Problem Set Angela

[Due May 2nd by 11:59 PM]

Question 1:

This question asks you to extend the medical-test example from class. Suppose there is a medical condition that afflicts 20% of the population. There is a test for this condition, and the reliability of this test can be characterized as follows:

- false-positive rate = 10% — for those who do not have the condition, 10% will test positive.
- false-negative rate = 5% — for those who have the condition, 5% will test negative.

(a) If you receive a positive test result, what is the likelihood that you have the condition? If you receive a negative test result, what is the likelihood that you do not have the condition?

(b) Repeat part (a), except now assume that the false-positive rate is 5% and the false-negative rate is 10%.

Question 2:

Suppose there are two types of dice, red dice and blue dice. Each red die has 4 H's and 2 M's, whereas each blue die has 2 H's and 4 M's. The proportion of all dice that are red is 80%.

For each of the scenarios below, discuss whether the person's intuitive judgment is consistent with (i) base-rate neglect, (ii) over-inference from small samples, and (iii) conservatism.

(a) A person is told that a die was rolled three times and came up MMH. When asked the likelihood that the die is red, the person responds $1/2$.

(b) A person is told that a die was rolled three times and came up HMH. When asked the likelihood that the die is red, the person responds $9/10$.

(c) A person is told that a die was rolled ten times and came up $MMHHMHMHHH$. When asked the likelihood that the die is red, the person responds $17/20$.

Question 3:

Suppose there are two types of coins, heads-biased coins and tails-biased coins. A heads-biased coin has a $3/4$ probability of a heads, while a tails-biased coin has a $1/4$ probability of heads. The proportion of all coins that are heads-biased is $1/7$.

Suppose that we flip a coin twice and it comes up HH.

(a) For a standard Bayesian information processor:

> (i) What is the person's posterior probability that the coin is heads-biased?
>
> (ii) What is the person's forecast for a third flip being H?

(b) For an ($N = 8$)-Freddy (as defined in class):

> (i) What is the person's posterior probability that the coin is heads-biased?
>
> (ii) What is the person's forecast for a third flip being H?

(c) Repeat parts (a) and (b) when the proportion of all coins that are heads-biased is $6/7$.

(d) How do Freddy's forecasts compare to a Bayesian's forecasts? Provide some intuition for your conclusions.

Question 4:

Suppose that Lisa and Maggie both have "social-welfare preferences" of the form introduced by Charness & Rabin (that we discussed in class). They differ, however, in that Lisa takes a utilitarian view of social welfare (she has $\delta = 0$) while Maggie takes a maximin view of social welfare (she has $\delta = 1$).

(a) Solve for Lisa and Maggie's behavior in the Prisoners' Dilemma for the case when they believe that their opponent is playing C (use the version of the Prisoners' Dilemma from class).

(b) Solve for Lisa and Maggie's behavior in the Dictator Game.

(c) Solve for Lisa and Maggie's behavior in the role of Player 2 in the Ultimatum Game when they are offered a share $s \leq 1/2$.

Note: For each game, you should specify how their behavior depends on their λ.

(d) To what extent can social-welfare preferences explain experimental results in the Prisoners' Dilemma, the Dictator Game, and the Ultimatum Game?

Question 5:

Suppose Lisa and Maggie have social-welfare preferences as in Question 3. In contrast, Bart has "inequity aversion" of the form introduced by Fehr & Schmidt (that we discussed in class).

(a) Consider the following modified dictator game: Player 1 divides 40 tokens between Player 1 and Player 2. Each token is worth \$3 to Player 1, and each token is worth \$5 to Player 2. How would Lisa, Maggie, and Bart behave in this game?

(b) Consider the following modified dictator game: Player 1 divides 40 BLUE tokens and 30 RED tokens between Player 1 and Player 2. Each BLUE token is worth \$2 to Player 1 and \$1 to Player 2. Each RED token is worth \$2 to Player 1 and \$3 to Player 2. How would Lisa, Maggie, and Bart behave in this game?

Note: For each game, you should specify how Lisa and Maggie's behavior depends on their λ, and how Bart's behavior depends on his α and β. Also, if you like, you may assume that Player 1 can choose non-integer divisions — e.g., Player 1 might keep 25.6 tokens and give 14.4 tokens.

Question 6:

Consider a simple dictator game in which Player 1 has 4 options from which to choose:

$$\text{(A) } (\$50, \$50) \qquad \text{(B) } (\$75, \$140) \qquad \text{(C) } (\$50, \$200) \qquad \text{(D) } (\$75, \$0)$$

How would Lisa, Maggie, and Bart behave in this game? Provide some intuition for your answers.

Note: You should specify how Lisa and Maggie's behavior depends on their λ, and how Bart's behavior depends on his α and β.

Marge has inequity aversion, but with the following non-linear form:

$$u^1(x_1,x_2) = \begin{cases} 2(x_1)^{1/2} - \alpha\,[x_2 - x_1] & \text{if } x_1 \le x_2 \\ \\ 2(x_1)^{1/2} - \beta\,[x_1 - x_2] & \text{if } x_1 \ge x_2 \end{cases}$$

(a) Suppose Marge plays a dictator game in which she must divide \$10 between herself and another person. As a function of her α and β, how will she behave?

Note: Rather than solve for the *share* that Marge offers (as we did in class), it is perhaps easier to solve for the *amount* that Marge offers — i.e., if she offers amount \$$z$, then she will keep \$$(10-z)$ for herself.

(b) In class, we discussed how the linear version of inequity aversion does not explain well the quantitative results in experimental dictator games. Does this non-linear version work better?

BONUS QUESTIONS for 5 replacement points (not extra, replacement).

Question 7:

This question asks you to reconsider the model of optimal sin taxes that we studied in class with a different distribution of types. Assume that everyone has $\rho = 65$ and $\gamma = 40$. Assume further that proportion ϕ of the population has $\beta = 0.85$ while proportion $1-\phi$ has $\beta = 1$ (both types have $\delta = 1$).

(a) As a function of ϕ and t, what is the uniform lump-sum transfer?

(b) As a function of ϕ and t, derive an expression for social welfare.

(c) Solve for the optimal tax.

(d) How does the optimal tax depend on ϕ? Provide some intuition for this answer.

Question 8:

This question asks you to reconsider the model of optimal sin taxes that we studied in class when there is heterogeneity in people's tastes for potato-chip consumption (in addition to heterogeneity in self-control problems). Suppose that everyone has $\gamma = 40$ (everyone has the same susceptibility to health consequences). Suppose that $1/2$ of the population has $\beta = 1$ while $1/2$ of the population has $\beta = 0.85$. Suppose further that $2/3$ of the population has $\rho = 75$ and the other $1/3$ of the population has $\rho = 45$, where the distributions of β and ρ are independent.

Note that there are four types: (i) people with $\beta = 1$ and $\rho = 75$; (ii) people with $\beta = 1$ and $\rho = 45$; (iii) people with $\beta = 0.85$ and $\rho = 75$; and (iv) people with $\beta = 0.85$ and $\rho = 45$.

(a) As a function of t, how many potato chips will each type consume?

(b) As a function of t, what is the uniform lump-sum transfer?

(c) For each type, compare people's utility for $t = 0\%$ vs. $t = 10\%$.

(d) Are all types better off when $t = 10\%$? Provide some intuition for this answer.

(e) Are the two types with $\beta = 1$ on average better off? Are the two types with $\beta = 0.85$ on average better off? Provide some intuition for this answer.