Polynomial inequalities with asymmetric weights[†]

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Abstract

We initiate the study of the Bernstein-Markov type inequalities for the so called *asymmetric weights* which may vanish at a given point with different rates. Bernstein type inequality is proved for asymmetric weights with power discrepancies. It turns out that the order of Bernstein factors in the asymmetric case is in general greater than $O(n)$.

Keywords: Remez, Schur and Bernstein type inequality, asymmetric weight.

MSC: 41A17.

§1. Introduction

Consider the space $\mathcal{P}_n$ of real algebraic polynomials of degree at most n. Let $K \subset \mathbb{R}$ be any compact set and $\|p\|_K := \sup_{x \in K} |p(x)|$ the usual supremum norm on K. The classical Bernstein problem consists in estimating the derivative of the polynomial $p'(x)$ for a given $p \in \mathcal{P}_n, \|p\|_K = 1$ and $x \in \text{Int } K$. Typically, this estimate is given in terms of the degree n of the polynomials and the distance of point $x \in \text{Int } K$ to the boundary ∂K of the compact K. This problem goes back to Bernstein [1] who showed that when $K = [a, b]$ we have the estimate

$$\|\sqrt{(x - a)(b - x)}p'_n(x)\|_{[a,b]} \leq n\|p_n\|_{[a,b]}. \qquad (1.1)$$