PHY 491, Fall 2024 - Homework 6

DUE: Friday 10/11/24, 11:59pm

Problem 3.1 Suppose identical solid spheres are distributed through space in such a way that their centers lie on the points of each of the following structures, and spheres on neighbouring points just touch, without overlapping (this is called a close-packing arrangement). Assuming that the spheres have unit density, show that the packing density per unit cell (packing fraction) for the following cubic structures is

3.1.1 fcc: $\sqrt{2}\pi/6 = 0.74$. (2 Points)

3.1.2 bcc: $\sqrt{3}\pi/8 = 0.68$. (2 Points)

3.1.3 cubic-P: $\pi/6 = 0.52$. (2 Points)

3.1.4 diamond (look up the structure): $\sqrt{3}\pi/16 = 0.34$. (3 Points)

Problem 3.2 Using the following primitive lattice vectors

$$\vec{a_1} = a\hat{x}, \vec{a_2} = \frac{a}{2}\hat{x} + \frac{a\sqrt{3}}{2}\hat{y}, \vec{a_3} = c\hat{z}$$

3.1.1 Calculate the reciprocal lattice vectors $\vec{b_1}, \vec{b_2}, \vec{b_3}$. (5 Points)

3.1.2 Calculate the angles between lattice vectors $\vec{a_1}$ and $\vec{b_1}$ of the direct and reciprocal lattice. (4 Points)

3.1.3 What conclusion can you draw about the shape and orientation of the reciprocal lattice with respect to the direct lattice? (2 points)