Fundamentals

Experiment One

Physics 191
Michigan State University

Before Lab

- Print a copy of this document and bring it to lab each week
- Carefully read the entire lab guide
- Watch a short Crash Course video on derivatives (recommended)
- Read about drag and terminal velocity in the free Openstax textbook
- **Due at the beginning of class (10 pt)**: answer the pre-lab theory questions in Section 1

Experiment Overview

This is a single-week experiment split into two parts:

- Part 1 (20 pt): perform hands-on measurements to determine the density of a copper block
- Part 2 (20 pt): practice analysis techniques using simulated terminal velocity data

You and your partner will submit a single report at the beginning of the next lab. Your report should be formatted as explained in the Lab Report Format Guide on the last page of this document. The main objectives of this lab are:

- Practice estimating and propagating uncertainties
- Implement a basic curve fitting method
- Compare a theoretical model with experimental data

Motivation

The purpose of the first lab is to practice fundamental skills used throughout experimental science. While all experiments in this class are based on classical physics, the techniques developed in the lab are ubiquitous in science and engineering. A key aspect of the course is error analysis: the quantitative study of measurement uncertainties. Finding and mitigating uncertainties requires critical thinking at every step in your experimental procedure.

Uncertainty is not a fault of the experimenter, nor is the word *error* a synonym for *mistake*. A careful scientist may fine tune an experiment to minimize uncertainty, but a complete elimination is impossible. Uncertainties arise outside of the lab, too. Mathematical theories are limited by fundamental assumptions, engineering products are produced within a tolerance, and computational models are limited by finite machine precision (rounding errors).

This document serves as a brief introduction to many concepts in error analysis that will be used in every experiment. Some of the material is quite dense, so it is worth rereading most sections after you have some experience with uncertainty propagation and curve fitting. A comprehensive discussion of all topics covered here can be found in *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements* by John R. Taylor. A PDF copy of this book can be obtained for free with a quick Google search.

Precision and Accuracy

The technical jargon used by scientists is essential for unambiguous communication. The distinction between *precision* and *accuracy* is important as the two words have fundamentally different meanings. Figure 1 illustrates the difference between the two concepts.

The scatter of a set of measured values indicates the **precision** of the measurement. Imagine using a stopwatch make repeated measurements of the period of a pendulum. If each trial yields a vastly different number, the measurements are of low precision. Conversely, if the obtained numbers are all very close, the measurements are of high precision. When successive measurements yield the *exact* same value, the precision is limited by the resolution of the measurement tool. In general, devices with smaller divisions correspond to higher precision. A digital kitchen scale that reads out two decimal places is more precise than a scale that balances weights.

An **accurate** measurement is one in which the result is close to a predicted or known value. Note that the concept of accuracy is linked to the knowledge of some predetermined, accepted value for the measured quantity.

Limitations of precision and accuracy are linked to specific types of errors. **Random** errors are observed when successive measurements yield a range of different values. Such random fluctuations are closely linked to the *precision* of the measurement. This type of error can be treated statistically through repetition and averaging, as you will see in Experiment Two.

Systematic errors cause a shift in measured values which is pervasive across all measurements. Most often, systematic errors result from issues related to the experimental apparatus. For example, a poorly calibrated scale will cause all measurements to be shifted in one direction. Systematic errors can be very difficult to identify, and they directly influence measurement *accuracy*.

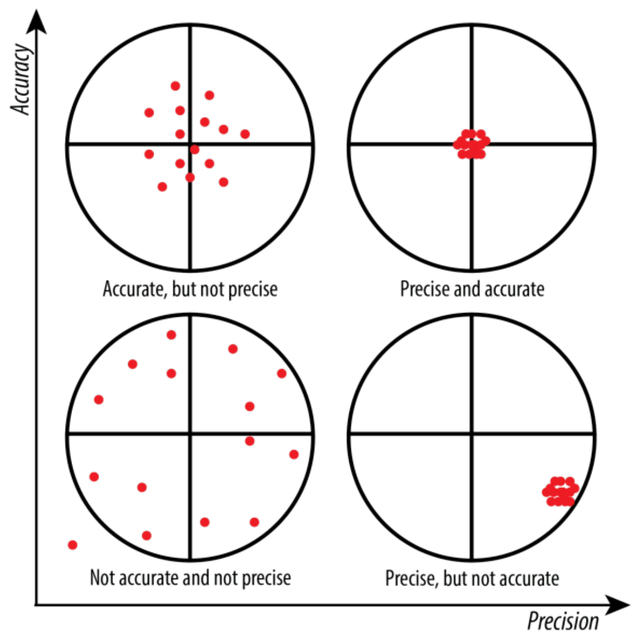


Figure 1: A visual representation of precision and accuracy using a target (image source: *Precision vs. Accuracy*, St. Olaf College)

Mistakes (human errors) are another type of error which, like systematics, can be difficult to detect. Miscalculations, misreading scales, or recording incorrect values are all unintended actions by the experimenter. There is an important distinction between human and random error. In the example of a stopwatch and pendulum, human reaction time when using the stopwatch would result in a spread of measured values. These variations are not mistakes; there is no way to know which measurement was "best." Fluctuations of this type are classified as random.

Estimating Minimum Uncertainties

A measurement should always be stated in the form "*measurement* $\pm$ *uncertainty*." The picture below shows a pencil oriented along a length scale. The end of the pencil is close to 5, but falls somewhere between 5 and 6; the exact number is impossible to determine.

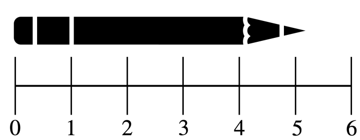


Figure 2: A pencil aligned with a unitless scale.

As long as the end of the pencil is carefully aligned, repeated measurements would yield the same number. The limiting factor in this measurement is the finite resolution of the scale; the difference between 5.1 and 5.2 cannot be resolved. For any analog measurement tool, the minimum uncertainty is *half* of the smallest division[1]. The scale in Figure 2 has divisions of length 1, giving an uncertainty of 0.5. Using this convention, we can say that the pencil is 5.5 ± 0.5 units long.

In contrast with simple analog scales, digital scales often come with a tolerance stated by the manufacturer. When this is not the case, we cannot be sure how the device rounds a number. Consider a digital scale that gives a measurement with a precision of 0.1. If the scale rounds in the conventional way, a number like 3.27 would display as 3.3; however, if the scale truncates the measurement, the value of 3.27 would display as 3.2. For this reason, we adapt the convention of using the *full* last digit as the minimum uncertainty.

The *minimum* uncertainty is determined by the scale on an instrument. This value can only be used as the measurement uncertainty when successive measurements yield identical values. In all other cases, the uncertainty will be larger to account for random fluctuations.

Rounding

In a quantitative lab, there are a few important rules to consider when stating the value of a measurement:

1. Numbers should be rounded *after* all calculations are finished to avoid introducing round-off error
2. Uncertainties should almost always be rounded to one significant figure
3. All numbers stated in a lab report must be rounded such that the number of digits agrees with the measurement scale
4. For any stated measurement, the last significant digit should be in the same decimal place as the uncertainty

Bad	**Good**
3.141 ± 0.1	3.1 ± 0.1
93.4 ± 3	93 ± 3
$1.6216 \cdot 10^{-19} \pm 5 \cdot 10^{-21}$	$(1.62 \pm 0.05) \cdot 10^{-19}$

Spreadsheet software typically displays calculated numbers with 9 decimal places. Such precision is far beyond any capabilities in the PHY 191/192 lab. Always round your numbers appropriately before including in a lab report.

Error and Uncertainty

You will often hear the words *error* and *uncertainty* used interchangeably, but there are technical distinctions between the two concepts. **Error** is the difference between a number and the true or accepted value. A *source of error* is something which causes a measurement to deviate from the true value.

We cannot know a *true* value in an exact sense. There may exist a true value for a measurement, but it is not possible to determine with absolute certainty. **Uncertainty** is a number assigned to a measurement describing the range in which the true value is *most likely* to be found. A lowercase Greek delta (δ) is the typical notation for uncertainty: interpret δx as "uncertainty in x" where x could be any measured quantity.

The uncertainty in a measurement can be represented graphically by **error bars** as shown in Figure 3. Let's return to the pencil example, in which the length was determined to be 5.5 ± 0.5. This measurement can be plotted along a number line with the uncertainty indicated by error bars of length $\delta x = 0.5$ extending in both directions from the measured value, labeled as x_m in Figure 3.

[1] The smallest division on a scale is often referred to as the *precision* of that tool.

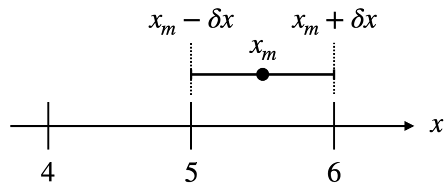


Figure 3: A point $x_m = 5.5$ plotted with error bars determined by the uncertainty, $\delta x = 0.5$.

Pay close attention to the plot above. The indicated number x_m is the best estimate at the true value, and the error bars indicate the range in which the true value is most likely to occur. The magnitude of this range has a length of exactly one division on the scale, which is the precision limit of the device; no numbers smaller than this value can be resolved. If the scale had smaller divisions, a more precise measurement could be made and the corresponding error bars would be smaller.

It is often useful to characterize a measurement by comparing to a theoretical or accepted (reference) value. The discrepancy between the measured and reference values is called the **absolute error**:

$$E_x = |x_{ref} - x_m| \tag{1}$$

where x_{ref} is the reference value and x_m is the measurement. Note that the error E_x has the same units as the measurement itself, and it is always a positive number. A more practical quantity is the **fractional error**, which gives the relative difference between the measured and reference values:

$$e_x = \left| \frac{x_{ref} - x_m}{x_{ref}} \right|. \tag{2}$$

The fractional error is a quantitative estimate of measurement accuracy. Another extremely useful relative quantity is the **fractional uncertainty**:

$$\text{fractional uncertainty} = \frac{\delta x}{|x|}. \tag{3}$$

Both fractional quantities are unitless numbers between 0 and 1 that can be multiplied by 100% to express the relationship in percent. Relative relationships are the easiest for readers to digest (see theory Question 3).

Key Concepts

- Measurements should always be stated in the form $x_m \pm \delta x$ where x_m is the measurement and δx is the uncertainty
- The **error** is a measure of how close a number is to a reference value and is related to **accuracy**
- The **uncertainty** indicates a range of probable values and is related to **precision**

Uncertainty Propagation

The discussion up to this point has been limited to uncertainties in direct measurements. In most cases, measured quantities are used to perform calculations. The subject of this section is to study how measurement uncertainties carry through calculations. Consider a situation where we directly measure three quantities:

$$x \pm \delta x, \quad y \pm \delta y, \quad z \pm \delta z,$$

where x, y, and z are any type of measurement and δx, δy, and δz are their measurement uncertainties. We are interested in calculating a quantity q which depends on x, y, and z. The uncertainties δx, δy, and

δz *must* cause an uncertainty in q. There is one formula which can be used to find the uncertainty in any calculated quantity:

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x} \cdot \delta x\right)^2 + \left(\frac{\partial q}{\partial y} \cdot \delta y\right)^2 + \left(\frac{\partial q}{\partial z} \cdot \delta z\right)^2}. \tag{4}$$

The derivatives in Equation 4 are called partial derivatives. $\partial q/\partial x$ is the derivative of q with respect to x, where y and z are treated as constants. The other partial derivatives behave the same way. Try not to be overwhelmed by all of the symbols! Remember, the quantities δx, δy, and δz are just numbers determined from the measurement process.

Uncertainty Propagation Example

A ball of mass (1.0 ± 0.1) kg moves in one dimension with velocity (-1.5 ± 0.2) m/s. Determine the ball's momentum $p = mv$, and use the uncertainty propagation formula to determine the momentum uncertainty δp.

The momentum of the ball is calculated by plugging in the measurements: $m = 1.0$ kg and $v = -1.5$ m/s.

$$p = mv = (1.0 \text{ kg})(-1.5 \text{ m/s}) = -1.5 \text{ kg} \cdot \text{m/s}$$

The momentum uncertainty is calculated using Equation 4 and the measurement uncertainties: $\delta m = 0.1$ kg and $\delta v = 0.2$ m/s.

It is important to recognize that the momentum p takes the place of the generalized function q defined above, while the mass and velocity take the place of generalized measurements x and y. This translates to

$$\delta p = \sqrt{\left(\frac{\partial p}{\partial m} \cdot \delta m\right)^2 + \left(\frac{\partial p}{\partial v} \cdot \delta v\right)^2}. \tag{5}$$

Starting with $p = mv$, we can evaluate mass derivative

$$\frac{\partial p}{\partial m} = \frac{\partial}{\partial m}(mv) = v$$

and the velocity derivative

$$\frac{\partial p}{\partial v} = \frac{\partial}{\partial v}(mv) = m.$$

The derivatives can be substituted back into Equation 5, giving

$$\delta p = \sqrt{(v \cdot \delta m)^2 + (m \cdot \delta v)^2} = \sqrt{((-1.5 \text{ m/s})(0.1 \text{ kg}))^2 + ((1.0 \text{ kg})(0.2 \text{ m/s}))^2} = 0.25 \text{ kg} \cdot \text{m/s}.$$

Using the measurements of mass and velocity, we have calculated the momentum of the ball to be

$$p \pm \delta p = (-1.5 \pm 0.3) \text{ kg} \cdot \text{m/s}$$

where the uncertainty has been rounded to one significant digit.

Useful Shortcuts

The uncertainty propagation formula applies to functions with any number of variables. In the case that q is a function of a single variable, the uncertainty propagation formula reduces to

$$\delta q = \left|\frac{dq}{dx}\right| \delta x. \tag{6}$$

If $q = Bx$ where B is an exact constant, then the uncertainty in q is the uncertainty in x multiplied by the same constant:

$$\delta q = |B| \delta x. \tag{7}$$

Power Rule for Derivatives

There is only one type of derivative that will be used in this class. You have likely evaluated derivatives of one-dimensional polynomials Calculus 1. This section extends the power rule to functions of more than one variable. First, consider a single-variable function $f(x) = x^n$ where n is a real number. The derivative of $f(x)$ is evaluated using the **power rule**,

$$\frac{df}{dx} = nx^{n-1}.$$

Consider a second function with two variables, $g(x, y) = x^n y^m$. Any function with more than one variable can be differentiated in more than one way: $g(x, y)$ can be differentiated with respect to either x or y. When differentiating multivariable functions, the differential symbol d is replaced by the *partial* symbol: $d \to \partial$. Do not be afraid of the fancy notation! It simply means to treat all variables other than the differentiation variable as constant. For the function $g(x, y) = x^n y^m$, the two *partial* derivatives are

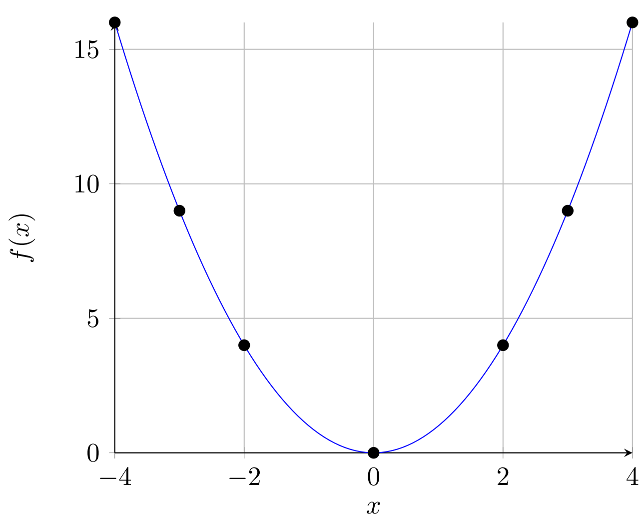
$$\frac{\partial g}{\partial x} = (n \cdot x^{n-1})y^m$$
$$\frac{\partial g}{\partial y} = x^n(m \cdot y^{m-1}).$$

Notice that the derivatives in the previous two equations are evaluated using the power rule just like the single-variable case. The distinction between d and ∂ is the absence of any implicit differentiation of the additional variables.

Curve Fitting

Consider a simple function $f(x) = x^2$. This function can be represented graphically by a curve, shown below in blue. Because $f(x)$ is known exactly, any number of ordered pairs (x_i, y_i) that fall directly on the curve can be easily computed. For example, $f(-3) = (-3)^2 = 9$ gives a point at (-3, 9).

x	$f(x)$
-4	16
-3	9
-2	4
0	0
2	4
3	9
4	16

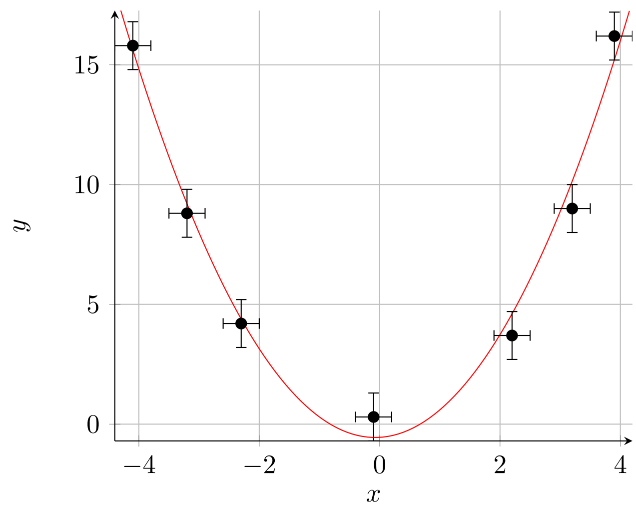


Case 1: Exact Data

Now consider a situation in which a set of discrete points is known, and we wish to find a function that best fits all of the points. This is a *much* harder problem to solve. The process of determining a function which most closely fits a set of data points is called **curve fitting** or more broadly, regression analysis. The full details of regression analysis are beyond the scope of this class, so we will implement web-based curve fitting software called curve.fit.

Case 2 shows a more realistic quadratic relationship where each point is subject to random error. The numbers from the table on the left are plotted with error bars representing measurement uncertainties.

x	y
-4.1	15.8
-3.2	8.8
-2.3	4.3
-0.1	0.3
2.2	4.2
3.2	9
3.9	16.2



Case 2: Experimental Data

Curve fitting software was used to find the best fit function which is represented by the red curve. Given the set of data points, the software finds values of the parameters a, b, and c such that a quadratic function $f(x) = ax^2 + bx + c$ best fits the set of points.

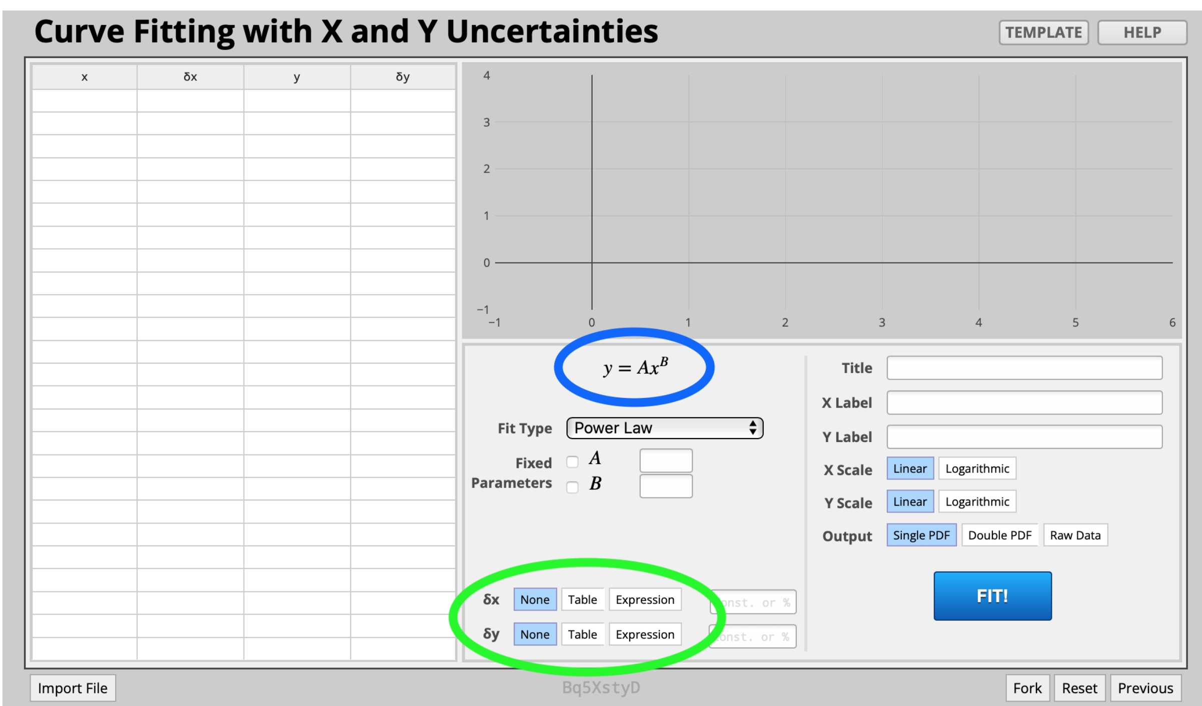


Figure 4: A screenshot of the curve.fit interface.

The fitting program used in this class operates entirely within your browser (website: curve.fit). The interface is shown in Figure 4. On the left is a table for the x and y values and their uncertainties. The uncertainties are represented by error bars on each point (δx = horizontal, δy = vertical). Circled in green are a series of buttons that tell the program where to find the uncertainties. There is a general procedure which can be followed to fit a curve to a dataset:

- Organize the *controlled* and *measured* quantities from your experiment into a spreadsheet.
  - controlled = independent = x-axis variable
  - measured = dependent = y-axis variable
- Refer to the theory to determine an expected relationship between the controlled and measured quantities in your experiment. This will guide your choice of "fit type," the functional form to which you will fit the data
- Copy-paste the values for x and y from your working spreadsheet to the data table in the browser.
- Choose a "fit type" that matches the form of the equation from the theory (circled in blue in Figure 4). The "fit" equations are in a generalized form. Some common examples:
  - Linear: $y = mx + b$
  - Quadratic: $y = ax^2 + bx + c$
  - Inverse: $y = A + \frac{B}{x}$
  - Power Law: $y = Ax^B$
  - Exponential: $y = Ae^{bx} + C$
- The quantities a, b, c, A, B, C, etc. are parameters (constants) that characterize your system. The variables are always labeled as x and y.
- *Important*: do not assume the parameter labels correspond to physical quantities. For example, in the linear equation, m does not mean mass; it is the slope of the line. In the quadratic equation, a is not an acceleration. These parameters characterize the curve, but you need to determine how they are related to the physical system.
- Add a title and axes labels. The axes must *always* have units indicated in parenthesis.
- Press "FIT!" and a PDF will be generated with the values of the fit parameters that determine the best fit curve.
- **Always** include the entire page generated by curve.fit in your lab report

General Analysis Strategy

Quantitative experiments are designed to examine how one quantity affects another. An experimenter must determine how to fix as many parameters as possible to directly identify cause-and-effect relationships. In this class, the labs are designed such that experimental results can be compared with a theoretical model. There is a set of steps which can, in general, be followed to perform and analyze an experiment:

1. Determine the cause-and-effect relationship to be studied
   - Define the **independent variable** $\longleftrightarrow$ varied quantity $\longleftrightarrow$ *the cause*
   - Define the **dependent variable** $\longleftrightarrow$ measured quantity $\longleftrightarrow$ *the effect*
2. Study the theoretical model:
   - Write out any relevant equations, especially those related to the dependent and independent variables
   - Manipulate the equations to get a single expression relating the dependent and independent variables
   - Formulate a hypothesis to test in the experiment
3. Sketch out the apparatus and any diagrams necessary to aid in explaining observed phenomena

4. Organize tables for data collection using spreadsheet software (Excel or Google Sheets)
5. Determine the best way to plot your data. Typically, you will plot *dependent variable* vs. *independent variable* (i.e. y vs. x), but there are cases when alternitive representations are easier to analyze
6. Carry out the experiment, recording the values for any fixed parameters or other relevant quantities
7. Generate a plot including axes labels with units, a title, and error bars corresponding to the uncertainties, and fit a curve (when applicable)
8. Analyze the results, considering all sources of error, and determine the uncertainties for any calculated quantities
9. Summarize results and draw conclusions from your experiment

Worked Example: Ideal Gas

Consider an experiment designed to test the relationship between the pressure and volume of an ideal gas. An apparatus is constructed with a fixed amount of gas confined to a vessel, and the gas is held at a constant temperature. The device allows the user to vary the volume and measure the pressure in the vessel. This defines the cause-and-effect variables:

- Independent (varied) variable = volume = V
- Dependent (measured) variable = pressure = P

The objective is to observe how changing V affects P. The ideal gas law gives a relationship between the quantities of interest,

$$PV = nRT, \tag{8}$$

where n, R, and T are constants. This equation should be manipulated to get the dependent variable isolated on the left side,

$$P = \frac{nRT}{V}. \tag{9}$$

According to the ideal gas law, the pressure should have an inverse dependance on the volume. This is the relationship we intend to verify.

The next step is to think about organizing and plotting data. Typically, the y-axis corresponds to the dependent variable and the x-axis corresponds to the independent variable. Keep in mind that curve.fit has no knowledge of the nature of our experiment. We must determine what to call x and y to generate the desired plot.

- *Varied* by user: $V = x \ \rightarrow\ \delta V = \delta x$
- *Measured* by user: $P = y \ \rightarrow\ \delta P = \delta y$

This step explicitly defines the values to feed curve.fit. With these substitutions, Equation 9 becomes

$$y = \frac{nRT}{x} \quad \longleftrightarrow \quad \text{dependent} = \frac{nRT}{\text{independent}}. \tag{10}$$

The relationship $1/x = x^{-1}$ can be used to write the previous equation as

$$y = (nRT)x^{-1}. \tag{11}$$

Recognize that this equation is identical to the ideal gas law in Equation 9, just expressed in a slightly different way. This manipulation was done to express the ideal gas equation in the form of a *fit type* called the "power law,"

$$y = Ax^{B}. \tag{12}$$

This is the form of the curve the program will attempt to fit to the pressure and volume data. The program finds values of A and B that produce a curve as close to the data as possible. If the experiment is in agreement with the theory, then the fit parameters should be $A = nRT$ and $B = -1$ (compare Equations 11 and 12 to verify this for yourself). The output from curve.fit is shown below.

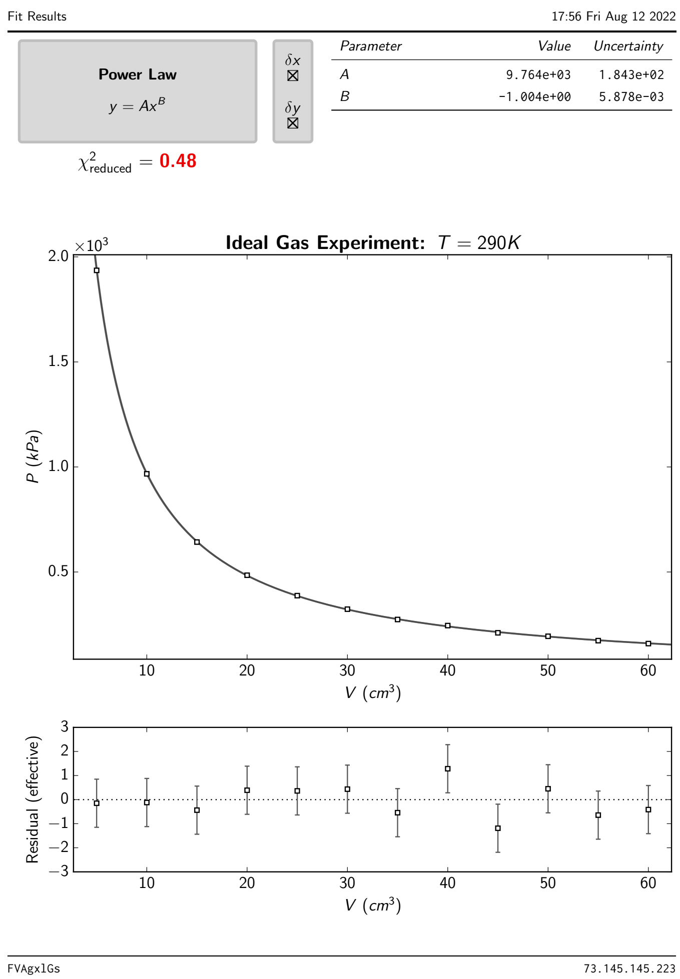


Figure 5: The PDF generated by curve.fit for the ideal gas example

Following the procedure outlined in the previous section, the pressure and volume data were plotted. The results are shown in Figure 5. The program found optimal values for A and B, giving $y = (9.764 \cdot 10^3)x^{-1.004}$, producing a curve that best fits all of the data. Key points for analysis:

- The numbers $A \pm \delta A$ and $B \pm \delta B$ are given on the top right of the page under "Value" and "Uncertainty"
- The values of A and B give an equation $y = Ax^B$ that best fits the data. This equation is just an alternative representation of $P = \frac{nRT}{V}$ from the ideal gas law.
- The experimental value of $B = -1.004$ is in good agreement with the expected value of -1, verifying the validity of the ideal gas law for describing the system. Using Equation 2, the relative error in B is found to be 0.4%
- The number $A = (97.64 \pm 1.843) \cdot 10^2$ should be related to n, R, and T by $A = nRT$. With knowledge of the ideal gas constant R and the temperature T, A can be used to calculate the amount of gas, n.

- There is a second plot of "residual" values related to the uncertainties (error bars). The error bars are not visible on the primary plot because their sizes are very small relative to the values of P.

Residual Plots

A **residual** is a measure of how far a given point is from falling directly on the curve. Figure 6 shows a set of data points (x_i, y_i) fit to a line, $f(x) = b + mx$ where b and m are the fit parameters (y-intercept and slope). The residual for each data point can be calculated as the difference

$$R_i = y_i - f(x_i). \tag{13}$$

where the subscript i is a count of which point you choose to examine. If the point falls directly on the line, this value will be zero. If the line is a very good fit, then the residuals will all be as small as possible. In the ideal gas example, there is a plot on the bottom of the page showing the residual with error bar for each point. If the error bar overlaps with 0.0, then the point is within the range of uncertainty of falling directly on the curve.

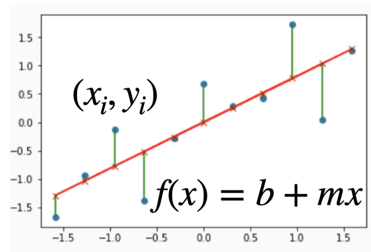


Figure 6

Residual plots are very useful for detecting both outliers and deviations from a theoretical model. This is illustrated in the three plots shown below. Figure 7 shows an example of a plot with an outlier, likely caused by a mistake or bad measurement. The curved trend in Figure 8 indicates that a fit type with more curvature should be used. If a data set is subject only to independent and random errors and the fit type is appropriate, the residuals should look like those in Figure 9.

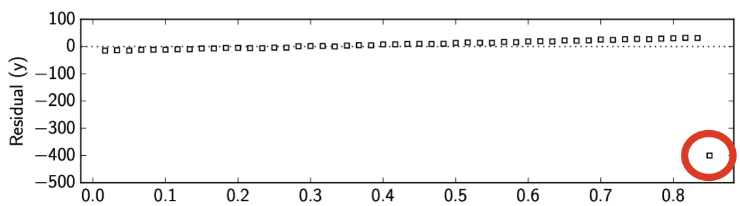


Figure 7: A significant outlier circled in red, indicating a mistake or bad data point

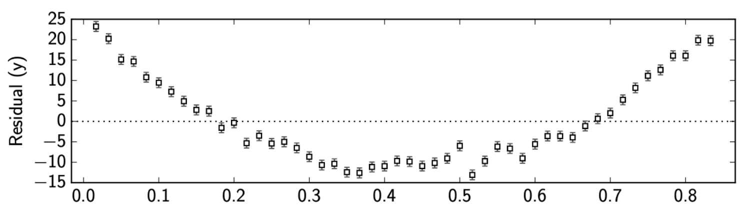


Figure 8: The curve indicates a nonlinear trend, a different fit type should be considered

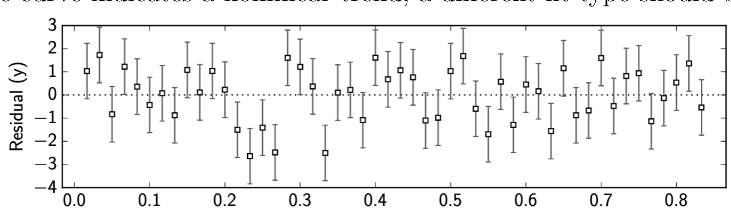


Figure 9: Evenly distributed random fluctuations do not indicate any underlying problems

1 Theory Questions

The theory questions are written to help you prepare for the lab. Your answers are due at the beginning of the experiment to ensure that you are ready to work with your partner. **You must show all of your work to receive full credit**.

1. (1 pt) Which equation(s) should be used to quantify the accuracy of a measurement? *Hint:* the notion of accuracy depends on the knowledge of a reference value.

2. (1 pt) Which equation should be used to quantify the precision of a measurement?

3. The accepted value for the electron charge is $q_{ref} = -1.60217 \cdot 10^{-19}$ C. Use this value for the following two questions:

   (a) (2 pt) A careful experimenter measures $q_m = -1.61819 \cdot 10^{-19}$ C for the electron charge. Use Equations 1 and 2 to quantify the error.

   (b) (1 pt) Student A has an absolute error of $4.80651 \cdot 10^{-21}$ C. Student B has a relative error of 2%. Which student made a more accurate measurement? *Hint:* use q_{ref} to find Student A's relative error.

4. (2 pt) Two quantities $x \pm \delta x$ and $y \pm \delta y$ are measured and used to calculate $q = x \cdot y$. Use Equation 4 to write the uncertainty δq in terms of x, δx, y, and δy.

5. (1 pt) What is the difference between an exponential function and a polynomial?

6. (1 pt) A steel ball is dropped into the ocean. It falls under the influence of gravity until it reaches the bottom. Along the journey, is the acceleration of the ball always equal to the gravitational acceleration, g? Is it ever equal to g? Explain your reasoning.

7. (1 pt) A doctor accidentally writes a prescription for a 0.1 g dose of a medication instead of the desired therapeutic dose of 10 mg. The drug causes serious side effects for any dose 20 times larger than the therapeutic dose. Will the doctor's unit conversion error lead to complications?

2 The Mass Density of a Block

The first part of the lab is designed to help you become familiar with the process of estimating and propagating uncertainties. You will measure the mass and the dimensions of a block to calculate its mass density. For a homogeneous object, the mass density is defined as

$$\rho = \frac{m}{V} \tag{14}$$

where m is the total mass and V is the volume of the object. The symbol ρ is a lowercase Greek "rho" and is the standard notation for density. For this and all other experiments, you must document your procedure such that your experiment could be reproduced by another experimenter.

2.1 Method 1: Ruler

1. Begin by organizing a table to store your measurements using Excel or Google Sheets using the table below as a template[2]

Method	m (g)	δm (g)	L (cm)	δL (cm)	W (cm)	δW (cm)	H (cm)	δH (cm)

[2]If you are unable to use Greek characters, a lowercase d can be used for "uncertainty in." For example, the uncertainty in volume can be expressed as dV. You can also type "rho" instead of ρ. Exponents are indicated using the caret symbol (^) and subscripts with an underscore (_).

2. (1 pt) Using the digital scale, determine the mass of your block and its uncertainty.
3. (1 pt) With a ruler, measure the physical dimensions of the block and determine the uncertainty for each measurement.
4. Make a second table to organize your calculations

Method	$\delta m/m$	V (cm^3)	δV (cm^3)	$\delta V/V$	ρ (g/cm^3)	$\delta\rho$ (g/cm^3)	$\delta\rho/\rho$

5. (5 pt) Calculate all of the quantities indicated in the table above and record with the appropriate number of significant figures. **Include all calculations in your lab report**.

2.2 Method 2: Digital Calipers

1. (2 pt) Follow the same procedure as above, but this time use the digital calipers for measuring the dimensions of the block.
2. (5 pt) Record the values in your tables with the appropriate number of significant figures. **Include all calculations in your lab report**.

2.3 Block Density Summary

The blocks are made of relatively high quality copper. The known density of pure copper is 8.96 g/cm^3 at room temperature.

1. (3 pt) Quantitatively compare your measured densities with theoretical density of copper by calculating the relative error in percent for each method. What can you say about the accuracy for each method?
2. (3 pt) Did you obtain a more precise result using the calipers? Be quantitative and explain your answer.

3 Terminal Velocity Experiment Analysis

This exercise is intended to help you develop the analytical tools you will use throughout the PHY 191/192 sequence. You will not perform any further measurements, the data have already been generated. Your job is to

- Study the theoretical model of an object falling in air
- Fit a curve to a simulated data using two different methods
- Extract information about the system using parameters determined by the curve fit
- Quantify uncertainties in calculated quantities

3.1 Theory

In physics, a fluid is a gas or liquid that continuously deforms under the influence of an applied force. An object moving through a fluid experiences a resistive force resulting from collisions with the surrounding particles. This effect is obvious when riding a bicycle. It is relatively easy to pedal at low speeds, but as you move faster, you must pedal much harder to compensate for wind resistance. Eventually, you reach a maximum speed at which you cannot pedal hard enough to continue accelerating.

This resistive force is called **drag**. From the bicycle example, it is evident that the magnitude of the drag force must increase with speed. Drag also depends on the size of the moving object; a large sail catches more wind than a small sail. In most cases, the drag force can be calculated as

$$\vec{F}_D = -\frac{1}{2} C_D \rho A_{cs} v^2 \hat{v} \tag{15}$$

where

- ρ is the density of the surrounding medium
- C_D is the drag coefficient, an empirically determined dimensionless number dependent on the object's shape
- A_{cs} is the cross-sectional area of the object (the area of a shadow projected onto the ground)
- v is the speed of the object relative to the surrounding medium
- $\hat{v} = \frac{\vec{v}}{|v|}$ is a unit vector indicating the *direction* of the object's velocity

A falling object will accelerate as a result of the gravitational force,

$$\vec{F}_g = m\vec{g} \tag{16}$$

where m is the mass of the object and $\vec{g}$ is the gravitational acceleration. From Newtonian mechanics, the net force acting on an object is always the sum of individual forces,

$$\vec{F}_{net} = \sum_i \vec{F}_i = m\vec{a} \tag{17}$$

where $\vec{a}$ is the acceleration of the object, itself. An object's **terminal velocity** is the maximum velocity reached when falling through a fluid. It is strongly dependent upon the character of the fluid and the shape of the object, as indicated in Equation (14).

3.2 Experiment

Consider an experiment designed to examine the relationship between mass and terminal velocity. A long tube is filled with glycerine and fixed to a stand. A series of smooth spheres with identical sizes but different densities are dropped in the fluid. A camera is mounted on a tripod, and a video of each sphere falling through the fluid is recorded. Digital object tracking software is used to determine the terminal velocity of each sphere from the videos. The data and fixed parameters are organized below.

m (g)	$\|v_T\|$ (cm/s)
5	24.22
10	34.01
15	42.87
20	49.28
25	55.47
30	58.44
35	63.45
40	68.03

- Glycerine density: $\rho = 1.25$ g/cm^3
- Terminal velocity uncertainty: $\frac{\delta(v_T)}{|v_T|} \cdot 100\% = 2.5\%$
- Absolute mass uncertainty: $\delta m = 0.5$ g
- Gravitational acceleration: $g = 981$ cm/s^2
- Cross-sectional area: $A_{cs} = 28 \pm 1$ cm^2

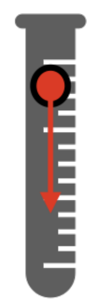


Figure 10: A simple diagram of the setup with the red arrow showing the direction of the sphere's velocity.

3.3 Theoretical Model

In this section, you will use concepts from your mechanics class to find an equation that predicts the behavior of the system. All of the relevant equations are given in the preceding sections. **Show your work**. If you need scratch paper, ask your TA.

1. (2 pt) Draw a free-body diagram showing the forces acting on a falling sphere, and use Equation 17 to get an expression for the sphere's acceleration in terms of C_D, ρ, A_{cs}, v, and m. *Hint:* this problem is one dimensional, so you can omit the vector symbols as long as the directions of any forces are indicated by positive or negative signs.

2. (2 pt) What is the acceleration of the sphere when it reaches terminal velocity?

3. (2 pt) Express the terminal velocity as a function of the mass.

4. (2 pt) To prepare for analysis, we want to translate this equation to the language of curve.fit by defining the x and y variables. The terminal velocity is the dependent variable, so define $y = |v_T|$. The independent variable is mass, so define $x = m$. Express your answer to the previous question in the form $y = Ax^B$. *Hint*: $\sqrt{x} = x^{0.5}$.

5. (2 pt) Write an equation for the fit parameter A in terms of g, ρ, C_D, and A_{cs}. What is your expected value of the fit parameter B?

3.4 Power Law Fit

The objective of this section is to determine if the experimental data are in agreement with the relationship between $|v_T|$ and m predicted by the theory. This procedure is identical to the ideal gas example, in which a power law fit type was used. By comparing the exponent B from the fit to the expected value, you can determine if the experiment agrees with the theory.

Tip: always carry out all algebra symbolically. Do not plug in numbers in place of symbols until you are done with any manipulation. Otherwise, you risk making calculation errors which can take a significant amount of lab time to find.

1. (3 pt) Use curve.fit to plot the terminal velocity data with "Power Law" as the fit type. Include error bars.

2. (2 pt) Compare the parameter B with the expected value determined by the theory. Report $B \pm \delta B$ and calculate the relative error e_B as a percent. Does the experiment agree with the theory?

3. (2 pt) Use the fit parameter A to calculate the drag coefficient C_D for a sphere.

4. (3 pt) Determine the uncertainty in the drag coefficient δC_D. Assume that the uncertainties in fluid density and gravitational acceleration are negligible. You will need to make use of Equation 4.

Lab Report Format Guide

Please try to organize your lab report according to the following outline. Lab reports do not need to be excessively long. The best lab reports are short and concise, containing only the information necessary to tell a complete story of your time in the lab. **Make sure that your report is written such that anyone could follow without referencing this document**.

1 Experiment Introduction

Write 3-5 sentences introducing the big-picture objectives of the lab. This sets the stage for a reader, making the subsequent sections easier to follow.

2 Block Density

Introduce what you did in this part of the lab. Include anything you find relevant from the theory section pertaining directly to this part of the lab. This is to give structure to your writing so that readers understand the purpose of this part of your experiment.

2.1 Ruler Method

Introduce any important details from the theory section relevant to this part of the experiment.

1. Methods (a *brief* list of steps completed in this part of the lab). Explain how instrument uncertainties were determined)
2. Data (in tables)
3. Calculations, Analysis, and Results (always show your work)

2.2 Digital Caliper Method

Follow the same format as previous section

2.3 Summary Questions

Answer the questions from §2.3

3 Terminal Velocity

Briefly introduce what you did in this part of the lab.

3.1 Theoretical Model

Answer the questions in §3.3

3.2 Power Law Fit

Explain the purpose of this part of the lab. Discuss your analysis and answer the questions. **Include your plot from curve.fit**. Write a short summary of what you learned in this experiment.