PHY 491, Fall 2024 - Homework 1

DUE: Friday 09/06/24, 11:59pm

Problem 1.1 Positronium is a hydrogen-like exotic atom consisting of an electron and a positron (anti-electron) orbiting each other. For this system in its ground state

1.1.1 Calculate the reduced mass, Bohr radius, and binding energy and compare to hydrogen. (4 Points)

1.1.2 Sketch the electron and positron positions relative to the center of mass (qualitatively) and compare this situation to hydrogen. (4 Points)

Problem 1.2 The variational principle is a method to find an upper bound for the energy of the lowest-energy eigenstate (ground state) of a quantum mechanical system for which we are unable to obtain an exact solution of its time-independent Schrödinger equation. Using any resources of your choice (textbooks, online resources, ...), familiarize yourself with the method. A good starting point including some examples can for example be *D. Griffiths, Introduction to Quantum Mechanics, Chapter 7.* Use this method to then

1.1.1 Calculate the ground state energy of the hydrogen atom using the variational principle. Assume that the variational wavefunction is a Gaussian of the form $Ne^{-(r/\alpha)^2}$, with a normalization constant N and variational parameter α. (10 points)

1.1.2 Compare your solution to the exact ground state energy of hydrogen. (2 point)

Use the following integrals:

$$\int_0^\infty xe^{-x^2}\,dx = \frac{1}{2}; \int_0^\infty x^2e^{-x^2}\,dx = \frac{\sqrt{\pi}}{4}; \int_0^\infty x^4e^{-x^2}\,dx = \frac{3\sqrt{\pi}}{8} \tag{1}$$