

# PHY 491, Fall 2024 - Homework 8

DUE: Friday 10/25/22, 11:59pm

**Problem 4.1** Consider electrons on a two dimensional square lattice with lattice constant  $a$  in the tight-binding approximation. Using one orbital per site and only nearest neighbor hopping and  $t=1$ , the resulting energy is  $E(k_x, k_y) = \epsilon_0[2 - \cos(ak_x) - \cos(ak_y)]$ .

- 4.1.1 What is the value of the energy at the  $\Gamma$  (center), X (middle of side face), and W (corner) high symmetry points of the first Brillouin zone? (3 points)
- 4.1.2 Sketch a few constant energy curves on the 2D Brillouin zone for  $k_x, k_y$  very close to the  $\Gamma$  and W points. (2 points)
- 4.1.3 Sketch the  $k_x, k_y$  curve corresponding to  $E(k_x, k_y) = 2\epsilon_0$ . (4 points)
- 4.1.4 Sketch  $E(|k|)$  along the  $\Gamma$ -X and W-X lines of the first Brillouin Zone. (4 points)
- 4.1.5 With one electron per site in this crystal, is this a metal or an insulator? Draw the Fermi surface. What is the Fermi energy? (4 points)
- 4.1.6 With two electrons per site in this crystal, is this a metal or an insulator? Draw the Fermi surface. What is the Fermi energy? (3 points)