Regression Assignment

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1. This measurement is unacceptable. There are several factors justifying this. The first is that as we know, we are never satisfied with just one measurement. Although we cannot measure again, we cannot use this as the entirety of our data. The second is that one point of data counts as a small set. This means that we will calculate σ_{n-1} rather than σ_n. Since $n = 1$, $n - 1 = 0$. We cannot divide by 0 no matter what we do. That is why such a measurement is unacceptable.

2. (a)

$$\begin{aligned}
f &= x_1 + x_2 \\
&= (10.005 \pm .003)\text{ cm} + (20.06 \pm .03)\text{ cm} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.003)^2 + (1)^2(.03)^2} \\
&= \sqrt{1(.000009) + 1(.0009)} \\
&= \sqrt{.000009 + .0009} \\
&= \sqrt{.0009} \\
&= .03 \\
f &= (10.005 \pm .003)\text{ cm} + (20.06 \pm .03)\text{ cm} \\
&= (10.005 + 20.06 \pm \sigma_f)\text{ cm} \\
&= (30.07 \pm .03)\text{ cm}
\end{aligned}$$

(b)

$$
\begin{aligned}
f &= x_1 - x_2 \\
&= (352.1 \pm .9) \text{ m} - (162.36 \pm .05) \text{ m} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.9)^2 + (-1)^2(.05)^2} \\
&= \sqrt{1(.8) + 1(.003)} \\
&= \sqrt{.8 + .003} \\
&= \sqrt{.8} \\
&= .9 \\
f &= (352.1 \pm .9) \text{ m} - (162.36 \pm .05) \text{ m} \\
&= (352.1 - 162.36 \pm \sigma_f) \text{ m} \\
&= (189.7 \pm .9) \text{ m}
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= x_1 + x_2 \\
&= (56.7 \pm .2) \text{ cm} + (93.48 \pm .01) \text{ m} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.2)^2 + (1)^2(1)^2} \\
&= \sqrt{1(.04) + 1(1)} \\
&= \sqrt{.04 + 1} \\
&= \sqrt{1} \\
&= 1 \\
f &= (56.7 \pm .2) \text{ cm} + (93.48 \pm .01) \text{ m} \\
&= (56.7 \pm .2) \text{ cm} + (9348 \pm 1) \text{ cm} \\
&= (56.7 + 9348 \pm \sigma_f) \text{ cm} \\
&= (9410 \pm 1) \text{ cm}
\end{aligned}
$$

(d)

$$
\begin{aligned}
f &= x_1 + x_2 - x_3 \\
&= (14.5 \pm .2) \text{ mm} \pm (14.5 + .2) \text{ mm } - (23.1 \pm .1) \text{ mm} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2 \sigma_{x_3}^2} \\
&= \sqrt{(1)^2(.2)^2 + (1)^2(.2)^2 + (-1)^2(.1)^2} \\
&= \sqrt{1(.04) + 1(.04) + 1(.01)} \\
&= \sqrt{.04 + .04 + .01} \\
&= \sqrt{.09} \\
&= .3 \\
f &= (14.5 \pm .2) \text{ mm} \pm (14.5 + .2) \text{ mm } - (23.1 \pm .1) \text{ mm} \\
&= (14.5 + 14.5 - 23.1 \pm \sigma_f) \text{ mm} \\
&= (5.9 \pm .3) \text{ mm}
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
f &= x_1 * x_2 \\
&= (23.56 \pm .05) \text{ kmh}^{-1} * (56.3 \pm .4) \text{ h} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(x_2)^2(.05)^2 + (x_1)^2(.4)^2} \\
&= \sqrt{(56.3)^2(.003) + (23.56)^2(.2)} \\
&= \sqrt{3170(.003) + 555.1(.2)} \\
&= \sqrt{10 + 100} \\
&= \sqrt{100} \\
&= 10 \\
f &= (23.56 \pm .05) \text{ kmh}^{-1} * (56.3 \pm .4) \text{ h} \\
&= (23.56 * 56.3 \pm \sigma_f) \text{ km} \\
&= (1330 \pm 10) \text{ km}
\end{aligned}
$$

(b)

$$
\begin{aligned}
f &= \frac{x_1}{x_2} \\
&= \frac{(15.745 \pm .006)\ \mathrm{m}}{(0.36 \pm .05)\ \mathrm{m}} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(\frac{1}{x_2})^2 (.006)^2 + (\frac{-x_1}{x_2^2})^2 (.05)^2} \\
&= \sqrt{(\frac{1}{0.36})^2 (.00004) + (\frac{-15.745}{0.36^2})^2 (.003)} \\
&= \sqrt{(2.8)^2 (.00004) + (\frac{-15.745}{0.13})^2 (.003)} \\
&= \sqrt{7.8(.00004) + (-120)^2 (.003)} \\
&= \sqrt{0.0003 + 14000(.003)} \\
&= \sqrt{0.0003 + 40} \\
&= \sqrt{40} \\
&= 6 \\
f &= \frac{(15.745 \pm .006)\ \mathrm{m}}{(0.36 \pm .05)\ \mathrm{m}} \\
&= (\frac{15.745}{0.36} \pm \sigma_f)\ \mathrm{m} \\
&= (44 \pm 6)\ \mathrm{m}
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= 2x_1 \\
&= 2(1.63 \pm .03)\ \mathrm{mm} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2} \\
&= 2(.03) \\
&= .06 \\
f &= 2(1.63 \pm .03)\ \mathrm{mm} \\
&= (2(1.63) \pm \sigma_f)\ \mathrm{mm} \\
&= (3.26 \pm .06)\ \mathrm{mm}
\end{aligned}
$$

(d)

$$
\begin{aligned}
f &= \frac{x_1 x_2}{x_3} \\
&= \frac{(1.23 \pm .02)\ \mathrm{ms}^{-1} * (2.637 \pm .003)\ \mathrm{ms}^{-1}}{(5.6 \pm .1)\ \mathrm{ms}^{-1}} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2 \sigma_{x_3}^2} \\
&= \sqrt{(\frac{x_2}{x_3})^2 (.02)^2 + (\frac{x_1}{x_3})^2 (.003)^2 + (\frac{-x_1 x_2}{x_3^2})^2 (.1)^2} \\
&= \sqrt{(\frac{2.637}{5.6})^2 (.0004) + (\frac{1.23}{5.6})^2 (.000009) + (\frac{-1.23(2.637)}{5.6^2})^2 (.01)} \\
&= \sqrt{(.47)^2 (.0004) + (.22)^2 (.000009) + (\frac{-3.24}{31.36})^2 (.01)} \\
&= \sqrt{.22(.0004) + .048(.000009) + (-.103)^2 (.01)} \\
&= \sqrt{.00009 + .0000004 + .0107)(.01)} \\
&= \sqrt{.00009 + .0001} \\
&= \sqrt{.0002} \\
&= .01 \\
f &= \frac{(1.23 \pm .02)\ \mathrm{ms}^{-1} * (2.637 \pm .003)\ \mathrm{ms}^{-1}}{(5.6 \pm .1)\ \mathrm{ms}^{-1}} \\
&= (\frac{1.23 * 2.637}{5.6} \pm \sigma_f)\ \mathrm{ms}^{-1} \\
&= (.58 \pm .01)\ \mathrm{ms}^{-1}
\end{aligned}
$$

4. (a)

$$
\begin{aligned}
f &= x_1 + \frac{2x_2}{x_3} \\
&= (61.12 \pm .05)\text{ cm} + \frac{2(45.23 \pm .06)\text{ cm}^2}{(1.03 \pm .04)\text{ cm}} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2\sigma_{x_3}^2} \\
&= \sqrt{(1)^2(.05)^2 + (\frac{2}{x_3})^2(.06)^2 + (\frac{-2x_2}{x_3^2})^2(.04)^2} \\
&= \sqrt{1(.003) + (\frac{2}{1.03})^2(.004) + (\frac{-2(45.23)}{1.03^2})^2(.002)} \\
&= \sqrt{.003 + (\frac{2}{1.03})^2(.004) + (\frac{-2(45.23)}{1.03^2})^2(.002)} \\
&= \sqrt{.003 + (2)^2(.004) + (\frac{-90.46}{1.06})^2(.002)} \\
&= \sqrt{.003 + 4(.004) + (-85.3)^2(.002)} \\
&= \sqrt{.003 + .02 + 7280(.002)} \\
&= \sqrt{.02 + 15} \\
&= \sqrt{15} \\
&= 3.9 \\
&= 4 \\
f &= (61.12 \pm .05)\text{ cm} + \frac{2(45.23 \pm .06)\text{ cm}^2}{(1.03 \pm .04)\text{ cm}} \\
&= (61.12 + \frac{2(45.23)}{1.03} \pm \sigma_f)\text{ cm} \\
&= (61.12 + \frac{90.46}{1.03} \pm 4)\text{ cm} \\
&= (61.21 + 87.8252 \pm 4)\text{ cm} \\
&= (148.9 \pm 4)\text{ cm} \\
&= (149 \pm 4)\text{ cm}
\end{aligned}
$$

(b)

$$
\begin{aligned}
f &= x_1 x_2 - x_3 \\
&= (1005.1 \pm .2)\ \mathrm{kmh}^{-1} * (3.93 \pm .02)\ \mathrm{kmh}^{-1} - (583.68 \pm .06)\ \mathrm{km}^2\mathrm{h}^{-2} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2 \sigma_{x_3}^2} \\
&= \sqrt{(x_2)^2(.2)^2 + (x_1)^2(.02)^2 + (-1)^2(.06)^2} \\
&= \sqrt{(3.93)^2(.2)^2 + (1005.1)^2(.02)^2 + 1(.0004)} \\
&= \sqrt{15.4(.04) + 1010200(.0004) + .0004} \\
&= \sqrt{.6 + 400 + .0004} \\
&= \sqrt{400} \\
&= 20 \\
f &= (1005.1 \pm .2)\ \mathrm{kmh}^{-1} * (3.93 \pm .02)\ \mathrm{kmh}^{-1} - (583.68 \pm .06)\ \mathrm{km}^2\mathrm{h}^{-2} \\
&= (1005.1 * 3.93 - 583.68 \pm \sigma_f)\ \mathrm{km}^2\mathrm{h}^{-2} \\
&= (3370 \pm 20)\ \mathrm{km}^2\mathrm{h}^{-2}
\end{aligned}
$$

5. (a)

$$
\begin{aligned}
y &= x_1^2 \sin(x_1 x_2) \\
&= (1.23 \pm .04)^2 \sin((1.23 \pm .04) * (1.99 \pm .01)) \\
\sigma_y &= \sqrt{(\frac{\partial y}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial y}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(2x_1 \sin(x_1 x_2) + x_1^2 x_2 \cos(x_1 x_2))^2 (.04)^2 + (x_1^3 \cos(x_1 x_2))^2 (.01)^2} \\
&= (.002(2(1.23)\sin(1.23(1.99)) + (1.23)^2 (1.99)\cos(1.23(1.99)))^2 + \\
&+ .0001((1.23)^3 \cos(1.23(1.99)))^2)^{\frac{1}{2}} \\
&= \sqrt{.002(2.46\sin(2.45) + 1.51(1.99)\cos(2.45))^2 + .0001(1.86\cos(2.45))^2} \\
&= \sqrt{.002(2.46(.638) + 3.00(-.770))^2 + .0001(1.86(-.770))^2} \\
&= \sqrt{.002(1.57 - 2.31)^2 + .0001(-1.43)^2} \\
&= \sqrt{.002(-.740)^2 + .0001(2.04)} \\
&= \sqrt{.002(.548) + .0002} \\
&= \sqrt{.001 + .0002} \\
&= \sqrt{.001} \\
&= .03 \\
y &= (1.23 \pm .04)^2 \sin((1.23 \pm .04) * (1.99 \pm .01)) \\
&= (1.23)^2 \sin(1.23(1.99)) \pm \sigma_y \\
&= 1.51 \sin(2.45) \pm .03 \\
&= 1.51(.638) \pm .03 \\
&= 0.963 \pm .03 \\
&= 1 \pm .03
\end{aligned}
$$

(b)

$$\begin{aligned}
y &= x_1^2 x_2^3 e^{x_1 x_2} + x_1^2 x_2 \\
&= (1.23 \pm .04)^2 * (1.99 \pm .01)^3 e^{(1.23 \pm .04)*(1.99 \pm .01)} + (1.23 \pm .04)^2 * (1.99 \pm .01) \\
\sigma_y &= \sqrt{(\frac{\partial y}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial y}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(2x_1 x_2^3 e^{x_1 x_2} + x_1^2 x_2^4 e^{x_1 x_2} + 2x_1 x_2)^2 (.04)^2 + (3x_1^2 x_2^2 e^{x_1 x_2} + x_1^3 x_2^3 e^{x_1 x_2} + x_1^2)^2 (.01)^2} \\
&= ((2(1.23)(1.99)^3 e^{1.23(1.99)} + (1.23)^2 (1.99)^4 e^{1.23(1.99)} + 2(1.23)(1.99))^2 (.002) + \\
&+ (3(1.23)^2 (1.99)^2 e^{1.23(1.99)} + (1.23)^3 (1.99)^3 e^{1.23(1.99)} + (1.23)^2)^2 (.0001))^{\frac{1}{2}} \\
&= ((2.46(7.88) e^{2.45} + 1.51(15.7) e^{2.45} + 4.90)^2 (.002) + (3(1.51)(3.96) e^{2.45} + \\
&+ 1.86(7.88) e^{2.45} + 1.51)^2 (.0001))^{\frac{1}{2}} \\
&= \sqrt{(19.4(11.6) + 23.7(11.6) + 4.90)^2 (.002) + (17.9(11.6) + 14.7(11.6) + 1.51)^2 (.0001)} \\
&= \sqrt{(225 + 275 + 4.90)^2 (.002) + (208 + 171 + 1.51)^2 (.0001)} \\
&= \sqrt{(505)^2 (.002) + (381)^2 (.0001)} \\
&= \sqrt{255000(.002) + 145000(.0001)} \\
&= \sqrt{500 + 10} \\
&= \sqrt{500} \\
&= 20 \\
y &= (1.23 \pm .04)^2 * (1.99 \pm .01)^3 e^{(1.23 \pm .04)*(1.99 \pm .01)} + (1.23 \pm .04)^2 * (1.99 \pm .01) \\
&= (1.23)^2 (1.99)^3 e^{1.23(1.99)} + 1.99(1.23)^2 \pm \sigma_y \\
&= 1.51(7.88) e^{2.45} + 1.99(1.51) \pm 20 \\
&= 138 + 3 \pm 20 \\
&= 141 \pm 20 \\
&= 140 \pm 20
\end{aligned}$$