

Rules of Inference

Examples

Double Negation Elimination

$$\frac{1. \neg\neg P}{\begin{array}{l} \circ \\ \circ \end{array} P}$$

Negation Introduction

$$\begin{array}{l} 1. P \Rightarrow Q \\ 2. \neg Q \\ 3. \text{Suppose } P \\ 4. \text{By (3) and (1), } Q \\ 5. Q \wedge \neg Q \text{ by (4) and (2)} \\ \hline \begin{array}{l} \circ \\ \circ \end{array} \neg P \text{ by (3) and Contradiction (5)} \end{array}$$

" \wedge "-Introduction

1. P

2. Q

$P \wedge Q$

" \wedge "-Elimination

1. $P \wedge Q$

$\begin{matrix} \circ \\ \circ \end{matrix} P$

" \Rightarrow "-Elimination

a.k.a. Modus Ponens

1. $P \Rightarrow Q$

2. P

$\begin{matrix} \circ \\ \circ \end{matrix} Q$

" \Rightarrow " - Introduction

1. $P \Rightarrow Q$

2. $Q \Rightarrow R$

3. Suppose P

4. Q by (3) & (1)

5. R by (4) & (2)

$\therefore P \Rightarrow R$ by (3) & (5)

" \vee " - Introduction

1. P

$\therefore P \vee Q$

"V" - Elimination

$$1. P \vee Q$$

$$2. P \Rightarrow R$$

$$3. Q \Rightarrow R$$

$$4a. \text{Suppose } P \mid 4b. \text{Suppose } Q$$

$$5a. R \text{ by } (4a) \& (2) \mid 5b. R \text{ by } (4b) \& (3)$$

$$\begin{array}{l} \text{0} \\ \text{00} \end{array} R \text{ by } (5a) \text{ and } (5b)$$

Modus Tollens

$$1. \neg Q$$

$$2. P \Rightarrow Q$$

$$\begin{array}{l} \text{0} \\ \text{00} \end{array} \neg P$$