PHY 481 - Fall 2024
Homework 05

Due Sunday October 6, 2024 - note the extra day

Preface

Homework 5 continues with exploring relationships between electric charge, electric field, electric potential as well as the boundary conditions for electric field and work-energy of a point charge distribution.

1 Finding V and ρ from E

Consider the field $\mathbf{E} = (2x^2 - 2xy - 2y^2)\hat{x} + (-x^2 - 4xy + y^2)\hat{y}$

a) Is it irrotational? If so, what is the potential function?

b) Calculate the volume charge density ρ.

2 Finding voltage from a charge distribution

We have found a number of ways of relating ρ, $\mathbf{E}$, and V. In this problem, you will use ρ to find V through the method of direct integration (i.e. using the integral expression for V).

1. Find a formula for the electrostatic potential $V(z)$ everywhere along the symmetry-axis of a charged disk (radius a, centered on the z-axis, with uniform surface charge density $+\sigma$ around the disk). Please use the method of direct integration to do this, and set your reference point to be $V(\infty) = 0$.

2. Sketch $V(z)$, how does $V(z)$ behave as $z \to \infty$? (Don't just say it goes to zero. How does it go to zero?) Does your answer make physical sense to you? Explain briefly.

3. Use your result from part 1 for $V(0, 0, z)$ to find the z-component of the electric field anywhere along the z-axis.

4. What is the voltage at the origin? What is the electric field at the origin? Do these results from V and $\mathbf{E}$ at the origin make physical sense to you? Briefly explain.

3 Surface charge and boundary conditions

It might seem to you that the results that the electric field is discontinuous by an amount σ/ϵ_0 isn't really a big deal. There's probably a question about how useful this result is. We will come back to this particularly when we get to fields in matter, and suffice it to say, it will help us a lot there. To get a flavor of what is coming, this problem will discuss this discontinuity in a familiar context.

1. Consider a cylindrical metal rod (radius r, length L) with a constant charge density σ distributed across its outer surface (as we will learn that is the only place the charge can be). Using Gauss' Law (far from the ends of the rod; assume it's long and skinny), determine the electric field inside and outside the rod.

2. Take the difference between the electric fields you determined in Part 1 (technically, the perpendicular component) across the outer surface of the metal rod to show you recover the the result that the all the charge lives on the surface.

3. Consider a similarly cylindrical plastic rod with a constant charge density ρ distributed over its entire volume. Again, using Gauss' Law (far from the ends of the rod; assume it's long and skinny), determine the electric field inside and outside the rod.

4. Again, take the difference between the electric fields you determined in Part 3 across the outer surface of the plastic rod. What do you find? Does your result make physical sense?

4 Energy of a point charge distribution

When studying crystal structures (e.g., in condensed matter physics), it is sometimes convenient to model those structures as rectangular grids of charged ions, this problem offers a starting point for such a model.

Imagine a small square (side a) with four point charges $+q$, one on each corner.

1. Calculate the total stored energy of this system (i.e. the amount of work required to assemble it).

2. Calculate how much work it takes to "neutralize" these charges by bringing in one more point charge ($-4q$) from far away and placing it right at the center of this square.