Alumni Networking Night

Wednesday Jan 31

6 - 7:30 PM PST

Ever wondered what happens after grad? Want to meet professionals from diverse careers?

Register now for SCI Team's annual Alumni Networking Night to explore career options and the chance to connect with UBC BSc alumni!

SCI TEAM

sciteam.ubc.ca

https://science.ubc.ca/students/events/alumni-networking-night

UBC Faculty of Science

For this week

Lab 1 due next week, before the start of your next lab session (see syllabus for date/time)

- Wednesday’s and Thursday’s labs (snowed out!) are rescheduled for Feb 14 & 15, and will be due one week later

Late submissions: you can still turn in Pre-reading 1 by Sunday, HW1 by Wednesday.

- *Do not change your answers after submission*
- We will have a practice version available after the due date.

For next week: Pre-reading 2 is due Sunday. Our first quiz is Wednesday covering material through today (no HW next week).

Quiz 1 on Wednesday!

Starts at the beginning of class (be here a bit early to start on time!)

20 minutes, 6 multiple choice questions.

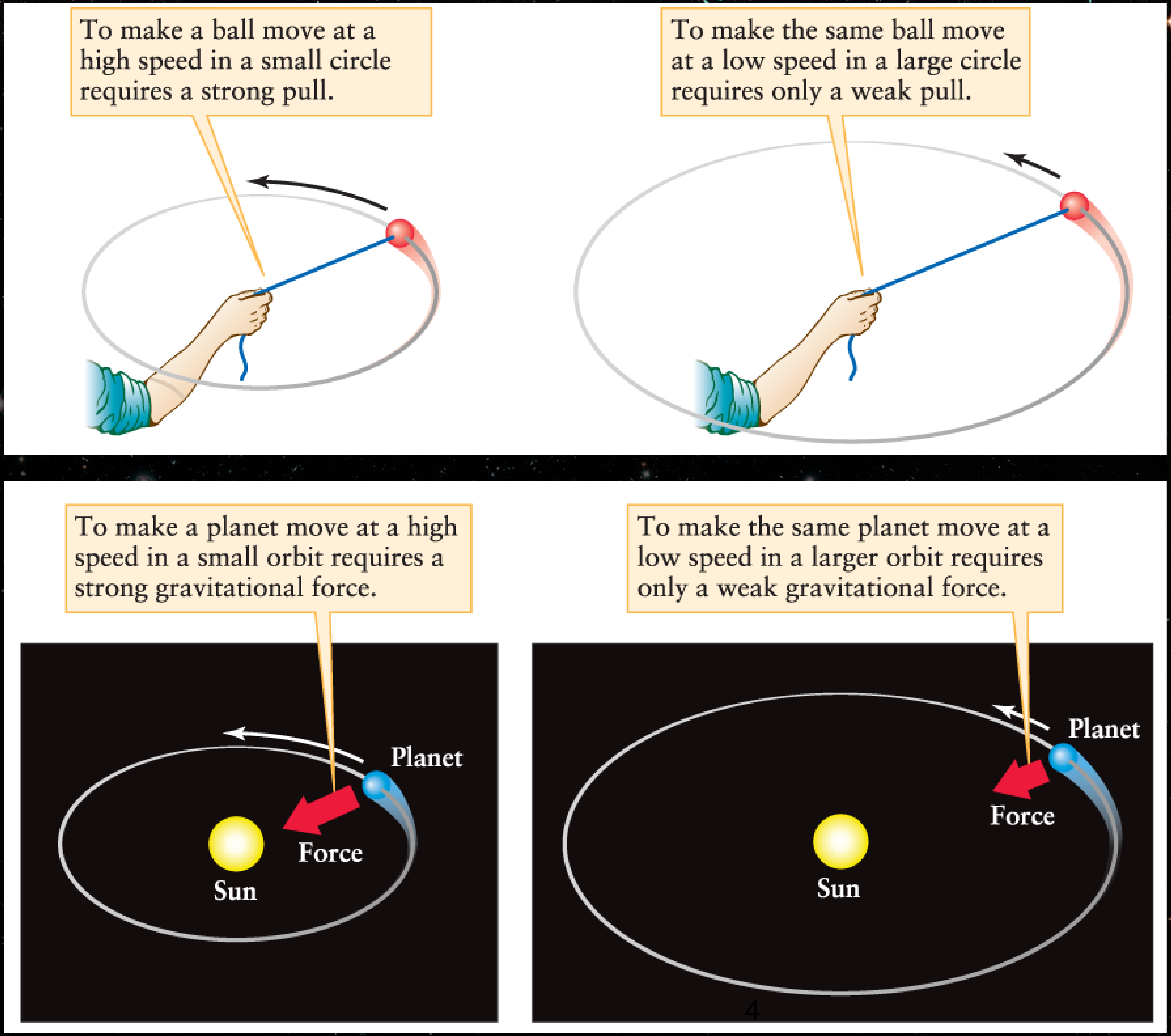
Bring a non-programmable calculator, a pencil, and a one-page formula sheet with anything you like on it.

Best ways to practice: review iClicker questions, homework questions.

Cut off point: end of today’s lecture

Gravitational force

We can tell how strong the force of gravity is from the orbit of two objects!



Universe, Geller et al.

Review: Newton's law of universal gravitation

Two objects attract each other with a force that is directly proportional to the mass of each object and inversely proportional to the square of the distance between them.

Mathematically, this is:

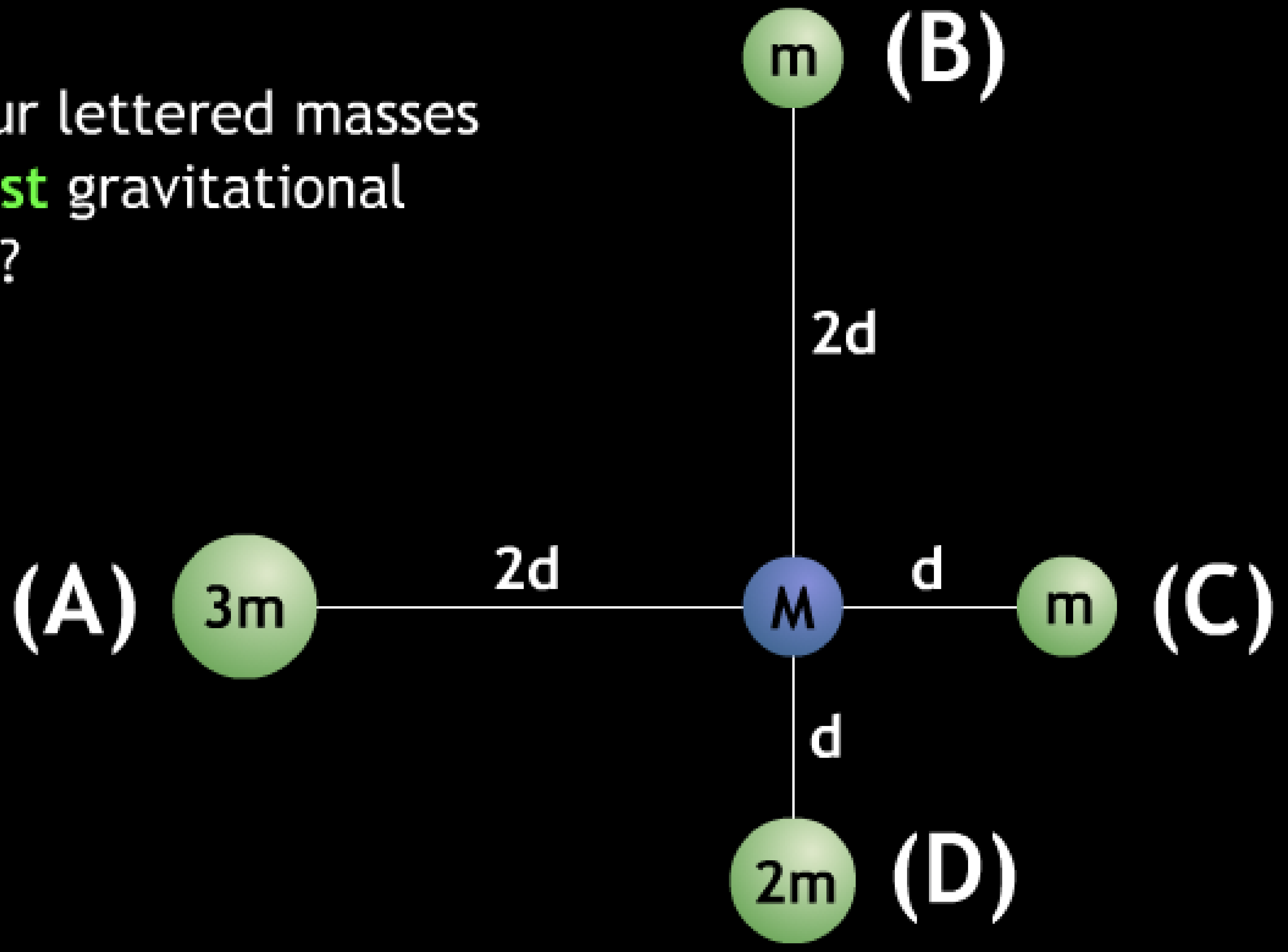
$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

where G is Newton's constant, m_1 and m_2 are the two masses, and r is the distance between the two objects.

$G = 6.67 \times 10^{-11}$ N m^2/kg^2

iClicker question

Which of the four lettered masses exerts the **largest** gravitational force on mass M?



Class Action (2020)

Let's weigh the Sun

An object moving in a circle experiences a centripetal force equal to

$$F_{centr} = \frac{mv^2}{r}$$

This must equal the gravitational force exerted by the Sun on the Earth:

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

How much does the Sun weigh?

G = 6.67 × 10^{-11} N m^2/kg^2 r = 1AU = 1.5×10^{11} m

First let's work out Earth's velocity

$$F_{centr} = \frac{mv^2}{r}$$

$$v = \frac{distance}{time} = \frac{2\pi r}{(365\ \mathrm{days})(86400\frac{\mathrm{s}}{\mathrm{day}})}$$

$$v = \frac{2\pi(1.5 \times 10^{11}\ \mathrm{m})}{(365\ \mathrm{days})(86400\,\frac{\mathrm{s}}{\mathrm{day}})} = 29{,}900\ \mathrm{m/s}$$

r = 1 AU = 1.5×10^{11} m

Now work out Sun's mass

An object moving in a circle experiences a centripetal force equal to

$$F_{centr} = F_{grav} \quad : \quad \frac{m_E v^2}{r} = G\left(\frac{m_E m_\odot}{r^2}\right)$$

$$m_\odot = \frac{v^2 r}{G} = \frac{(29{,}900\ \mathrm{m/s})^2 (1.5 \times 10^{11}\mathrm{m})}{6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}}$$

$$m_\odot = 2\times10^{30}\ \mathrm{kg}$$

Compare to $m_E = 6\times10^{24}$kg

Let's weigh the Milky Way

Stars at the edge of the Milky Way orbit the galaxy at 220 km/s.
They are 18 kpc from the centre.

$$F_{centr} = F_{grav} \quad : \quad \frac{m_* v^2}{r} = G\left(\frac{m_* m_{MW}}{r^2}\right)$$

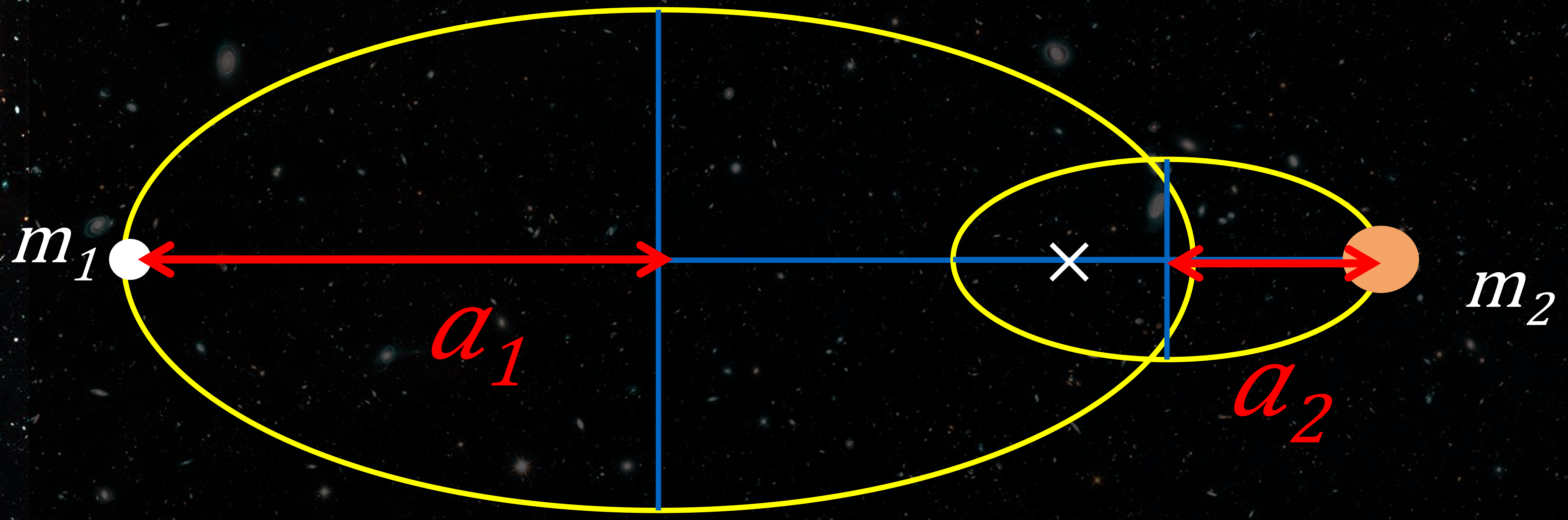
Work out the answer with a friend.

Useful numbers:

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 1 pc = 3.1×10^{16} m

Both bodies orbit their centre of mass

When two bodies orbit each other, you can relate the orbital period T to the semi-major axes a of the ellipses of their orbits:

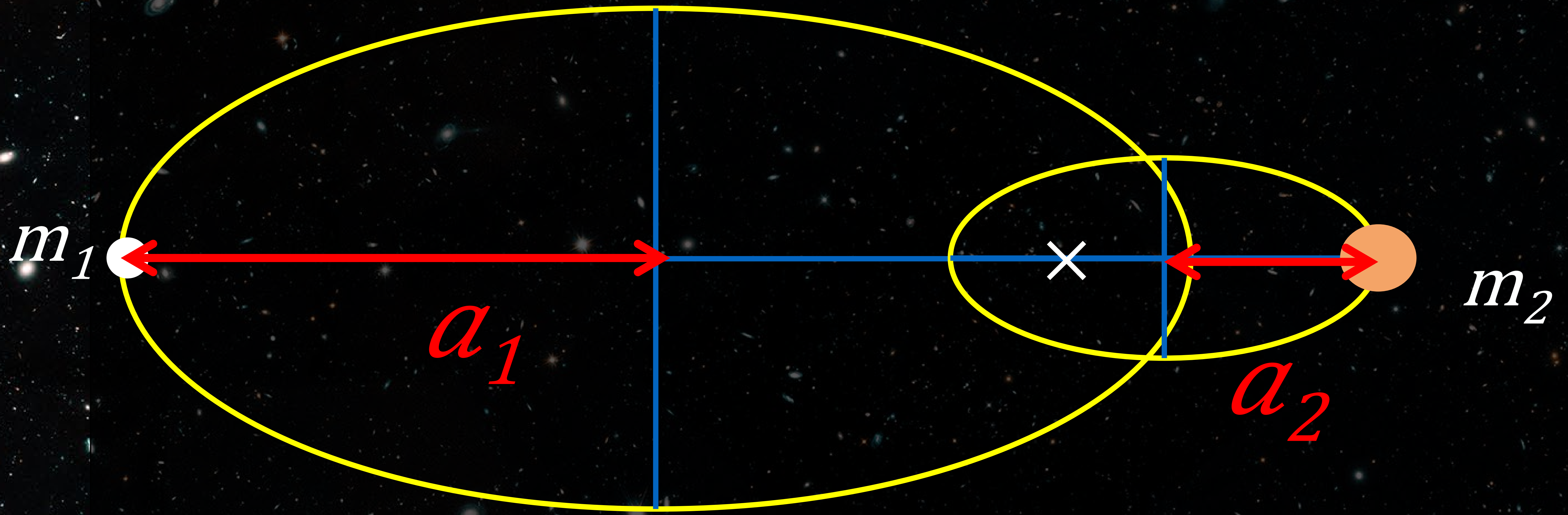


$\times$ = location of centre of mass

Relating orbital radius to period

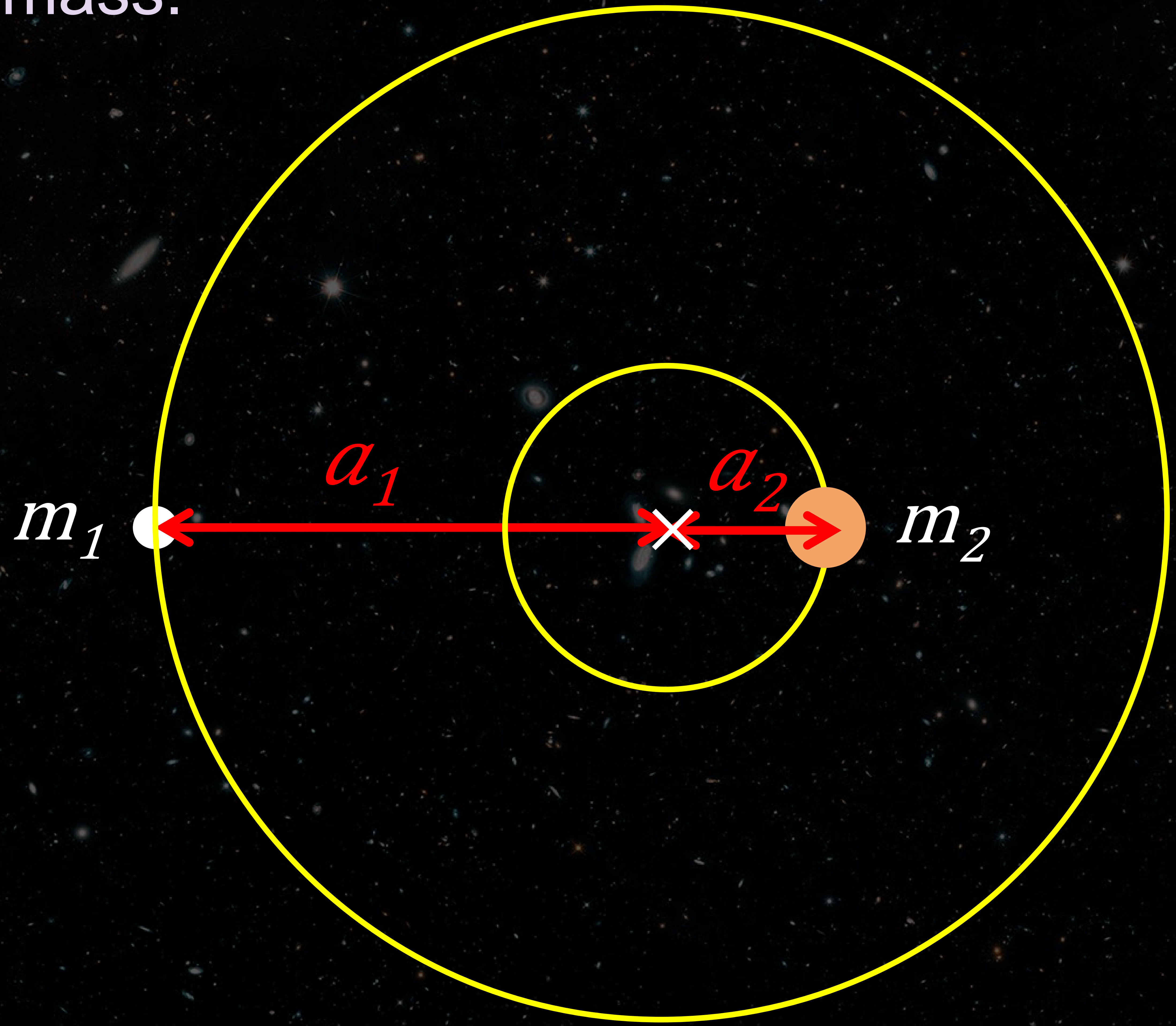
When two bodies orbit each other, you can relate the orbital period T to the semi-major axes a of the ellipses of the orbit:

$$\frac{(a_1 + a_2)^3}{T^2} = \frac{G(m_1 + m_2)}{4\pi^2}$$



Objects each orbit their common centre of mass.

$$\frac{(a_1 + a_2)^3}{T^2} = \frac{G(m_1 + m_2)}{4\pi^2}$$



For circular orbits,
$a_1 + a_2 = R$
(distance between objects),
and if one object is much heavier than the other,
then $m_1 + m_2 \approx m_2$

https://phet.colorado.edu/sims/html/my-solar-system/latest/my-solar-system_all.html

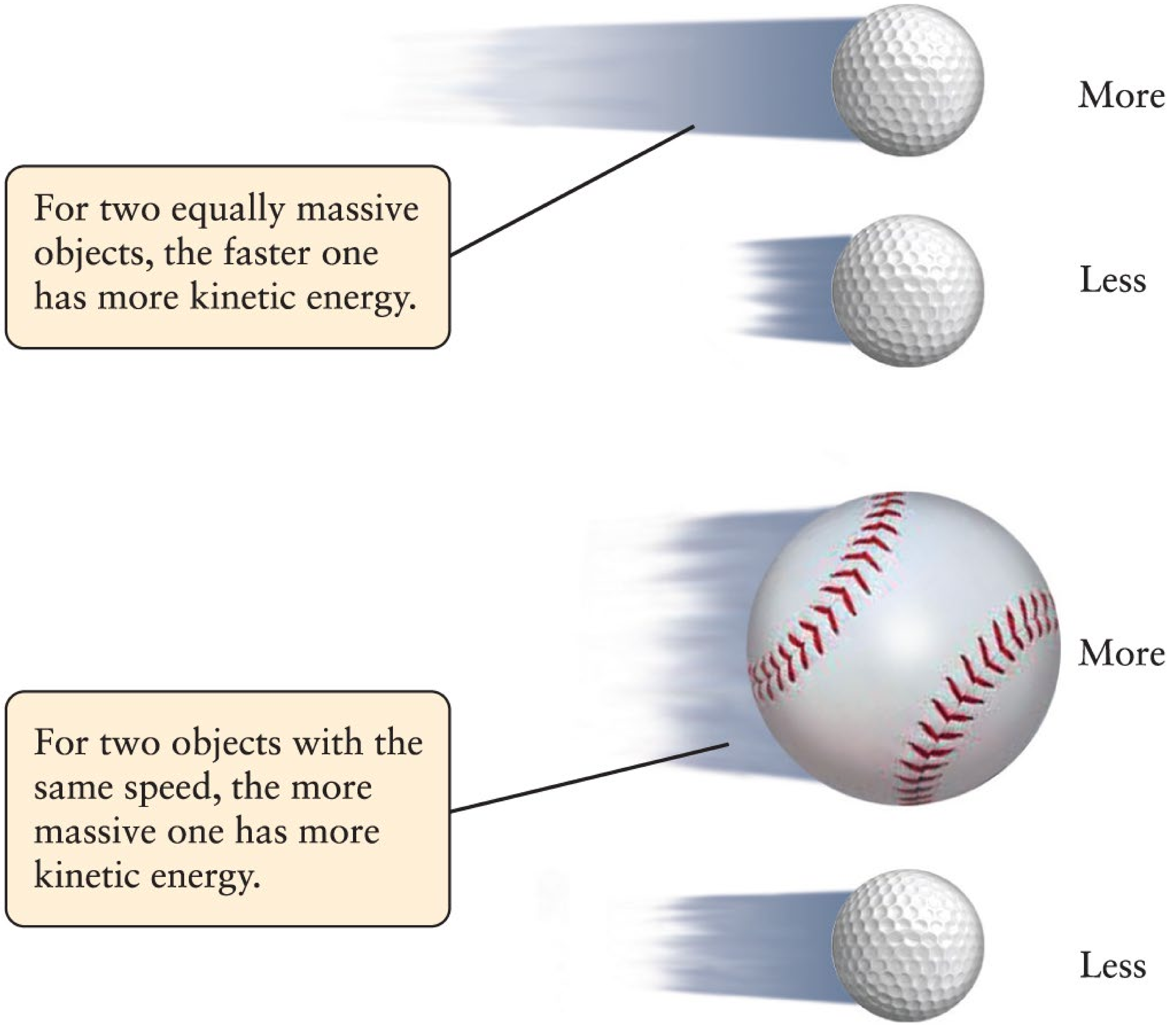
Forms of energy: kinetic

Energy can take many forms. While it can change from one form to another, it cannot be created or destroyed; this is the law of **conservation of energy**.

One familiar form of energy is kinetic energy, the energy of motion.

$$K = \frac{1}{2}mv^2$$

This is proportional to the mass of an object and to the square of its velocity.



Geller et al., *Universe*, 11e, © 2019 W. H. Freeman and Company

Forms of energy: potential

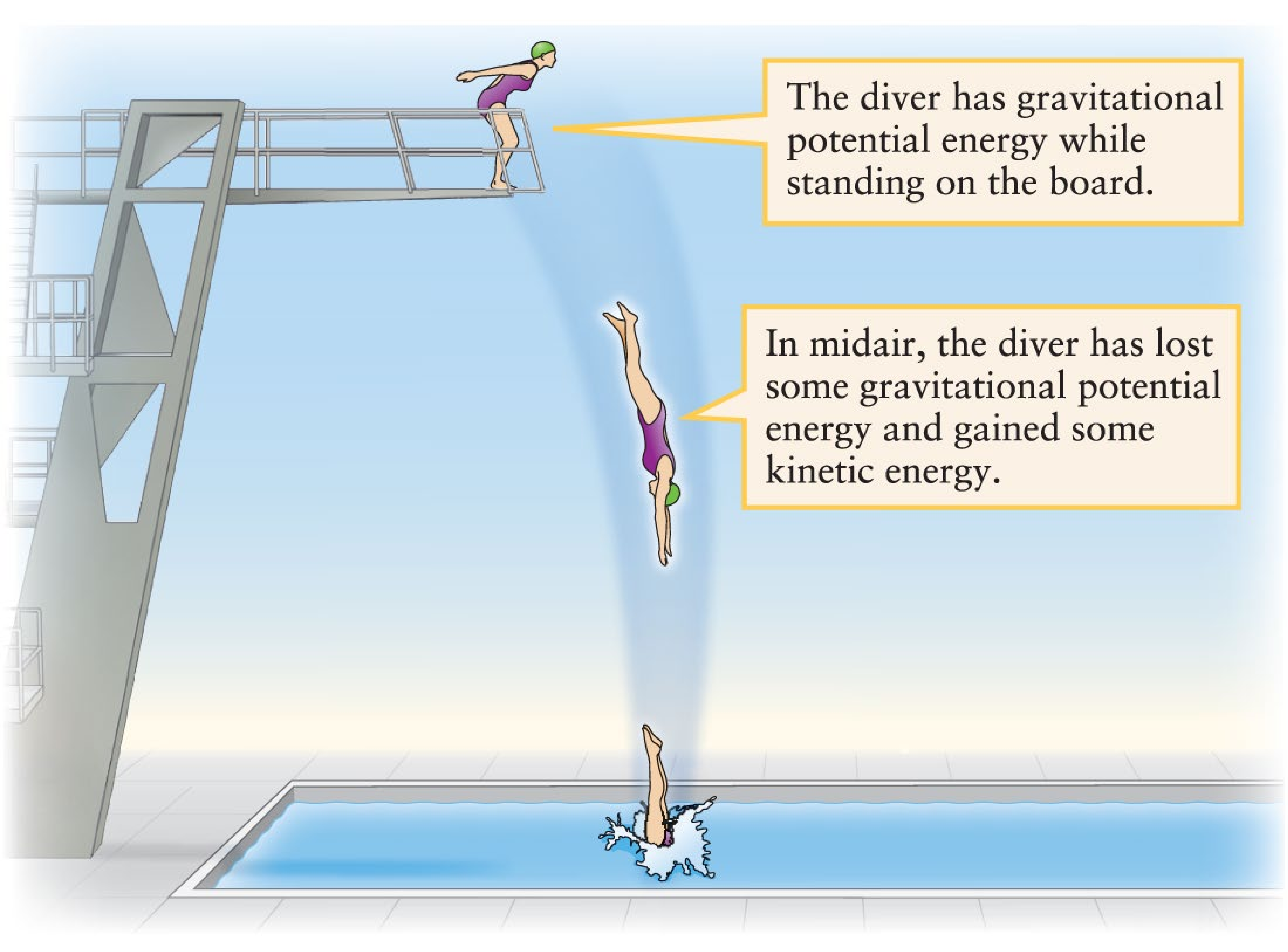
Energy can also be stored, as **potential energy**. For example, food and batteries store chemical energy.

Another common form of potential energy is gravitational potential energy. This occurs when gravity has the potential to cause an object to accelerate.

At the surface of the Earth, gravitational potential energy is given by

$$U = mgh$$

where m is the mass of an object, h is its height above the Earth's surface, and g is the acceleration due to the Earth's gravity, 9.8 m/s^2.



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Energy and orbits

The same principles apply to orbiting objects.

Planets, moons, and satellites also have both kinetic and gravitational potential energy.

For example, the faster a satellite orbits the Earth, the greater its kinetic energy, and if it falls back to the Earth, it loses gravitational potential energy.

Energy and orbits

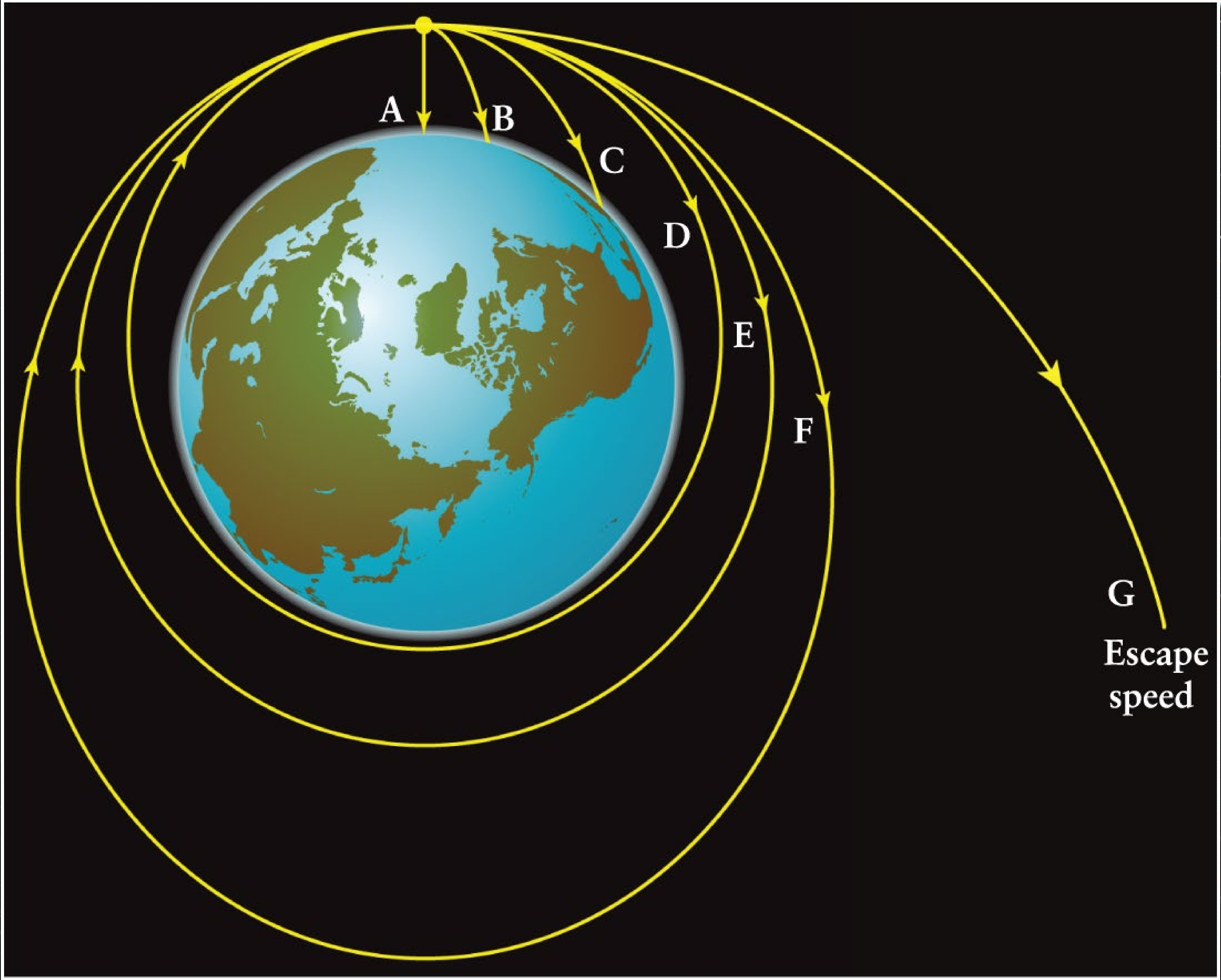
An object bound in orbit has kinetic energy $K = \frac{1}{2}mv^2$

and gravitational potential energy $U = -\frac{GMm}{r}$

The sum of these two is negative (for a bound orbit) and is conserved.

Let's look at different types of trajectories and orbits.

If a ball is dropped from a great height above Earth's surface, it falls straight down (A).
If the ball is thrown with some horizontal speed, it follows a curved path before hitting the ground (B, C). If thrown with just the right speed (E), the ball goes into circular orbit; the ball's path curves but it never gets any closer to Earth's surface.



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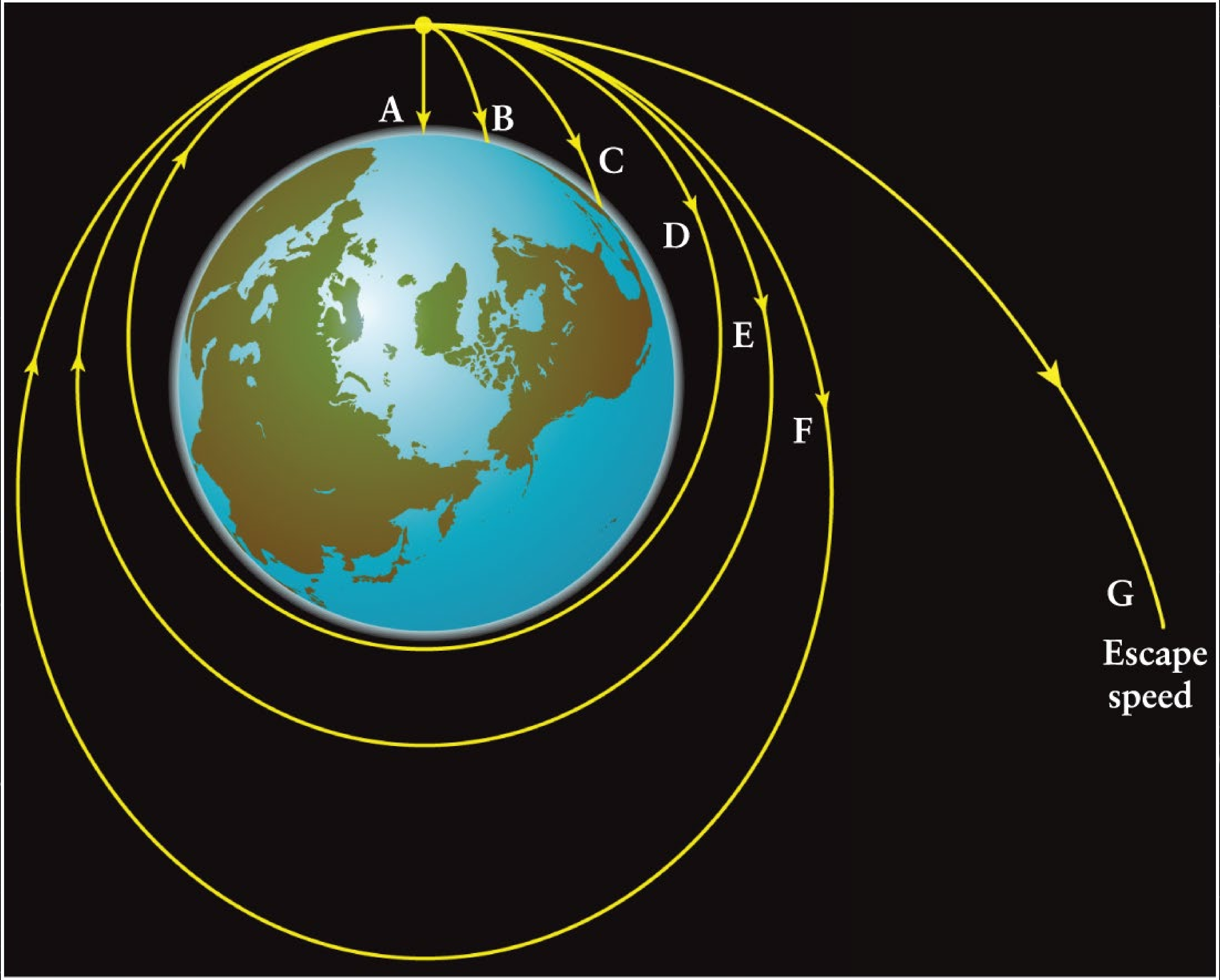
If the ball is thrown with a speed that is slightly less (D) or slightly more (F) than the speed for a circular orbit (E), the ball's orbit is an ellipse.

A-F are all bound orbits.



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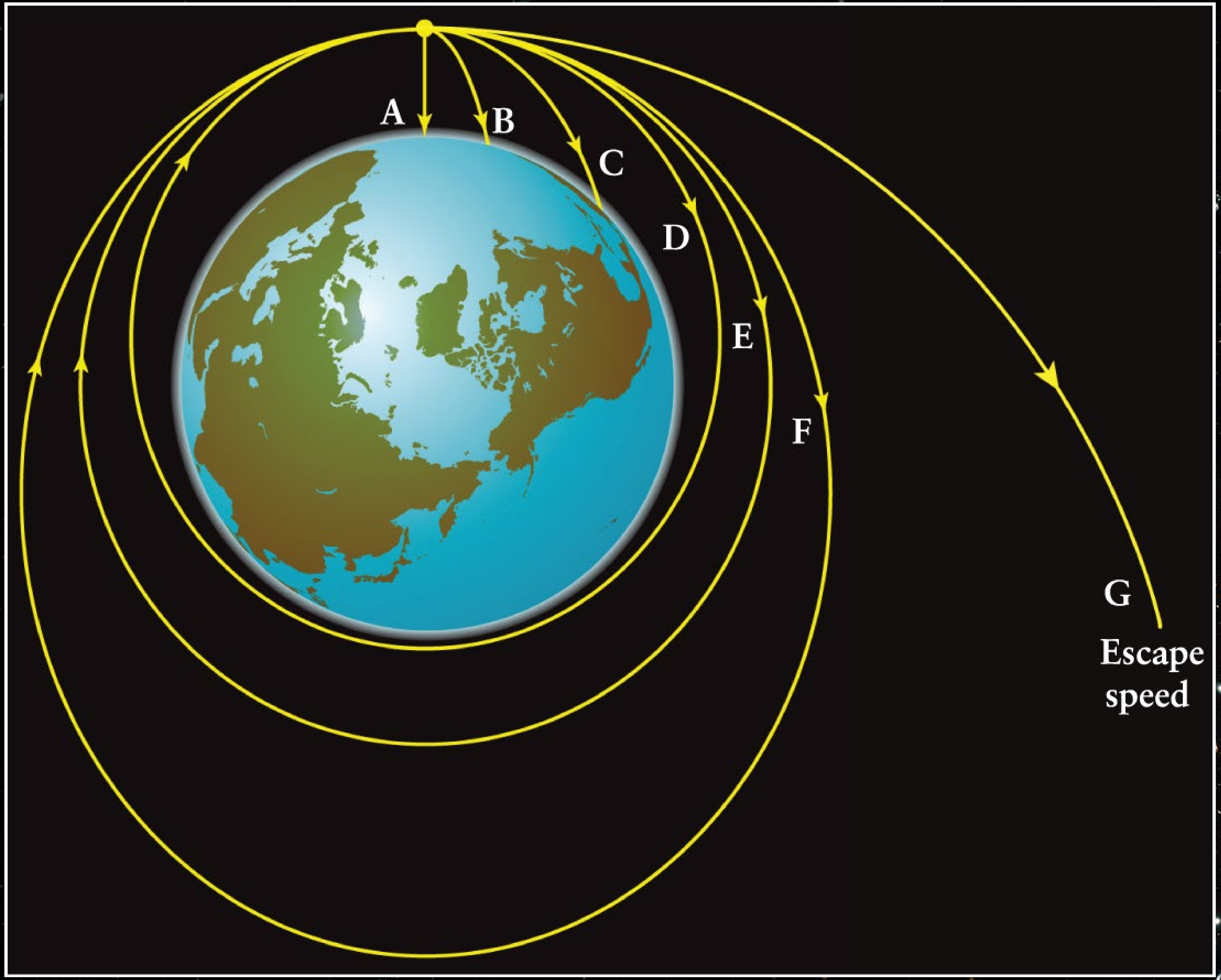
If it is thrown faster than the escape speed (G), the ball will leave Earth and never return.

The escape speed is:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

where *M* and *r* are the mass and radius of the planet, respectively. This can be derived by setting K+U = 0 to unbind the object. For the Earth, the escape speed is 11.2 km/s.



Universe, Geller et al.

Example

The Earth's mass is M = 6×10^{24} kg. What would the radius of the Earth have to be in order for the escape velocity from its surface to be equal to the speed of light, 300,000 km/s? *You will need:* $G = 6.67 \times 10^{-11}\ m^3 kg^{-1}\ s^{-2}$

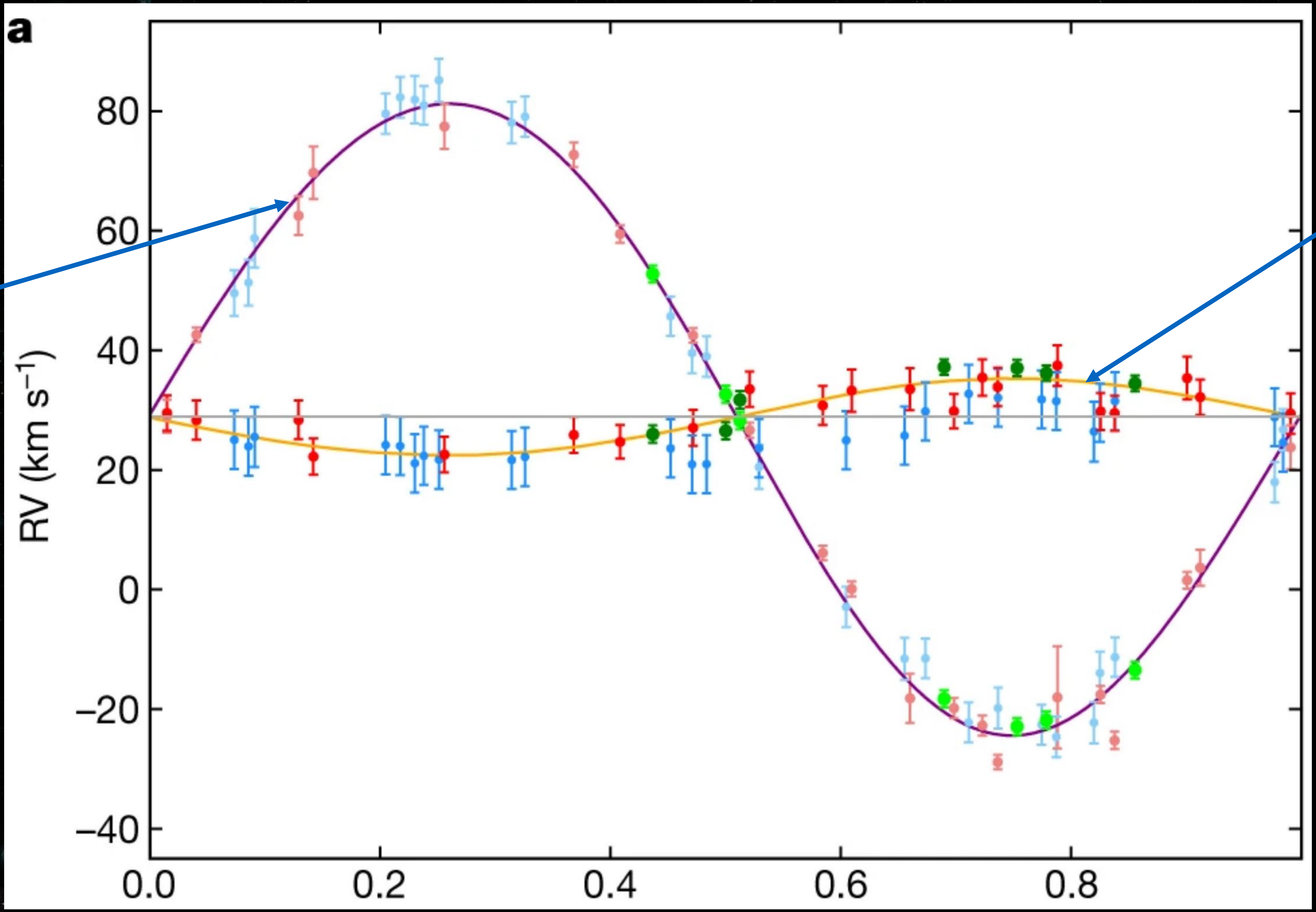
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

A) 9 mm
B) 9 cm
C) 9 m
D) 9 km

A wide star–black-hole binary system from radial-velocity measurements

by Jifeng Liu et al. (Nature 575, 618-621, 2019.)

Radial velocity curve of star



Radial velocity curve of Balmer line of hydrogen (Hα)

Can we figure out the masses?

$$\frac{a^3}{T^2} = \frac{G(m_1 + m_2)}{4\pi^2}$$

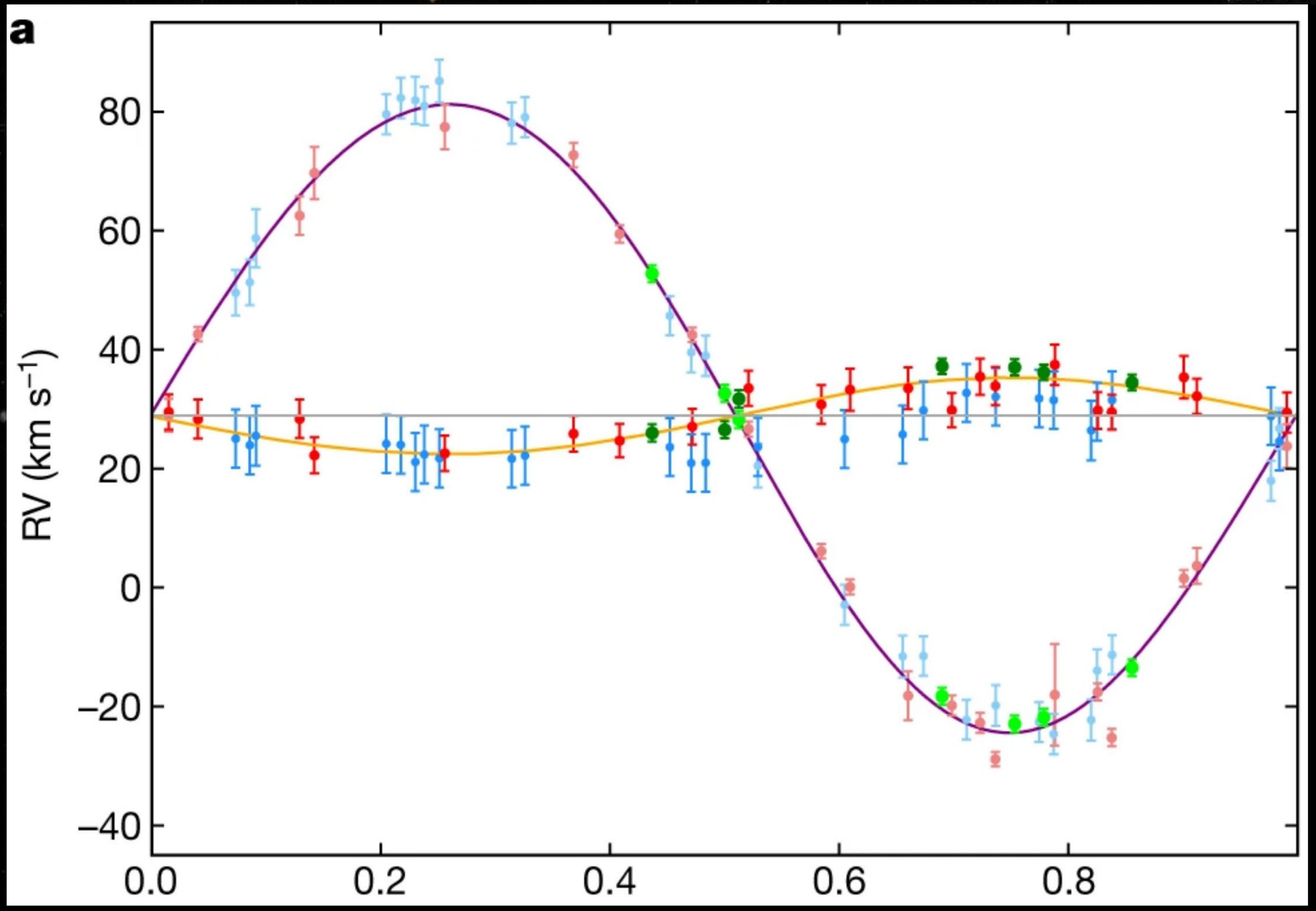
T is measured to be 79 days orbital period.

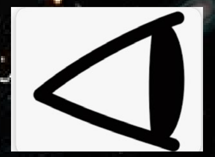
Need to figure out distance between objects, a.

Radial velocity of star varies by ± 52 km/s.

If we are viewing the system edge-on, then the orbital velocity is exactly 52 km/s.

Multiply by the period (79 days) to get the circumference of the orbit.





Do the same thing with the dim emission line, which varies much less. This means it has a much smaller orbit.

IF system is edge-on:

Distance between objects: 6.4×10^{10} m

Total mass = $1.7 M_{\odot}$

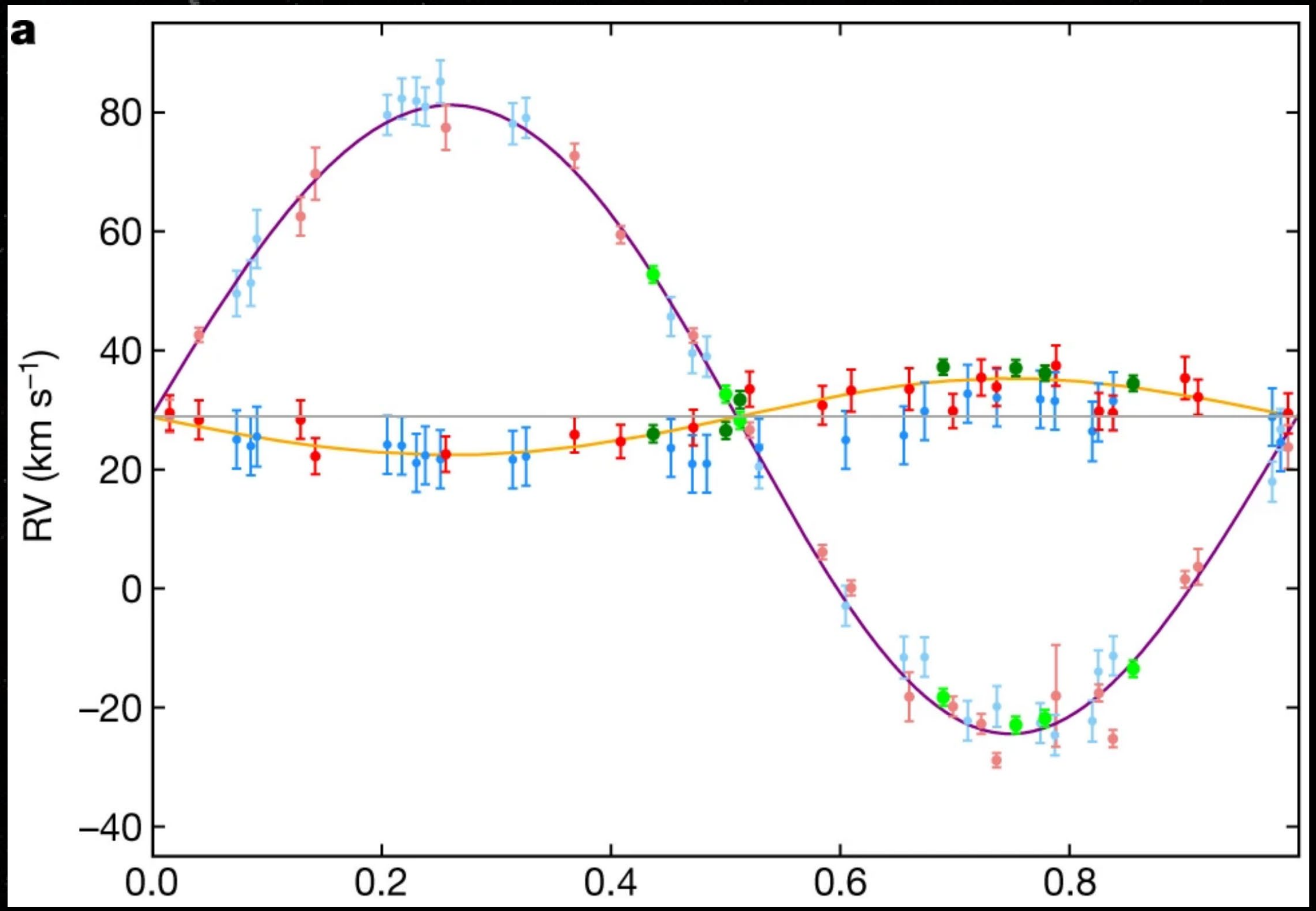
But what if we’re not looking at it edge-on?

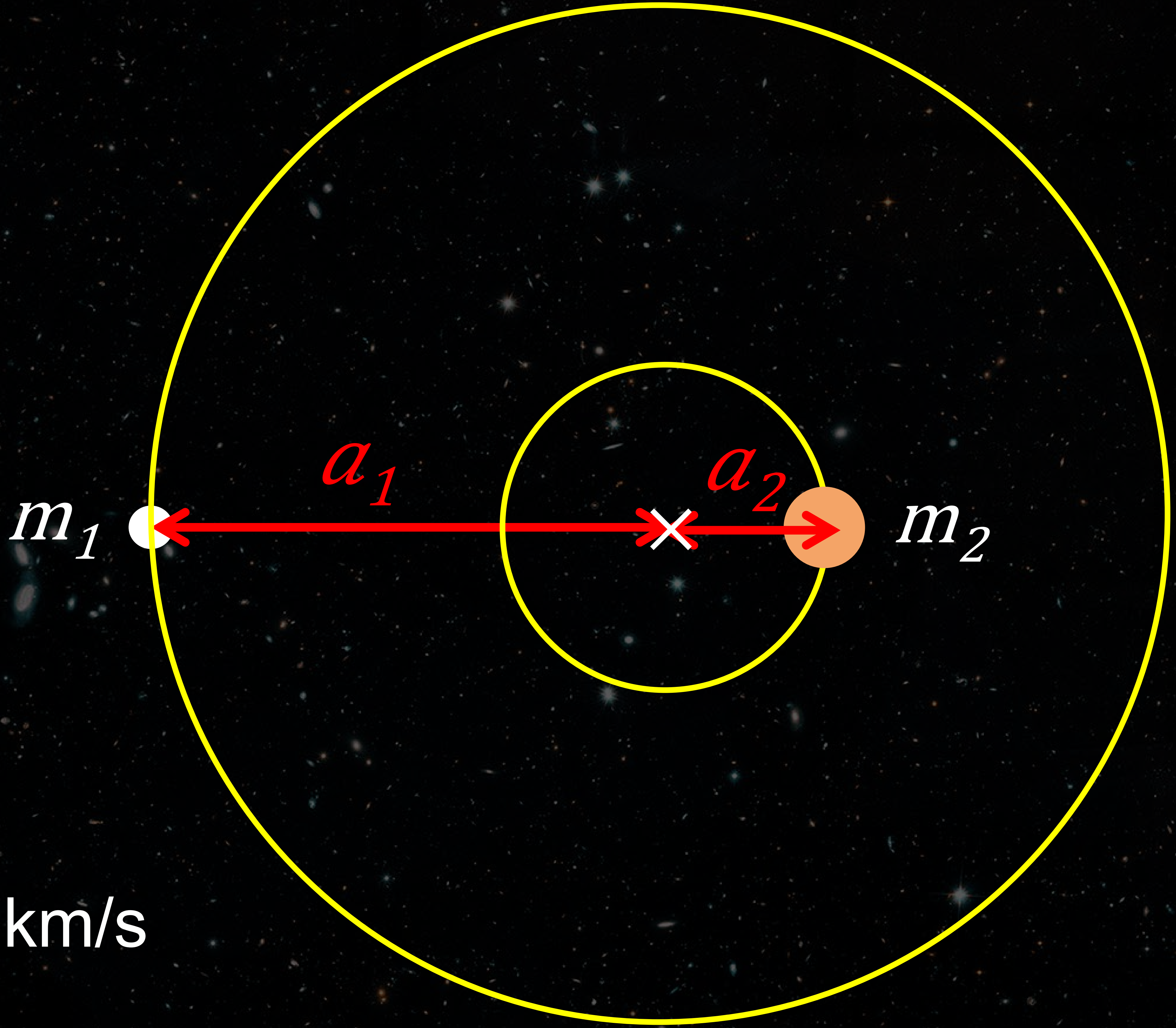
Edge-on (90° inclination): maximum velocity towards us (radial velocity)

Tilted: velocity towards us (radial velocity) is less than the total true velocity (reduced by sine of inclination angle)

If system is not edge-on, true velocity (and so true radius and mass) would be larger

Ratio of masses can be inferred from relative amplitudes of radial velocity curves





Star's radial velocity: 52.8 km/s
Dark companion's radial velocity: 6.4 km/s

$$\frac{m_2}{m_1} = \frac{v_1}{v_2} = \frac{52.8}{6.4} = 8.25$$

$$m_1 v_1 = m_2 v_2$$

By comparing light from visible star to other known stars, astronomers estimate its mass to be $8.2M_{\odot}$.

Mass of companion = $8.2M_{\odot} \times 8.25 = 68M_{\odot}$!

Something that heavy that doesn't produce significant light? Gotta be a black hole!

Distance between star and black hole: 1.5 AU

iClicker question

An astronaut in the International Space Station reports feeling “weightless”. Which best explains her experience?

For reference: the ISS orbits the Earth ~400 km above the Earth.

A) She is too far away to feel the pull of the Earth’s gravity

B) In her trip to the ISS, she exceeded the escape velocity of the Earth

C) She is falling around the Earth at the same rate as the ISS, and experiences no net force

Weightlessness in orbit

An astronaut on board an orbiting spacecraft feels “weightless”.

However, this is not because she is “beyond the pull of gravity”. The astronaut is herself an independent satellite of Earth, and Earth’s gravitational pull is what holds her in orbit.

She feels “weightless” because she and her spacecraft are falling together around Earth (in freefall), so there is nothing pushing her against any of the spacecraft walls.

You feel the same “weightless” sensation whenever you are falling, such as when you jump off a diving board.