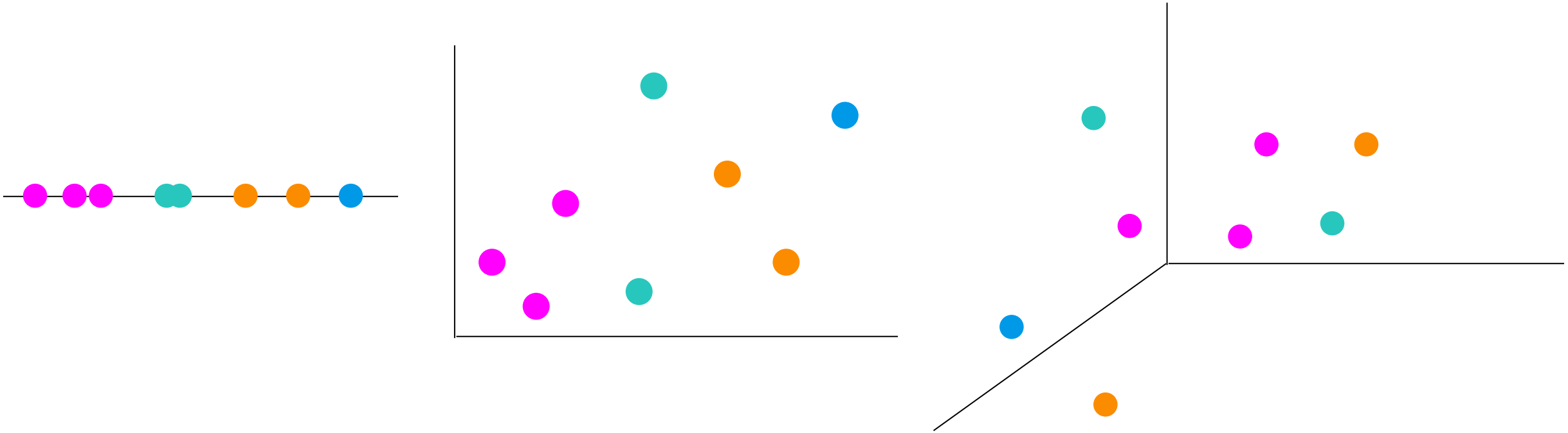
The Curse of Dimensionality

More data is always better, right?

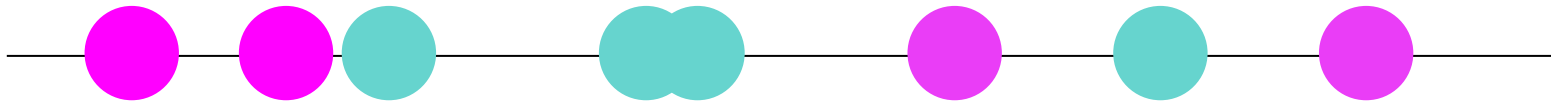
- Not always! Depends on whether you're adding **observations** or **dimensions**
- If we hold fixed the number of observations, adding more dimensions is not always good
- Issues from having too many dimensions:
  - Overfitting
  - Model that's harder to interpret
- This problem is called **the curse of dimensionality** and it's not necessarily intuitive
- This applies both to classification and regression, but it'll be easier to visualize for classification

Data is sparser in higher dimensions



Several dimensions are better than one

- Let’s try a classification task in one dimension



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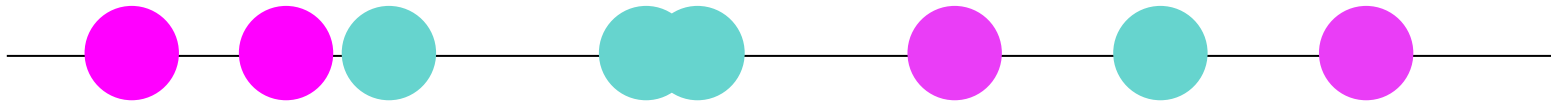

- We can’t get separability in this case
- Let’s try adding another dimension

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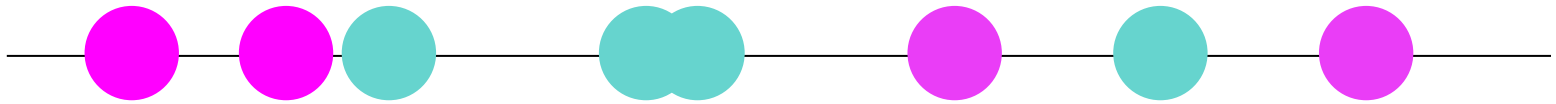

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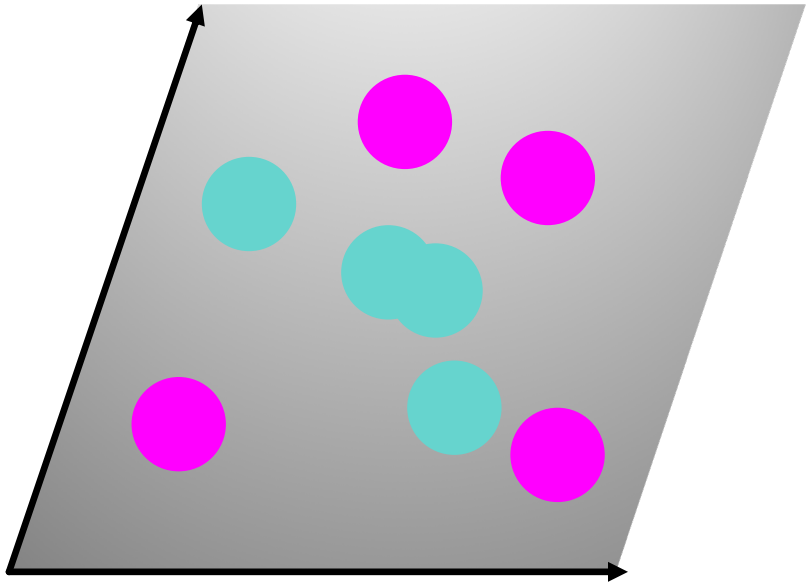

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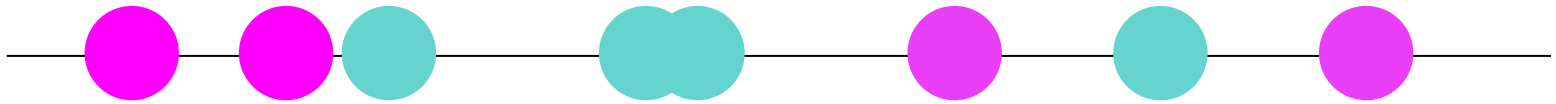

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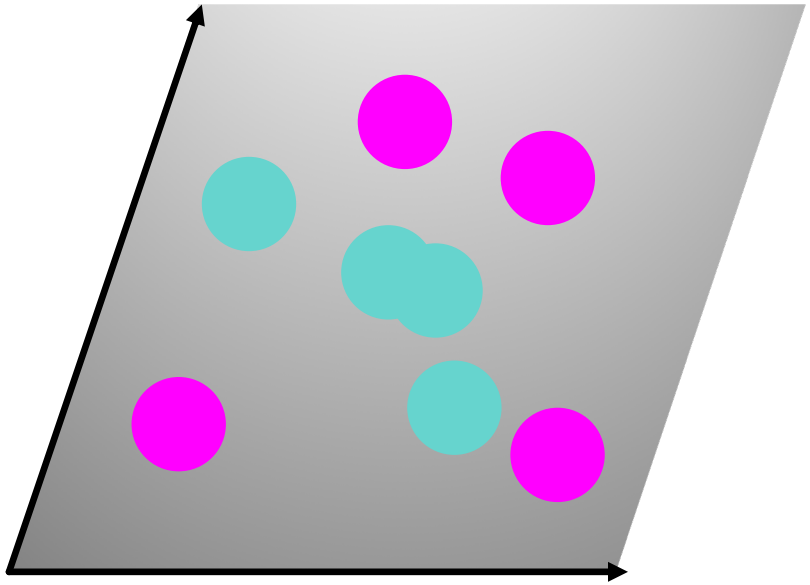
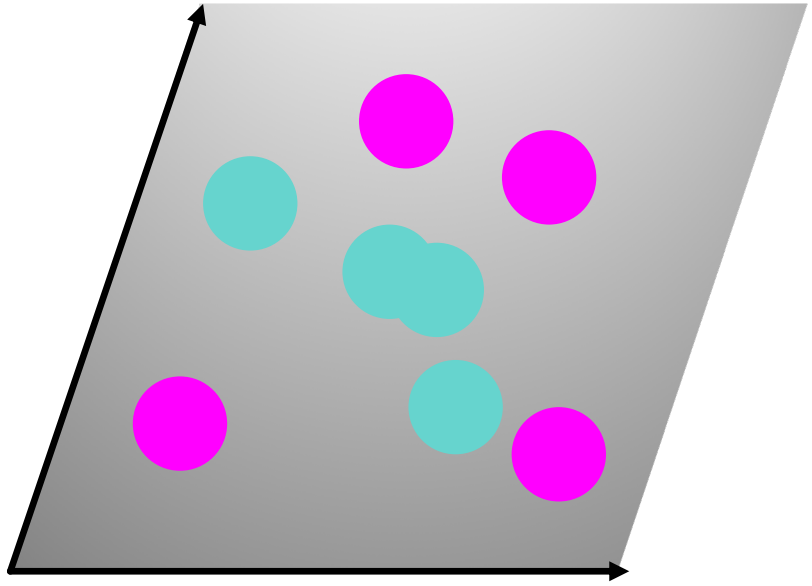
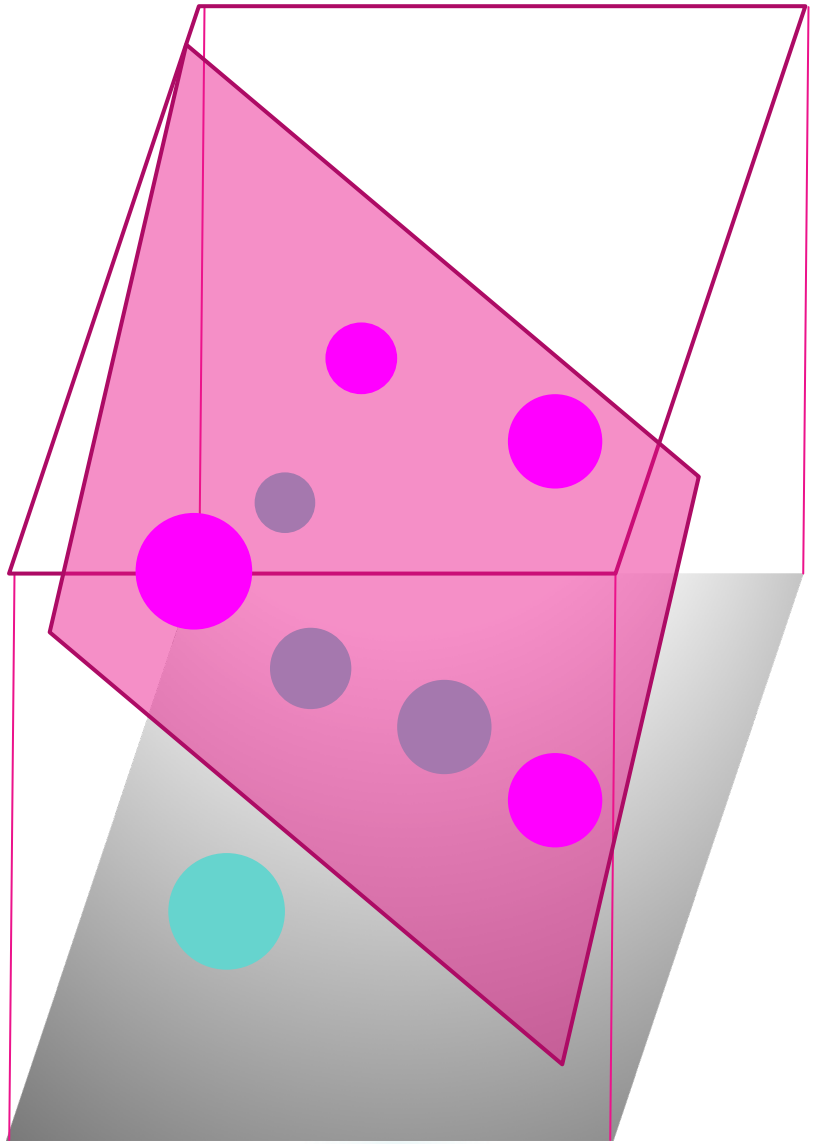


- Hmm, still not great
- What if we add a third dimension?

Several dimensions are better than one

- With a third dimension, we can now linearly separate our classes





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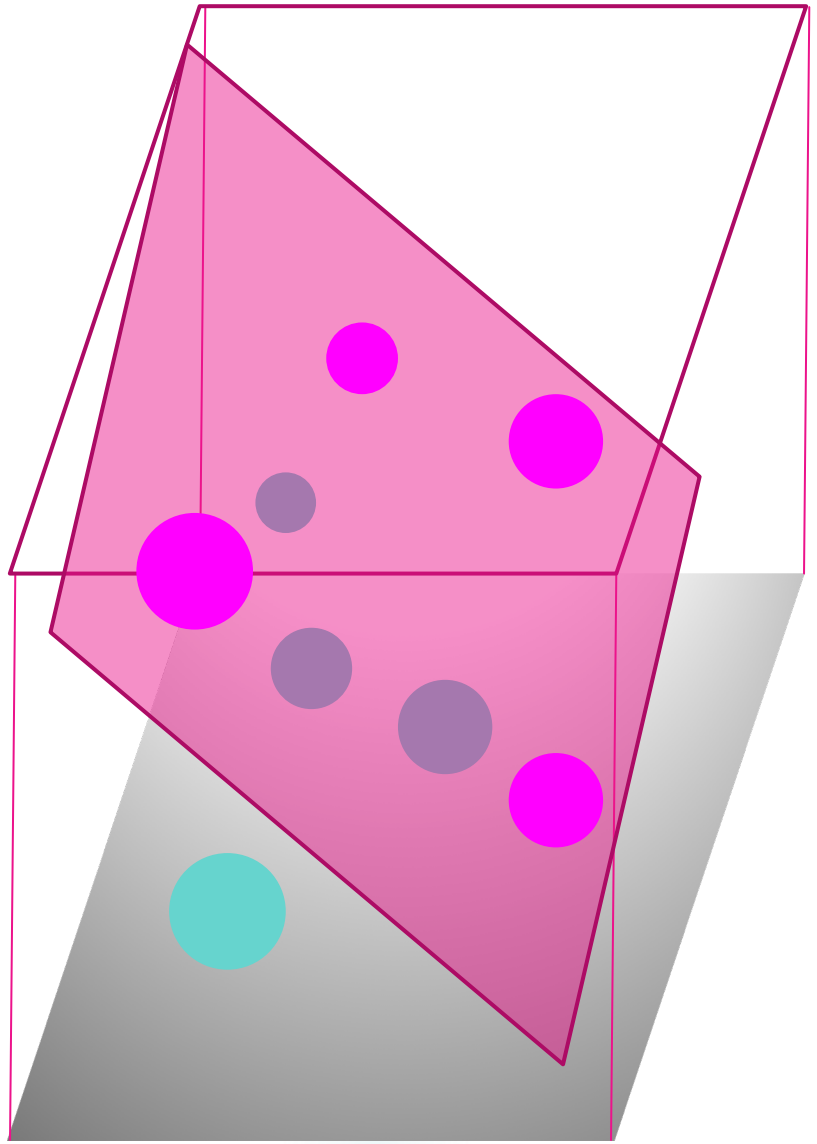
- With a third dimension, we can now linearly separate our classes
- Does this mean we should keep adding dimensions forever?

Check for understanding

What are some things we notice about our observations as we add more dimensions?



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- With a third dimension, we can now linearly separate our classes
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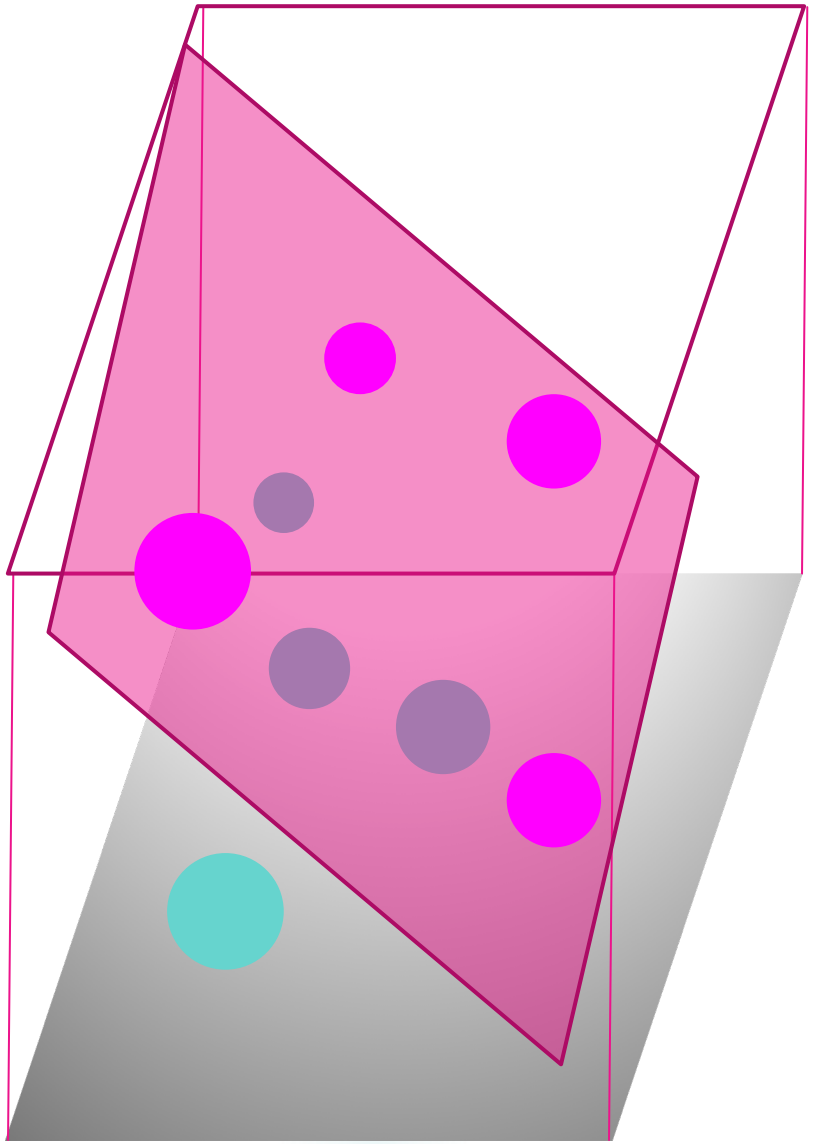
Check for understanding

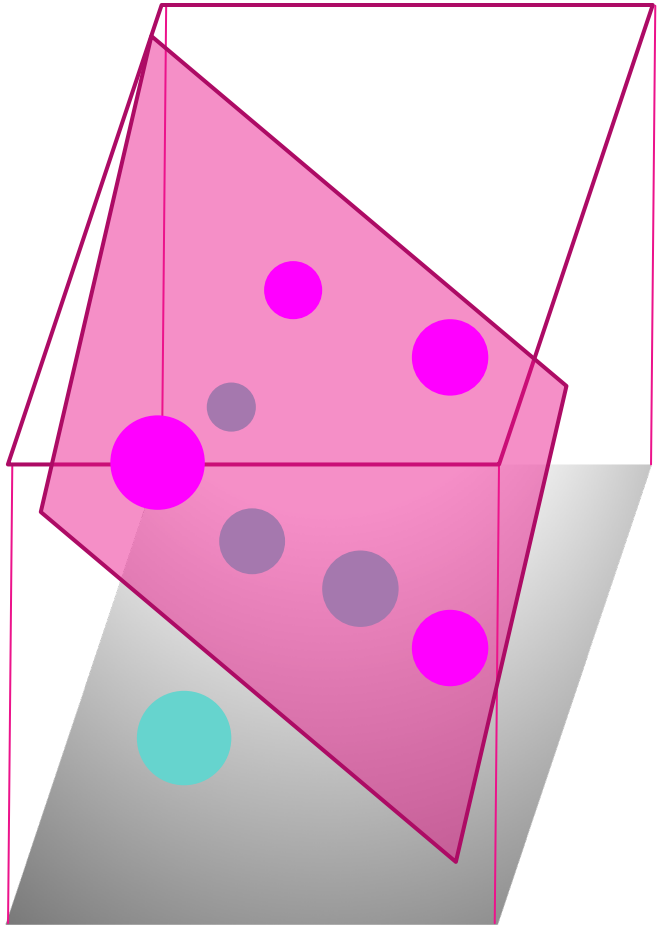
What are some things we notice about our observations as we add more dimensions?

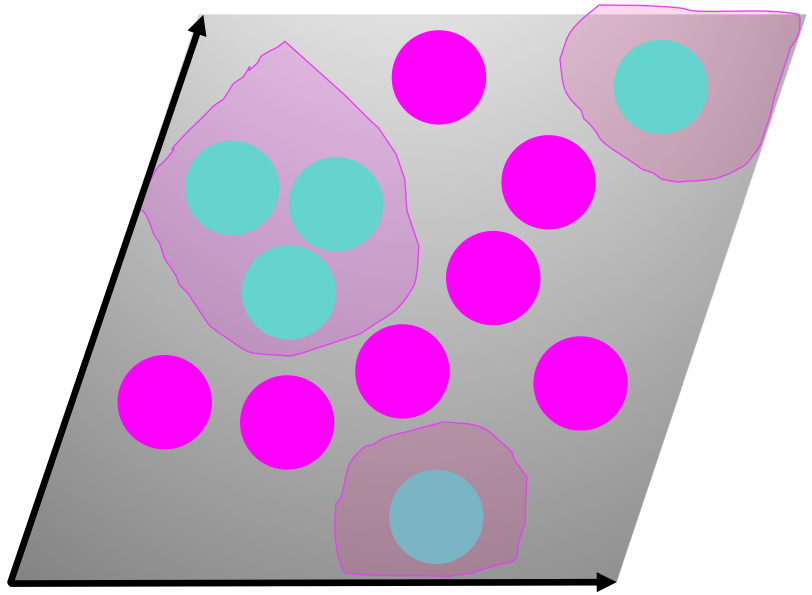
- Points are farther away from each other
- More dimensions make it possible to linearly separate our observations



Adding dimensions is good, up to a limit

- What happens if we squish our observations and separating plane down to two dimensions?





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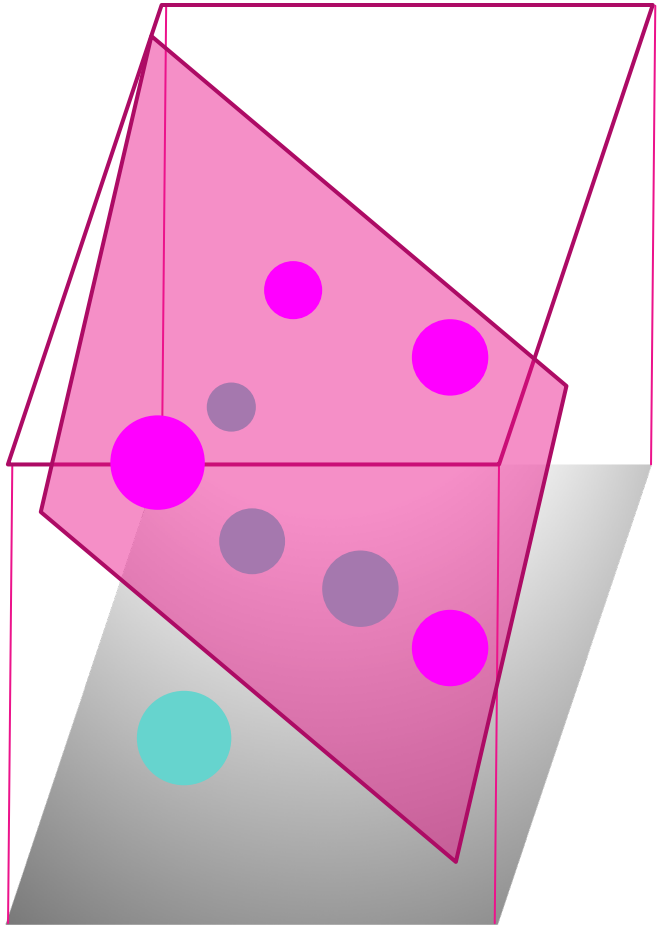
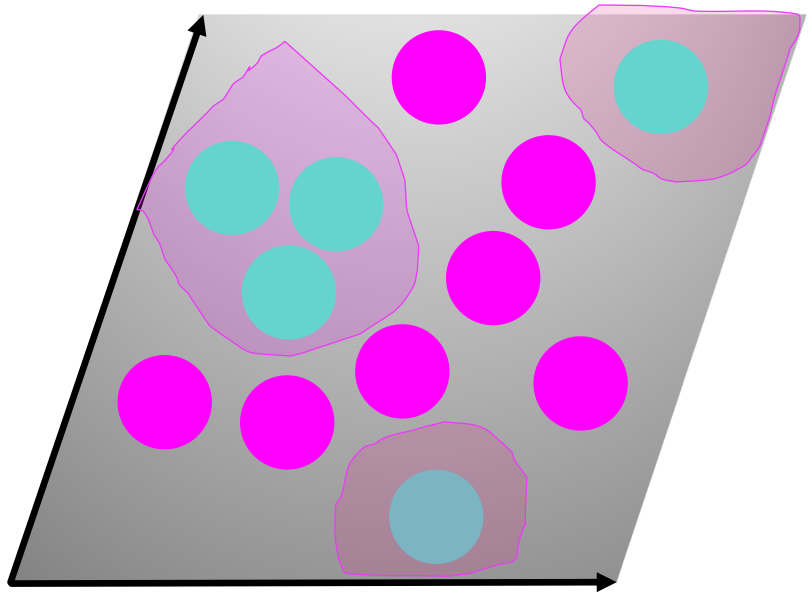


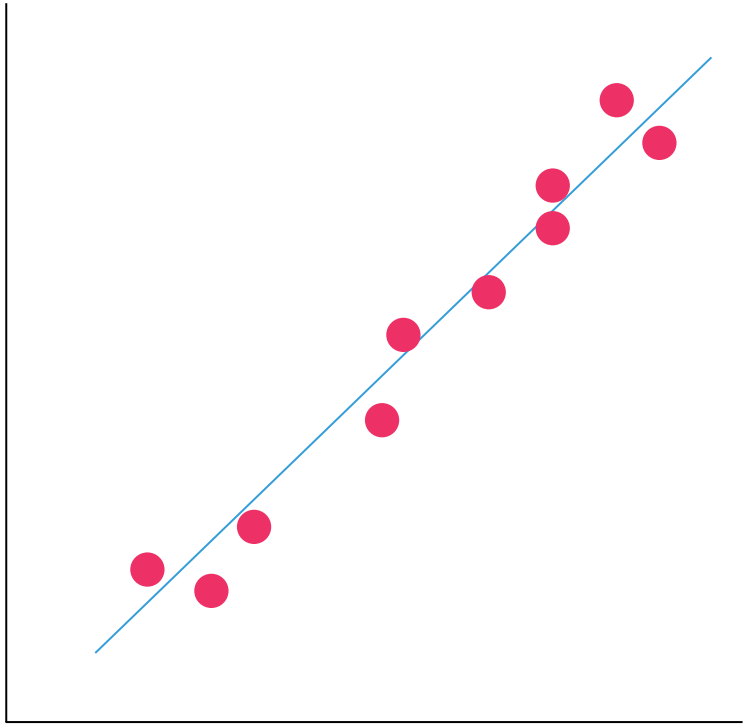


- Yikes, it looks like we've **over-fit**

The curse of dimensionality

- Observations are farther apart in higher dimensions
- As our data becomes more sparse, clustering and classification tasks become harder
  - As density decreases, there are fewer observations in any given neighborhood, meaning we have to travel farther to find a neighbor to be part of our cluster/class
  - If all of our neighbors are far away, what does it even mean to belong to the same cluster/class?
  - If our observations are spread out, we need more data to decrease our variance

Dimensionality reduction



- To deal with the curse of dimensionality, we can reduce our dimensions
- We could represent our pink dots in two dimensions, or we could collapse all points to be on the line and accept some minor error
- What we gain from reducing dimensionality:
  - Fewer dimensions to deal with, and comes at relatively cheap cost of minor error
  - Remove redundancy and noise
  - Can be used for visualization

More on this in the next lecture!

In the next lecture, we'll look at some methods we can use to fight the curse of dimensionality