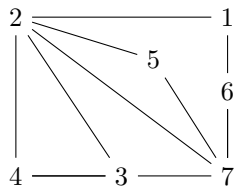
Homework 2
Justify all your answers
due on Fr 9/20/24 at 11:30AM in A236WH

Exercise 1. *Find a simple cycle of maximal length in the graph*



Exercise 2. *Construct a tree representation of the ternary code using the alphabet* $T = \{0, 1, 2\}$ *and codewords* 20, 121, 102, 001 *and* 000. *Is it possible to extend this code without destroying the PF property?*

Exercise 3. *Design a PF binary code* $c : \{1, 2, 3, 4, 5, 6\} \to \mathbb{B}^*$ *such that the sum of the lengths of the codewords is as small as possible. Construct the tree representation of such a code.*

Exercise 4. *Does there exist a prefix-free ternary code with the following parameter:*
(a) $(0, 1, 3, 10)$ *(b)* $(0, 0, 1, 3, 39)$.

Exercise 5. *A code is called* complete *if it is PF and* $K = 1$. *Show that if there exists a complete b-nary code for an alphabet of size* m, *then* $b - 1$ *divides* $m - 1$.

Exercise 6. *Let* $S = \{a, b, c, d, e, f, g\}$ *and let* $c : S \to \mathbb{B}^*$ *be the binary code with*

$$a \mapsto 00, \quad b \mapsto 010, \quad c \mapsto 011, \quad d \mapsto 1000, \quad e \mapsto 1001, \quad f \mapsto 1101, \quad g \mapsto 1111.$$

For a positive integer i *define* $Q_i(x) = Q_{c^i}(x)$.

(a) *Write down* $Q_1(x)$ *and then compute* $Q_2(x)$ *and* $Q_3(x)$.

(b) *What do the coefficients of* x^7 *in* $Q_2(x)$ *and* $Q_3(x)$ *represent? Verify your answers by making a list of the corresponding messages in* S^*.