Homework3
Justify all your answers
due on Fr 10/04/24 at 11:30AM in A236WH

Exercise 1. *Consider the probability distributions* $(0.6, 0.3, 0.1)$ *and* $(0.5, 0.3, 0.2)$ *on the alphabet* $S = (a, b, c)$. *Which one has larger entropy? What probability distribution on* S *produces the largest entropy?*

Exercise 2. *Construct a binary Shannon-Fano code with respect to the probability distribution* $(0.5, 0.3, 0.2)$. *Is this code an optimal binary PF-code?*

Exercise 3. *Let* $p = (\alpha, \beta, \gamma)$ *be a probability distribution on the ordered alphabet* $S = (a, b, c)$ *with* $\alpha > \beta > \gamma$. *Show that the average codeword length of an optimal binary UD-code for* S *with respect to* p *is* $2 - \alpha$.

Exercise 4. *Find a Huffman code for the probability distribution* $(0.4, 0.3, 0.1, 0.1, 0.06, 0.04)$.

Exercise 5. *While computing a Huffman code, in each application of the Huffman rule* **H1** *a new probability* $\tilde{p}(e)$ *is introduced. Show that the average length* L *of the Huffman code is the sum of these* $\tilde{p}(e)$.

Exercise 6. *For a code* c *let* $\mathrm{TL}(c)$ *be the sum of the lengths of the codewords and* $\mathrm{M}(c)$ *the maximal length of a codeword.*

(a) *Let* $\tilde{c}$ *and* c *be codes as in Huffman rule* **H2**. *Show that*

$$\mathrm{M}(c) \leq \mathrm{M}(\tilde{c}) + 1 \qquad \textit{and} \qquad \mathrm{TL}(c) \leq \mathrm{TL}(\tilde{c}) + M(\tilde{c}) + 2.$$

(b) *If* c *is a Huffman code on an alphabet of size* N, *show that*

$$\mathrm{M}(c) \leq N - 1 \qquad \textit{and} \qquad \mathrm{TL}(c) \leq \frac{N^2 + N - 2}{2}.$$

Exercise 7. *Let* $(\mathbb{B}, P)$ *be a source. Suppose that*

(i) $P(000) = \frac{1}{12}$;

(ii) $p^{(l)}(0) = p^{(l)}(1)$ *for all integers* l *with* $1 \leq l \leq 3$.

(iii) *whenever* (l_1, l_2) *is a pair of integers with* $1 \leq l_1 < l_2 \leq 3$, *then* $p^{(l_1)}$ *and* $p^{(l_2)}$ *are independent with respect to* $p^{(l_1, l_2)}$.

(a) *Compute* $p^{(l)}$ *for* $1 \leq l \leq 3$.

(b) *Compute* $p^{(l_1, l_2)}$ *for* $1 \leq l_1 < l_2 \leq 3$.

(c) *Show that, for* $x_1, x_2, x_3 \in \mathbb{B}$,

$$P(x_1 x_2 x_3) = \begin{cases} \frac{1}{12} & \textit{if } x_1 + x_2 + x_3 \textit{ is even} \\ \frac{1}{6} & \textit{if } x_1 + x_2 + x_3 \textit{ is odd} \end{cases}$$

(d) *Show that* $p^{(1)}$ *and* $p^{(2,3)}$ *are not independent with respect to* p^3.

(e) *Show that* $(\mathbb{B}, P)$ *is not memoryless.*