DeMorgan's Laws

To prove DeMorgan's Laws we need to establish a proof in BOTH directions

$$\neg(P \lor Q) \equiv \neg P \land \neg Q$$

"$\vdash$"

1. $\neg(P \lor Q)$

2a. Assume P	2b. Assume Q
3a. $P \lor Q$ by (2a)	3b. $P \lor Q$ by (2b)
4a. $\neg P$ by (1) & (3a)	4b. $\neg Q$ by (1) & (3b)

$\therefore \neg P \land \neg Q$

"$\dashv$"

1. $\neg P \land \neg Q$
2. Assume $P \lor Q$

3a. P	3b. Q
4a. $\neg P$ by (1)	4b. $\neg Q$ by (1)
5a. $\neg(P \lor Q)$ by (3a) & (4a)	5b. $\neg(P \lor Q)$ by (3b) & (4b)

$\therefore \neg(P \lor Q)$

if we can prove that

$$\frac{P}{\therefore Q} \quad \text{and} \quad \frac{Q}{\therefore P}$$

Then $P \equiv Q$

Therefore we have proven

$$\neg(P \lor Q) \equiv \neg P \land \neg Q$$

This way of proving Equivalence does <u>not</u> rely on a truth table.

In words:
If we take a Proposition P and from P we can prove Q, AND
If we take Q and we can prove P, then P and Q are <u>Equivalent</u>

The other DeMorgan's Law proof

"$\vdash$" $\neg(P \land Q) \equiv \neg P \lor \neg Q$

1. $\neg(P \land Q)$
2. Assume $\neg(\neg P \lor \neg Q)$

3a. Assume $\neg P$	3b. Assume $\neg Q$
4a. $\neg P \lor \neg Q$ by (3a)	4b. $\neg P \lor \neg Q$ by (3b)
5a. P by (2) & (4a)	5b. Q by (2) & (4b)

6. $P \land Q$ by (5a) & (5b)
7. $\neg P \lor \neg Q$ by (2) & (6)

We then go the other way

“$\neg I$”

1. $\neg P \vee \neg Q$

2. Assume $P \wedge Q$

3a. $\neg P$ by (1)	3b. $\neg Q$ by (1)
4a. P by (2)	4b. Q by (2)
5a. $\neg(P \wedge Q)$	5b. $\neg(P \wedge Q)$

$\therefore\ \neg(P \wedge Q)$

Thus we have proven DeMorgan's Laws purely by inference.