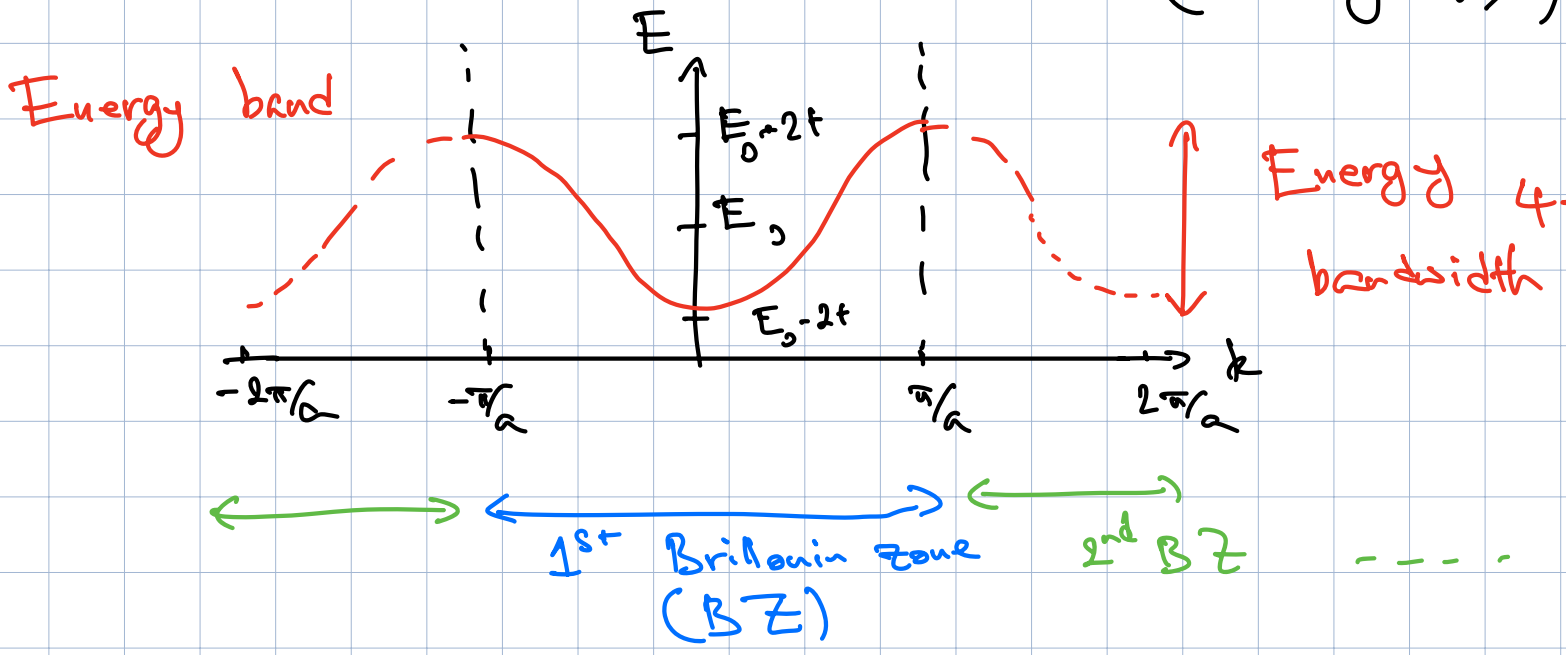
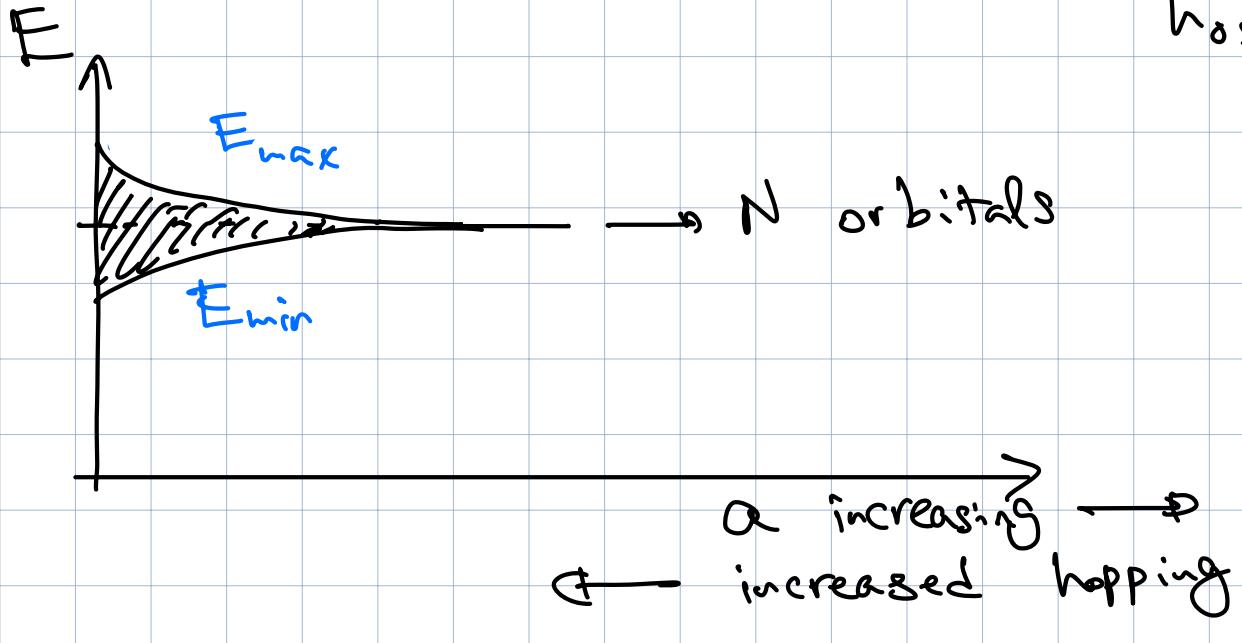
Dispersion relation $E = E_0 - 2t\cos(ka)$

(assuming $t>0$)



Momentum $k \sim k + \frac{2\pi}{a}$ → corresponding to same solution

Bandwidth as a function of lattice spacing / hopping



$E_{max} - E_{min} = 4|t|$

1st BZ: points in $\vec{k}$ space that are closer to $\vec{k}=0$ than any other reciprocal lattice pt.

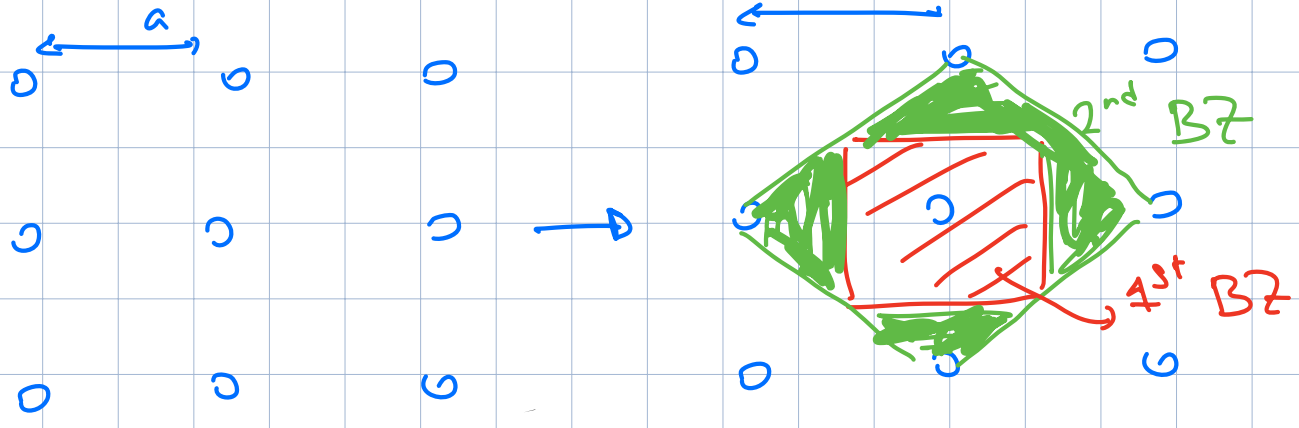
2nd BZ: points where $\vec{k}=0$ is the second closest recip. lattice pt.

....

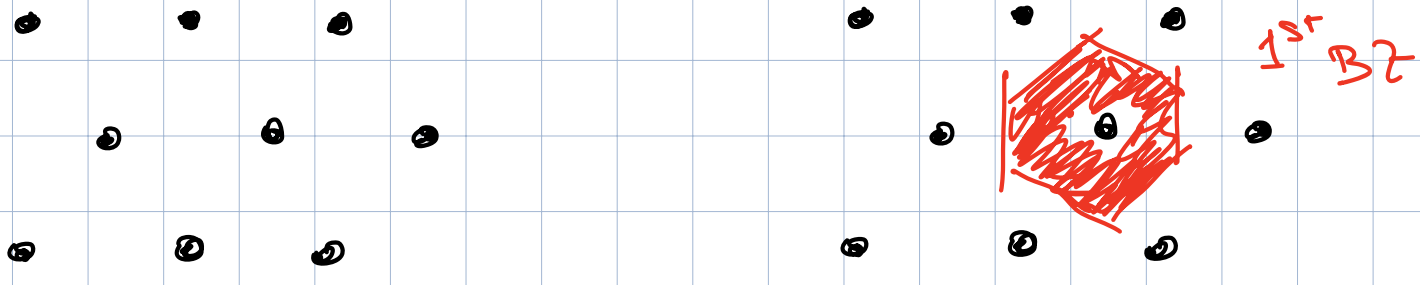
Some examples:

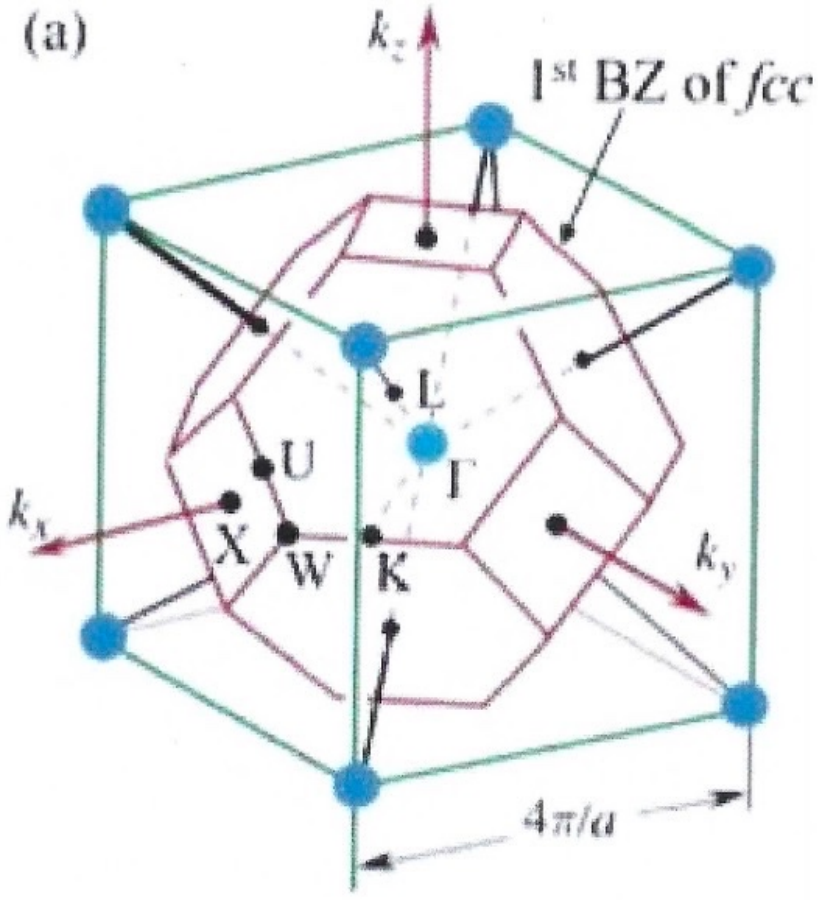
Direct lattice | Recip. lattice

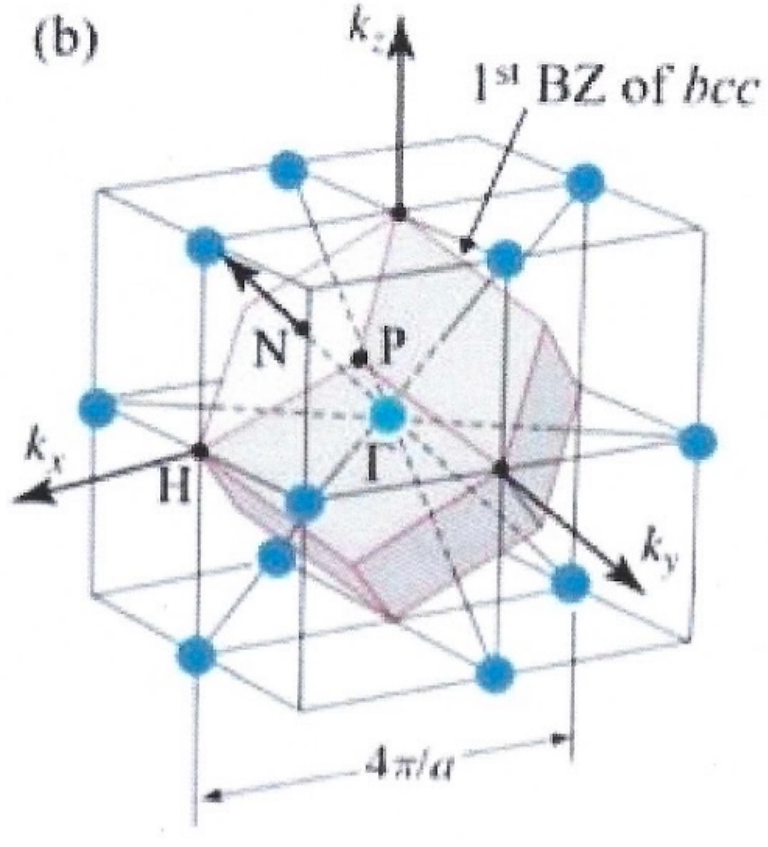
Square lattice



Triangular lattice

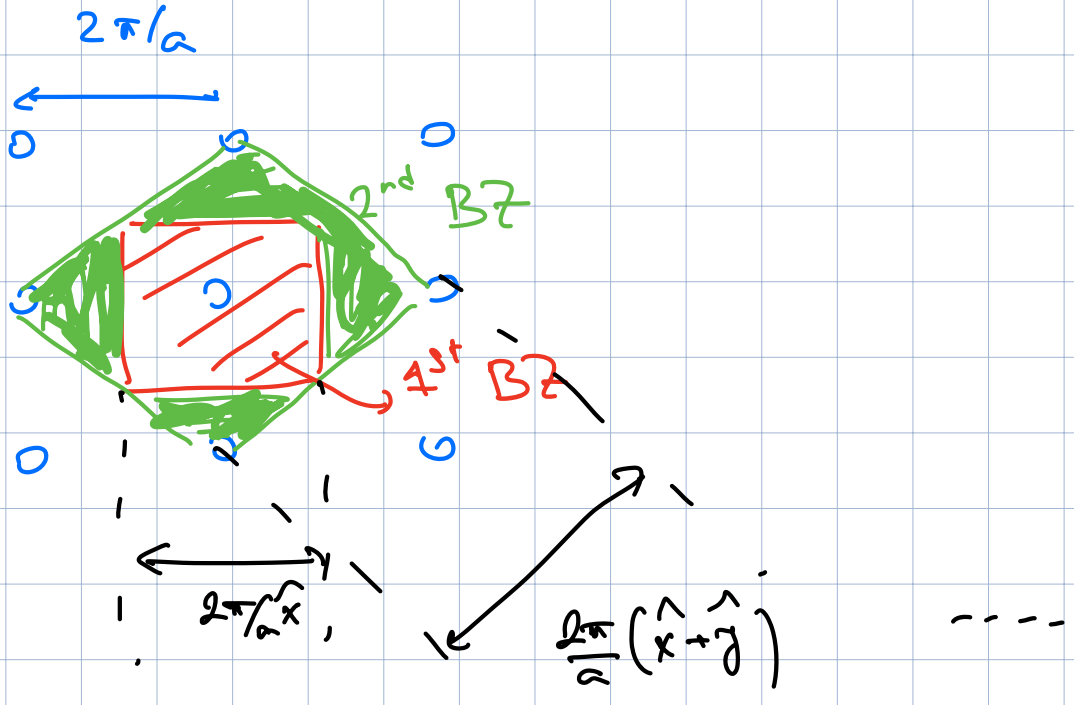



(a)
k_z
1st BZ of *fcc*
L
U
Γ
k_x
X
W
K
k_y
$4\pi/a$


(b)
k_z
1st BZ of *bcc*
N
P
k_x
H
Γ
k_y
$4\pi/a$

Facts:

1. BZ boundary to opposite boundary is a reciprocal lattice vector



2. Each BZ (1st, 2nd, ...) has the same area in k space.

3. Number of k states in each BZ equals # unit cells in system

N nuclei in chain:

Length of system $L = N \cdot a$

Allowed k values $k = \frac{2\pi}{L} \cdot p \rightarrow \in \mathbb{Z}$

different k_z $\frac{2\pi/a}{2\pi/L} = \frac{L}{a} = N$ → range of k in BZ

↳ An example of this is the tight-binding model