Homework 5
Justify all your answers
due on Fr 11/01/24 at 11:30AM in A236WH

Exercise 1. *Find a Minimum Distance decision rule for the binary code*

$$C := \{0000, 1100, 0011, 1111\}$$

Exercise 2. *Let* $D := \{00000, 11100, 10011\}$. *Find all* $a \in \mathbb{B}^5 \setminus D$ *such that* $D \cup \{a\}$ *is a 1-error-correcting binary code.*

Exercise 3. *Let* C *be a* 3*-error-correcting code with* $C \subseteq \mathbb{B}^{12}$ *and* $|C| = 8$. *Determine* $|N_3(C)|$.

Exercise 4. *Let* n *and* r *be positive integers, let* $D \subseteq \mathbb{B}^n$ *be an* r*-error-correcting code and let* $a \in \mathbb{B}^n \setminus D$. *Show that* $D \cup \{a\}$ *is an* r*-error-correcting code if and only if* $a \notin N_{2r}(D)$.

Exercise 5. *Let* $n \in \mathbb{N}$ *and suppose* $C \subseteq \mathbb{B}^n$ *is a perfect,* 1*-error-correcting binary code. Show that there exists* $l \in \mathbb{N}$ *such that* $n = 2^l - 1$ *and* $|C| = 2^{2^l - l - 1}$.

Exercise 6. *Let* $n \in \mathbb{N}$.

(a) *Let* $a, b, c \in \mathbb{B}^n$. *Show that* $d(a, b) + d(b, c) + d(a, c) \leq 2n$.

(b) *Let* $C \subseteq \mathbb{B}^n$ *be a binary code with minimum distance* δ. *Suppose that* $|C| \geq 3$. *Show that* $d(a, b) \leq 2(n - \delta)$ *for all* $a, b \in C$.

Exercise 7. *Which of the following subsets of* $\mathbb{F}_2^5$ *are linear codes:*

(a) $C_1 := \{00000, 11000, 10011, 11111\}$.

(b) $C_2 := \{00000, 11000, 00111, 11111, 01010, 10010, 01101, 10100\}$.

(c) $C_3 := \{x \in \mathbb{F}_2^5 \mid x_1 + x_2 + x_5 = 0\}$.