

PSEUDOCODE

```
A[n]
d ← n
bool run ← true
while run is true
    run ← false
    for x ← 0 to d-2
        if A[x] > A[x+1]
            swap A[x] and A[x+1]
            run ← true
    d ← d-1
```

```
swap A[x] and A[y]
temp = A[x]
A[x] = A[y]
A[y] = temp
```

PROVING CORRECTNESS

Invariant:

The invariant is that any element below A[d] should be \leq any element above or equal to A[d]

Initialization:

In the first loop ($d = \text{size}$) it brings the largest element to the end of the array. This means all indexes larger than $d-1$ are sorted

Maintenance:

The sub-array of A[d] through A[size-1] is sorted and if we keep decrementing d by one and pushing back all elements smaller than the new largest elements from the unsorted smaller sub-array (A[0] through A[d-1]) in order to place this new element in the respective place in the sorted array above A[d] then, we will be left with a sub-array of A[d] through A[size-1] where all elements above A[d] are larger than those below the invariant is maintained.

Termination:

The final iteration will yield A[0] through A[size-1] sorted which means all elements below one another are smaller than that element. The invariant is withheld.

BEST CASE

Already sorted is the best case. This will run

```
1 A[n]
1 d ← n
1 bool run ← true
1 while run is true
1     run ← false
1     for x ← 0 to d-2
d-1         if A[x] > A[x+1]
0             swap A[x] and A[x+1]
0             run ← true
```

```
1      d ← d-1
```

```
0 swap A[x] and A[y]
0     temp = A[x]
0     A[x] = A[y]
0     A[y] = temp
```

A total of $d+6$ lines will run. Since d is a factor of n , this program is $\Theta(n)$.

WORST CASE

A reverse sorted algorithm is the worst case.

```
1 A[n]
1 d ← n
1 bool run ← true
d-1 while run is true
d-1     run ← false
d-1     for x ← 0 to d-2
(d-1)+(d-2)+.. if A[x] > A[x+1]
(d-1)+(d-2)+.. swap A[x] and A[x+1]
(d-1)+(d-2)+.. run ← true
d-1     d ← d-1
```

```
(d-1)+(d-2)+...      swap A[x] and A[y]
(d-1)+(d-2)+...      temp = A[x]
(d-1)+(d-2)+...      A[x] = A[y]
(d-1)+(d-2)+...      A[y] = temp
```

Since the highest factor here is $(d-1) + (d-2) + \dots$ which equals $n(n+1)/2 = (n^2 + n)/2$ which yields a $\Theta(n^2)$ as the worst case.

PROBLEM 2

a) Is $2^{(2n)} = \Theta(2^n)$, $\Omega(2^n)$, or $O(2^n)$?

no because $2^{(n*2)} = (2^n)^2$ which is not $O(2^n)$

b) Is $2^{(n+1)} = \Theta(2^n)$, $\Omega(2^n)$, or $O(2^n)$?

Yes because $2^{(n+1)} = (2^n)*2 = (2^n)*c = O(2^n)$

PROBLEM 3

$10^{12} * 60 * 60 * 2 = \text{operations in 2 hours} = 7.2 * 10^{15} = \text{totalCalcs}$

a) $200n^2 + 5n + 40000 = 7.2 * 10^{15}$

divide both sides 200

$n = \text{quadratic formula}$

$n = 5,999,999$

b) $n^3 + 3 = 7.2 * 10^{15}$

$n = \text{cube root}(\text{totalCalcs} - 3)$

$n = 193,097$

c) $10n^2 + 50000 = \text{totalCalcs}$

$$n = \text{Squareroot}(\text{totalCalcs}/10 - 5000)$$

$$n = 26,832,815$$

d) $n \log_2(n) = \text{totalCalcs}$

Really hard to solve without just pretending $\log_2(n)$ is close enough to 1 compared to n lets just guess it is like 50 because \log_2 of totalCalcs is 52 so we know its gonna be less but pretty close

$$50n = \text{totalCalcs} = 1.44 * 10^{14}$$

pretty close as the log base 2 of this is 47.03\

e)

$$2^{(2n)} = \text{totalCalcs}$$

$$2^n = \text{sqrt}(\text{totalCalcs})$$

$$n = \log_2(\text{sqrt}(\text{TotalCalcs}))$$

$$n = 26.3$$