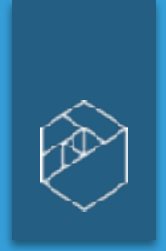
PERMUTATIONS & COMBINATIONS

By: METIS

AGENDA

- Permutations
  - With Repetition
  - Without Repetition
- Combinations
- Class Exercises
  - Challenge #1
  - Challenge #2



PERMUTATIONS

WITH REPETITION

PERMUTATION
WITH REPETITION

How many ways can you fill this with numbers between 0 and 9?

_ _

It’s just all numbers between 0 and 99, which is **100 ways**.

PERMUTATION
WITH REPETITION

How many ways can you fill this with numbers between 0 and 9?

_ _ _ _

10 ways x 10 ways x 10 ways x 10 ways = **10,000 ways**

PERMUTATION
WITH REPETITION

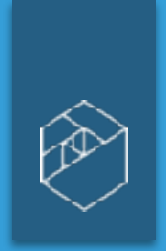
10 ways x 10 ways x 10 ways x 10 ways = **10,000 ways**

In this case, we had **N = 10** numbers to choose from,

and we chose **r = 4** of them.

Repetition is allowed and order matters.

Total Number of Permutations = N^r



PERMUTATIONS

WITHOUT REPETITION

PERMUTATION

WITHOUT REPETITION

How many ways can you fill this with numbers between 0 and 9?
(without repetition)

_ _

It’s all numbers between 0 and 99, except repeating numbers (00, 11, 22, etc.), which ends up being 100 - 10 = **90 ways**.

PERMUTATION

WITHOUT REPETITION

How many ways can you fill this with numbers between 0 and 9?
(without repetition)

_ _ _ _

10 ways x 9 ways x 8 ways x 7 ways = **5,040 ways**

PERMUTATION

WITHOUT REPETITION

10 ways x 9 ways x 8 ways x 7 ways = **5,040 ways**

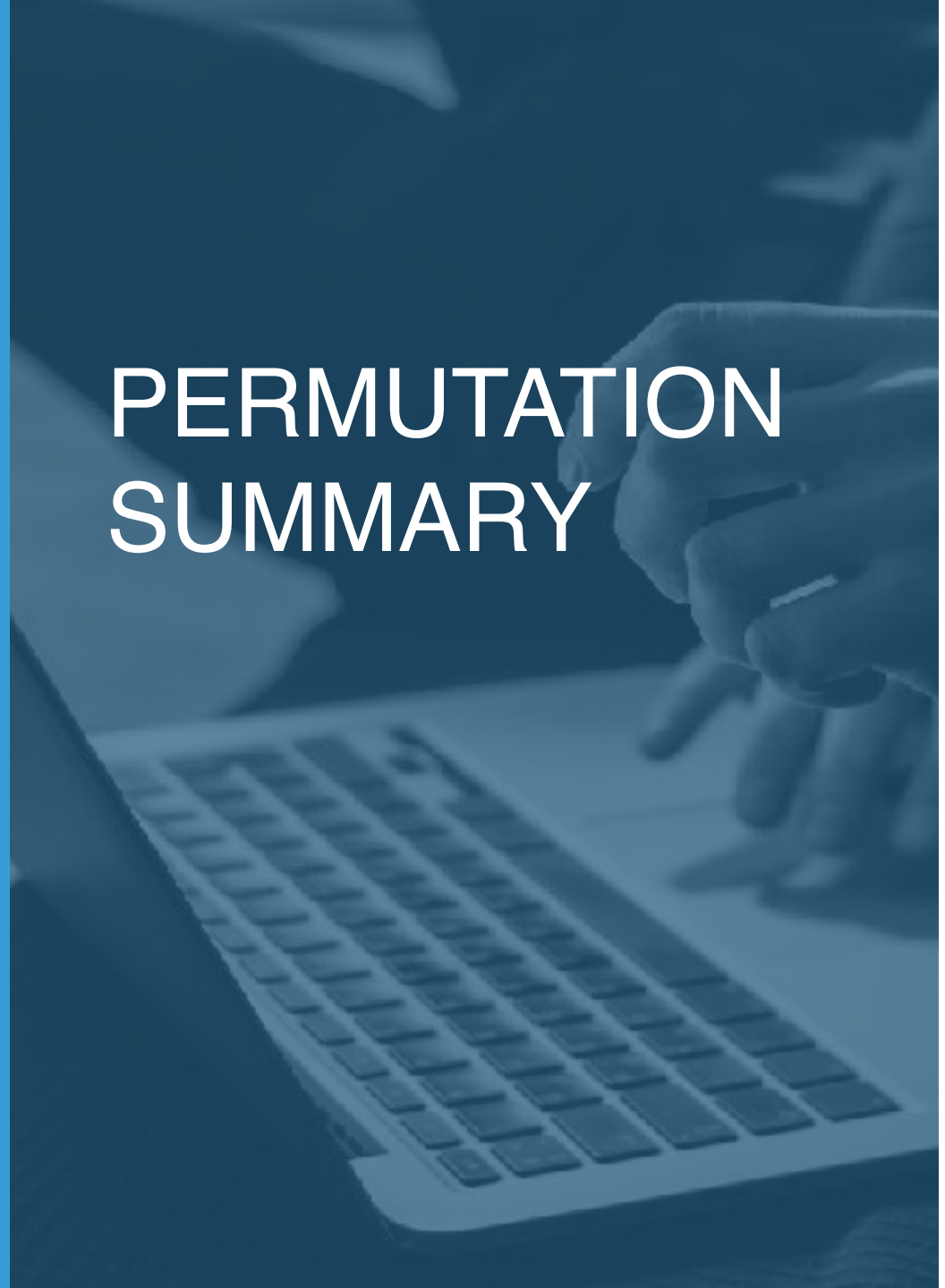
In this case, we had **N = 10** numbers to choose from,

and we chose **r = 4** of them.

Repetition is **not** allowed and order matters.

Total Number of Permutations = $${}_{N}P_{r} = \frac{N!}{(N-r)!}$$

$$\frac{10 \times 9 \times 8 \times 7 \times \cancel{6} \times \cancel{5} \ldots}{\cancel{6} \times \cancel{5} \times \cancel{4} \times \ldots}$$



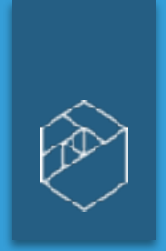
PERMUTATION SUMMARY

PERMUTATION
WITH REPETITION

Total Number of Permutations = N^r

PERMUTATION
WITHOUT REPETITION

Total Number of Permutations = $_NP_r = \frac{N!}{(N-r)!}$



COMBINATIONS

COMBINATION

ORDER OF THE RESULTS DO NOT MATTER

How many ways can you fill this with numbers from 0 - 9 without repetition?
Order doesn't matter, so 1234, 2134, 4213 are all the same.

— — — —

In this case, we need to uncount the various orderings.
How many ways can you order **r = 4** numbers? **r! ways** or 4! = 24 ways

COMBINATION

ORDER OF THE RESULTS DO NOT MATTER

How many ways can you fill this with numbers from 0 - 9 without repetition?
Order doesn't matter, so 1234, 2134, 4213 are all the same.

— — — —

We had 5,040 numbers from before. Let's remove the 24 orderings.
5,040 / 24 = **210 ways**

COMBINATION

ORDER OF THE RESULTS DO NOT MATTER

5,040 / 24 = **210 ways**

In this case, we had **N = 10** numbers to choose from,

and we chose **r = 4** of them.

Repetition is **not** allowed and order **does not** matter.

Total Number of Combinations = $${}_{N}C_{r} = \frac{N!}{(N-r)!r!}$$

CLASS EXERCISES

CHALLENGE #1

There are 10 kids who what to order ice cream. Each one can choose between chocolate, vanilla or strawberry.

You'll collect their choices and make one order, something like: 5 chocolate, 3 vanilla and 2 strawberry.

How many possible orders can you make, where every flavor is represented (8 chocolate, 1 vanilla and 1 strawberry is valid but 8 chocolate, 2 vanilla and 0 strawberry is not valid)?

CHALLENGE #2

Six individuals are sitting in a circle. I have six cards, three red and three blue.

If I randomly hand each one draw a card, what is the probability that the colors would perfectly alternate, that is no two adjacent people have the same color?

QUESTIONS?