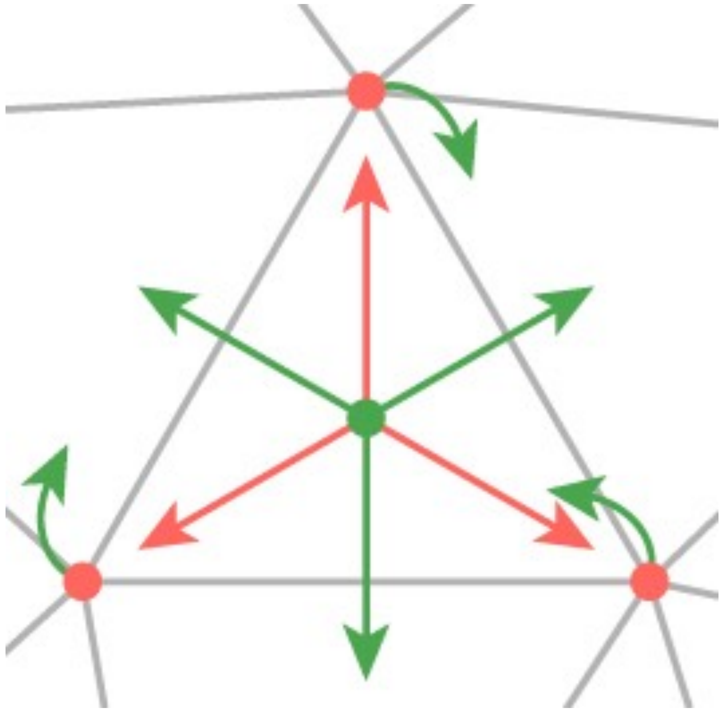
Triangle meshes 2

CS 4620 Lecture 3

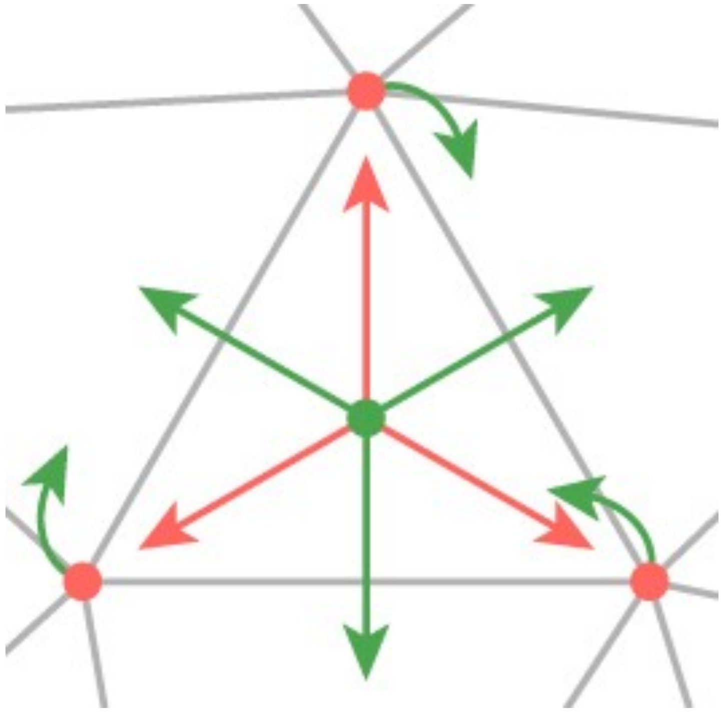
Triangle neighbor structure

- **Extension to indexed triangle set**
- **Triangle points to its three neighboring triangles**
- **Vertex points to a single neighboring triangle**
- **Can now enumerate triangles around a vertex**



Triangle neighbor structure

```
Triangle {
   Triangle nbr[3];
   Vertex vertex[3];
   }

// t.neighbor[i] is adjacent
// across the edge from i to i+1

Vertex {
   // ... per-vertex data ...
   Triangle t;  // any adjacent tri
   }

// ... or ...

Mesh {
   // ... per-vertex data ...
   int tInd[nt][3];  // vertex indices
   int tNbr[nt][3]; // indices of neighbor triangles
   int vTri[nv]; // index of any adjacent triangle
   }
```

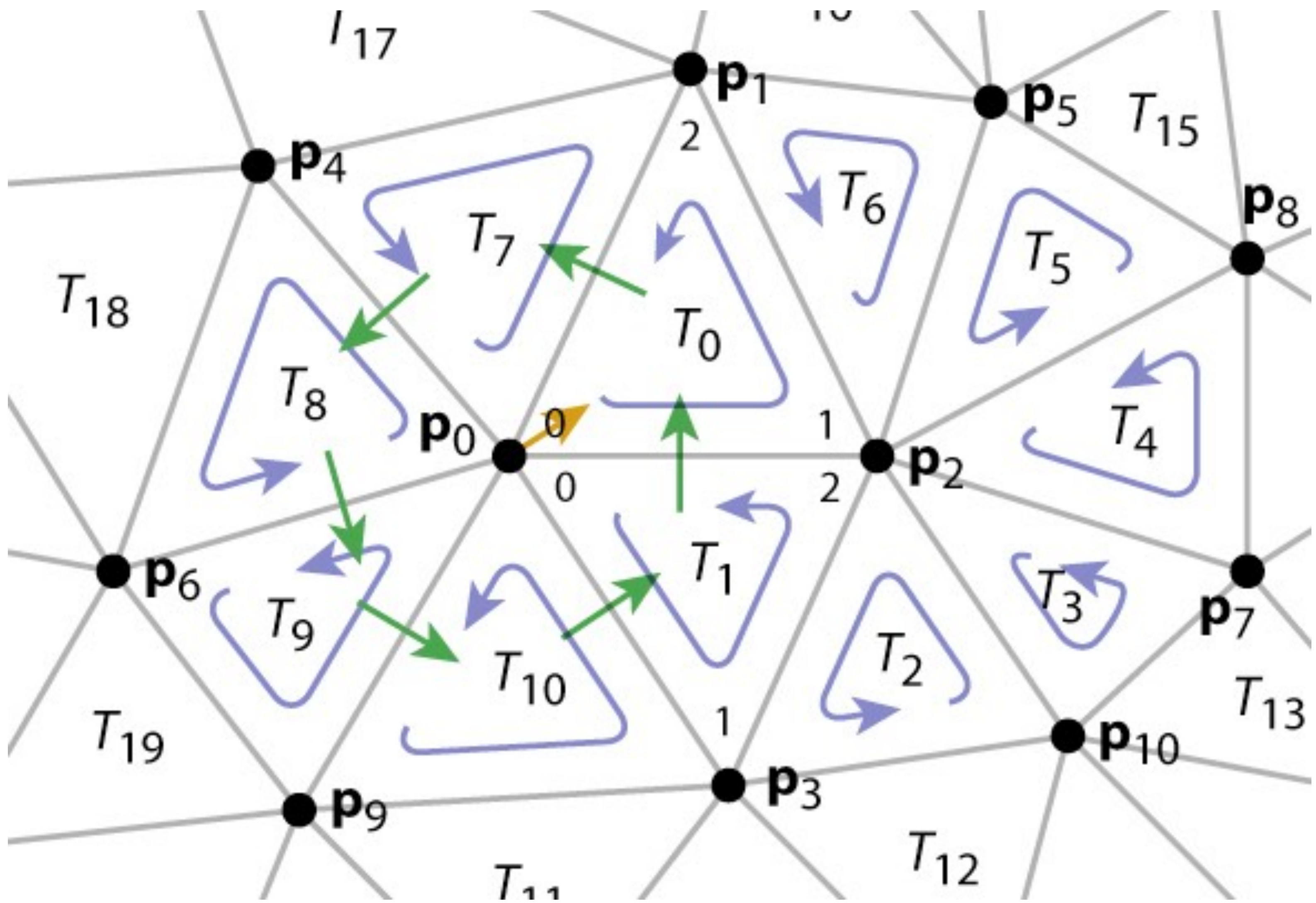


Triangle neighbor structure

vTri[0]	0
vTri[1]	6
vTri[2]	1
vTri[3]	1
	⋮

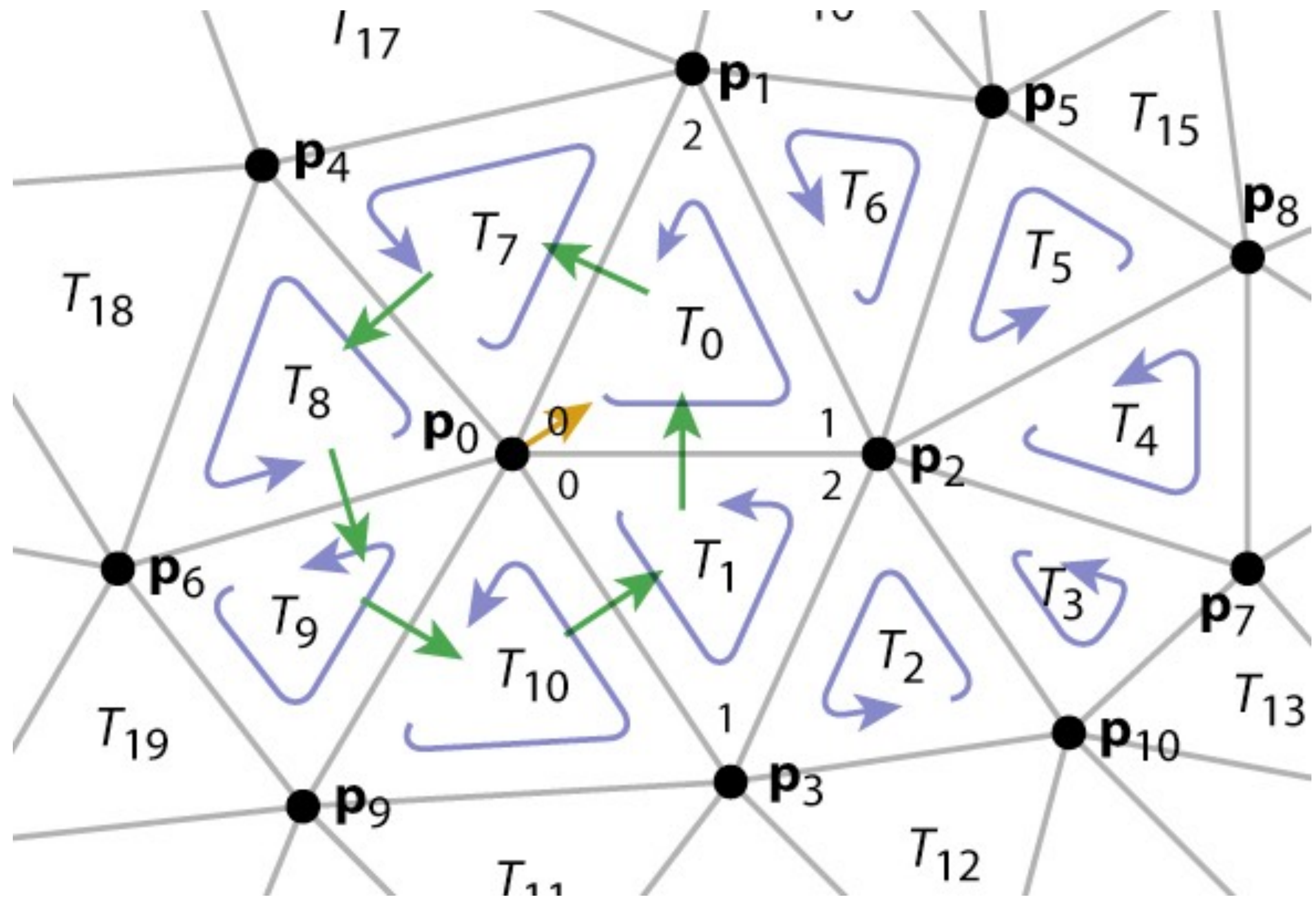
tNbr[0]	1, 6, 7
tNbr[1]	10, 2, 0
tNbr[2]	3, 1, 12
tNbr[3]	2, 13, 4
	⋮

tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
tInd[2]	10, 2, 3
tInd[3]	2,10,7
	⋮



Triangle neighbor structure

```
TrianglesOfVertex(v) {
   t = v.t;
   do {
      find t.vertex[i] == v;
      t = t.nbr[pred(i)];
      } while (t != v.t);
   }

pred(i) = (i+2) % 3;
succ(i) = (i+1) % 3;
```

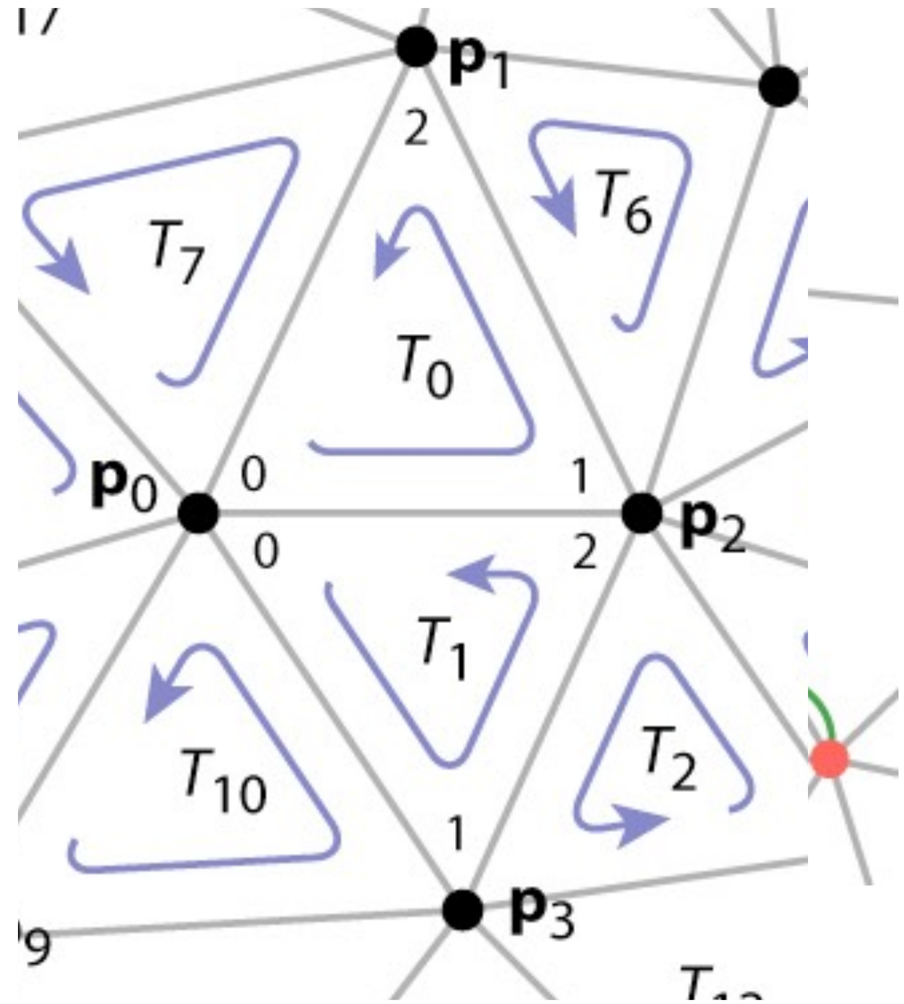


Triangle neighbor structure

- **indexed mesh was 36 bytes per vertex**
- **add an array of triples of indices (per triangle)**
  - int[n_T][3]: about 24 bytes per vertex
    - 2 triangles per vertex (on average)
    - (3 indices x 4 bytes) per triangle
- **add an array of representative triangle per vertex**
  - int[n_V]: 4 bytes per vertex
- **total storage: 64 bytes per vertex**
  - still not as much as separate triangles

Triangle neighbor structure—refined

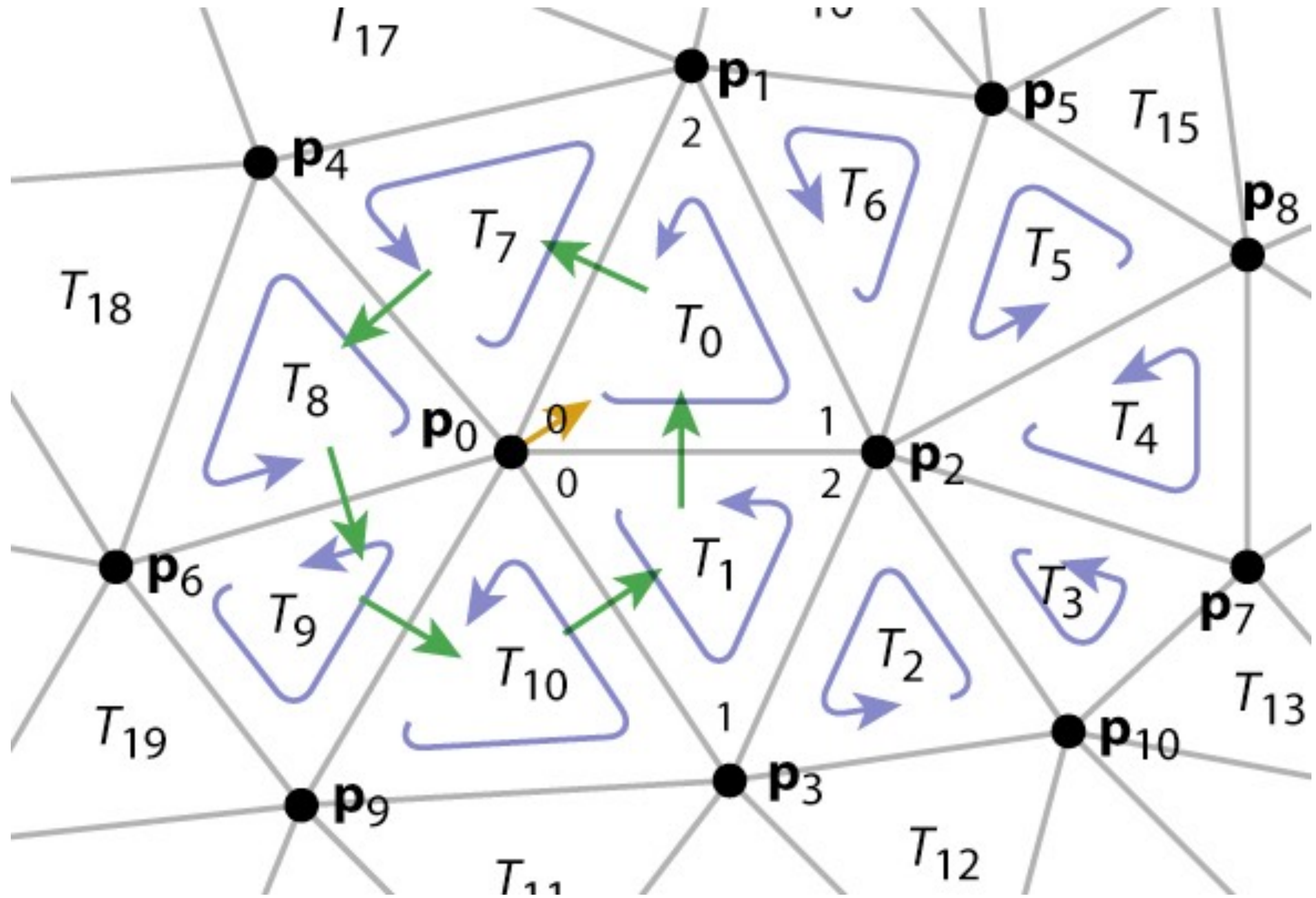
```
Triangle {
   Edge nbr[3];
   Vertex vertex[3];
   }

// if t.nbr[i].i == j
// then t.nbr[i].t.nbr[j] == t

Edge {
   // the i-th edge of triangle t
   Triangle t;
   int i;  // in {0,1,2}
   // in practice t and i share 32 bits
   }

Vertex {
   // ... per-vertex data ...
   Edge e;  // any edge leaving vertex
   }
```



$T_0.\text{nbr}[0] = \{ T_1, 2 \}$
$T_1.\text{nbr}[2] = \{ T_0, 0 \}$
$V_0.\text{e} = \{ T_1, 0 \}$

Triangle neighbor structure

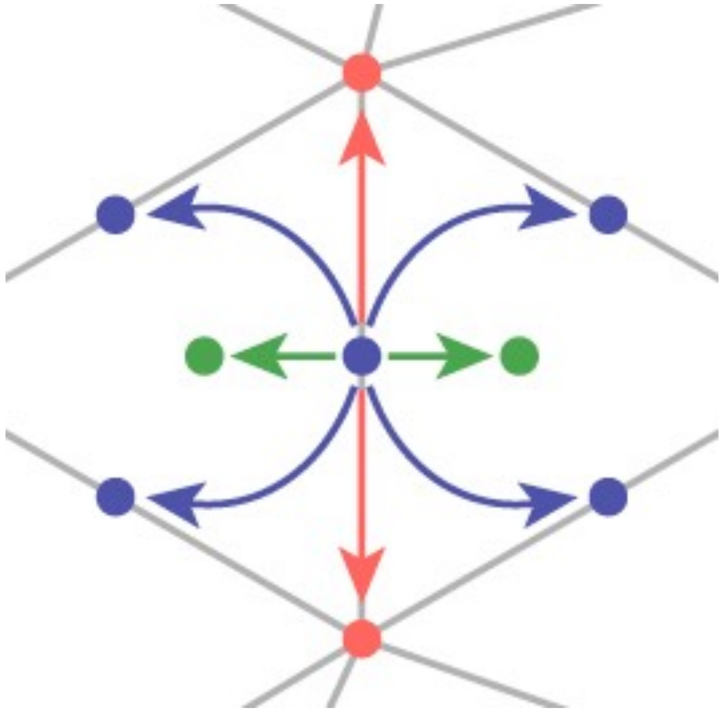
```
TrianglesOfVertex(v) {
  {t, i} = v.e;
  do {
    {t, i} = t.nbr[pred(i)];
    } while (t != v.t);
  }

pred(i) = (i+2) % 3;
succ(i) = (i+1) % 3;
```



T_0.nbr[0] = { T_1, 2 }
T_1.nbr[2] = { T_0, 0 }
V_0.e = { T_1, 0 }

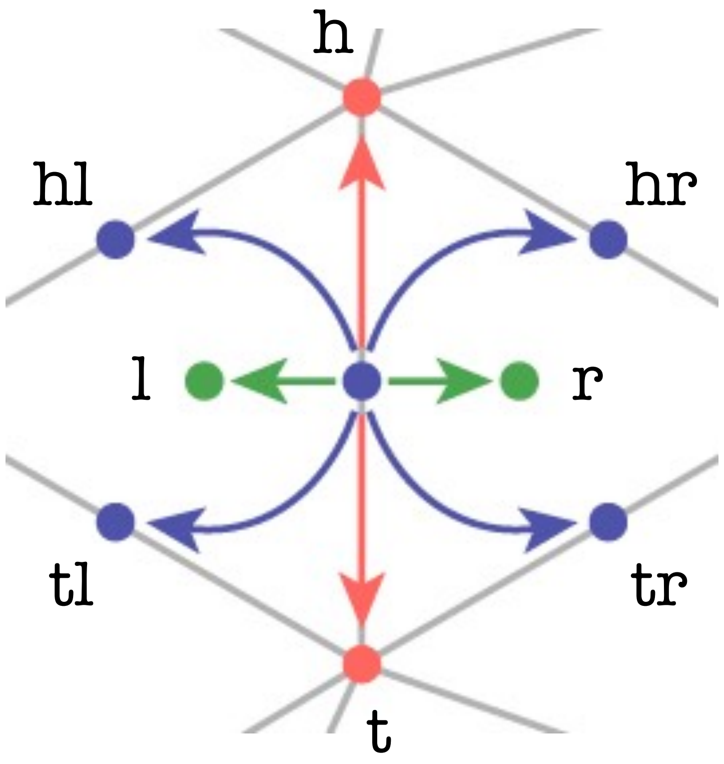
Winged-edge mesh

- **Edge-centric rather than face-centric**
  - therefore also works for polygon meshes
- **Each (oriented) edge points to:**
  - left and right forward edges
  - left and right backward edges
  - front and back vertices
  - left and right faces
- **Each face or vertex points to one edge**



Winged-edge mesh

```
Edge {
  Edge hl, hr, tl, tr;
  Vertex h, t;
  Face l, r;
  }

Face {
  // per-face data
  Edge e;  // any adjacent edge
  }

Vertex {
  // per-vertex data
  Edge e;  // any incident edge
  }
```

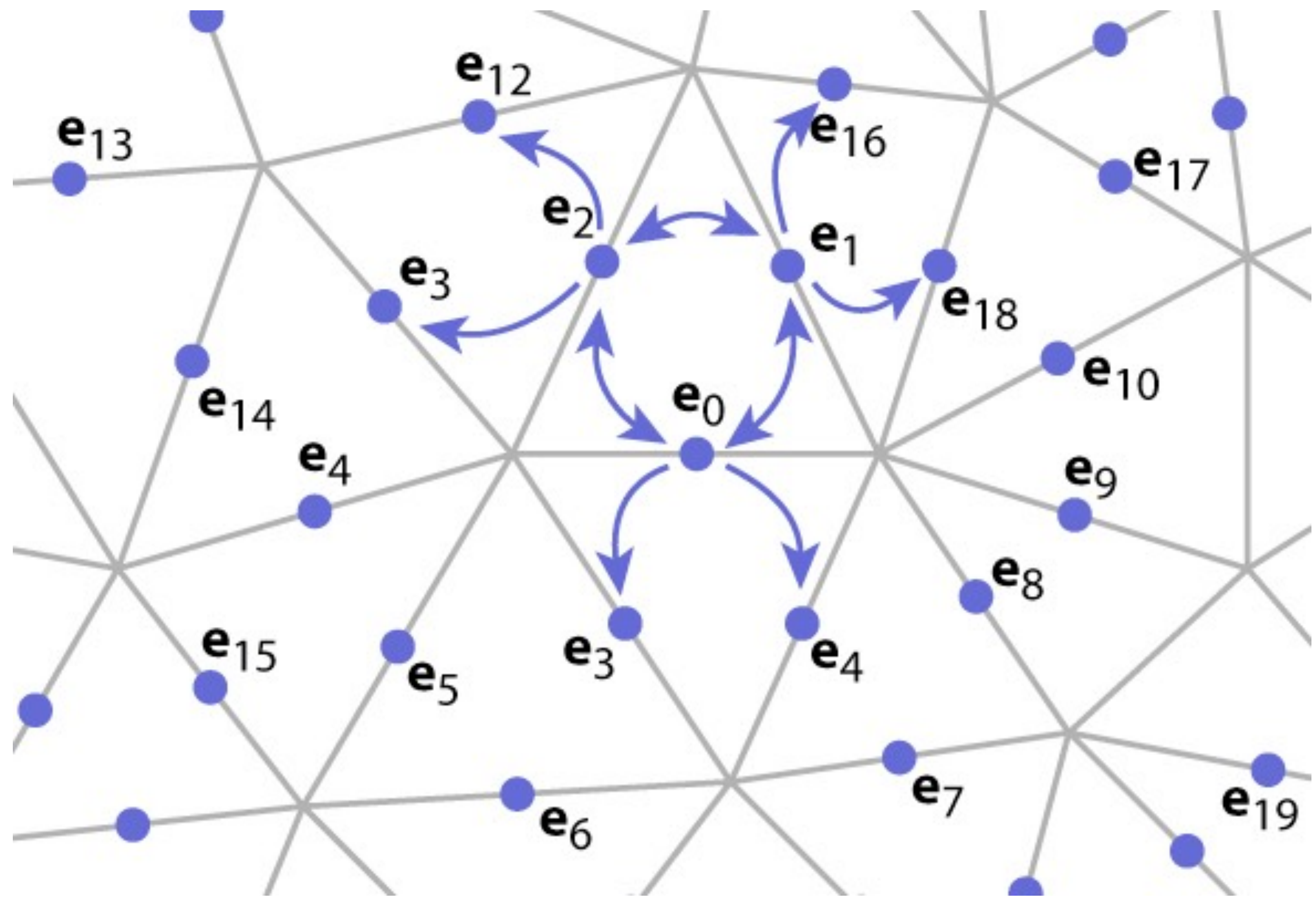


Winged-edge structure

```
EdgesOfFace(f) {
  e = f.e;
  do {
    if (e.l == f)
      e = e.hl;
    else
      e = e.tr;
  } while (e != f.e);
}
```

```
EdgesOfVertex(v) {
  e = v.e;
  do {
    if (e.t == v)
      e = e.tl;
    else
      e = e.hr;
  } while (e != v.e);
}
```

	hl	hr	tl	tr
edge[0]	1	4	2	3
edge[1]	18	0	16	2
edge[2]	12	1	3	0
		⋮		

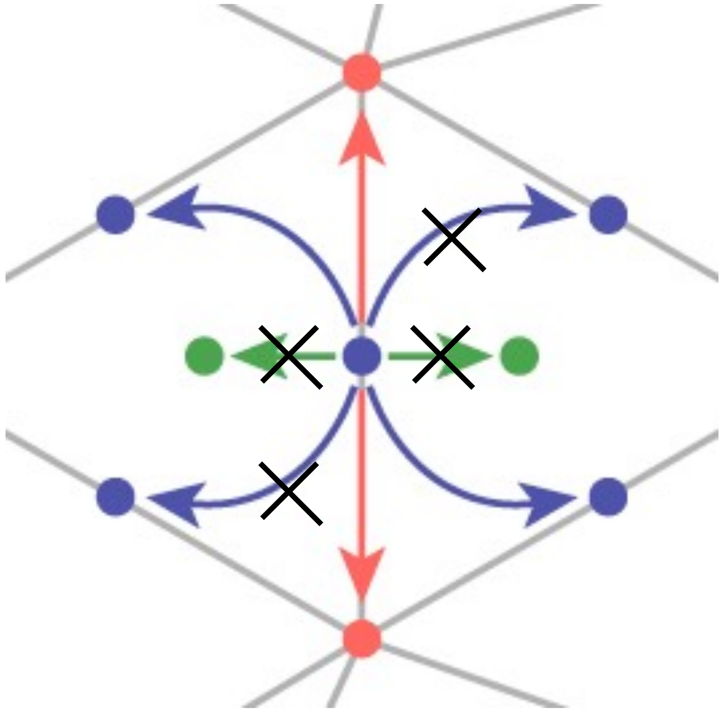


Winged-edge structure

- **array of vertex positions: 12 bytes/vert**
- **array of 8-tuples of indices (per edge)**
  - head/tail left/right edges + head/tail verts + left/right tris
  - int[n_E][8]: about 96 bytes per vertex
    - 3 edges per vertex (on average)
    - (8 indices x 4 bytes) per edge
- **add a representative edge per vertex**
  - int[n_V]: 4 bytes per vertex
- **total storage: 112 bytes per vertex**
  - but it is cleaner and generalizes to polygon meshes

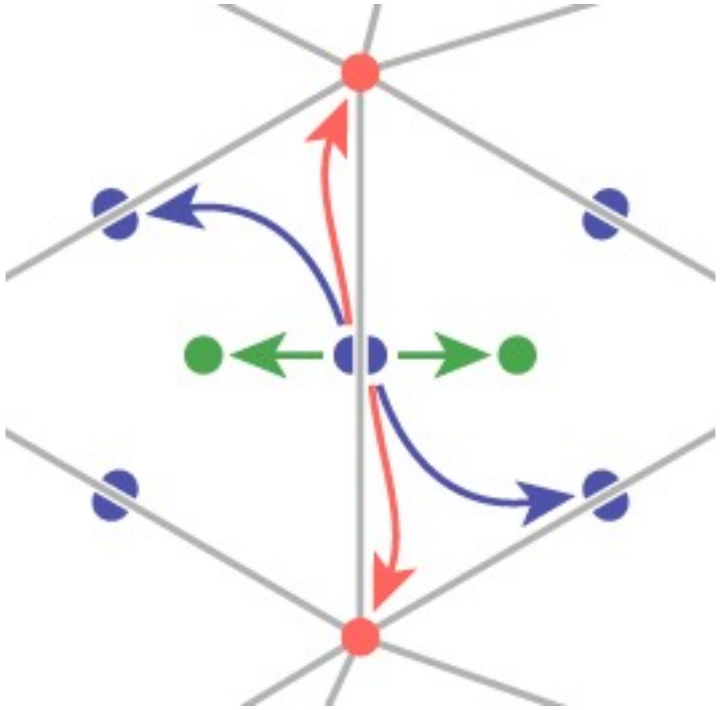
Winged-edge optimizations

- **Omit faces if not needed**
- **Omit one edge pointer on each side**
  - results in one-way traversal



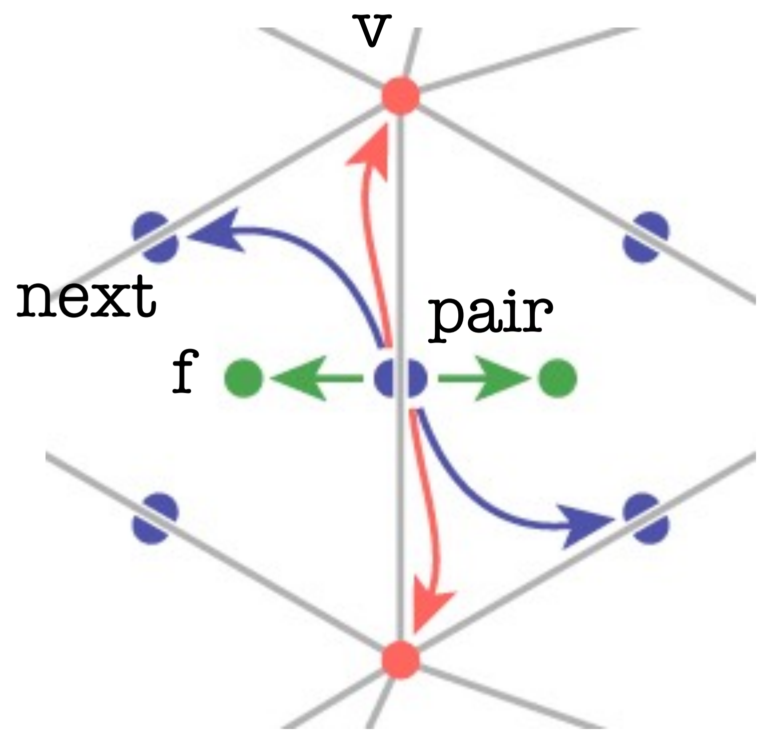
Half-edge structure

- **Simplifies, cleans up winged edge**
  - still works for polygon meshes
- **Each half-edge points to:**
  - next edge (left forward)
  - next vertex (front)
  - the face (left)
  - the opposite half-edge
- **Each face or vertex points to one half-edge**



Half-edge structure

```
HEdge {
  HEdge pair, next;
  Vertex v;
  Face f;
  }

Face {
  // per-face data
  HEdge h;  // any adjacent h-edge
  }

Vertex {
  // per-vertex data
  HEdge h;  // any incident h-edge
  }
```

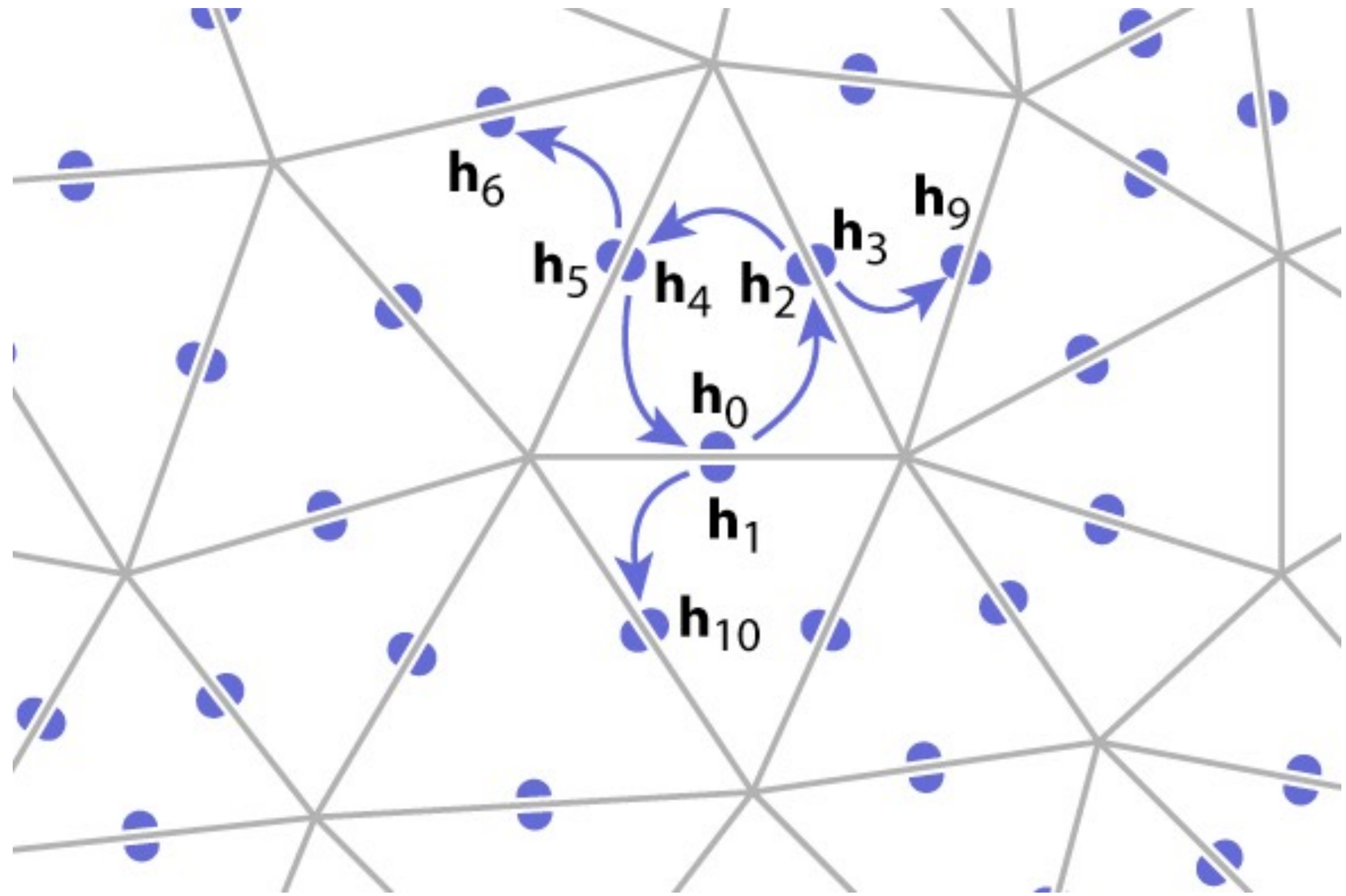


Half-edge structure

```
EdgesOfFace(f) {
  h = f.h;
  do {
      h = h.next;
    } while (h != f.h);
  }
```

```
EdgesOfVertex(v) {
  h = v.h;
  do {
      h = h.pair;
    } while (h != v.h);
  }
```

	pair	next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



Half-edge structure

- **array of vertex positions: 12 bytes/vert**
- **array of 4-tuples of indices (per h-edge)**
  - next, pair h-edges + head vert + left tri
  - int[$2n_E$][4]: about 96 bytes per vertex
    - 6 h-edges per vertex (on average)
    - (4 indices x 4 bytes) per h-edge
- **add a representative h-edge per vertex**
  - int[n_V]: 4 bytes per vertex
- **total storage: 112 bytes per vertex**

Half-edge optimizations

- **Omit faces if not needed**
- **Use implicit pair pointers**
  – they are allocated in pairs
  – they are even and odd in an array
- **Result: 2 indices per HEdge**
  – HEdges are 48 bytes/vertex
  – total 64 bytes/vertex
  (same as triangle neighbor)