Uncertainty Assignement - SPH 3U3

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1.

2. (a)

$$
\begin{aligned}
f &= x_1 + x_2 \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2} \\
&= \sqrt{(1)^2\sigma_{x_1}^2 + (1)^2\sigma_{x_2}^2} \\
&= \sqrt{1\sigma_{x_1}^2 + 1\sigma_{x_2}^2} \\
&= \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} \\
f &= (10.005 \pm .003)\text{ cm} + (20.06 \pm .03)\text{ cm} \\
&= (10.005 + 20.06 \pm \sigma_f)\text{ cm} \\
&= (30.065 \pm \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2})\text{ cm} \\
&= (30.065 \pm \sqrt{.003^2 + .03^2})\text{ cm} \\
&= (30.065 \pm \sqrt{.000009 + .0009})\text{ cm} \\
&= (30.065 \pm \sqrt{.000909})\text{ cm} \\
&= (30.065 \pm .03)\text{ cm} \\
&= (30.07 \pm .03)\text{ cm}
\end{aligned}
$$

(b)

$$
\begin{aligned}
f &= x_1 - x_2 \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(1)^2 \sigma_{x_1}^2 + (-1)^2 \sigma_{x_2}^2} \\
&= \sqrt{1\sigma_{x_1}^2 + 1\sigma_{x_2}^2} \\
&= \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} \\
f &= (352.1 \pm .9)\ \mathrm{m} - (162.36 \pm .05)\ \mathrm{m} \\
&= (352.1 - 162.36 \pm \sigma_f)\ \mathrm{m} \\
&= (189.74 \pm \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2})\ \mathrm{m} \\
&= (189.74 \pm \sqrt{.9^2 + .05^2})\ \mathrm{m} \\
&= (189.74 \pm \sqrt{.81 + .0025})\ \mathrm{m} \\
&= (189.74 \pm \sqrt{.8125})\ \mathrm{m} \\
&= (189.74 \pm 0.9)\ \mathrm{m} \\
&= (189.7 \pm 0.9)\ \mathrm{m}
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= x_1 + x_2 \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2} \\
&= \sqrt{(1)^2\sigma_{x_1}^2 + (1)^2\sigma_{x_2}^2} \\
&= \sqrt{1\sigma_{x_1}^2 + 1\sigma_{x_2}^2} \\
&= \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} \\
f &= (56.7 \pm .2)\text{ cm} + (93.48 \pm .01)\text{ m} \\
&= (56.7 \pm .2)\text{ cm} + (9348 \pm 1)\text{ cm} \\
&= (56.7 + 9348 \pm \sigma_f)\text{ cm} \\
&= (9404.7 \pm \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2})\text{ cm} \\
&= (9404.7 \pm \sqrt{.2^2 + .01^2})\text{ cm} \\
&= (9404.7 \pm \sqrt{.04 + .0001})\text{ cm} \\
&= (9404.7 \pm \sqrt{.0401})\text{ cm} \\
&= (9404.7 \pm 0.2)\text{ cm}
\end{aligned}
$$

(d)

$$
\begin{aligned}
f &= x_1 \pm x_2 \pm x_3 \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2\sigma_{x_3}^2} \\
&= \sqrt{(1)^2\sigma_{x_1}^2 + (\pm 1)^2\sigma_{x_2}^2 + (\pm 1)^2\sigma_{x_3}^2} \\
&= \sqrt{1\sigma_{x_1}^2 + 1\sigma_{x_2}^2 + 1\sigma_{x_3}^2} \\
&= \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2} \\
f &= (14.5 + .2)\text{ mm} + (14.5 + .2)\text{ mm }(23.1 + .1)\text{ mm} \\
&= (14.5 + 14.5 - 23.1 \pm \sigma_f)\text{ mm} \\
&= (5.9 \pm \sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}^2})\text{ mm} \\
&= (5.9 \pm \sqrt{.2^2 + .2^2 + .1^2})\text{ mm} \\
&= (5.9 \pm \sqrt{.04 + .04 + .01})\text{ mm} \\
&= (5.9 \pm \sqrt{.09})\text{ mm} \\
&= (5.9 \pm .3)\text{ mm}
\end{aligned}
$$

3.

4.

5.

2. Perform the following operations using the rules for adding and subtracting with

uncertainties. [3 marks each]

a) (10.005 + .003) cm + (20.06 + .03) cm

b) (352.1 + .9) m (162.36 + .05) m

c) (56.7 + .2) cm + (93.48 + .01) m

d) (14.5 + .2) mm + (14.5 + .2) mm (23.1 + .1) mm

The first thing to realize about LaTeX is that it is not "WYSIWYG". In other words, it isn't a word processor; what you type into your .tex file is not what you'll see in your .dvi file. For example, LaTeX will completely ignore extra spaces within a line of your .tex file. Pressing return in the middle of a line will not register in your .dvi file. However, a double carriage-return is read as a paragraph break.

Like this. But any carriage-returns after the first two will be completely ignored; in other words, you

can't

add

more

space

between

lines, no matter how many times you press return in your .tex file.

In order to add vertical space you have to use "vspace"; for example, you could add an inch of space by typing `\vspace{1in}`, like this:

To get three lines of space you would type `\vspace{3pc}` ("pc" stands for "pica", a font-relative size), like this:

Notice that LaTeX commands are always preceeded by a backslash. Some commands, like `\vspace`, take arguments (here, a length) in curly brackets.

The second important thing to notice about LaTeX is that you type in various "environments"...so far we've just been typing regular text (except for a few inescapable usages of `\verb` and the centered, smallcaps, large title). There are basically two ways that you can enter and/or exit an environment;

this is the first way...

this is the second way.

Actually there is one more way, used above; for example, THIS WAY. The way that you get in and out of environment varies depending on which kind of environment you want; for example, you use `\underline` "outside", but `\it` "inside"; notice this versus *this*.

The real power of LaTeX (for us) is in the math environment. You push and pop out of the math environment by typing `$`. For example, $2x^3 - 1 = 5$ is typed between dollar signs as `$2x^3 - 1 = 5$`. Perhaps a more interesting example is $\lim_{N\to\infty} \sum_{k=1}^{N} f(t_k)\Delta t$.

You can get a fancier, display-style math environment by enclosing your equation with double dollar signs. This will center your equation, and display sub- and super-scripts in a more readable fashion:

$$\lim_{N\to\infty} \sum_{k=1}^{N} f(t_k)\Delta t.$$

If you don't want your equation to be centered, but you want the nice indicies and all that, you can use `\displaystyle` and get your formula "in-line"; using our example this is $\displaystyle\lim_{N\to\infty} \sum_{k=1}^{N} f(t_k)\Delta t$. Of course this can screw up your line spacing a little bit.

There are many more things to know about LaTeX and we can't possibly talk about them all here. You can use LaTeX to get tables, commutative diagrams, figures, aligned equations, cross-references, labels, matrices, and all manner of strange things into your documents. You can control margins, spacing, alignment, *et cetera* to higher degrees of accuracy than the human eye can percieve. You can waste entire days typesetting documents to be "just so". In short, LaTeX rules.

The best way to learn LaTeX is by example. Get yourself a bunch of .tex files, see what kind of output they produce, and figure out how to modify them to do what you want. There are many template and sample files on the department LaTeX page and in real life in the big binder that should be in the computer lab somewhere. Good luck!