Free Fall

Experiment Three

Physics 191
Michigan State University

Before Lab

- **Due at the beginning of class (10 pt)**: answer pre-lab theory questions in Section 1
- Watch a short Crash Course video on motion in one dimension: https://youtu.be/ZM8ECpBuQYE
- Bring a copy of the Fundamentals lab guide to class and review error analysis and curve fitting concepts

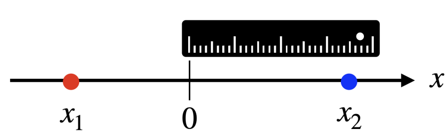
Experiment Overview

In this two-week experiment, you will explore one-dimensional motion under a specific set of assumptions that define free fall. You will conduct your first experiment on physical motion and apply techniques from the first two labs to analyze measurements.

- Conduct an experiment using a *Behr free fall apparatus* to record the position of an object falling under the influence of gravity
- Use curve.fit to test the theoretical free-fall model and determine parameters characterizing motion
- **Produce a professionally formatted lab report**:
  - Communicate the details of your experiment to a general scientific reader
  - Introduce each section to provide context for your work
  - Include captions describing and interpreting plots
  - Show all relevant calculations (handwritten or typed)

Kinematics

All of the experiments in this class explore mechanics, the branch of physics describing relationships between matter, forces, and motion. **Kinematics** is a subfield of mechanics concerned only with physical movement, independent from any forces responsible for motion. The time dependance of an object's position, velocity, and acceleration completely characterize its motion. Understanding kinematics amounts to answering: where is it? is it moving? is its movement changing?



The first step in measuring kinematic quantities is to choose a coordinate system. In a lab, the experimenter must choose a fixed location from which to make measurements. Once a coordinate system is chosen, the location of an object can be measured. The figure above shows a simple one-dimensional coordinate system with the position 0 defining the origin and an arrow indicating the positive direction. The point x_1 is left of of the origin, so the measured value would be assigned a negative sign. Conversely, the point x_2 would take a positive value. Motion from point x_1 to x_2 is said to be in the *positive direction*.

The definitions of the kinematic quantities are given in Table 1. If a smooth function $x(t)$ describing the position as a function of time is known, then the *instantaneous* velocity and acceleration can be determined by differentiation. Average values can be obtained from discrete points (t_i, x_i) without explicit knowledge of a function for position.

Quantity	Instantaneous	Average
Position	x	$\bar{x}$
Velocity	$v_x = \frac{dx}{dt}$	$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{\Delta t}$
Acceleration	$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{x2} - v_{x1}}{\Delta t}$

Table 1: Definitions of kinematic quantities in one dimension. Δ should be read is "change in."

Free Fall

A dynamic description of motion involves the analysis of forces responsible for motion. An object in **free fall** is acted on *only* by the gravitational force. In 1687, Isaac Newton published a mathematical description of the gravitational force. **Newton's law of gravitation** states that the magnitude of the gravitational force between two objects of mass m_1 and m_2 is

$$F_G = G\frac{m_1 m_2}{r^2} \tag{1}$$

where r is the distance between their centers of mass and G is a number called the gravitational constant (not to be confused with the gravitational acceleration).

The dependance on $1/r^2$ indicates that the strength of the gravitational force decreases as the separation increases. Figure 1 shows the earth with grossly exaggerated mountains peaked at $\vec{r}_2$. According to Equation 1, the gravitational force acting on a person at the top of the mountain should be less than the force experienced at the base, because $|\vec{r}_2| > |\vec{r}_1|$.

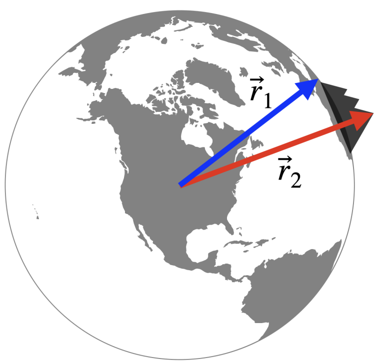


Figure 1

The gravitational force experienced by an object is responsible for **weight**. Apparently, your weight decreases as you move higher in elevation. In everyday experience, this difference is imperceptible. In contrast to Figure 1, the real difference between the highest and lowest points on Earth is only 0.2% of the radius, amounting to a 0.4% variation in weight.

An alternative expression for the magnitude of the gravitational force is used for objects on the surface of the earth:

$$F_g = mg \tag{2}$$

where m is the mass of the object and g is the free fall acceleration, or **acceleration due to gravity**.

To understand the significance of g, imagine dropping a steel ball from the top of a building on a planet with no atmosphere. Newton's second law gives the net force acting on the ball,

$$F_{net} = ma = F_g \;\Rightarrow\; ma = mg$$

where a is its acceleration. The previous line shows that the mass This leads to the **free-fall condition**,

$$a = g. \tag{3}$$

The acceleration of an object in free fall is equal to the acceleration due to gravity, a value which is effectively constant. This is a special case of motion, in which any other forces, such as drag, are neglected.

The Kinematics of Free Fall

This section presents a derivation of the position and velocity kinematic relationships for an object in free fall. A key aspect allowing g to be accurately measured in this experiment is that it does not change with time. There is no instrument in the lab to directly measure acceleration. Instead, we can measure the position at fixed times. In order to determine the acceleration, a relationship between position and acceleration must be established. This is accomplished directly from the free-fall condition

$$a = \frac{dv}{dt} = g \tag{4}$$

where the relationship between velocity and acceleration has been included[1]. This is the simplest type of **ordinary differential equation**. While it may sound intimidating, a differential equation is simply a relationship between a function and its derivatives.

Table 1 shows that position, velocity, and acceleration are related by derivatives. The first step is to start with the acceleration, a constant, and integrate to find the velocity as a function of time:

$$a = \frac{dv}{dt} = g \quad \rightarrow \quad dv = g \cdot dt \quad \rightarrow \quad \int dv = \int g \cdot dt$$

If the initial time is $t = 0$ and the initial velocity is $v(t = 0) = v_0$, this integral can be solved explicitly as

$$\int_{v_0}^{v} dv' = g \int_0^t dt'$$

where the constant g is pulled outside of the integral, and the variables are changed to $v \rightarrow v'$ and $t \rightarrow t'$ in order to avoid confusion with the integration limits. This gives

$$v - v_0 = gt - 0 \quad \rightarrow \quad v(t) = v_0 + gt.$$

The last equation should look familiar: it is the velocity kinematic equation. By understanding this procedure, you may forego any memorization of kinematic relationships. This process can be performed a second time to determine the position as a function of time, the actual quantity measured in the lab.

$$v = \frac{dy}{dt} = v_0 + gt \quad \rightarrow \quad dy = (v_0 + gt)\, dt \quad \rightarrow \int dy = \int (v_0 + gt)\, dt$$

Once again, an initial condition is required to evaluate the definite integral. The position at $t = 0$ is defined to be $y(t = 0) = y_0$, and the resulting integral is

$$\int_{y_0}^{y} dy' = \int_0^t (v_0 + gt')\, dt'$$
$$y - y_0 = v_0 t + \frac{1}{2} g t^2$$

giving the familiar position kinematic equation $y(t) = y_0 + v_0 t + \frac{1}{2} g t^2$. The functions $v(t)$ and $y(t)$ define the theoretical models you will use in this experiment.

$$\boxed{\begin{aligned} y(t) &= y_0 + v_0 t + \frac{1}{2} g t^2 \quad (5) \\ v(t) &= v_0 + gt \quad (6) \end{aligned}}$$

[1] The vector component symbols may be dropped for motion in one dimension. Just make sure you realize that $v = v_y$ (i.e. a vector component, not a magnitude). The sign of v depends on the direction of motion.

1 Theory Questions

1. (1 pt) Under the following approximations, estimate the gravitational acceleration g on the surface of the earth (*Hint*: start with $F_{net} = ma = F_G$).
   - Earth may be approximated as a sphere with uniform density
   - The average radius of the earth is $\bar{r} = 6.371 \times 10^6$ m
   - The estimated mass of the earth is: $M = 5.972 \times 10^{24}$ kg
   - Gravitational constant: $G = 6.674 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
2. (1 pt) The established standard acceleration due to gravity is $g_0 = 9.807$ m/s^2. Is this the exact number found in Question 1? What factors may cause a discrepancy between the calculated and standard values?
3. (1 pt) The International Space Station (ISS) orbits the earth at an altitude of about 410 km. Use Equation 1 to calculate the gravitational force acting on an 80 kg astronaut. Is there gravity on the space station?
4. (1 pt) What prevents ISS and other satellites from crashing into the earth?

Common Fit Equations

$$\text{Linear} : y = mx + b$$
$$\text{Quadratic} : y = ax^2 + bx + c$$
$$\text{Inverse} : y = A + \frac{B}{x}$$
$$\text{Power Law} : y = Ax^B$$
$$\text{Exponential} : y = Ae^{bx} + C$$

5. In the lab, you will measure the position of an object as it falls. According to the theory, a plot of your data should agree with $y(t)$ in Equation 5.
   (a) (1 pt) Sketch a graph of $y(t)$.
   (b) (1 pt) Which fit type represents this relationship?
   (c) (1 pt) Recall that the fit parameters are the constants in the fit equation which are tuned to generate a best-fit curve. How are the parameters from the fit equation related to the physical quantities y_0, v_0, and g?
6. You will use your measured positions to calculate the average velocity at various points as the object falls. According to the theory, your velocity plot should agree with $v(t)$ in Equation 6.
   (a) (1 pt) Sketch a graph of $v(t)$.
   (b) (1 pt) Which fit type represents this relationship?
   (c) (1 pt) How are the parameters from the fit equation related to the physical quantities v_0 and g?

The Behr Free Fall Apparatus

Before beginning, your TA will demonstrate how to use the device. The Behr free fall apparatus records the position of a metal cylinder as it falls. Initially, the cylinder is suspended at the top of the device by an electromagnet. When the electromagnet is switched off, the cylinder drops and an electric spark is generated at precise intervals of $\Delta t = 1/60$ s. The spark travels through the cylinder, leaving small burn marks on a piece of tape as shown on the right side of Figure 2.

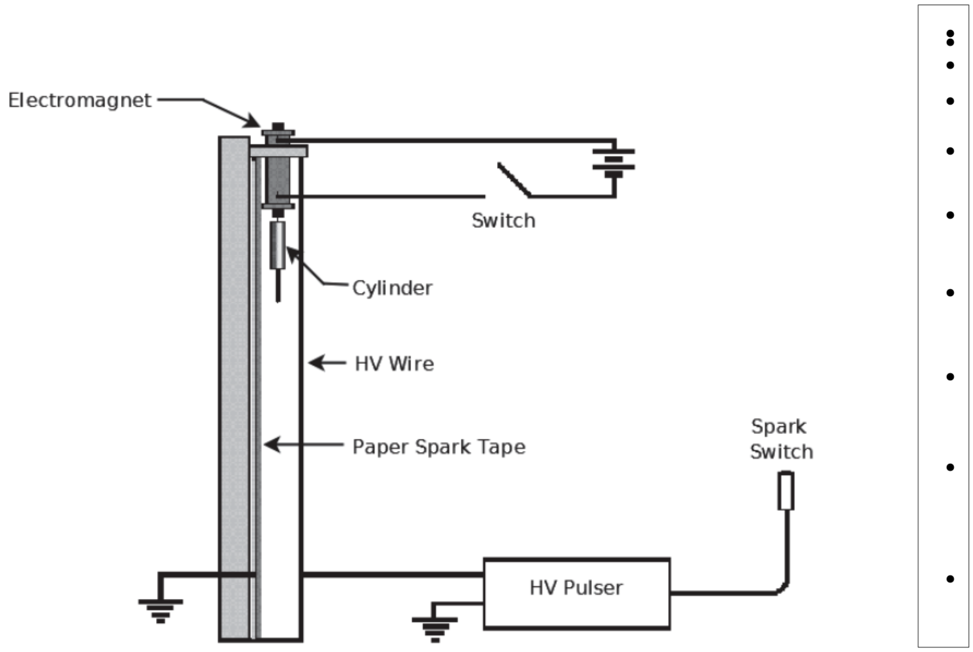


Figure 2: On the left is a schematic of the Behr free fall apparatus. The tape generated with black spots produced by the electric spark is shown on the right. Acceleration of the cylinder causes the distance between sparks to increase at it falls.

You and your partner should produce one tape. While the burn marks are very faint, close inspection should reveal about 30 small gray spots. The distances between marks can be measured directly using a ruler or meter stick. Your objective is to measure the positions of the marks and use the information to determine $g \pm \delta g$ using two different models.

2 Position Model

In this section, you will model your measurements of position as a function of time. The analytical procedure should emulate the terminal velocity analysis practiced in Experiment One. Your measurements can be plotted as a discrete set of data points. With the appropriate choice of fit type, curve.fit will find a continuous function that best models your data. The parameters and uncertainties determined by your fit can be used to calculate $g \pm \delta g$. Before starting your measurements, consider the following:

- Notation:
  - Initial position: y_0
  - The i^{th} position measured from the initial position: y_i (e.g. y_3 is the third mark from y_0)
  - The distance between mark i and the previous mark is Δy_i (e.g. Δy_2 is the distance between y_1 and y_2, or $\Delta y_2 = y_2 - y_1$)
  - In your report and spreadsheets, Δ can be replaced with D
- Choose the first and last marks to include in your measurements. Think about why you may not want to include the very first or last discernible marks.
- Each mark is separated by a time interval Δt. The **total elapsed time** at position y_i is $t_i = i \cdot \Delta t$.
- Construct a table to organize your measurements. This should include columns for t_i, y_i, δy_i, and any other values you find pertinent.

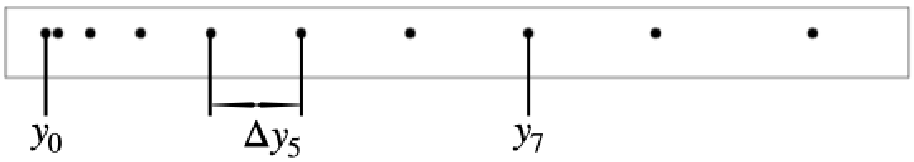


Figure 3: An example of a tape with the initial mark y_0, the displacement $\Delta y_5 = y_5 - y_4$, and an additional mark y_7 shown.

Think carefully about how to measure the positions. There are two different methods with which the positions y_i can be determined.

Method A: Measure the total distance to each point directly from y_0 using a meter stick. For example, you would determine y_3 by directly measuring the distance from y_0.

Method B: Measure the separation between points using a ruler and sum up individual distances between points. For example, $y_3 = \Delta y_1 + \Delta y_2 + \Delta y_3$.

You do not need to use both methods; instead, measure using the method that you think is best. In order to help you decide, answer the following questions. As always, write your answers in complete sentences.

1. (1 pt) To make any distance measurement, you need to align your scale with two adjacent marks. Determine your uncertainty associated with this type of measurement.
2. (1 pt) Following Method A, what is the uncertainty in a measurement of the position y_3?
3. (1 pt) Following Method B, what is the uncertainty in a measurement of the position y_3?
4. (2 pt) Which method is better? How are you defining "better"?

You are now in a position to perform your measurements. The remainder of your lab report should be written like a technical document. Try to be organized and methodological, you do not need to rush through this experiment. **Explain your thought process** and show your work[2].

5. (2 pt) In your own words, write a short paragraph explaining your objective for this part of the experiment.
6. (2 pt) Choose the appropriate fit equation to model your data. Write the relationships between the physical quantities y_0, v_0, and g and the parameters a, b, and c.
7. (1 pt) Construct a table with columns for t_i, y_i, and δy_i. Include units in the column headers. Assume the relative uncertainty in the spark timer is negligible. *Note*: t_i is the total elapsed time at the i^{th} point, $t_i = i \cdot \delta t$.
8. (3 pt) Generate a plot including vertical error bars, a descriptive title, axes labels with units, the appropriate fit, and the residual plot as generated by curve.fit
9. (2 pt) Interpret your residual plot. If there are outliers, remove the points and make a new plot. Explain your reasoning for removing any points.
10. (2 pt) Use the results from your fit to determine $y_0 \pm \delta y_0$ and $v_0 \pm \delta v_0$. Explain why these values may not be zero.
11. (2 pt) Use the results from your fit to determine $g \pm \delta g$.

[2] When doing math, avoid skipping steps. You are more likely to introduce *human error* (mistakes) when you do not write out your work. Finding calculation errors can be very tedious and time consuming.

3 Velocity Model

There are two kinematic relationships derived in the theory section. With the position model, you plotted points which were *directly measured* and modeled by $y(t)$. The objective of this section is to follow a similar procedure to model $v(t)$ (Equation 6). There is no way to directly measure the instantaneous velocity in this experiment; instead, an indirect procedure can be used by *calculating* the velocities using the *measured* positions. The average velocity over an interval can be calculated from measurements of the position using

$$\bar{v}_i = \frac{\Delta y_i}{\Delta t}. \tag{7}$$

This is straight forward to calculate: you measure the distance between two marks and divide by the time interval. For example, the average velocity over the fifth interval ($i = 5$) shown in Figure 3 is

$$\bar{v}_5 = \frac{\Delta y_5}{\Delta t} = \frac{y_5 - y_4}{\Delta t}.$$

We would like to use the average velocities to approximate the instantaneous velocity at a specific moment in time. According to Figure 3, it would be a poor estimation to say $\bar{v}_1 \approx v(t_1)$. It is more likely that $\bar{v}_1 \approx v(t_1/2)$. This amounts to shifting the total elapsed time by $-\Delta t/2$.

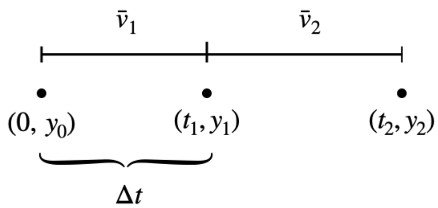


Figure 4: Average velocities shown for the first 2 complete intervals, with a total elapsed time of $t_2 = 2 * \Delta t$.

The table below shows how the total elapsed time t can be calculated in terms of the spark interval $\Delta t = 1/60$ s. Organized in this way, each average velocity is approximated to occur at the center of each time interval.

t_i (s)	$\bar{v}_i$ (cm/s)
$\Delta t/2$	$\bar{v}_1$
$\Delta t/2 + \Delta t$	$\bar{v}_2$
$\Delta t/2 + 2\Delta t$	$\bar{v}_3$
$\vdots$	$\vdots$

You are now in a position to begin your measurements and analysis. You may want to make an additional set of measurements using a different method to minimize your uncertainty. Once again, consider the following measurement methods:

Method A: Directly measure the separation Δy_i between two marks using a ruler.

Method B: Measure the total distance to each mark from y_0 using a meter stick. Then calculate $\Delta y_i = y_i - y_{i-1}$ using the two measured values.

Once again, you should use the best measurement method. In general, error is minimized in measurement processes requiring the fewest calculations. Remember, you are making this measurement so you can calculate the average velocity using Equation 7

1. (2 pt) Think about the propagation of uncertainties, and choose the method you will use to measure the distances Δy_i. Explain your decision.

2. (2 pt) In your own words, write a short paragraph explaining your objective for this part of the experiment.

3. (2 pt) Use uncertainty propagation to find $\delta\bar{v}_i$. Assume the relative uncertainty in the spark timer is negligible. *Hint*: start by writing down the equation for $\bar{v}_i$.

4. (2 pt) Choose the appropriate fit equation to model your data. Write the relationships between the physical quantities v_0 and g and the parameters from your fit equation.

5. (1 pt) Construct a table with columns for t_i, $\bar{v}_i$, and $\delta\bar{v}_i$. Include units in the column headers.

6. (3 pt) Generate a plot including vertical error bars, a descriptive title, axes labels with units, the appropriate fit, and the residual plot as generated by curve.fit

7. (2 pt) Interpret your residual plot. If there are outliers, remove the points and make a new plot. Explain your reasoning for removing any points.

8. (2 pt) Use the results from your fit to determine $g \pm \delta g$.

4 Analytical Summary

Your summary should include the details of your experimental findings. Try to think critically about your results. All questions must be answered in complete sentences. You will need the equation for the statistical t-test,

$$t = \frac{|x_{ref} - x_m|}{\delta\bar{x}} \tag{8}$$

where x_m refers to the measured quantity and $\delta\bar{x}$ is its uncertainty.

1. (2 pt) Include a summary table comparing each model. Compare your results with the standard value of $g_0 = 980.7$ cm/s^2). Note that $e_g(\%)$ is the relative error in percent.

Model	g (cm/s^2)	δg (cm/s^2)	$\delta g/|g|$	$e_g(\%)$	t-test
Position					
Velocity					

2. (1 pt) Discuss which model (position or velocity) yielded a more precise result.

3. (1 pt) Discuss which model yielded a more accurate result.

4. (1 pt) A discrepancy is considered significant if $t \geq 2$. Do your t-test results from either method indicate a significant discrepancy? If so, what could be the cause?