Clase 6 Expresiones Regulares

- AF → Lenguajes Regulares.
- ER → forma declarativa
  LR
  Expresión algebraica

$$0(0+1)^*1$$

Regex

Operadores

1. Union L y M L. Regulares.

$L = \{001, 10, 111\}$ y $M = \{\varepsilon, 001\}$

$L \cup M = \{\varepsilon, 10, 001, 111\}$

2. concatención

LM ó $L \cdot M$

×

$$LM = \{\underbrace{001, 10, 111}, \underbrace{001001, 10001, 111001}_{001}\}$$

$L\varepsilon$

$LM \neq ML$

3. Clausura de Kleene

L^*

$L = \{0, 1\} \Rightarrow L^*$ son todas las cadenas con 0s y 1s

$L = \{0, 11\} \Rightarrow L^*$ todas cadenas que tienen 1 en pares

011, 11110, ...

$L^* = \bigcup_{i \geq 0} L^i$

$L^0 = \{\varepsilon\}$, $L^1 = L$, ...

$L^i = LLL \ldots L$ (i veces)

Ej. $L = \{0, 11\}$

$L^0 = \{\varepsilon\}$

$L^1 = \{\varepsilon, 0, 11\}$

$L^2 = \{00, 011, 110, 1111\}$

$\vdots$

$L^i \sim$ infinito

Ej: L ~ conjunto de tiras 0

$$\mathbf{L} \to \infty$$

$$L^* \to \infty$$

$$L^1 = \mathbf{L}$$

$$L^2 = \mathbf{L}$$

$$L^3 = \mathbf{L}$$

Ej: $L = \emptyset \to \emptyset = L^*$

$$\{\varepsilon\} \neq \emptyset$$

Construcción de Expresiones Regulares.

$L(E)$ ~ Lenguaje descrito por E = ER

$L(A)$ ~ lenguaje descrito por A = AFD

Base: 1. ε y $\emptyset$ son ER

$$L(\varepsilon) = \{\varepsilon\}$$

$$L(\emptyset) = \emptyset$$

2. si a es un símbolo

$\underline{a}$ es un ER

negrita $\quad L(\mathbf{a}) = \{a\}$

3. L representa un lenguaje.

Paso Inductivo:

1. E y F ER

$$L(E+F) = L(E) \cup L(F)$$

2. $L(EF) = L(E)L(F)$

3. $L(E^*) = (L(E))^*$

4. $L((E)) = L(E)$

Ej.

$$\underline{0}\,\underline{1}$$

$L(\underline{0}) = \{0\}$ $\quad$ $L(\underline{1}) = \{1\}$

01

- cadena 01 ?

$01^* \neq (01)^*$ ← 1 o más veces

01^*	$(01)^*$
01	01 ✓
011	0101 ✓
01111	010101 ✓
⋮	⋮

no: acepta ...

1010

- 0 y 1 alternados

$$(\varepsilon + 1)(01)^*(\varepsilon + 0)$$

ó

1 o más veces

agrupan

101

010

$$(\varepsilon + 0)^* = \{\varepsilon, 0, 00, 000, 0000, \dots\}$$

$$\{101, 010, 1010, 0101, \dots\}$$

$$L((\varepsilon+1)(01)^*(\varepsilon+0)) =$$

$$\{\varepsilon\} \rightarrow L^0 \cup L^1 \cup L^2 \cup \dots$$

Precedencia

$$* > \cdot > +$$

cuantas veces

concatenación

"o"

$$(01)^* \qquad xy^2 \neq (xy)^2$$

$$0.1^* = 0(1^*) = 0(1)^*$$

despues

$1^* = \{\varepsilon, 1, 11, 111\}$

$11^* = \varepsilon \cancel{\phi} \quad 11^*$

$1^* = \{\varepsilon, 1, 11, 111, \ldots\}$

$11^* = \{1, 11, 111, 1111, \ldots\}$

$(\varepsilon + a + b + c)^* \overset{a}{(a+b)} (\varepsilon + a + b + c)^* \overset{b}{(a+b)} (\varepsilon + a + b + c)^*$

$(\varepsilon + 11)(01)^*$

$0^* (10)^* 0 (\varepsilon + 11)\underbrace{0^* (01)^*}_{\varepsilon} 0^*$

↑ 1 vez 2

01
01
010

$(0^* 1)^*$

$(0^* (01)^*)^*$

11

0110

0101101 01

$0^* (01)^*$

010110001 $\{0, 01, 001,$

0 $0101, \ldots$

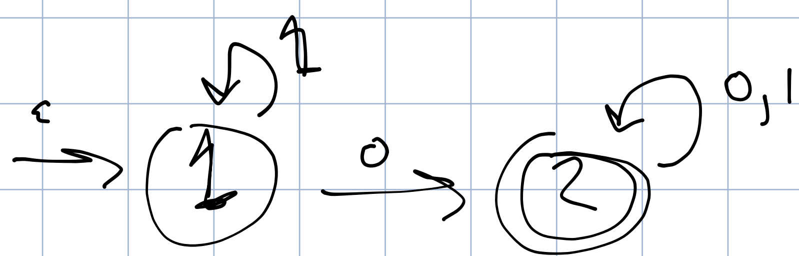
11000010100000

001001

AFD $\longleftrightarrow$ AFN $\longleftrightarrow$ AFN-ε $\longleftrightarrow$ ER

Lenguaje Regular

AFD $\longrightarrow$ ER (3 métodos)



cadena con al menos 0

R^{k}_{ij} ~ ER de i a j que tiene k estados intermedios

$R^{0}_{i,j}$

1. R^{0}_{ij} ~ i a j

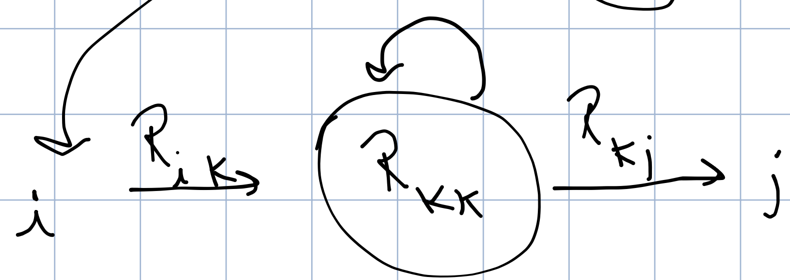
$$R^0_{11} = \varepsilon + 1$$

$$R^0_{12} = 0$$

$$R^0_{21} = \emptyset$$

$$R^0_{22} = \varepsilon + 0 + 1$$

2. Inductivo $R^k_{ij} = R^{k-1}_{ij} + R^{k-1}_{ik}\underbrace{\left(R^{k-1}_{kk}\right)^*}\, R^{k-1}_{kj}$



$$R^1_{ij} = R^0_{ij} + R^0_{i1}\left(R^0_{11}\right) R^0_{1j}$$

R^1_{11}	$R^0_{11} + R^0_{11}\left(R^0_{11}\right)^* R^0_{11} = \varepsilon+1 + (\varepsilon+1)(\varepsilon+1)^*(\varepsilon+1) = \varepsilon+1+(\varepsilon+1)^*$
	$(\varepsilon+1)^* = 1^*$
R^1_{12}	$R^0_{12} + R^0_{11}\left(R^0_{11}\right)^* R^0_{12} = 0 + (\varepsilon+1)(\varepsilon+1)^* 0 = 0 + 1^* 0 = 1^* 0$
R^1_{21}	$\emptyset + \emptyset(\varepsilon+1)^*(\varepsilon+1) = \emptyset$

R_{22} $\ldots \varepsilon + 0 + 1 + \emptyset(\varepsilon+1)^*\emptyset = \varepsilon + 0 + 1$

$$(\varepsilon + R)^* = R^*, \quad \emptyset R = R\emptyset = \emptyset$$

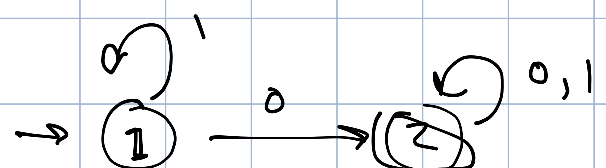
$$0 + 1^* 0 \Rightarrow \{0, 10, 110, 1110, \ldots\}$$

$$R^2_{ij} = R^1_{ij} + R^1_{i2}(R^1_{22})R^1_{2j}$$

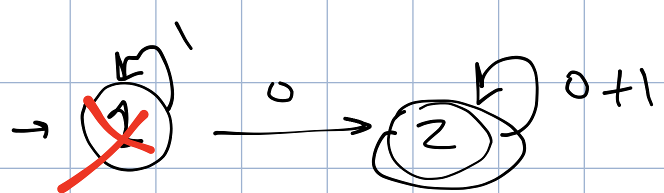
R^2_{11}	1^*
$\rightarrow R^2_{12}$	$1^*0(0+1)^*$
R^2_{21}	$\emptyset$
R^2_{22}	$(0+1)^*$

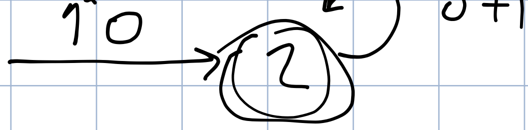
$$L = 1^*0(0+1)^*$$

Eliminación de estados



$0,1 = 0+1$





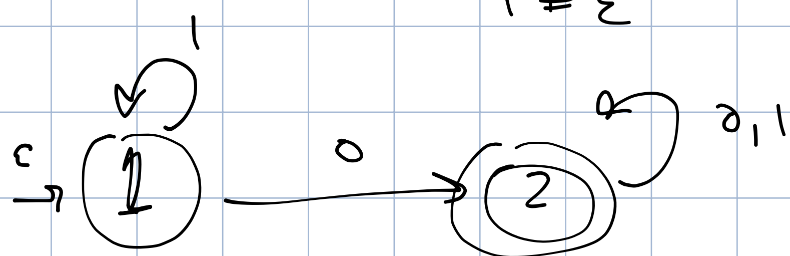
$1^*0(0+1)^*$

Método Arden

teorema Arden ~ R, Q, P ~ ER

$$R = Q + RP = QP^*$$

$$P \neq \varepsilon$$



$$\overset{Q}{q_1 = \varepsilon} + \overset{RP}{q_1 1} \rightarrow q_1 = \varepsilon(1)^*$$

$$q_2 = q_1 0 + q_2 0 + q_2 1 = \underbrace{q_1 0 + q_2 (0+1)}_{Q + RP = QP^*}$$

$$q_2 = q_1 0 (0+1)^*$$

$$q_2 = 1^* 0 (0+1)^*$$

ER