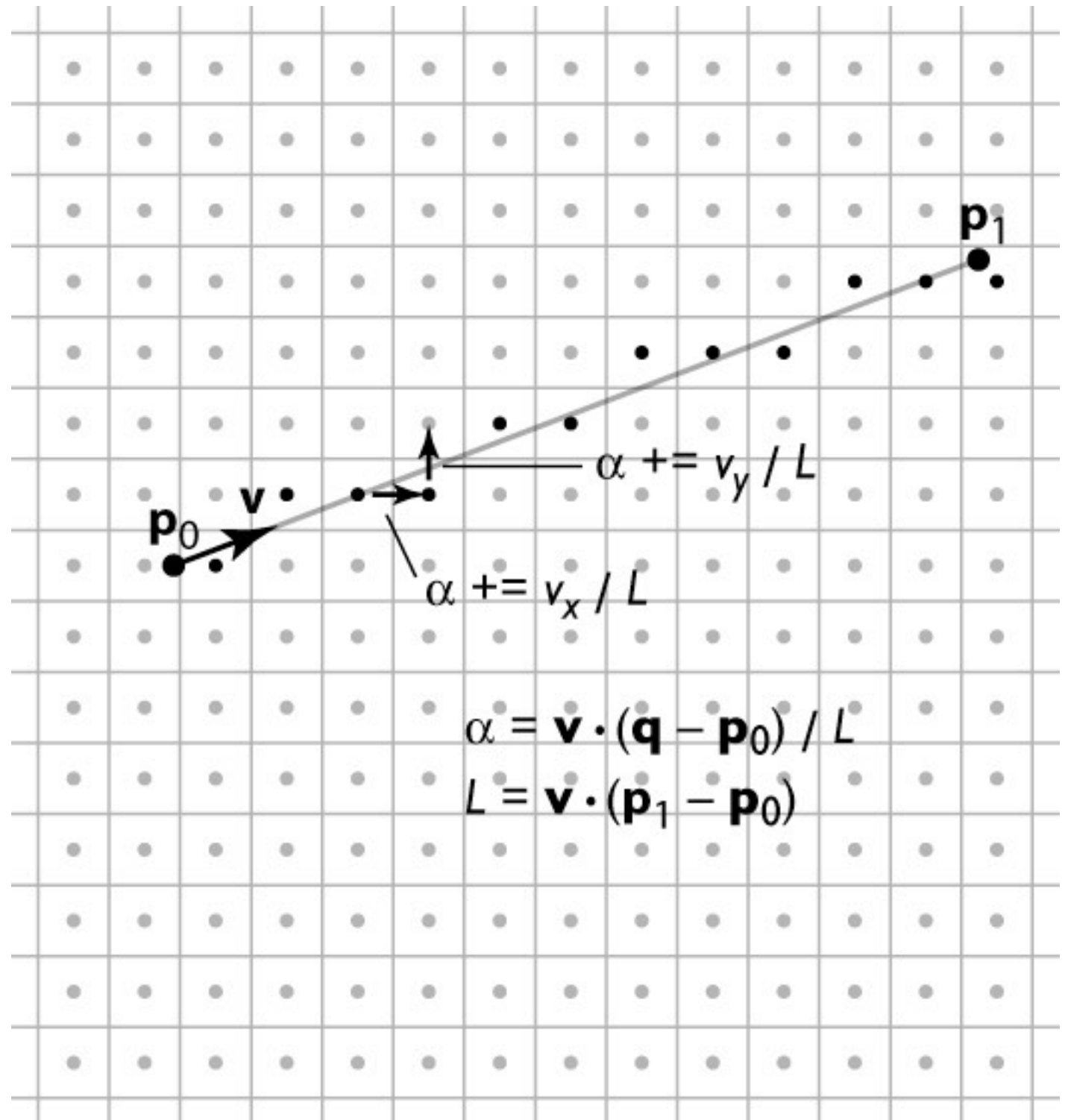
# Linear interpolation

- **Pixels are not exactly on the line**
- **Define 2D function by projection on line**
  - this is linear in 2D
  - therefore can use DDA to interpolate



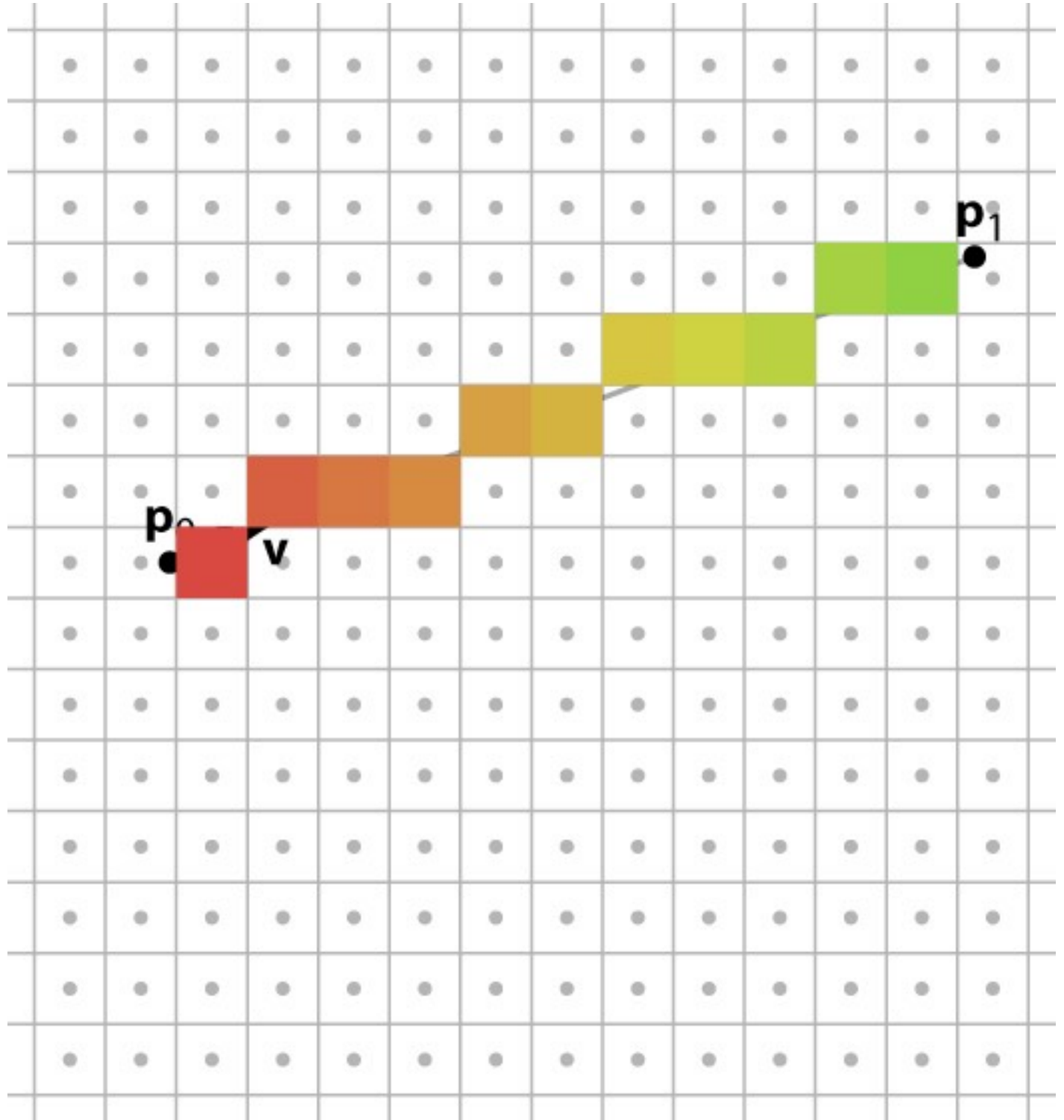
# Linear interpolation

- **Pixels are not exactly on the line**
- **Define 2D function by projection on line**
  - this is linear in 2D
  - therefore can use DDA to interpolate



# Linear interpolation

- **Pixels are not exactly on the line**
- **Define 2D function by projection on line**
  - this is linear in 2D
  - therefore can use DDA to interpolate



# Alternate interpretation

- **We are updating  $d$  and  $\alpha$  as we step from pixel to pixel**
  - $d$  tells us how far from the line we are
  - $\alpha$  tells us how far along the line we are
- **So  $d$  and  $\alpha$  are coordinates in a coordinate system oriented to the line**

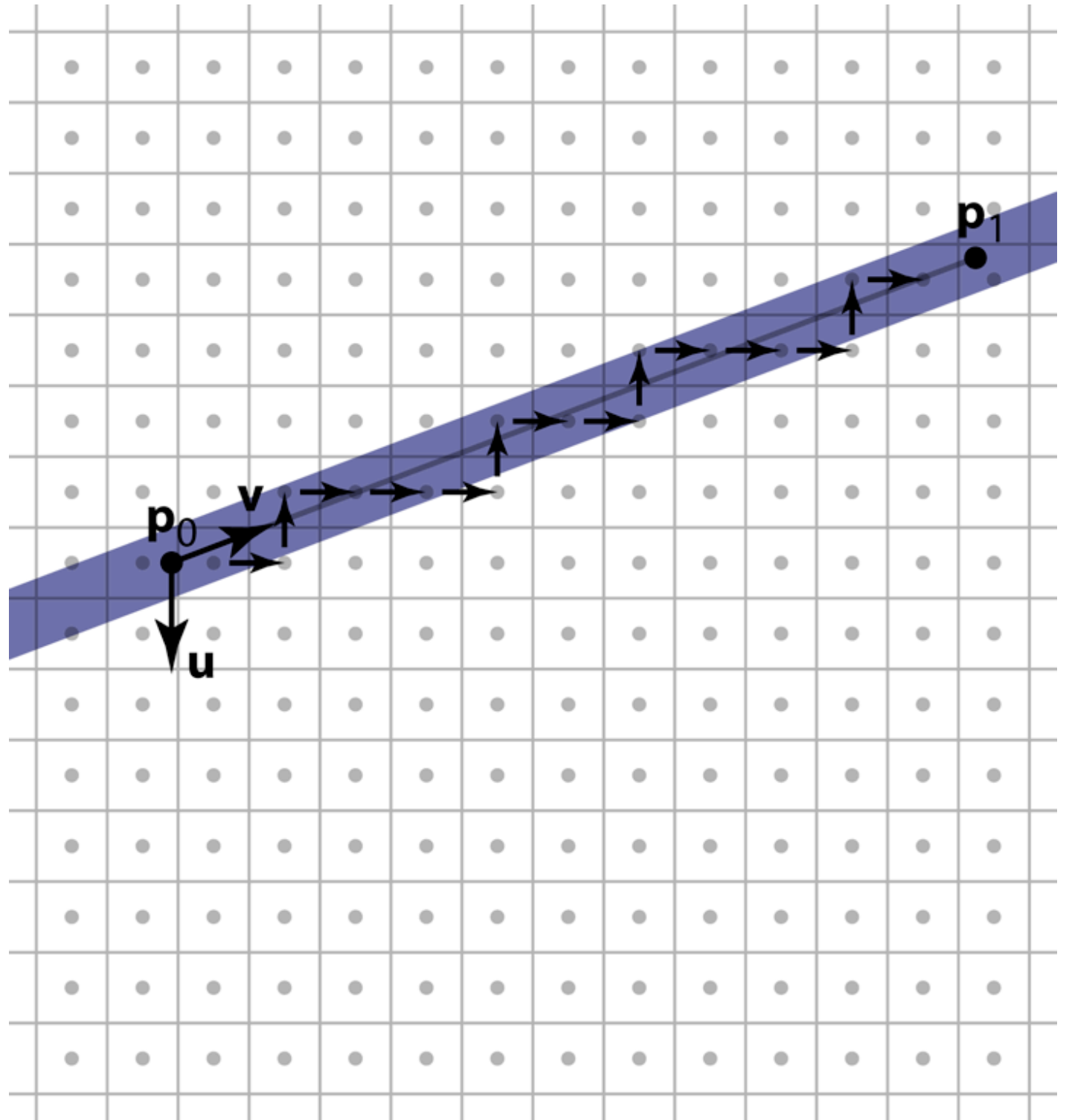
# Alternate interpretation

- **View loop as visiting all pixels the line passes through**

Interpolate  $d$  and  $\alpha$   
for each pixel

Only output frag.  
if pixel is in band

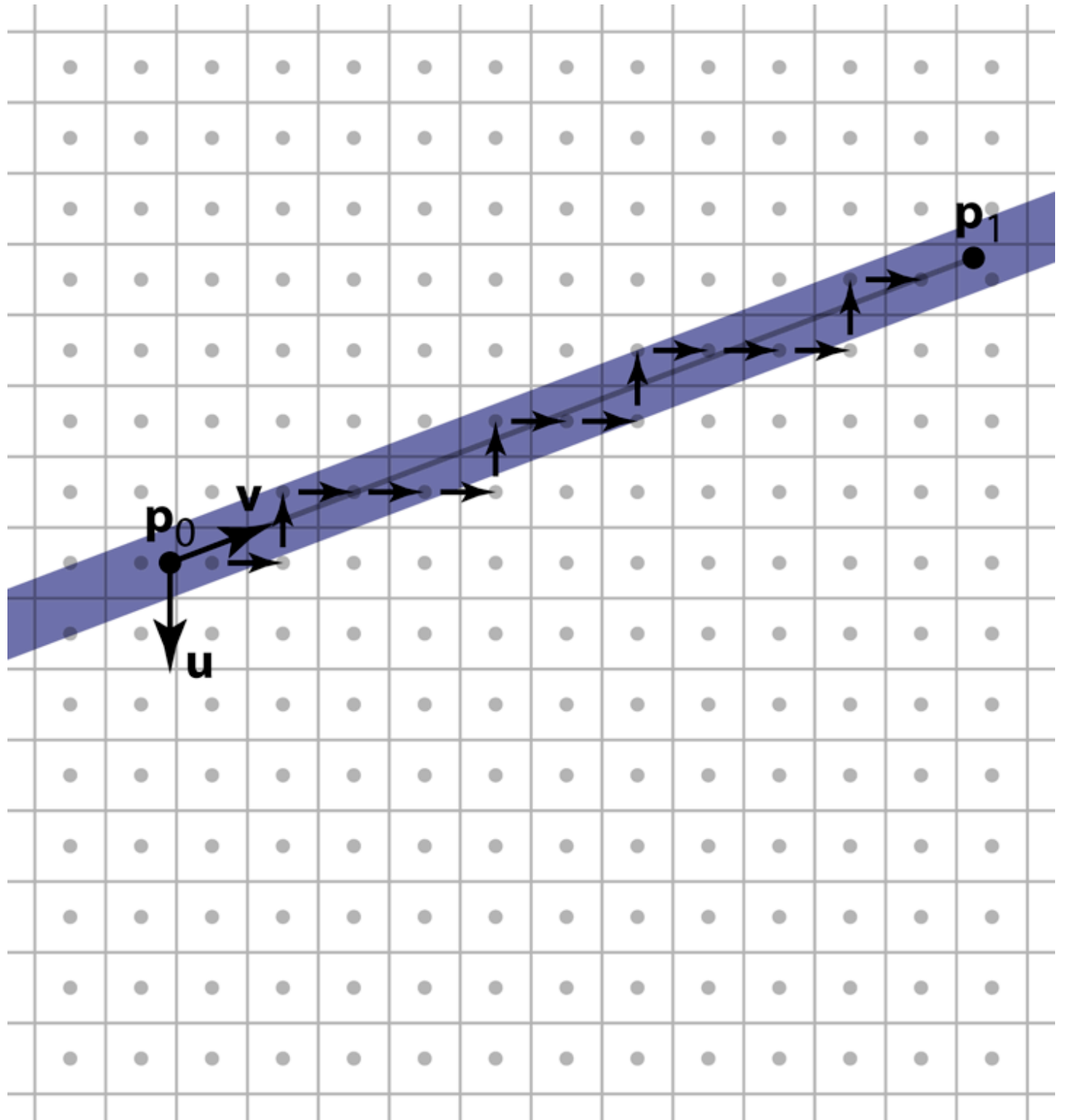
- **This makes linear interpolation the primary operation**





# Pixel-walk line rasterization

```
x = ceil(x0)
y = round(m * x + b)
d = m * x + b - y
while x < floor(x1)
  if d > 0.5
    y += 1; d -= 1;
  else
    x += 1; d += m;
  if -0.5 < d ≤ 0.5
    output(x, y)
```



# Rasterizing triangles

- **The most common case in most applications**
  - with good antialiasing can be the only case
  - some systems render a line as two skinny triangles
- **Triangle represented by three vertices**
- **Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization**
  - walk from pixel to pixel over (at least) the polygon's area
  - evaluate linear functions as you go
  - use those functions to decide which pixels are inside

# Rasterizing triangles

- **Input:**

- three 2D points (the triangle's vertices in pixel space)
  - $(x_0, y_0); (x_1, y_1); (x_2, y_2)$
- parameter values at each vertex
  - $q_{00}, \dots, q_{0n}; q_{10}, \dots, q_{1n}; q_{20}, \dots, q_{2n}$

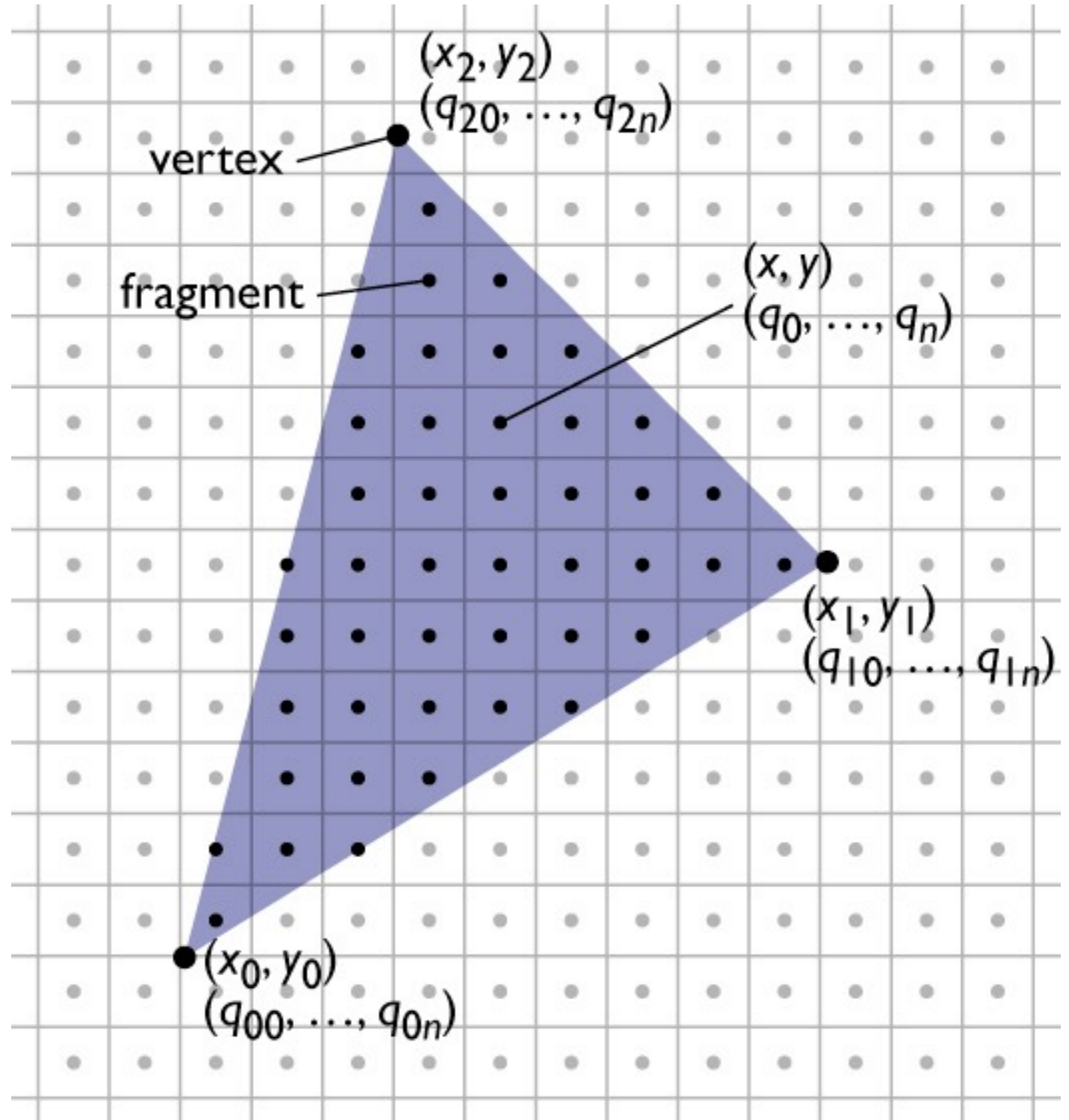
- **Output: a list of fragments, each with**

- the integer pixel coordinates  $(x, y)$
- interpolated parameter values  $q_0, \dots, q_n$

# Rasterizing triangles

- **Summary**

- 1 evaluation of linear functions on pixel grid
- 2 functions defined by parameter values at vertices
- 3 using extra parameters to determine fragment set



# Incremental linear evaluation

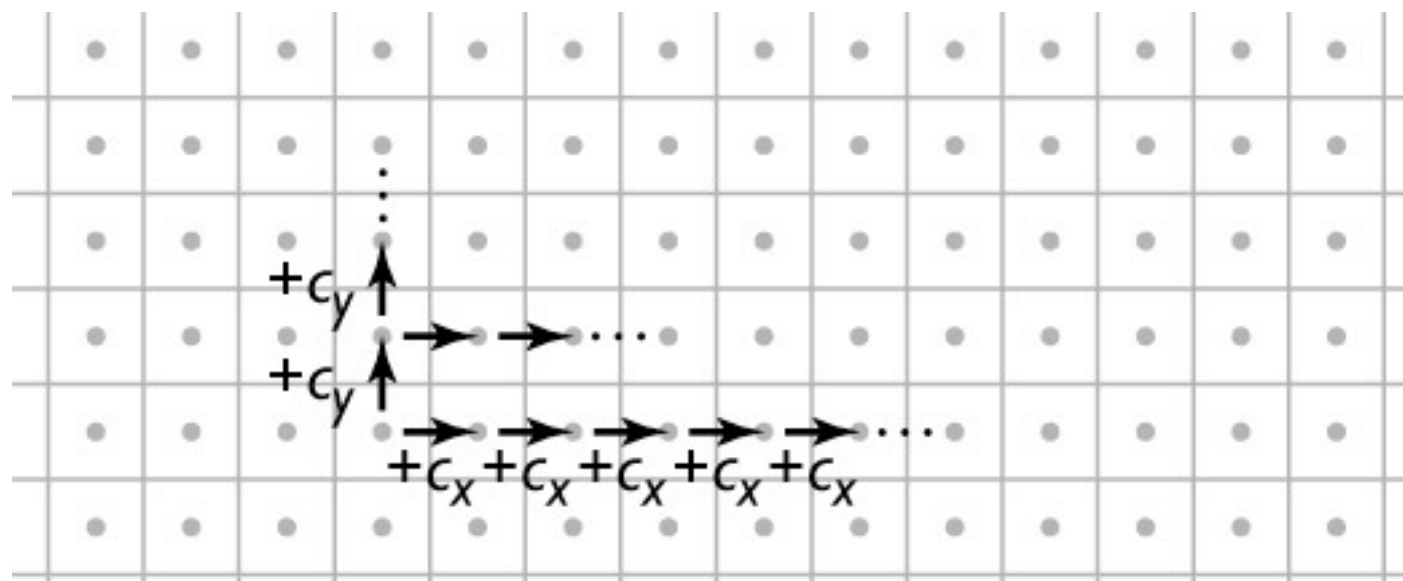
- **A linear (affine, really) function on the plane is:**

$$q(x, y) = c_x x + c_y y + c_k$$

- **Linear functions are efficient to evaluate on a grid:**

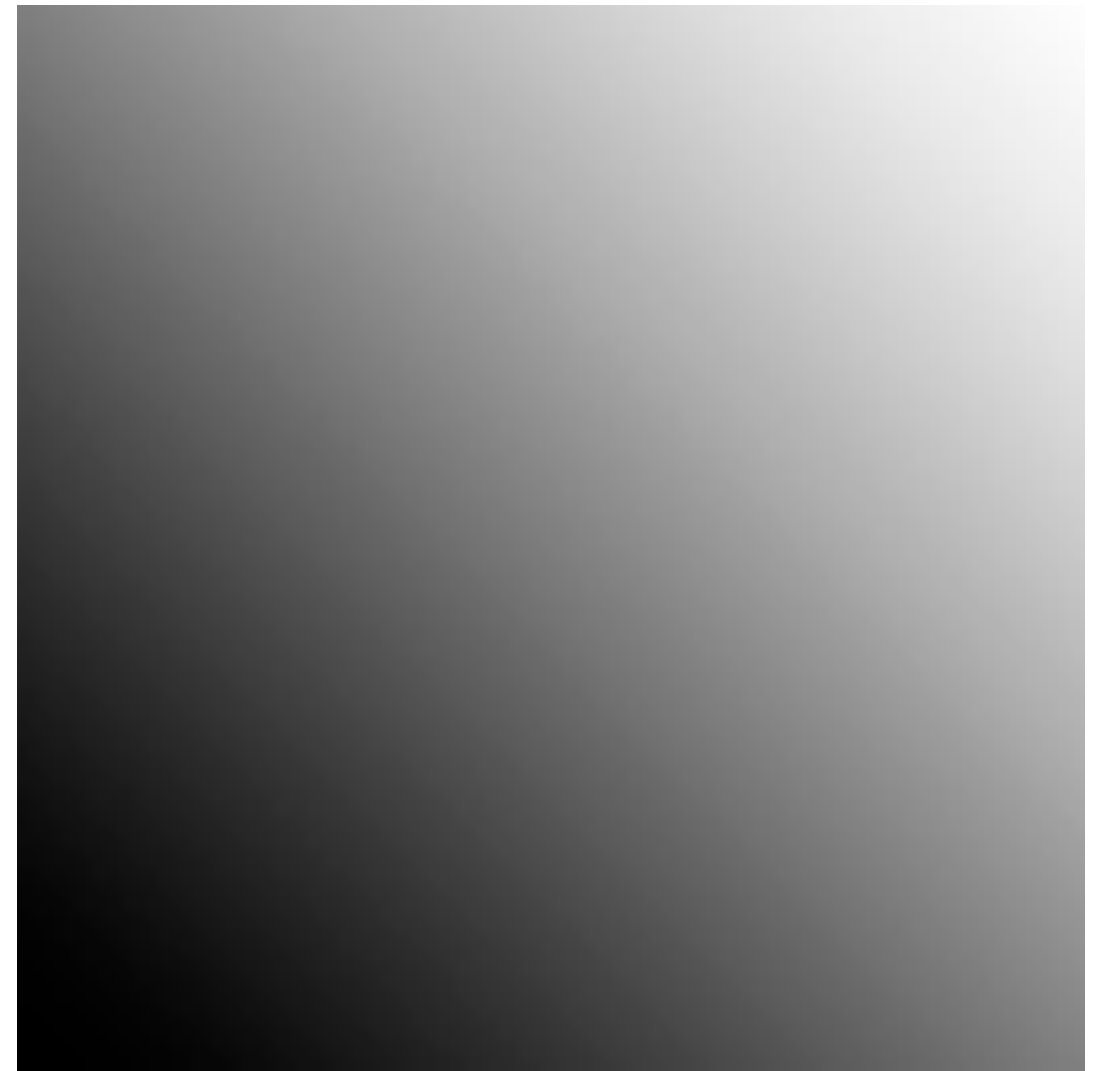
$$q(x + 1, y) = c_x(x + 1) + c_y y + c_k = q(x, y) + c_x$$

$$q(x, y + 1) = c_x x + c_y(y + 1) + c_k = q(x, y) + c_y$$



# Incremental linear evaluation

```
linEval(xm, xM, ym, yM, cx, cy, ck) {  
  
    // setup  
    qRow = cx*xm + cy*ym + ck;  
  
    // traversal  
    for y = ym to yM {  
        qPix = qRow;  
        for x = xm to xM {  
            output(x, y, qPix);  
            qPix += cx;  
        }  
        qRow += cy;  
    }  
}
```

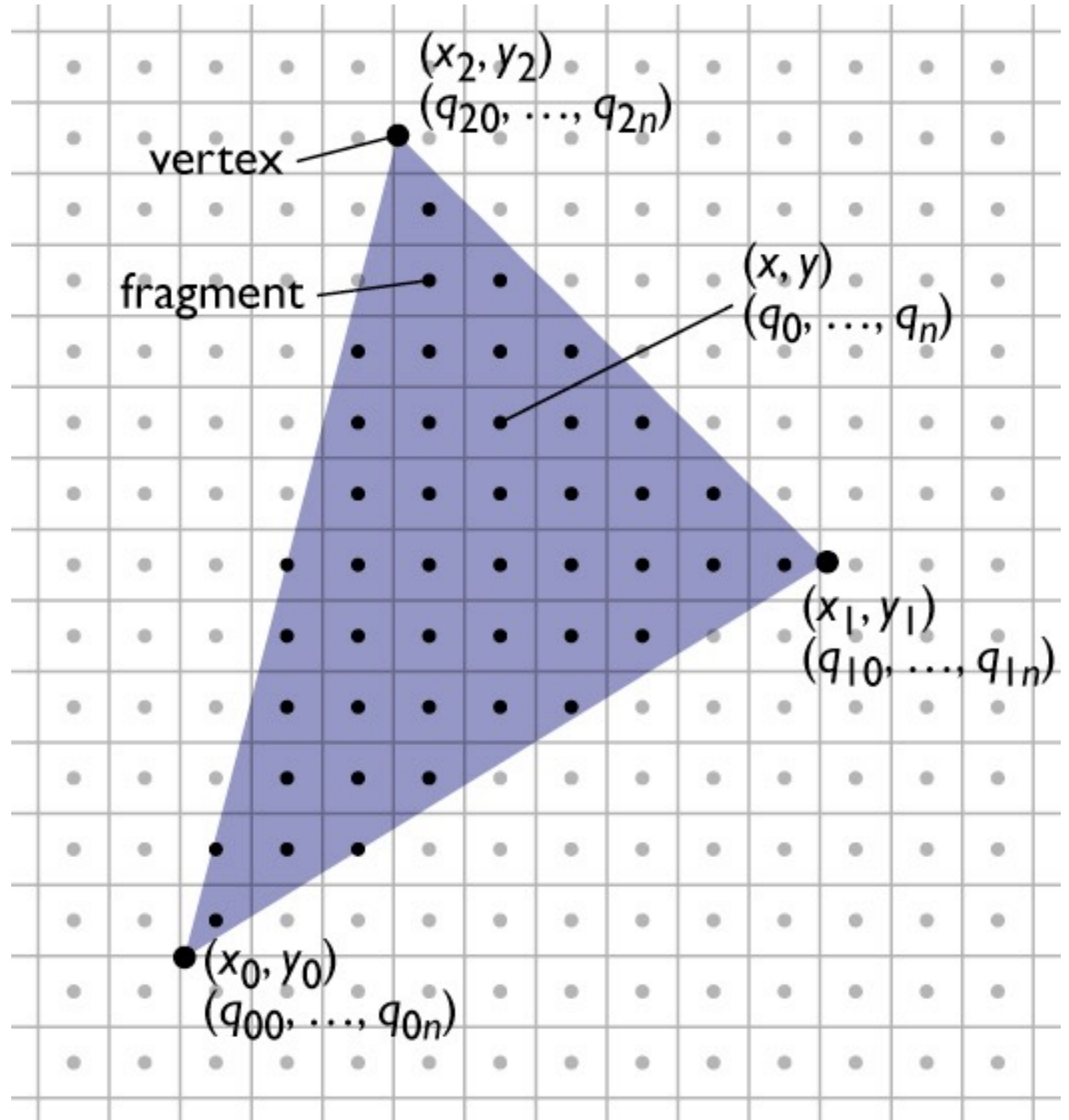


$c_x = .005; c_y = .005; c_k = 0$   
(image size 100x100)

# Rasterizing triangles

- **Summary**

- 1 evaluation of linear functions on pixel grid
- 2 functions defined by parameter values at vertices
- 3 using extra parameters to determine fragment set



# Defining parameter functions

- **To interpolate parameters across a triangle we need to find the  $c_x$ ,  $c_y$ , and  $c_k$  that define the (unique) linear function that matches the given values at all 3 vertices**

– this is 3 constraints on 3 unknown coefficients:

$$c_x x_0 + c_y y_0 + c_k = q_0$$

$$c_x x_1 + c_y y_1 + c_k = q_1$$

$$c_x x_2 + c_y y_2 + c_k = q_2$$

(each states that the function agrees with the given value at one vertex)

– leading to a 3x3 matrix equation for the coefficients:

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_k \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}$$

(singular iff triangle is degenerate)



# Defining parameter functions

- **More efficient version: shift origin to  $(x_0, y_0)$**

$$q(x, y) = c_x(x - x_0) + c_y(y - y_0) + q_0$$

$$q(x_1, y_1) = c_x(x_1 - x_0) + c_y(y_1 - y_0) + q_0 = q_1$$

$$q(x_2, y_2) = c_x(x_2 - x_0) + c_y(y_2 - y_0) + q_0 = q_2$$

- now this is a 2x2 linear system (since  $q_0$  falls out):

$$\begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} q_1 - q_0 \\ q_2 - q_0 \end{bmatrix}$$

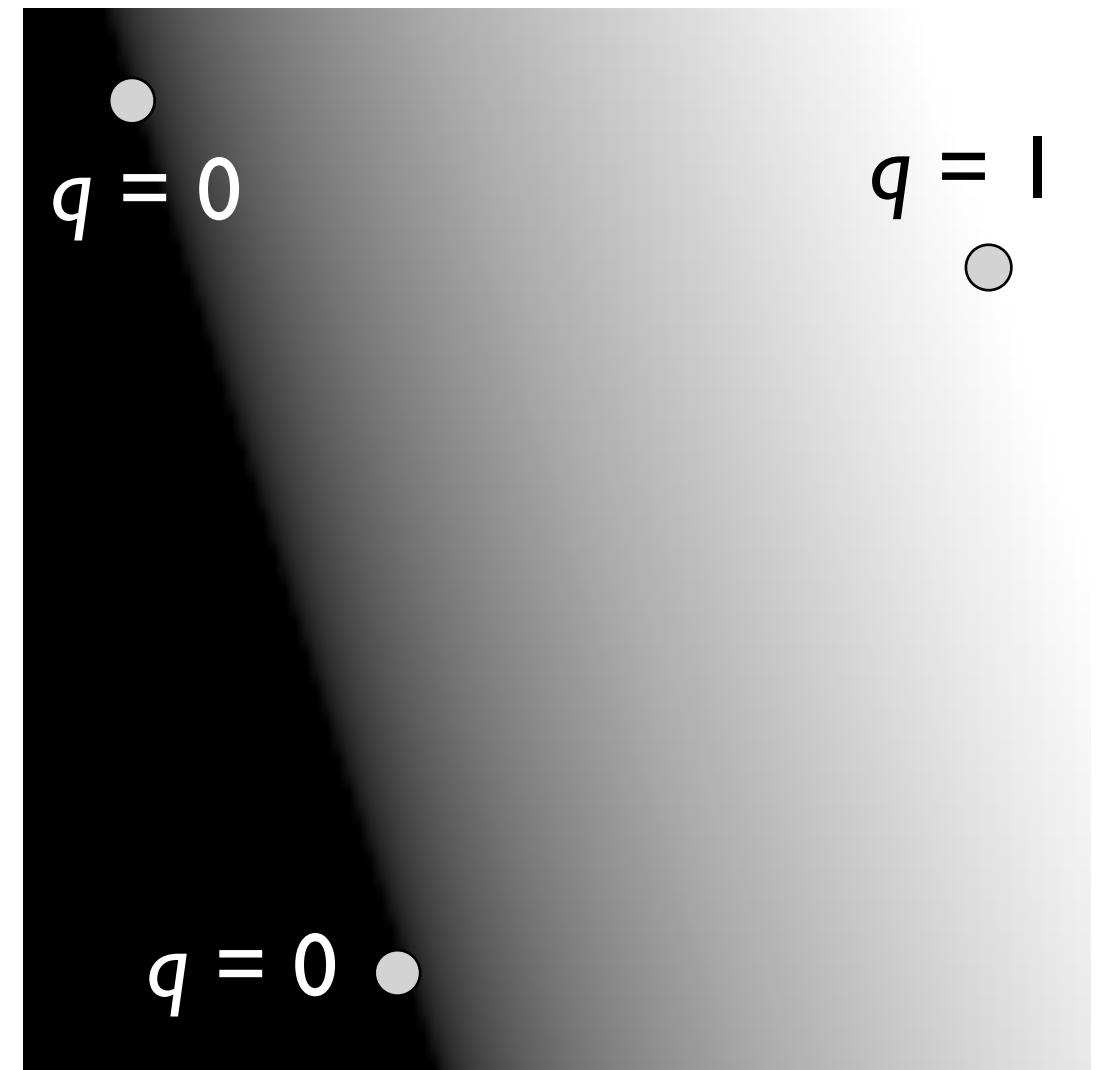
- solve using Cramer's rule (see Shirley):

$$c_x = (\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1) / (\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

$$c_y = (\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2) / (\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

# Defining parameter functions

```
linInterp(xm, xM, ym, yM, x0, y0, q0,  
          x1, y1, q1, x2, y2, q2) {  
  
    // setup  
    det = (x1-x0)*(y2-y0) - (x2-x0)*(y1-y0);  
    cx = ((q1-q0)*(y2-y0) - (q2-q0)*(y1-y0)) / det;  
    cy = ((q2-q0)*(x1-x0) - (q1-q0)*(x2-x0)) / det;  
    qRow = cx*(xm-x0) + cy*(ym-y0) + q0;  
  
    // traversal (same as before)  
    for y = ym to yM {  
        qPix = qRow;  
        for x = xm to xM {  
            output(x, y, qPix);  
            qPix += cx;  
        }  
        qRow += cy;  
    }  
}
```

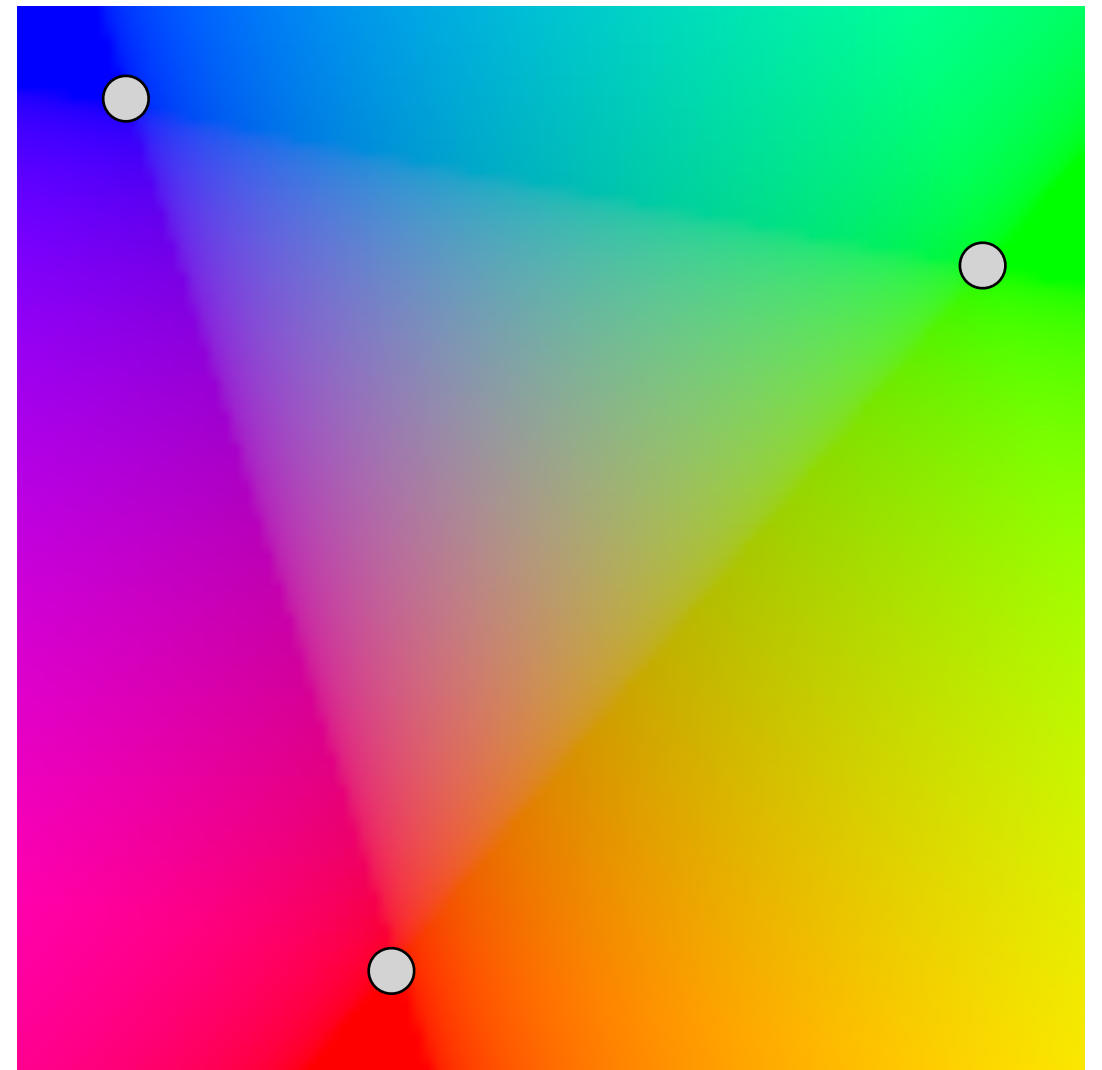


# Interpolating several parameters

```
linInterp(xm, xM, ym, yM, n, x0, y0, q0[],
          x1, y1, q1[], x2, y2, q2[]) {

    // setup
    for k = 0 to n-1
        // compute cx[k], cy[k], qRow[k]
        // from q0[k], q1[k], q2[k]

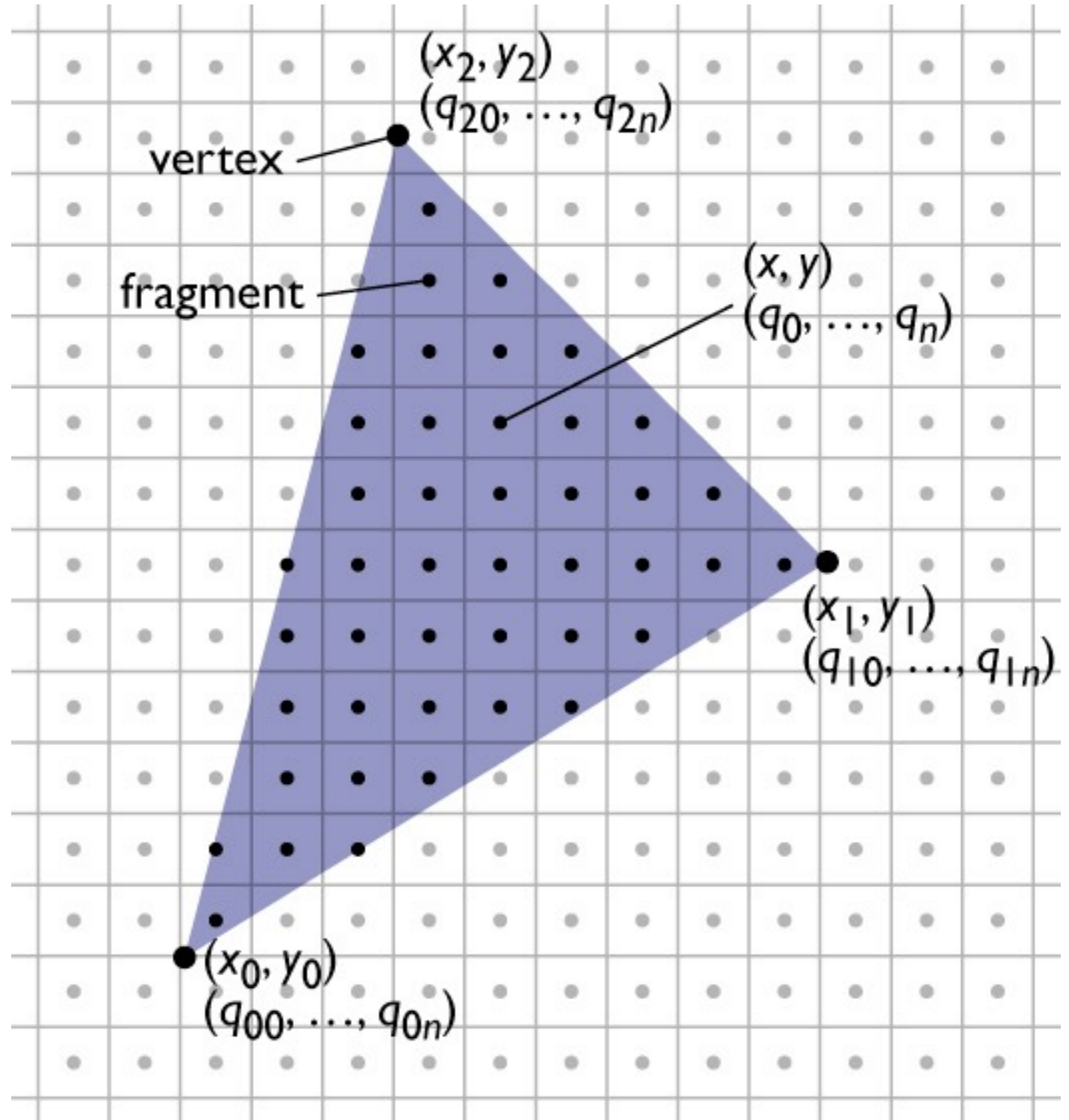
    // traversal
    for y = ym to yM {
        for k = 1 to n, qPix[k] = qRow[k];
        for x = xm to xM {
            output(x, y, qPix);
            for k = 1 to n, qPix[k] += cx[k];
        }
        for k = 1 to n, qRow[k] += cy[k];
    }
}
```



# Rasterizing triangles

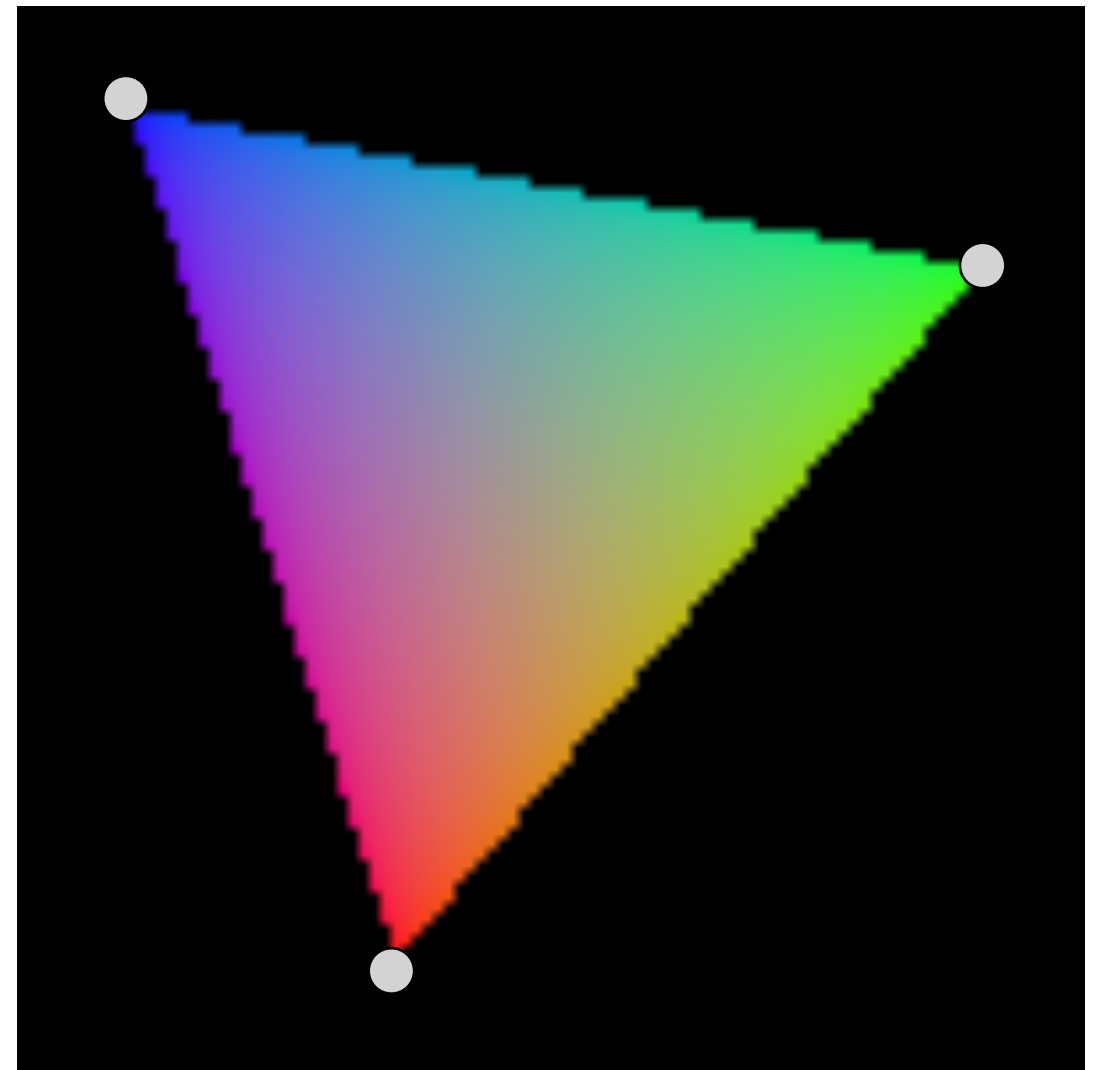
- **Summary**

- 1 evaluation of linear functions on pixel grid
- 2 functions defined by parameter values at vertices
- 3 using extra parameters to determine fragment set



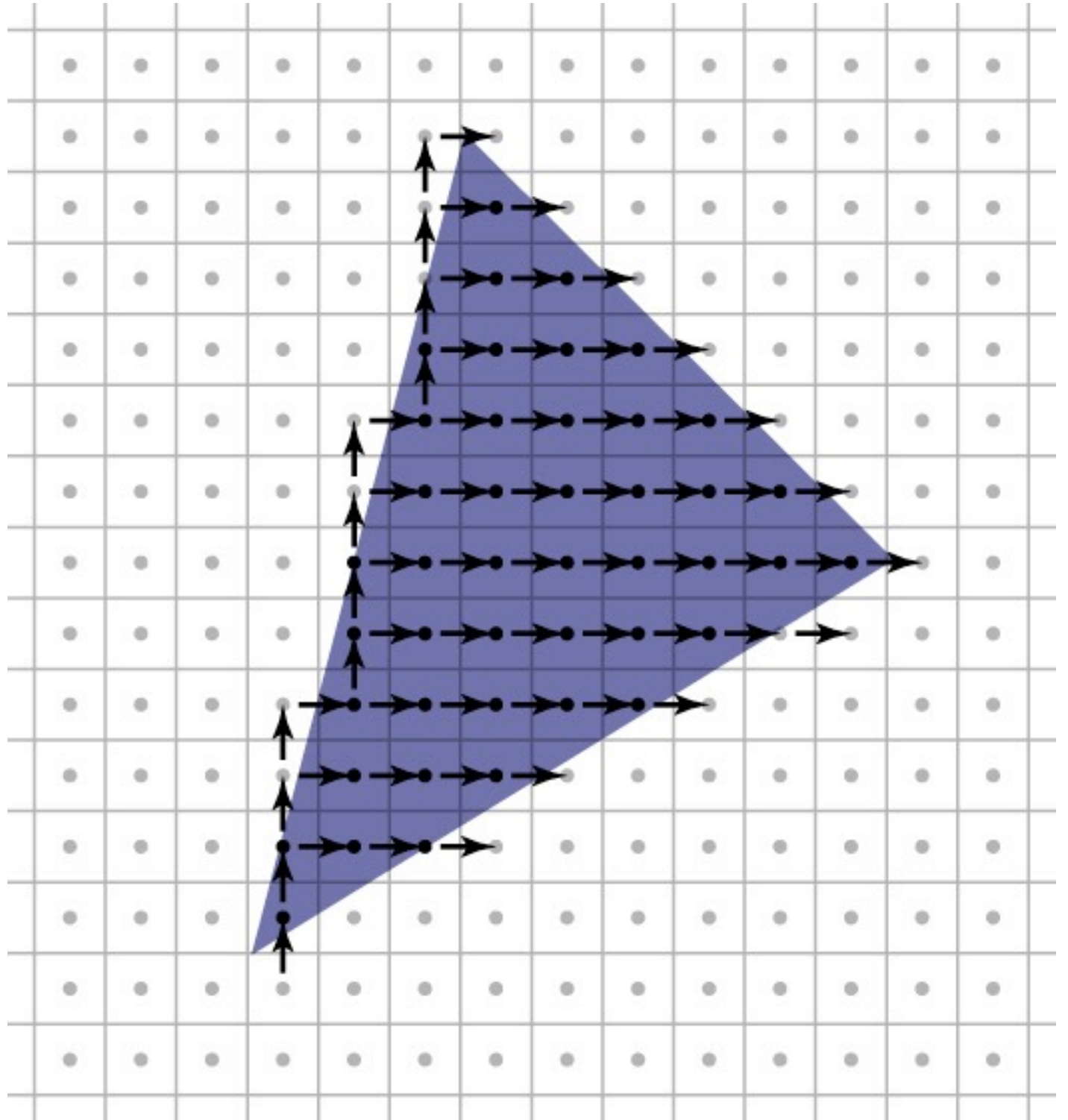
# Clipping to the triangle

- **Interpolate three barycentric coordinates across the plane**
  - recall each barycentric coord is 1 at one vert. and 0 at the other two
- **Output fragments only when all three are  $> 0$ .**



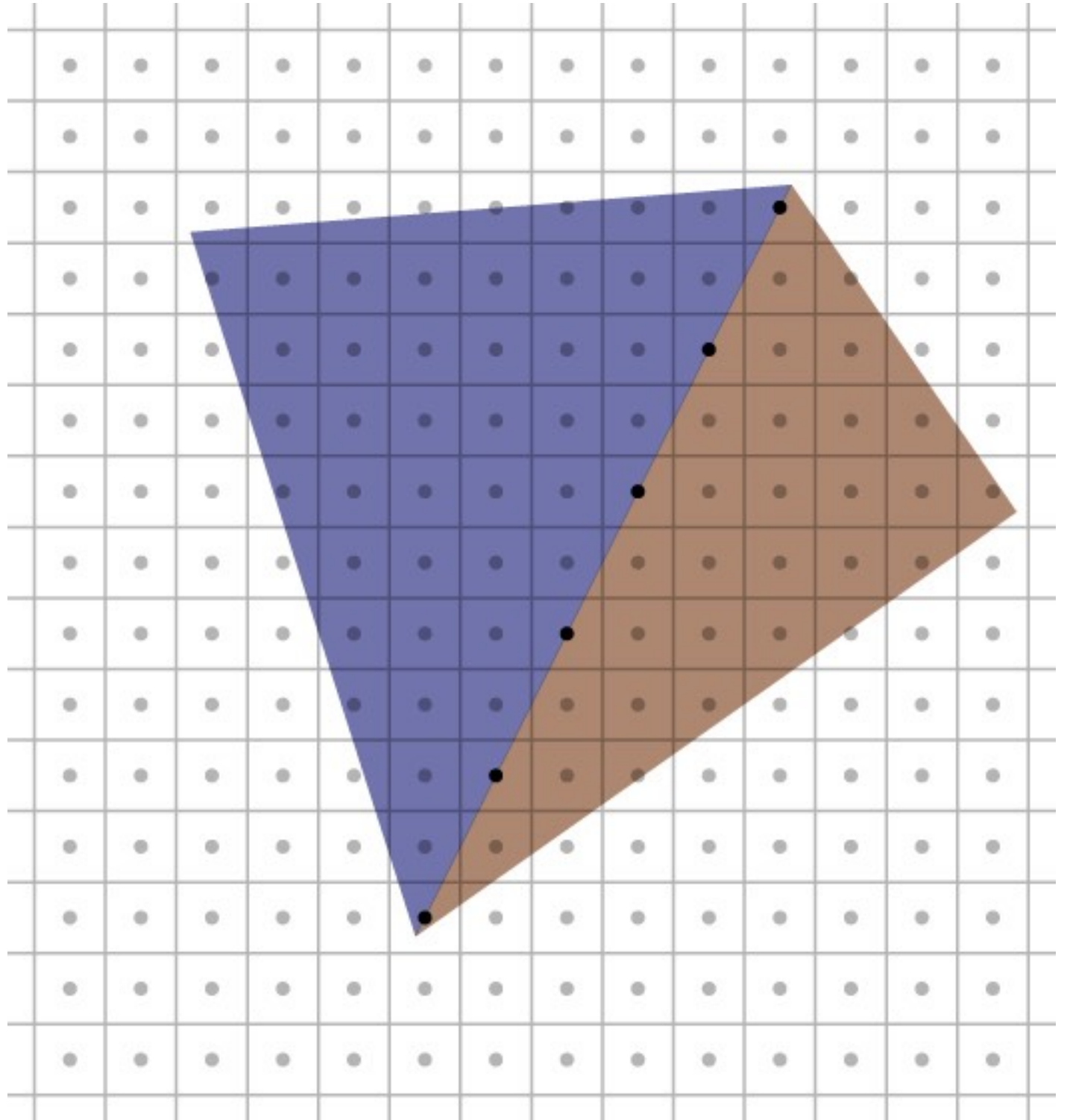
# Pixel-walk (Pineda) rasterization

- **Conservatively visit a superset of the pixels you want**
- **Interpolate linear functions**
- **Use those functions to determine when to emit a fragment**



# Rasterizing triangles

- **Exercise caution with rounding and arbitrary decisions**
  - need to visit these pixels once
  - but it's important not to visit them twice!



# Clipping

- **Rasterizer tends to assume triangles are on screen**
  - particularly problematic to have triangles crossing the plane  $z = 0$
- **After projection, before perspective divide**
  - clip against the planes  $x, y, z = 1, -1$  (6 planes)
  - primitive operation: clip triangle against axis-aligned plane



# Clipping a triangle against a plane

- **4 cases, based on sidedness of vertices**
  - all in (keep)
  - all out (discard)
  - one in, two out (one clipped triangle)
  - two in, one out (two clipped triangles)

