EC404 ps6

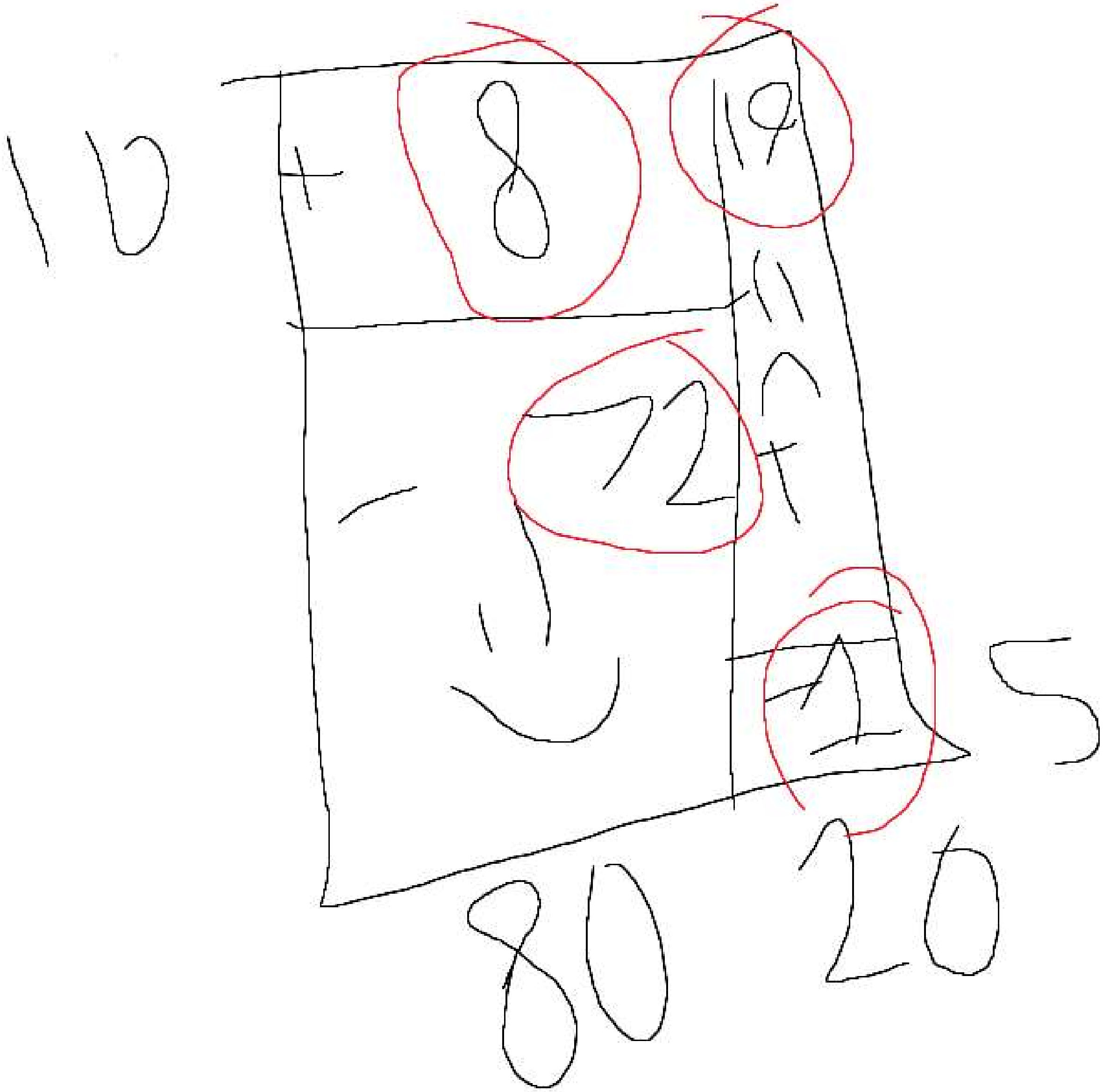
Question 1:

This question asks you to extend the medical-test example from class. Suppose there is a medical condition that afflicts 20% of the population. There is a test for this condition, and the reliability of this test can be characterized as follows:

- false-positive rate = 10% — for those who do not have the condition, 10% will test positive.
- false-negative rate = 5% — for those who have the condition, 5% will test negative.

(a) If you receive a positive test result, what is the likelihood that you have the condition? If you receive a negative test result, what is the likelihood that you do not have the condition?

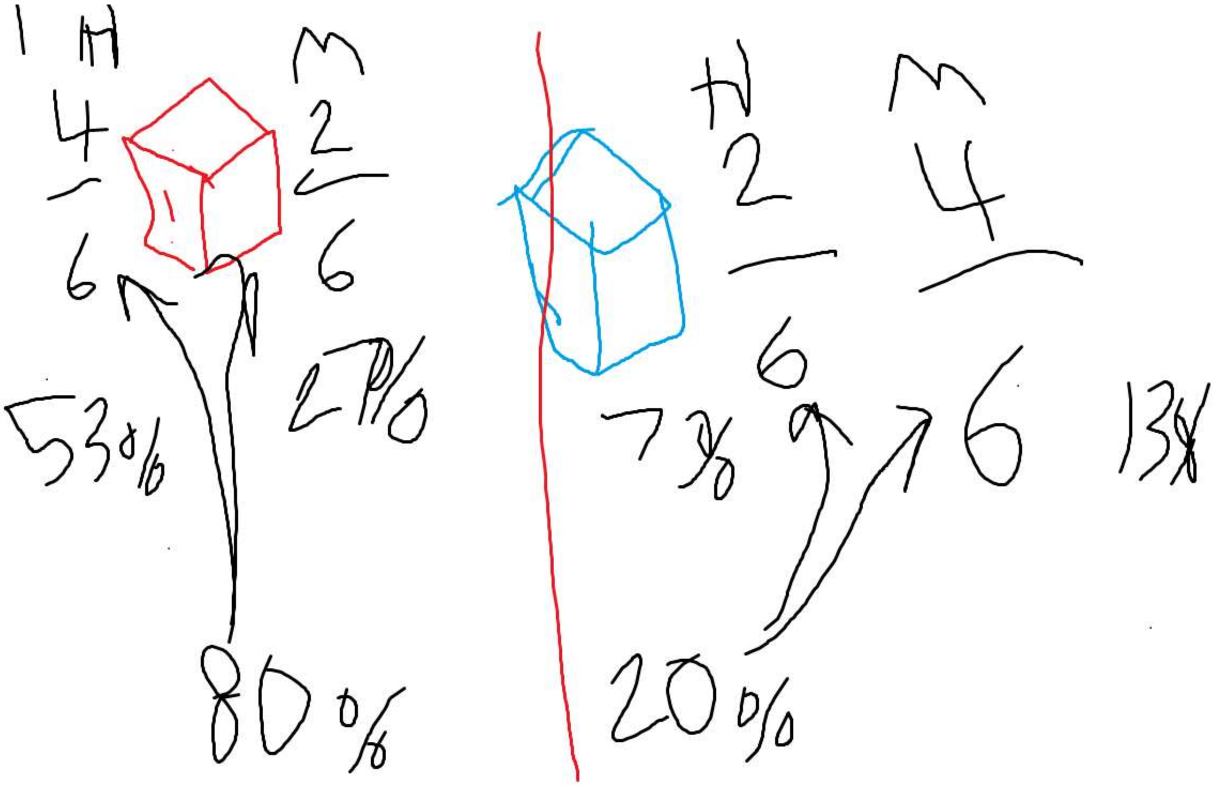
(b) Repeat part (a), except now assume that the false-positive rate is 5% and the false-negative rate is 10%.



(a) P(sick|+) = P(sick)P(+|sick)/P(+) = .19/(.08+.19) = 70%

(b) = .18/(.04+.18) = 81%

Q2:



(a) Certainly not over-infrence, since 50/50 is the most conservative result. This is conservativism as this outcome is more common for red and the base rate is already 80%. Still not the worst answer, since the small sample size leaves things close to 50/50

(b) This person judges very accurately. After 3 rolls, 2H and 1 M has a 14.8% chance of occcuring for red and 7.4% chance of occuring for blue, yielding a 14.8/22.2 = 66% chance of occuring when the base rate is 50/50. Alongside the base rate of 80% red, the actual probability of this outcome should be even greater than 80%. Perhaps there is some over-infrence from small samples, but they're in the right direction.

(c) This is likely an example of base rate neglect. Even with a 50/50 base rate this has a $(2/6)^4(4/6)^6$ portion for red and $(4/6)^4(2/6)^2$ portion for blue, yielding an 80% chance of it being red. In reality the likelyhood of it being red is much higher than 9/10

Question 3:

Suppose there are two types of coins, heads-biased coins and tails-biased coins. A heads-biased coin has a $3/4$ probability of a heads, while a tails-biased coin has a $1/4$ probability of heads. The proportion of all coins that are heads-biased is $1/7$.

Suppose that we flip a coin twice and it comes up HH.

(a) For a standard Bayesian information processor:

(i) What is the person's posterior probability that the coin is heads-biased?

(ii) What is the person's forecast for a third flip being H?

Prior = 1/7 HB and 6/7 TB Posterior = after result

Probability of HB getting this result: $9/16 = .56$
Probability of TB getting this result: $1/16 = .07$
Probability of this outcome: $1/7 * 9/16 + 6/7 * 1/16 = 15/112$
Posterior Probability of HB: $\dfrac{1/7 * 9/16}{15/112} = 3/5 = .60$

Prob of H = Prob of H from HB + Prob of H from TB = $3/5 * 3/4 + 2/5 * 1/4 = 11/20 = 0.55$

(b) For an ($N = 8$)-Freddy (as defined in class):

(i) What is the person's posterior probability that the coin is heads-biased?

(ii) What is the person's forecast for a third flip being H?

More generally, on n^{th} draw (where $n \geq 2$):

- If $(n-1)^{\text{st}}$ draw is x, N-Freddy believes $\Pr(x) = \frac{pN-1}{N-1}$.
- If $(n-1)^{\text{st}}$ draw NOT x, N-Freddy believes $\Pr(x) = \frac{pN}{N-1}$.

N-Freddy only updates probability based on the previous draw.

For (i) we do bayes to update probabilites

HH|HB = $3/4 * \Pr(3/4) = 3/4 * \frac{3/4(8)-1}{7} = 15/28 = .54$ not too far off
HH|TB = $1/4 * \Pr(1/4) = 1/4 * \frac{1/4(8)-1}{7} = 1/28 = .04$

Freddy's Bayes prob of HB: $\frac{.54*1/7}{.04*6/7+.54*1/7} = 0.69$

Third H: $.69 * \frac{3/4(8)-1}{7} + (1-.69)\frac{1/4(8)-1}{7} = 0.54$ Suprisingly accurate

(c) Repeat parts (a) and (b) when the proportion of all coins that are heads-biased is 6/7.

(d) How do Freddy's forecasts compare to a Bayesian's forecasts? Provide some intuition for your conclusions.

(c)
Bayes: Probability of this outcome: $6/7 * 9/16 + 1/7 * 1/16 = 55/112$
Posterior probability of HB: $\frac{6/7*9/16}{55/112} = 54/55 = .98 < br > Prob(3H|2H)$:3/4 * .98 + 1/4 * .02 \approx 3/4 $

Freddy: Probability of this outcome: $6/7 * (3/4 * 5/7) + 1/7 * (1/4 * 1/7) = 13/28$
Posterior probability of HB: $\frac{6/7*(3/4*5/7)}{13/28} = 90/91 = .98$ Same answer!
Prob(3H|2H): $5/7 * .98 + 1/7 * .02 = .72$ A bit lower than the actual value

(d)
8-Freddy's forecasts aren't too far off compared to the actual values, but in the original scenario he doesn't properly account for base-rates and overadjusts his belief that the die is HB. 8-Freddy tends to undervalue the third heads each time since his gambler's fallacy pushes him to expect a tails next.

Question 1:

Suppose that Lisa and Maggie both have "social-welfare preferences" of the form introduced by Charness & Rabin (that we discussed in class). They differ, however, in that Lisa takes a utilitarian view of social welfare (she has $\delta = 0$) while Maggie takes a maximin view of social welfare (she has $\delta = 1$).

(a) Solve for Lisa and Maggie's behavior in the Prisoners' Dilemma for the case when they believe that their opponent is playing C (use the version of the Prisoners' Dilemma from class).

$$\delta = 0 \implies u^1(x_1, x_2) = x_1 + \lambda * (x_1 + x_2)$$
$$\delta = 1 \implies u^1(x_1, x_2) = x_1 + \lambda * \min\{x_1, x_2\}$$

We'll use the standard prisoners dilema with $3,3;0,4;4,0;1,1$.

Other agent chooses C: $\delta = 0$ Lisa has $u(c) = 3 + \lambda(3+3) = 3 + \lambda 6$ vs $u(D) = 4 + \lambda 4$
Comparing the two yields the indiffrence point $3 + \lambda 6 = 4 + \lambda 4 \implies \lambda = 1$
For $\lambda > 1$, Lisa will choose C

$\delta = 1$ Maggie has $u(c) = 3 + \lambda(3)$ vs $u(D) = 4$
As such, Maggie chooses C for $\lambda > 1/3$

(b) Solve for Lisa and Maggie's behavior in the Dictator Game.

Without loss of generality, lets say P = 1

Lisa $u(s) = (1 - s) + \lambda$ Lisa doesn't share $s = 0$

Maggie $u(s) = (1 - s) + \lambda X(s)$ where $X(s) = \begin{cases} s & s \leq 0.5 \\ 1 - s & s > 0.5 \end{cases}$
The minimum Maggie can earn is 1, and earns the most due to sharing when $0.5 + \lambda 0.5$, Maggie will shre for $\lambda \geq 1$

(c) Solve for Lisa and Maggie's behavior in the role of Player 2 in the Ultimatum Game when they are offered a share $s \leq 1/2$.

- Player 1 (the proposer) offers a share $s \in [0, 1]$.
- Player 2 (the responder) then accepts or rejects this offer.
- If Player 2 accepts, payoffs are $x_1 = (1 - s)P$ and $x_2 = sP$; otherwise $x_1 = x_2 = 0$.

With $P = 1$, Lisa's earnings are $s + \lambda$ for accepting and 0 for rejecting, so she always accepts

Maggie earns $s + \lambda s$ for accepting and 0 for rejecting, so she always accepts

(d) To what extent can social-welfare preferences explain experimental results in the Prisoners' Dilemma, the Dictator Game, and the Ultimatum Game?

Prisoner's and Dictators seems to align well with the real world. Different outcomes in Dictator Game and Prisoners Dilema are well explained by varying the λ value, as a variety of choices make sense. The Ultimatum game includes a bit more revenge than what these models explain.

Question 2: SKIPPED!

Question 3:

Consider a simple dictator game in which Player 1 has 4 options from which to choose:

$$(A)\ (\$50, \$50) \qquad (B)\ (\$75, \$140) \qquad (C)\ (\$50, \$200) \qquad (D)\ (\$75, \$0)$$

How would Lisa, Maggie, and Bart behave in this game? Provide some intuition for your answers.

Note: You should specify how Lisa and Maggie's behavior depends on their λ, and how Bart's behavior depends on his α and β.

We can calculate Lisa and Maggie's behavior in this game, but no matter what Lisa will choose B or C depending on her λ and Maggie will choose B, since she wants to maximizer the smallest number and her own utility.

Bart does something

Question 4 SKIPPED. Sorry, ran out of time. I did most of it though!