What is condensed matter physics?

- $\frac{1}{3}$ of all physics
- large overlap with other fields
    - chemistry
    - bio
    - atomic physics
    - HE physics

      ⋮

why study it?

- It is the world!
- need it to explain properties of most materials
- it is technol. important

electr. properties → electronics → iPhone → ?

- it is fundamental!
- it is playground to study e.g. QM, HE physics...

our approach: start with individual atoms and build up

H atom → multi-e^- atoms → molecules → solids

Chapter 1 : Atoms and Molecules

Hydrogen Atom

McIntyre
Chap 7, 8, (9)

→ use H as a building block

interested in gross structure of H-like atoms

$q_n = Ze$

→ only electrostatics
(no spin
no relativistic effects
no magnetism)

find stationary states

$$H|E\rangle = E|E\rangle$$

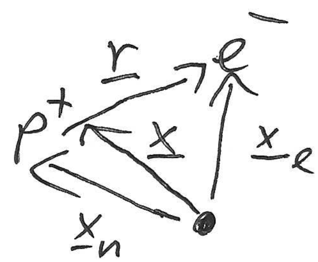
$$H = \underbrace{\frac{p_n^2}{2m_n}}_{\substack{E_{kin}\\ \text{nucleus}}} + \underbrace{\frac{p_e^2}{2m_e}}_{\substack{E_{kin}\\ e^-}} \underbrace{- \frac{Ze^2}{4\pi\varepsilon_0 |\underline{x}_e - \underline{x}_n|}}_{\text{interaction}}$$

$$= \left(-\frac{\hbar^2 \nabla_{\underline{x}_n}^2}{2m_n} - \frac{\hbar^2 \nabla_{\underline{x}_e}^2}{2m_e} - \frac{Z e^2}{4\pi\varepsilon_0 (\underline{x}_e - \underline{x}_n)} \right)$$

Key: choose clever coordinates!

new coordinates:

$$\underline{X} = \frac{m_e \underline{x}_e + m_n \underline{x}_n}{m_e + m_n} \qquad \text{center of mass}$$



$$\underline{r} = \underline{x}_e - \underline{x}_n \qquad \text{separation}$$

remove $\underline{x}_e, \underline{x}_n$ and replace with $\underline{X}, \underline{r}$

$$\frac{\partial}{\partial \underline{x}_e} = \frac{\partial \underline{X}}{\partial \underline{x}_e} \cdot \frac{\partial}{\partial \underline{X}} + \frac{\partial \underline{r}}{\partial \underline{x}_e} \cdot \frac{\partial}{\partial \underline{r}} \qquad \text{(chain rule)}$$

$$= \frac{m_e}{m_e + m_n} \frac{\partial}{\partial \underline{X}} + \frac{\partial}{\partial \underline{r}}$$

$$\nabla_{x_e}^2 = \left(\frac{m_e}{m_e + m_n} \right)^2 \nabla_X^2 + \nabla_r^2 + \frac{2m_e}{m_e + m_n} \frac{\partial^2}{\partial \underline{X} \, \partial \underline{r}}$$

$$\frac{\partial}{\partial \underline{x}_n} = \frac{\partial \underline{x}}{\partial \underline{x}_n}\frac{\partial}{\partial \underline{x}} + \frac{\partial \underline{r}}{\partial x_n}\frac{\partial}{\partial \underline{r}}$$

$$= \frac{m_n}{m_e + m_n}\frac{\partial}{\partial \underline{x}} - \frac{\partial}{\partial \underline{r}}$$

$$\nabla^2_{x_n} = \left(\frac{m_n}{m_e + m_n}\right)^2 \nabla_x^2 + \nabla_r^2 - \frac{2 m_n}{m_e + m_n}\frac{\partial^2}{\partial \underline{x}\,\partial \underline{r}}$$

$$\frac{1}{m_e}\nabla_{x_e}^2 + \frac{1}{m_n}\nabla_{x_n}^2$$

$$= \frac{m_e + m_n}{(m_e + m_n)^2}\nabla_x^2 + \left(\frac{1}{m_e} + \frac{1}{m_n}\right)\nabla_r^2$$

$$= \frac{1}{m_e + m_n}\nabla_x^2 + \frac{1}{\mu}\nabla_r^2 \qquad \mu = \frac{m_e m_n}{m_e + m_n}$$

(reduced mass)

Hamiltonian in new coordinates:

$$H = \underbrace{-\frac{\hbar^2}{2(m_e+m_n)}\nabla_x^2}_{H_k} \underbrace{-\frac{\hbar^2}{2\mu}\nabla_r^2 - \frac{Ze^2}{4\pi\varepsilon_0 |\underline{r}|}}_{H_r}$$

we find: $[H_k, H_r] = 0$

$[H_k, H] = 0$

$[H_r, H] = 0$

$\rightarrow$ means that there is a complete set of mutual eigenstates of H, H_k and H_r

H_k is kinetic energy of free particle $(e^- + p^+)$ (atom travelling through space)

$\rightarrow$ boring!

$H_r |E_r\rangle = E_r |E_r\rangle$ internal energy of atom

should have derived in PHY 471

$$-\nabla_r^2 = \underbrace{\frac{p_r^2}{\hbar^2}}_{\substack{\text{radial} \\ \text{kin } E}} + \underbrace{\frac{L^2}{r^2}}_{\substack{\text{tangential} \\ \text{kin } E}}$$

$\leftarrow$ total orb. angular momentum operator

radial momentum

$$p_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$$