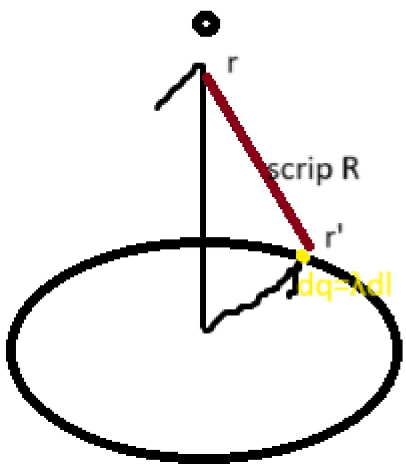
Phy 481 HW 3

1. Ring of charge

We'll use spherical coordinates

(1) Drawing



(2)

$$\vec{r} = z\hat{z}$$
$$\vec{r}' = R\hat{s}$$
$$\text{script r } \vec{\mathcal{R}} = z\hat{z} - R\hat{s}$$
$$\hat{\mathcal{R}} = \frac{z\hat{z} - R\hat{s}}{\sqrt{z^2 + R^2}}$$
$$dl = Rd\theta$$

(3) Z is common between the two

$$\hat{\mathcal{R}} = \frac{z\hat{z} - R(\cos\theta\hat{x} + \sin\theta\hat{y})}{\sqrt{z^2 + R^2}}$$

(4)

$$\vec{E} = k\int \frac{\lambda}{\mathcal{R}^2}\hat{\mathcal{R}}dl$$
$$= \lambda k \int \frac{z\hat{z} - R(\cos\theta\hat{x} + \sin\theta\hat{y})}{(z^2 + R^2)^{3/2}} R d\theta$$

(5) For x and y the integral evaluates to 0 since there is a $\cos$ or $\sin$ term that evaluates to zero when integrated across the domain. This makes sense physically since each charge has an opposite at -x, -y

$$E_z = k\lambda \frac{zR}{\left(z^2 + R^2\right)^{3/2}} \int_0^{2\pi} d\theta$$
$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda z R * 2\pi}{(z^2 + R^2)^{3/2}}$$

2. Checking result

(1) dimensional analysis

$$\frac{1}{4\pi\epsilon_0} = \frac{Nm^2}{C^2}$$
$$\lambda = \frac{C}{m}$$
$$[Nm^2C^{-2}][Cm^{-1}][m^2][m^{-3}] = [NC^{-1}] = \frac{N}{C}$$

(2)

Z=0 should be zero according to argument from question 1.5, since there is no z-component and x/y components cancel themselves.

Z>>R=0 since electric field decays to zero with long distances

(3)

$Z = 0$:

$$E = \frac{\lambda 2\pi R}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}}$$

Approaches $0/R^3$✓

$Z >> R$:

$$E = \frac{\lambda 2\pi R}{4\pi\epsilon_0}\frac{z}{(z^2+R^2)^{3/2}} \approx \frac{\lambda 2\pi R}{4\pi\epsilon_0}\frac{z}{(z^2)^{3/2}}\frac{\lambda 2\pi R}{4\pi\epsilon_0}\frac{z}{z^3}$$

Which goes to $0/z^2$ as $z \to \infty$

Since these checks align with my expectations I believe this is the correct answer

3. Disk of charge.

(1) Use same formulas only with area integral and s instead of R

$$\vec{r}' = s'\hat{s} = -s'\cos\theta\hat{x} - s'\sin\theta\hat{y}$$
$$\mathcal{R} = z\hat{z} - s'\cos\theta\hat{x} - s'\sin\theta\hat{y}$$
$$E = k\int_0^{2\pi}\int_0^R(-\sigma)\left(\frac{\mathcal{R}}{|\mathcal{R}|^3}\right)sdsd\theta$$
$$= -k\sigma\int_0^{2\pi}\int_0^R\left(\frac{z\hat{z} - s'\cos\theta\hat{x} - s'\sin\theta\hat{y}}{(z^2+s^2)^{3/2}}\right)sdsd\theta$$

Consider only E_z since cos and sin terms go to zero when integrating 0 to 2π

$$E_z = -k\sigma\int_0^R\frac{zsds}{(z^2+s^2)^{3/2}} * 2\pi$$

By the power of looking up integrals, we have

$$\int_0^R\frac{zsds}{(z^2+s^2)^{3/2}} = -\frac{z}{\sqrt{z^2+s^2}}\Bigg|_0^R = \frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{R^2+Z^2}}$$

$$E(z) = -\frac{\sigma}{2\epsilon_0}\left(\frac{z}{|z|} - \frac{z}{\sqrt{R^2+Z^2}}\right)\hat{z}$$

(2) $z >> R$

$$E(z) \approx \frac{z}{|z|} - \frac{z}{z\sqrt{\frac{R^2}{z^2+1}}}$$

$$= 1 - \frac{1}{\sqrt{\frac{R^2}{z^2}+1}} = \frac{\sqrt{\frac{R^2}{z^2}+1}-1}{\sqrt{\frac{R^2}{z^2}+1}}$$

$$\frac{1}{z^2} \to 0$$

$$E(z) \approx \frac{\sqrt{0+1}-1}{0+1} = 1 - 1 = 0$$

a function of distance." So, don't just say "it goes to zero" (if that's what you think happens). Tell us how, functionally it vanishes (like $1/z$? like e^{-z}? Something else?).

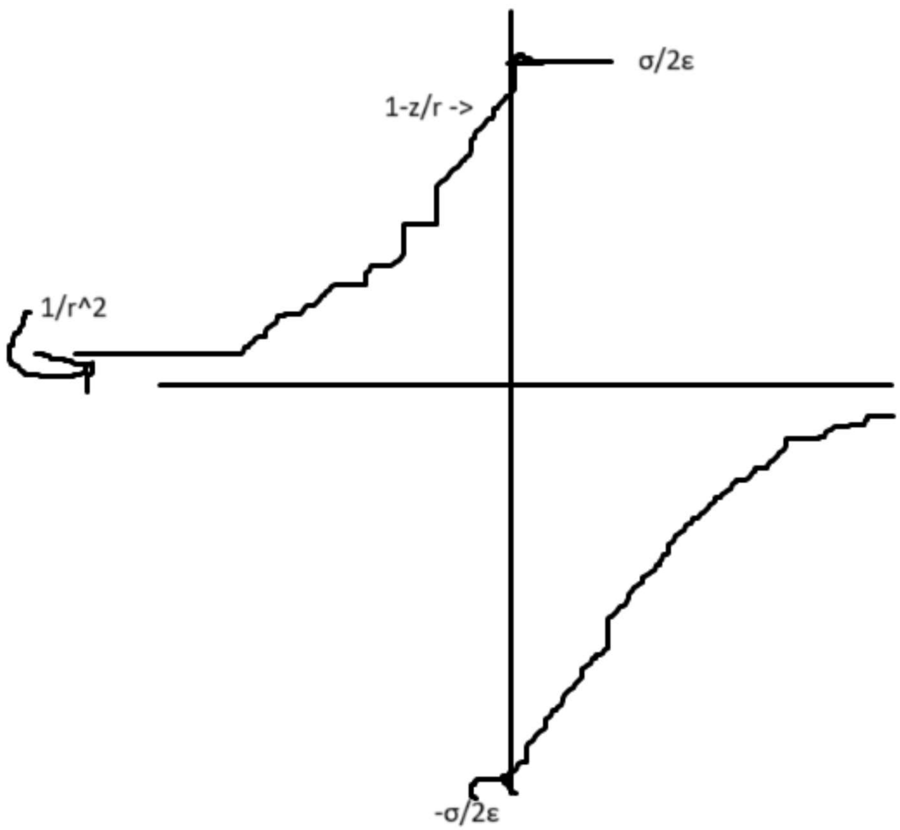
This function isn't exactly pretty, but I know for a fact the answer should be something along the lines of $\frac{1}{z^2}$ since it should become a point charge as R becomes small with respect to z

(3) $z \to 0$

$$E(z \to 0) \approx \frac{z}{|z|} - \frac{z}{\sqrt{R^2}}$$
$$\to \approx 1 - \frac{z}{r}$$

Since $-\frac{\sigma}{2\epsilon_0}$ is our result for a flat infinite field and z/R goes to zero, the disk should look like an infinite field to the observation point and have an even, constant field

(4)



Python:

ak_HW3_Calculate_Electric_Field.ipynb