Homework 4
Justify all your answers
due on Fr 10/18/24 at 11:30AM in A236WH

Exercise 1. *Given a stationary source (S,P) with entropy H. Let n,q,r,s be integers with $n = qr+s$, $0 \le s < r$, $n \ge 1$ and $q \ge 1$. Show that*

(a)
$$\frac{H(p^n)}{n} \le \frac{H(p^r)}{r}\left(1-\frac{s}{n}\right)+\frac{H(p^s)}{n}$$

(b)
$$\lim_{n\to\infty}\frac{H(p^n)}{n} = H$$

(Hint: Given $\epsilon > 0$, show that there exists $r \in \mathbb{Z}^+$ such that $\frac{H(p^r)}{r} < H + \frac{\epsilon}{2}$. Let K be the maximum value of $H(p^s)$ for $0 \le s < r$ and choose a positive integer n_0 such that $\frac{K}{n_0} < \frac{\epsilon}{2}$.)

Exercise 2. *Given a memoryless source $(\mathbb{B},P)$ with probability distribution $p = (0.2, 0.8)$.*

(a) *Compute the entropy $H_2(P)$ of the source to the base 2.*

(b) *Use the Coding Theorem for Memoryless Sources 4.4.10 to find a positive integer n such that there exists a prefix-free binary code for $\mathbb{B}^n$ with average word-length L such that $\frac{L}{n}$ is within 10% of $H_2(P)$.*

Exercise 3. *Compute $0_* z$ for $z = 00100101$ and for $z = 10111001$.*

Exercise 4. *Consider a source (S,P) such that $S = \{a,b\}$ and p^2 is given by*

p^2	a	b
a	0.1	0.2
b	0.3	0.4

Let c_2 be the arithmetic code on S^2 with respect to the ordering $bb < ba < ab < aa$ and the probability distribution p^2. Compute $c_2(ba)$.

For exercises 5 and 6 let S be an ordered alphabet and (S,P) a memoryless source. For $r \in \mathbb{N}$ view S^r as an ordered alphabet via the lexicographic order. So if $x,y \in S^r$ then $x < y$ if there exists $1 \le k \le r$ with $x_i = y_i$ for $1 \le i < k$ and $x_k < y_k$.) Let c_r be the arithmetic code for S^r with respect to p^r.

Exercise 5. *Let $y = y_1 \dots y_r \in S^r$ and $s \in S$. Prove that*

$$\alpha(ys) = \alpha(y) + P(y)\alpha(s) = \alpha(y) + p(y_1)p(y_2)\cdots p(y_r)\alpha(s).$$

Exercise 6. *Suppose $S = (a,b,c,d)$ and $p = (0.4, 0.3, 0.2, 0.1)$.*

(a) *Compute the average codeword length L of c_2 with respect to p^2 and compare $\frac{L}{2}$ with $H_2(p)$.*

(b) *Compute $c_2(ac)$ and $c_3(acd)$.*

Exercise 7. *Use the LZW-decoding rules for the ordered alphabet $S = (\sqcup, B, D, E, N, O, P, R, S, T)$ to decode*

10.6.1.2.4.1.6.8.1.5.6.10.1.11.13.4

Exercise 8. *Compute the LZW-encoding of MISSISSIPPI for ordered alphabet $S = (I, M, P, S)$.*