Degeneracy

For $n=1 \rightarrow \mathcal{L}=n-1=0$

$n=2 \rightarrow \mathcal{L}=1$, $l=0$

$l=1$ ($m_l=1,0,-1$)

$n=3 \rightarrow \mathcal{L}=2$ $l=0$ (1st)

$l=1$ (3st)

$l=2$ (5st)

Hydrogen eigen functions

- building blocks for atomic structure

want to know the amplitude at $\underline{x}$ when atom is in state $|E,l\rangle$

$$\langle \underline{x}|E,l\rangle = \underbrace{u_n^l(r)}_{\text{radial dep.}} \cdot \underbrace{Y_l^m(\theta,\varphi)}_{\text{spherical harmonics}}$$

$Y_l^m = N e^{im\varphi} P_l^m(\cos\theta)$ Legendre poly.

$$u_n^\ell = -A_n^\ell e^{-\rho} 2\rho^\ell L_{n+\ell}^{2\ell+1}(2\rho)$$

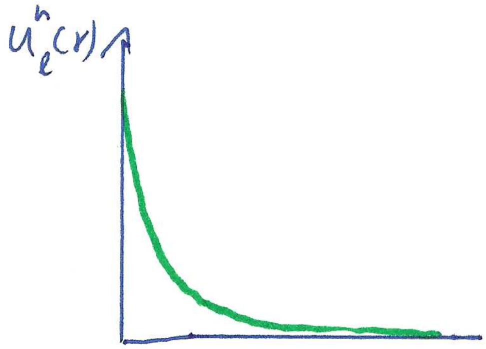
with $\rho = \frac{Zr}{n a_0}$

and

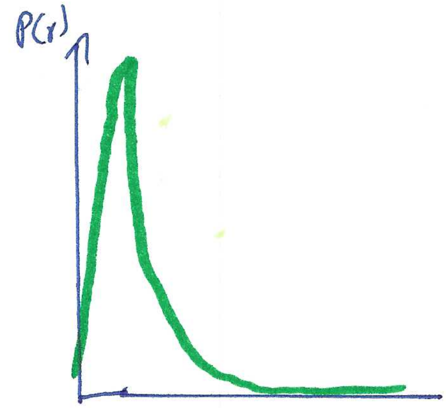
$$A_n^\ell = \left(\frac{2\rho}{r}\right)^{3/2} \sqrt{\frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}}$$

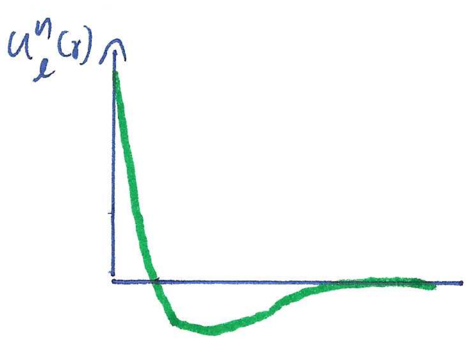
and Laguerre Polynomials

$$L_{n+\ell}^{2\ell+1} = \frac{e^r r^{-2(\ell+1)}}{(n+\ell)!} \frac{d^{n+\ell}}{dr^{n+\ell}} \left(e^{-r} r^{(n+\ell)+(2\ell+1)}\right)$$



1s





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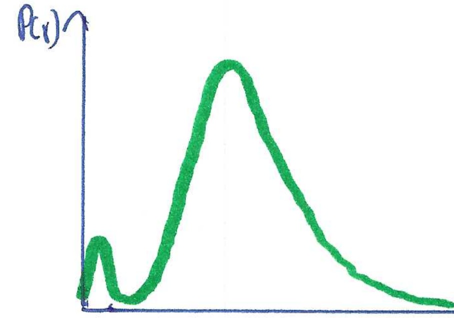

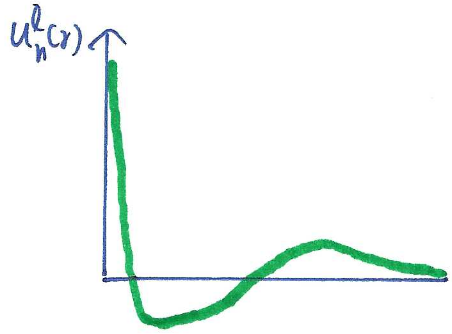

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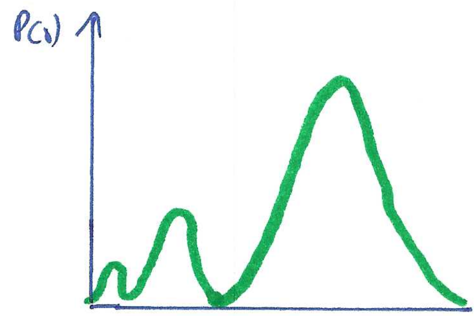
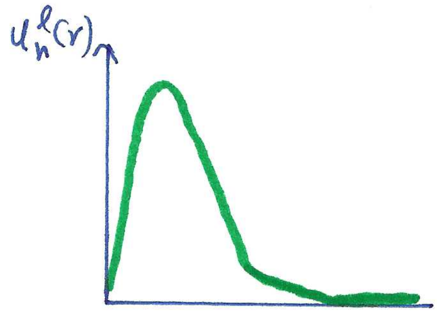
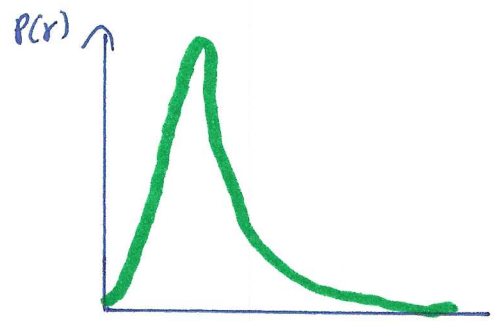




2p



with $P(r) = \int |u^n_\ell(r)|^2 r^2 \, dr$

A note on units

- convenient to set many fundamental constants to one as possible

- In non-relativistic QM: Hartree units

  $e = \hbar = m_e = 1$

  which means that $c = \frac{1}{\alpha} = 137$

- In relativistic QM: Lorentz-Heaviside units

  $c = \hbar = 1$, $e = \sqrt{\alpha}$

Hydrogen-like systems

- exotic atoms: Muonium, Hadronic hydrogen, positronium

- quasiparticles: excitons (bound state of e^- and a hole h^+ in semiconductor)

- highly ionized systems: regular atom with all but one e^- removed.

(- multi-e^- systems)