Rules of Inference Examples

Double Negation Elimination

1. $\neg\neg P$

$\therefore P$

Negation Introduction

1. $P \Rightarrow Q$
2. $\neg Q$
3. Suppose P
4. By (3) and (1), Q
5. $Q \wedge \neg Q$ by (4) and (2)

$\therefore \neg P$ by (3) and contradiction (5)

"∧" - Introduction

1. P
2. Q

P ∧ Q

"∧" - Elimination

1. P ∧ Q

P

"⇒" - Elimination
a.k.a. Modus Ponens

1. P ⇒ Q
2. P

Q

"⇒" - Introduction

1. $P \Rightarrow Q$
2. $Q \Rightarrow R$
3. Suppose P
4. Q by (3) & (1)
5. R by (4) & (2)

$\therefore P \Rightarrow R$ by (3) & (5)

"∨" - Introduction

1. P

$\therefore P \vee Q$

"$\vee$" - Elimination

1. $P \vee Q$
2. $P \Rightarrow R$
3. $Q \Rightarrow R$

4a. Suppose P	4b. Suppose Q
5a. R by (4a) & (2)	5b. R by (4b) & (3)

$\therefore R$ by (5a) and (5b)

Modus Tollens

1. $\neg Q$
2. $P \Rightarrow Q$

$\therefore \neg P$