Uncertainty Assignement - SPH 3U3

Andrey Khesin and Noah Cassidy

1.

2. (a)

$$\begin{aligned}
f &= x_1 + x_2 \\
&= (10.005 \pm .003)\text{ cm} + (20.06 \pm .03)\text{ cm} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.003)^2 + (1)^2(.03)^2} \\
&= \sqrt{1(.000009) + 1(.0009)} \\
&= \sqrt{.000009 + .0009} \\
&= \sqrt{.000909} \\
&= .03 \\
f &= (10.005 \pm .003)\text{ cm} + (20.06 \pm .03)\text{ cm} \\
&= (10.005 + 20.06 \pm \sigma_f)\text{ cm} \\
&= (30.065 \pm .03)\text{ cm} \\
&= (30.07 \pm .03)\text{ cm}
\end{aligned}$$

(b)

$$\begin{aligned}
f &= x_1 - x_2 \\
&= (352.1 \pm .9)\text{ m} - (162.36 \pm .05)\text{ m} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2\sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2\sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.9)^2 + (-1)^2(.05)^2} \\
&= \sqrt{1(.81) + 1(.0025)} \\
&= \sqrt{.81 + .0025} \\
&= \sqrt{.8125} \\
&= .9 \\
f &= (352.1 \pm .9)\text{ m} - (162.36 \pm .05)\text{ m} \\
&= (352.1 - 162.36 \pm \sigma_f)\text{ m} \\
&= (189.74 \pm .9)\text{ m} \\
&= (189.7 \pm .9)\text{ m}
\end{aligned}$$

(c)

$$
\begin{aligned}
f &= x_1 + x_2 \\
&= (56.7 \pm .2)\ \text{cm} + (93.48 \pm .01)\ \text{m} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(1)^2(.2)^2 + (1)^2(.01)^2} \\
&= \sqrt{1(.04) + 1(.0001)} \\
&= \sqrt{.04 + .0001} \\
&= \sqrt{.0401} \\
&= .2 \\
f &= (56.7 \pm .2)\ \text{cm} + (93.48 \pm .01)\ \text{m} \\
&= (56.7 \pm .2)\ \text{cm} + (9348 \pm 1)\ \text{cm} \\
&= (56.7 + 9348 \pm \sigma_f)\ \text{cm} \\
&= (9404.7 \pm 0.2)\ \text{cm}
\end{aligned}
$$

(d)

$$
\begin{aligned}
f &= x_1 + x_2 - x_3 \\
&= (14.5 \pm .2)\ \text{mm} \pm (14.5 + .2)\ \text{mm}\ - (23.1 \pm .1)\ \text{mm} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2 \sigma_{x_3}^2} \\
&= \sqrt{(1)^2(.2)^2 + (1)^2(.2)^2 + (-1)^2(.1)^2} \\
&= \sqrt{1(.04) + 1(.04) + 1(.01)} \\
&= \sqrt{.04 + .04 + .01} \\
&= \sqrt{.09} \\
&= .3 \\
f &= (14.5 \pm .2)\ \text{mm} \pm (14.5 + .2)\ \text{mm}\ - (23.1 \pm .1)\ \text{mm} \\
&= (14.5 + 14.5 - 23.1 \pm \sigma_f)\ \text{mm} \\
&= (5.9 \pm .3)\ \text{mm}
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
f &= x_1 * x_2 \\
&= (23.56 \pm .05)\ \text{kmh}^{-1} * (56.3 \pm .4)\ \text{h} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(x_2)^2(.05)^2 + (x_1)^2(.4)^2} \\
&= \sqrt{(56.3)^2(.0025) + (23.56)^2(.16)} \\
&= \sqrt{3169.69(.0025) + 555.0736(.16)} \\
&= \sqrt{1.387684 + 88.811776} \\
&= \sqrt{90.19946} \\
&= 9 \\
f &= (23.56 \pm .05)\ \text{kmh}^{-1} * (56.3 \pm .4)\ \text{h} \\
&= (23.56 * 56.3 \pm \sigma_f)\ \text{km} \\
&= (1326.428 \pm 9)\ \text{km} \\
&= (1326 \pm 9)\ \text{km}
\end{aligned}
$$

(b)

$$
\begin{aligned}
f &= \frac{x_1}{x_2} \\
&= \frac{(15.745 \pm .006)\ \text{m}}{(0.36 \pm .05)\ \text{m}} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2} \\
&= \sqrt{(\frac{1}{x_2})^2 (.006)^2 + (\frac{-x_1}{x_2^2})^2 (.05)^2} \\
&= \sqrt{(\frac{1}{0.36})^2 (.000036) + (\frac{-15.745}{0.36^2})^2 (.0025)} \\
&= \sqrt{(2.77778)^2 (.000036) + (\frac{-15.745}{0.1296})^2 (.0025)} \\
&= \sqrt{7.71695(.000036) + (-121.489)^2 (.0025)} \\
&= \sqrt{0.000277778 + 14759.6(.0025)} \\
&= \sqrt{0.000277778 + 36.8991} \\
&= \sqrt{36.8994} \\
&= 6 \\
f &= \frac{(15.745 \pm .006)\ \text{m}}{(0.36 \pm .05)\ \text{m}} \\
&= \frac{15.745}{0.36} \pm \sigma_f \\
&= 43.7361 \pm 6 \\
&= 44 \pm 6
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= 2x_1 \\
&= 2(1.63 \pm .03)\ \text{mm} \\
\sigma_f &= \frac{\partial f}{\partial x_1} \sigma_{x_1} \\
&= 2(.03) \\
&= .06 \\
f &= 2(1.63 \pm .03)\ \text{mm} \\
&= 2(1.63) \pm \sigma_f\ \text{mm} \\
&= 3.26 \pm .06\ \text{mm}
\end{aligned}
$$

(d)

$$
\begin{aligned}
f &= \frac{x_1 x_2}{x_3} \\
&= \frac{(1.23 \pm .02)\ \text{ms}^{-1} * (2.637 \pm .003)\ \text{ms}^{-1}}{(5.6 \pm .1)\ \text{ms}^{-1}} \\
\sigma_f &= \sqrt{(\frac{\partial f}{\partial x_1})^2 \sigma_{x_1}^2 + (\frac{\partial f}{\partial x_2})^2 \sigma_{x_2}^2 + (\frac{\partial f}{\partial x_3})^2 \sigma_{x_3}^2} \\
&= \sqrt{(\frac{x_2}{x_3})^2 (.02)^2 + (\frac{x_1}{x_3})^2 (.003)^2 + (\frac{-x_1 x_2}{x_3^2})^2 (.1)^2} \\
&= \sqrt{(\frac{2.637}{5.6})^2 (.0004) + (\frac{1.23}{5.6})^2 (.000009) + (\frac{-1.23(2.637)}{5.6^2})^2 (.01)} \\
&= \sqrt{(0.470893)^2 (.0004) + (0.219643)^2 (.000009) + (\frac{-3.24351}{31.36})^2 (.01)} \\
&= \sqrt{0.22174(.0004) + 0.048243(.000009) + (-0.103428)^2 (.01)} \\
&= \sqrt{0.000088696 + 0.000000434187 + 0.0106974)(.01)} \\
&= \sqrt{0.0000891302 + 0.000106974} \\
&= \sqrt{0.000196104} \\
&= .01 \\
f &= \frac{(1.23 \pm .02)\ \text{ms}^{-1} * (2.637 \pm .003)\ \text{ms}^{-1}}{(5.6 \pm .1)\ \text{ms}^{-1}} \\
&= (\frac{1.23 * 2.637}{5.6} \pm \sigma_f)\ \text{ms}^{-1} \\
&= (0.579198 \pm .01)\ \text{ms}^{-1} \\
&= (0.58 \pm .01)\ \text{ms}^{-1}
\end{aligned}
$$

4. (a)

$$
f = \frac{x_1 + 2x_2}{x_3} \qquad = \frac{(61.12 \pm .05)\ \text{cm} + 2(45.23 \pm .06)\ \text{cm}^2}{(1.03 \pm .04)\ \text{cm}}
$$

(b)

5.

a) (61.12 +.05) cm + 2(45.23 + .06) cm2 (1.03 .04) cm
b) (1005.1 + .2) kmh-1 x (3.93 + .02) kmh-1 (583.68 + .06) km2h-2