

INTERFEROMETRY

Introduction

Interferometers are devices employed in the study of interference patterns produced by various light sources. They are conveniently divided into two main classes: those based on *division of wavefront*, and those based on *division of amplitude*.

Michelson interferometer: theory¹

Throughout this experiment we will mostly be using the Michelson interferometer, which employs a division of amplitude scheme. It can be used to carry out the following principal measurements:

1. Width and fine structure of spectral lines;
2. Lengths or displacements in terms of wavelengths of light;
3. Refractive indices of transparent solids;
4. Differences in the velocity of light along 2 different directions.

It operates as follows: we “divide” the wave amplitude by partial reflection (with a beam splitter² G_1), with the two resulting wave fronts maintaining the original width by having reduced amplitudes [1]. The two beams obtained by amplitude division are sent in different directions against plane mirrors, then reflected back along their same respected paths to the beam splitter to form an interference pattern. The core optical setup, which is labelled in fig. 1, consist of two highly polished plates, A_1 and A_2 , acting as the above-mentioned mirrors, and two parallel plates of glass G_1 and G_2 — one is the beam splitter, and the other is a compensating plate, whose purpose will be described below. The light reflected normally from mirror A_1 passes through G_1 and reaches the eye. The light reflected from the mirror A_2 passes back through G_2 for a second time, is reflected from the surface of G_1 and into the eye.

¹The experiments related to the Michelson interferometer constitute one to three (1-3) weights: actual weight distribution will be described later.

²A beam splitter is nothing more than a plate of glass, which is made partially reflective: as such, the splitting occurs because part of the light is reflected off of the surface, and part is transmitted through it.

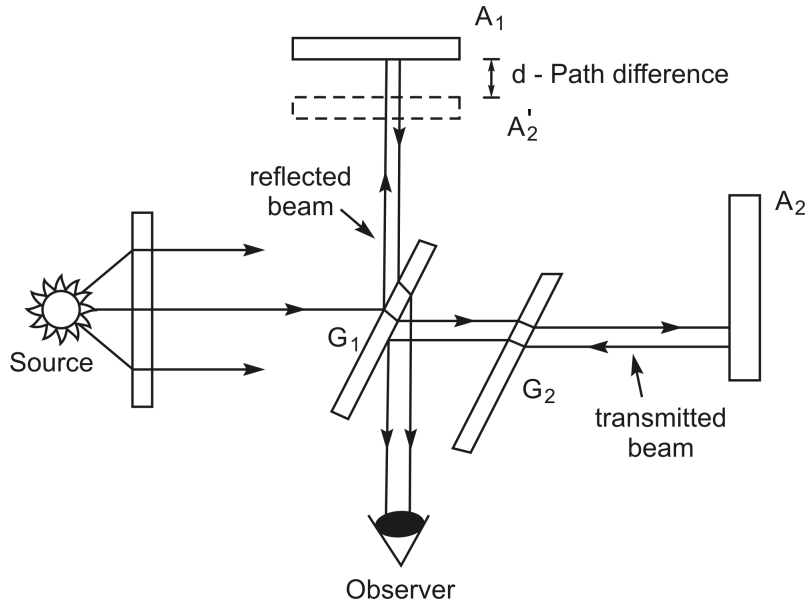


Figure 1: A schematic diagram of the Michelson interferometer.

The purpose of the compensating plate G_2 is to render the path in glass of the two rays equal [1]. This is not essential for producing effective, sharp, and clear fringes in monochromatic light, but it is crucial for producing such fringes in white light (a reason will be given in the “White Light Fringes” section). The mirror A_1 is mounted on a carriage, whose position can be adjusted with a micrometer. To obtain fringes, the mirrors A_1 and A_2 are made exactly perpendicular to each other by means of the calibration screws (seen in fig. 1), controlling the tilt of A_2 .

There are two very important requirements that need to be satisfied along with the above set up in order for interference fringes to appear:

1. One is advised to use an *extended light source*. The point here is purely one of illumination: if the source is a point, there is not much space for you to see the fringes on. You can convince yourself of the usefulness of using an extended source by positioning a variable size aperture in front of an extended source and shrinking its radius to the minimum possible (thus effectively converting it to a point source). As you can see, the field of view over which you can see the fringes shrinks right with it. Hence, it is in your best interest to use as big of a source as possible (a diffusion screen is of further great aid here).
2. The light must be *monochromatic*, or nearly so. This is especially important if the distances of A_1 and A_2 from G_1 are appreciably different. The spacing of fringes for a given colour of light is linearly proportional to the wavelength of that light: hence the fringes will only coincide near the region where the path difference is zero. The solid line here corresponds to the intensity of interference pattern of green light, and the dashed curve — to that of red light. We can see that only around

zero path difference will the colours remain relatively pure: as we move farther away from that region, colours will start to mix and become impure, unsaturated — already about 8-10 fringes away the colours mix back into white light, making fringes indistinguishable. Hence the region where fringes are visible is very narrow and hard to find with non-monochromatic light.

Some of the light sources suitable for the Michelson interferometer are a sodium flame, or a mercury arc. If you use a small source bulb instead, a ground-glass diffusing screen in front of the source will do the job; looking at the mirror A_1 through the plate G_1 , you then see the whole field of view filled with light.

Circular Fringes

To view circular fringes with monochromatic light, the mirrors must be almost perfectly perpendicular to each other. The origin of the circular fringes is understood from fig. 2³. Since light in the interferometer gets reflected many times, we can think of the extended source as being at L , where L is behind the observer as seen in fig. 2; L forms 2 virtual images, L_1 and L_2 , in mirrors A_1 and A'_2 , respectively. The virtual sources in L_1 and L_2 are said to be in phase with each other (such sources are called *coherent sources*), in that the phases of corresponding points in the two are exactly the same at all times. If d is the separation of A_1 and A'_2 , the virtual sources are then separated by $2d$, as can be seen in the diagram (fig. 2).

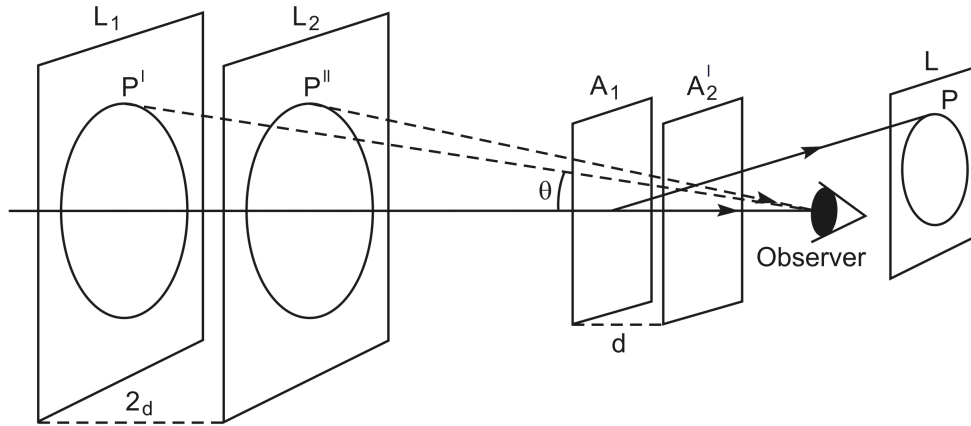


Figure 2: Virtual images from the two mirrors created by the light source and the beam splitter in the Michelson interferometer.

When d is exactly an integer number of half wavelengths⁴, every ray that is reflected

³The real mirror A_2 has been replaced by its virtual image A'_2 formed by the reflection in G_1 : hence A'_2 is parallel to A_1 .

⁴The path difference, $2d$, must then be an integer number of wavelengths.

normal to the mirrors A_1 and A'_2 will always be in phase. Rays of light that are reflected at other angles will not, in general, be in phase. This means that the path difference between two incoming rays from points P' and P'' will be $2d \cos \theta$, where θ is the angle between the viewing axis and the incoming ray. We can say that θ is the same for the two rays when A_1 and A'_2 are parallel, which implies that the rays themselves are parallel⁵. The parallel rays will interfere with each other, creating a fringe pattern of maxima and minima for which the following relation is satisfied:

$$2d \cos \theta = m\lambda \quad (1)$$

where d is the separation of A_1 and A'_2 , m is the fringe order, λ is the wavelength of the source of light used, θ is as above (if the two are nearly collinear, we, of course, have $\theta \approx 0$ — this is the case for the fringes in the very centre of the field of view).

Since, for a given m , λ , and d the angle θ is constant the maxima and minima lie on a circular plane about the foot of the perpendicular axis stretching from the eye to the mirrors. As was mentioned before, the Michelson interferometer uses division by amplitude scheme: hence the resultant amplitudes of the waves, a_1 and a_2 , are fractions of the original amplitude A , with respective phases α_1 and α_2 . We can calculate the phase difference between the two beams based on the respective mirror separation. If the path difference is $2d \cos \theta$, then the difference in phase δ for light of wavelength λ is simply

$$\delta = 2\pi \frac{2d \cos \theta}{\lambda} \quad (2)$$

Here the ratio of the path difference to the wavelength tells you what fraction of a wavelength have you passed, and multiplication by 2π makes it a fraction of the full period of a sinusoid, thus giving you exactly the sought phase difference.

By starting with A_1 a few centimetres beyond A'_2 , the fringe system will have the general appearance which is shown in fig. 3, where the rings of the system are very closely spaced. As the distance between A_1 and A'_2 decreases, the fringe pattern evolves, growing at first until the point of zero path difference is reached, and then shrinking again, as that point is passed.

⁵Since the eye focused to receive the parallel rays, it is more convenient to use a telescope lens, especially for looking at interference patterns with large values of d .

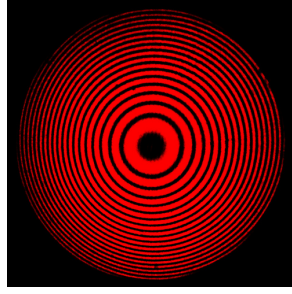


Figure 3: The circular fringe interference pattern produced by a Michelson interferometer.

This implies that a given ring, characterized by a given value of the fringe order m , must have a decreasing radius in order for (2) to remain true. The rings therefore shrink and vanish at the centre, where a ring will disappear each time $2d$ decreases by λ . This is because at the centre, $\cos \theta = 1$, and so we have the simplified version of equation (2),

$$2d = m\lambda \quad (3)$$

From here we see that the fringe order changes by 1 precisely when $2d$ changes by λ , hence for a fringe to disappear we need to decrease $2d$ by λ , as claimed above.

Localized fringes

In case when the mirrors are not exactly parallel, fringes can still be observed for path differences not much greater than a few millimetres (using monochromatic light, of course). The space between the mirrors is wedge-shaped, as can be seen in fig. 4: thus the two rays reaching the eye from the mirrors are no longer parallel and appear to diverge.

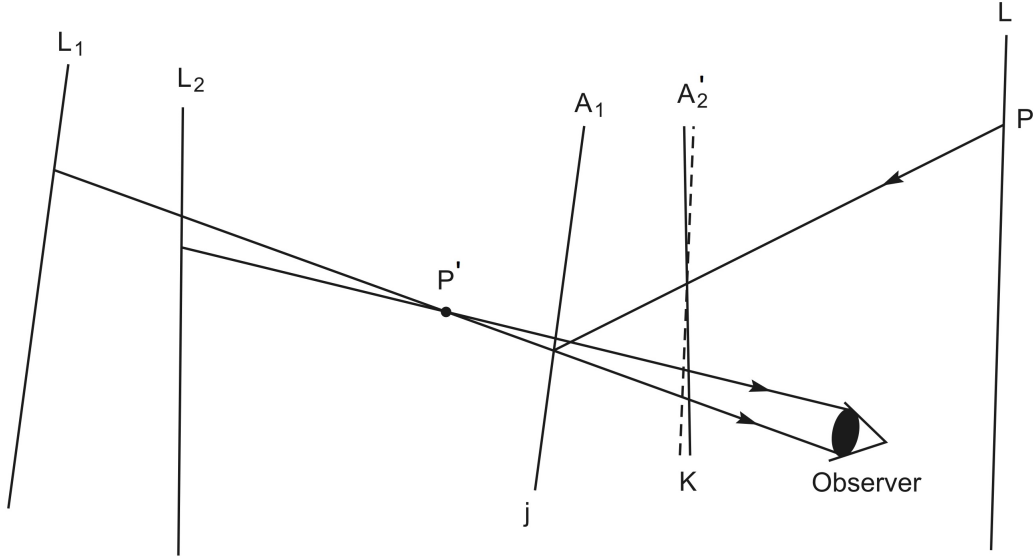


Figure 4: Formation of localized fringes with non-perpendicular mirrors — the air wedge is clearly seen.

Hence, the interference picture will be more like that of fig. 5: the fringes are now semi-circles, with the centre lying outside the field of view — such fringes are often called *localized fringes*. The reason these fringes are almost straight is primarily because of the variation of the thickness of air in the wedge, as that is now the main reason for the variation of the path difference between the two beams across the field of view. One would expect all fringes to be perfectly straight, parallel to the edge of the wedge: however, that is not the case, as the path difference still does vary somewhat with the angle θ , especially if d is large. Depending on the magnitude of d , we can observe different interference patterns: as we change the path difference, the fringes become straighter, until we hit point of zero path difference. At that point, if we were looking at circular fringes, they would fill the whole field of view, become very large circles — that means that localized fringes would become parallel lines, as if there were small sections of the circumferences of very large circles.

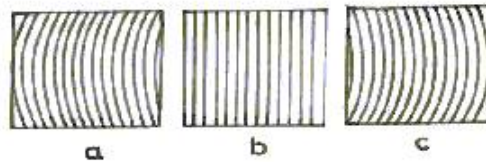


Figure 5: The localized fringe interference patterns produced by a Michelson interferometer: (a) and (c) are depictions of curved fringes, implying the mirror is far from the region of zero path difference; and (b) shows straight, parallel fringes — this must be at or very near the point of zero path difference.

The association “large circular fringes — parallel localized fringes” will be important in the next section, when we use it to locate white light fringes.

White Light Fringes

If instead of using monochromatic light, we wish to study the fringes created by white light, no fringes will be seen at all except for when the path difference between A_1 and A'_2 does not exceed a couple of wavelengths⁶. This is well-demonstrated in fig. 6: the dashed line corresponds to the intensity of the interference pattern of green light, while the solid line — to that of yellow light. As you can see from the diagram, the patterns only overlap over the narrow range of zero path difference between the two incoming beams: now if there are many different wavelengths involved in an interference process, as is the case for white light, one can conclude that anywhere too far away from the region of zero path difference the colours will mix back up into white light and no fringes will be visible.

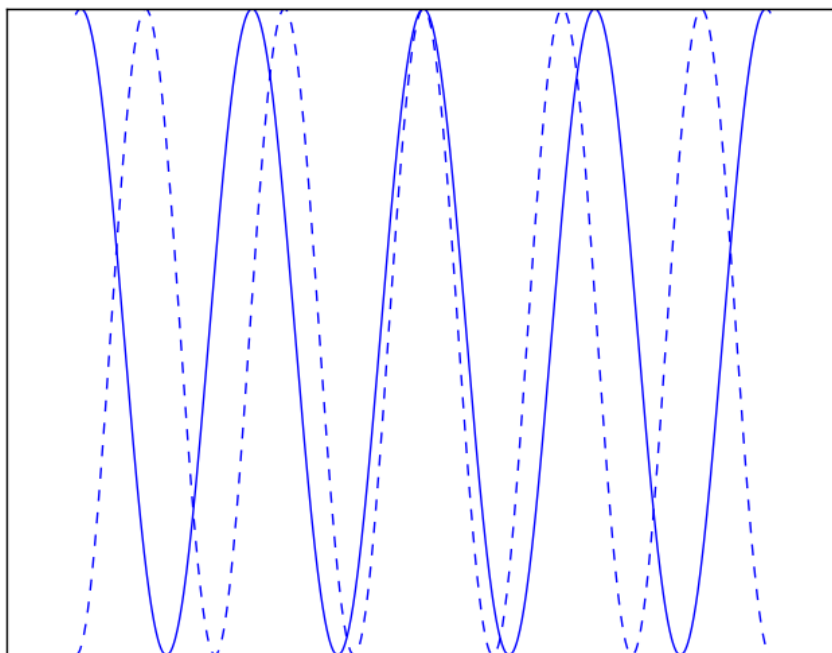


Figure 6: The intensity curves of interference of green light (dashed line) and yellow light (solid line). The dispersion of the interference patterns away from the region of zero path difference is readily observed.

With white light, there will be a central dark fringe, bordered on either side by 8 or 10 coloured fringes. Since the region over which the white light fringes are visible is so narrow, trying to search for it with white light alone is too time-consuming. Instead you

⁶They are extremely difficult to find, so *have patience* — they do exist.

can first approximate its location by using monochromatic light and finding the region of zero path difference. It will correspond to one of two regions: (a) if the mirrors are perfectly parallel and we are observing circular fringes, the region with the largest circular fringes is the region of zero path difference; (b) if the mirrors are almost parallel and we are observing localized fringes, then the region with straight, parallel fringes will be the region of zero path difference. Once we have approximately found the right region, we switch back to white light, and move VERY slowly through the region: the bright fringes should come into view. These fringes will only occur over a very narrow range of path difference values, corresponding to about a 20 degree turn on the micrometer — hence the need to move slowly, otherwise you can miss them. We advise that, upon finding the fringes, you mark the approximate position of the micrometer, to simplify future search (as the position of white light fringes will be needed for other experiments).

Fabry-Perót interferometer: theory⁷

Another commonplace division-of-amplitude interferometer is the Fabry-Perót interferometer, which uses a principle similar to that of the Michelson interferometer to produce interference fringes. The core of such a device consists of two parallel flat glass plates, one movable, one fixed, the inner surfaces of which are coated with a partially reflective metallic layer (see fig. 7).

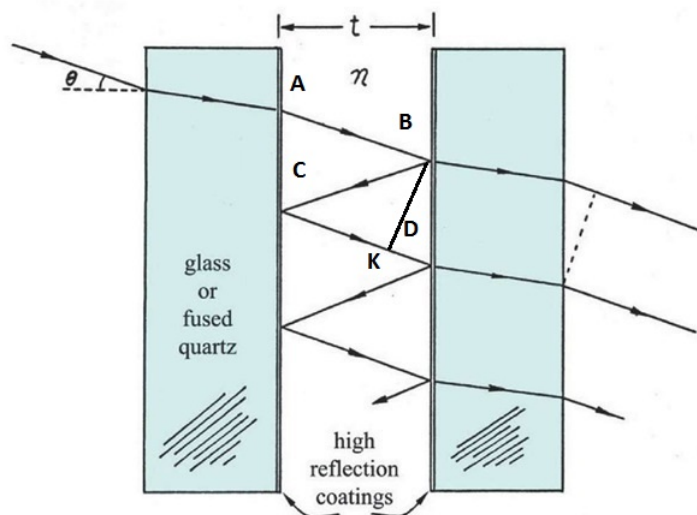


Figure 7: The reflected and transmitted beams of light going through the two glass surfaces of a Fabry-Perót interferometer (letters indicate points of reflection/refraction). Source: <http://what-when-how.com/radial-velocities-in-the-zodiacal-dust-cloud/preparations-and-experimental-details-1971-1974-zodiacal-dust-cloud-part-2/>

Due to the coating, a beam of light incident on the first plate at an angle θ to the horizontal produces a series of beams passing through to the other side, as each

⁷The experiments related to the Fabry-Perót interferometer constitute one (1) weight.

continuously gets either transmitted through the second plate to go on to the observer, or bounces back and forth between the inner surfaces until it does (it could also potentially come back out from the side the original beam entered the arrangement, but those rays are of no consequence to us). Each of the beams arrives at the point of observation with a path difference of δ with the one before and after it: thus they reinforce each other and produce an interference pattern. Let the distance between the plates be t . We see from fig. 7 that the path difference δ between the rays exiting at B and D is exactly

$$\delta = BC + CK$$

In the diagram, the line BK is normal to CD . The angle between BC and CK is 2θ , and the triangle BCK is a right angle one. Hence we may write

$$CK = BC \cos 2\theta$$

Moreover, we can relate the hypotenuse BC to the distance between the plates via

$$BC \cos \theta = t$$

Thus we have, for the path difference

$$\delta = BC + CK = BC(1 + \cos 2\theta) = 2BC \cos^2 \theta = 2t \cos \theta \quad (4)$$

The condition for constructive interference is, as always

$$n\lambda = 2t \cos \theta \quad (5)$$

where n is the fringe order, and λ is, of course, the wavelength. We can vary the separation between the glass plates and watch the fringes disappear in the centre of the field of view, thus allowing us to do almost exactly the same measurements as we could with a Michelson interferometer. The advantage of the Fabry-Perót is its high resolving power: it makes it a valuable tool in the study of the Zeeman effect and the hyperfine structure of certain spectral lines.

One point has to be made concerning this device: since the interference only occurs for light incident on the plate as an angle θ , a perfectly parallel beam of light may not produce fringes: hence we must once again use an extended light source to remedy this problem.

Apparatus

NOTE: Most mirrors in the apparatus are front surface aluminized. Do not touch the surfaces, nor wipe them. They can easily be permanently damaged.

Michelson interferometer: setup

The Michelson Interferometer is a fundamental design of a large variety of two-beam interferometer configurations. In this experiment we will use the most basic apparatus (see fig. 8). Light from a source unit N (a mercury or sodium lamp, in this experiment), passing through a diffusing screen/filter holder unit D^8 , is incident on the plane-parallel beam splitter plate with compensating plate (they are together in one whole unit C) and is divided into two beams, the axes of which, 1 and 2, fall normally on the mirrors A and B , respectively. The returned beams re-unite at the semi-reflecting surface of C . The interference pattern can be viewed directly with the naked eye or by means of a telescope at the viewing position.

⁸The diffusing screen is a piece of ground glass used to spread out (or *diffuse*) the light across the field of view, to get soft light. It is important for the same reason it is to make use of an extended light source — one needs to illuminate as large a part of the field of view as possible, to simplify fringe observation.

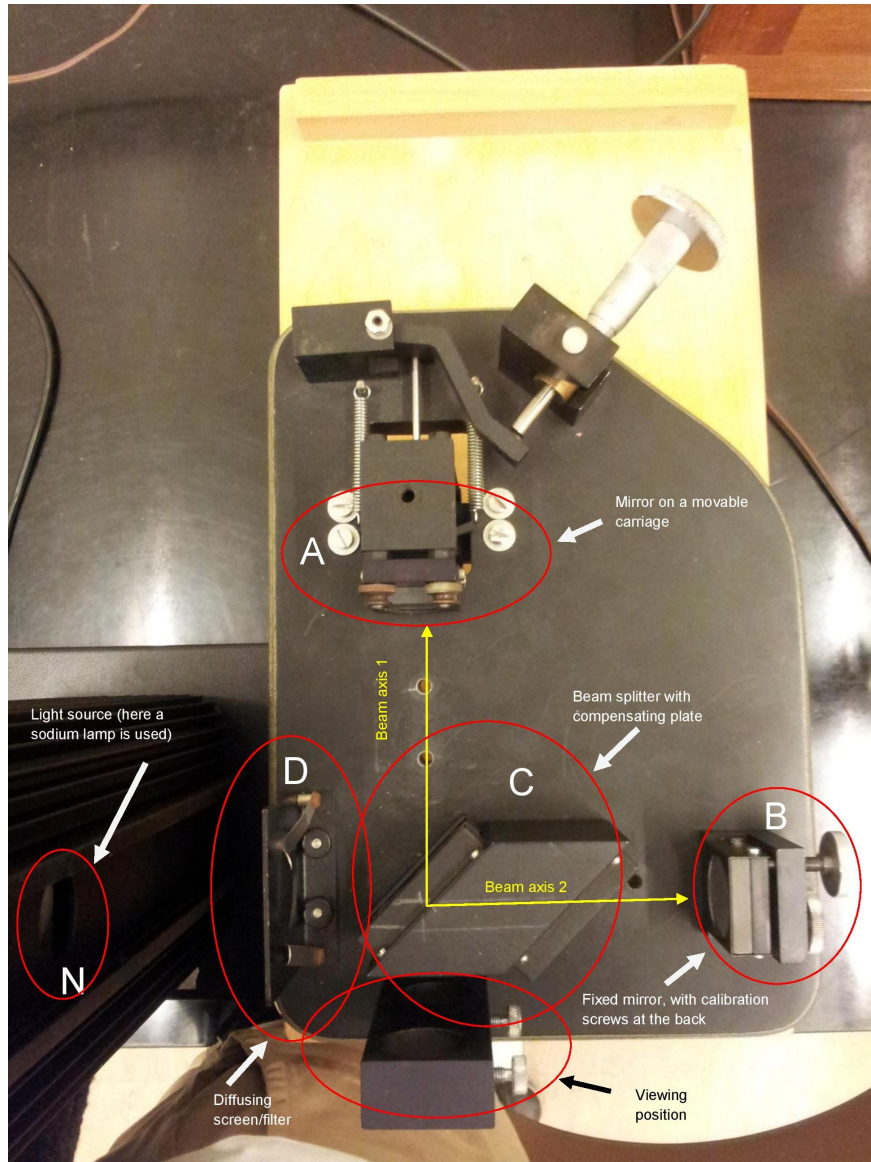


Figure 8: The Michelson interferometer setup used in this experiment — letters indicate units, as noted in the explanation above.

The compensating plate at C is identical in thickness to the beam splitter plate and is set accurately parallel to it. Its insertion then equalizes the glass paths in the two beams, as mentioned earlier. When the mirrors A and B are perpendicular, and A is slightly closer than B , the image from A will fall in front of that from B and a series of interference fringes will be seen. When the mirrors are equidistant and perpendicular, the interference field will be covered by one large circular fringe. When the surfaces B and A are not precisely parallel and the separation distance is very small, a series of fringes in shapes of approximately straight lines will be seen. For a non-laser source, fringe contrast increases as distance apart is reduced.

Fabry-Perót interferometer: setup

The Fabry-Perót interferometer used in this experiment is depicted in fig. 9. Light originates from an extended source in the back of the setup (in the diagram a sodium lamp is used), passes through the first mirror A , installed in the top position, and into the second mirror E . It continues along the viewing axis and into the telescope L , clamped in a holder H with a screw. Adjust the telescope magnifying unit until the light source is in focus. Do not use any collector lenses, as they only obstruct the view.



Figure 9: The Fabry-Perot interferometer setup used in this experiment — letters indicate units, as noted in the explanation above.

Vacuum pump

For one part of this experiment, you will need to use a vacuum pump to find the index of refraction of a gas at normal atmospheric temperature and pressure. Specifically, you will be using an Edward High Vacuum Ltd. rotary vacuum pump (parts of the technical manual are available in the resource room, 229). The pump apparatus consists of several parts:

1. The pump itself, housing the rotor and the oil chamber: attached to its vacuum connection (see the manual for a diagram) is the main access valve, separating the gas cell from the insides of the pump;
2. A gas cell, connected to the pump via three reinforced plastic tubes;
3. A tall dial gauge, indicating the pressure inside the gas cell, in mm Hg (operates between 1 and 760 mm Hg);
4. A release valve, also connected to the gas cell, to control the pressure level.

One operates the pump as follows:

1. Ensure the gas cell is properly connected to the pump.
2. Connect the pump to a wall outlet.
3. Slowly open the main access valve and establish a pressure of 760 mm Hg in the gas cell.
4. Use the release valve to vary the pressure inside the gas cell.

Be sure to keep the release valve open when you unplug the vacuum pump to let the pressure slowly leak from the pump. When you turn on the pump, it will make ungodly noise — that is to be expected.

Experiment and Procedure

With the Michelson interferometer (1-3 wt)

In this part of the experiment you will learn, on specific examples, the standard measurement techniques which utilize the Michelson interferometer: in particular, you will

1. Learn how to set up the device and align the mirrors;
2. Observe the interference fringes with both monochromatic and white light;
3. How to use it to determine the index of refraction of transparent solids and of gases.

The arrangement of the interferometer outlined in the section “Apparatus: Michelson interferometer: setup” will be the arrangement used for the entire experiment (except for when you will need to switch between the light sources used).

Initial adjustments, observation of fringes, and calibration (1 wt)

Dim the room lights. Position the sodium lamp (it will take some time for it to warm up after you turn it on) in front of the diffusing screen holder D , and insert the diffusing screen into the holder. Looking through the opening at the viewing position (usually the closer, the better the view), you will observe dark fringes on a yellow/orange background: they will most likely be localized fringes (if you do not see any fringes at all, try rotating the micrometer screw until they appear). Adjust the calibration screws on mirror B to make it perpendicular to mirror A : you will know they are perpendicular when you see complete circular fringes, with the centre of the fringes right in the centre of your field of view.

Once you have observed the fringes, locate the region where the path difference between the two beams of light is close to zero. Recall that when viewing circular fringes, this region is the region where the fringes observed are largest, covering the entire field of view; whereas when viewing localized fringes, this region corresponds to the region where the fringes are parallel to each other. It is advisable to use the latter to locate the region of zero path difference. Mark the approximate location of the region by noting the micrometer reading to speed up the procedure for next time.

Switch the source of light to white light. By rotating the micrometer and moving the mirror carriage very slowly through this region, you can observe the elusive white fringes. As was noted earlier, these fringes are only observable over a range of about a 20 degree rotation of the micrometer head: the range is only about 20 fringes wide, so be sure to rotate the micrometer very slowly. The fringes in white light can only be viewed when the path difference $2d \cos \theta \approx 0$.

Now switch the light source back to the sodium lamp, and adjust mirror B until you see circular fringes of medium to large size. You are going to set up a calibration curve between the motion of the micrometer screw and the actual displacement of mirror A . Since there is a rather non-trivial system of levers connecting the mirror carriage with the micrometer screw-head, not all of the motion of the micrometer is translated into the motion of the carriage: we need to determine the exact relationship. To do so, we will make use of equation (3) and our earlier observation that if the distance $2d$ changes by the wavelength λ , then one fringe passes out of the field of view. Hence if we count the number of fringes that disappeared from the field of view in a given distance moved by the micrometer, we can directly relate the two, as, using the number of fringes and equation (3), we can calculate how much the mirror actually moved, and relate that to what we took down for the motion of the micrometer.

NOTE: The mirror carriage should always be driven towards the observer

when making readings. Overshoot cannot be easily corrected by reversing the direction of rotation of the micrometer screw because of backlash between the screw thread and the carriage (i.e. reversal of the direction of rotation of the micrometer screw does NOT result in immediate reversal of motion of the carriage: over an angle of a few degrees, the screw rotates without moving the micrometer at all). Ask the demonstrator to explain this point if it is not clear to you.

There are multiple ways to build a calibration curve, but we propose the following scheme:

1. Note the initial position of the micrometer;
2. Slowly rotate the micrometer and count the number of fringes that disappear in the middle of the field of view (you need to be moving the mirror toward you);
3. Record the micrometer position for every 50 fringes as well as the total number of fringes you've moved and proceed until you have counted 1000 fringes;
4. Plot the micrometer displacement values as a function of fringes and, using equation (3), calculate the path distance moved per 50 fringes (you may use the mean wavelength value of the sodium spectral line $\lambda = 589.3 \text{ nm}$);
5. Finally, plot the micrometer reading against the corresponding actual motion of the carriage, and perform a linear fit. Consequently, the slope of the fit can be used to convert micrometer readings into actual distance moved by the mirror carriage;
6. Remark on the uniformity of the micrometer screw (i.e. on the linearity of the data).

Refractive index of a transparent solid (1 wt)

As mentioned earlier, one of the possible uses of a Michelson interferometer is to measure the index of refraction of a transparent solid. By inserting such a solid into the optical path of the beam aligned with the movable mirror, you displace the interference pattern (since the path difference is now increased due to the fact that the index of refraction of the solid is different from that of air): by making note of exactly how much the interference pattern was displaced, you can figure out how much the path difference increased and, consequently, the index of refraction of the material.⁹ In particular, in this part of the experiment we will do so for a small microscope slide, provided along with the other components in the box.

⁹There are limitations to this technique: for example, if the added path difference is too large (if, say, the solid is too thick), then the displacement of the interference pattern will be too large to account for — no motion of the mirror will restore the original picture. Thus the limitations of using the method depend on the bounds of motion of the mirror carriage — that is, on the size of the interferometer.

Consider a thin parallel plate solid, with index of refraction μ , flat on both sides and sufficiently transparent (we will also assume it is uniform, otherwise the index of refraction would vary on the exact place where the beam passes through the solid), of thickness t . If we place it in the optical path of the beam going toward mirror A , the path length of that beam will increase by $\delta = 2d(\mu - 1)$ ¹⁰. Since the displacement of one complete fringe is equivalent to changing the path difference $2t(\mu - 1)$ by one wavelength λ , for m fringes we will have

$$2t(\mu - 1) = m\lambda$$

or

$$\mu = \frac{m\lambda}{2d} + 1 \quad (6)$$

Thus, using a light of known wavelength, along with the thickness of the solid, and noting the amount of fringes that the pattern was displaced by, we can find the index of refraction of the solid.

Refractive index of a transparent solid: procedure

In practice, it is slightly more complicated. First of all, you want to make the solid parallel to the mirror to make sure the distance it travels through the solid a distance equal to the thickness (and not longer — if, say, the beam was incident on the plate at an angle). To achieve this, try to make sure the solid is as parallel to the mirror as possible: it is advisable that you use a holder, which you can screw to the optical bench — then put a simple white light source into position, and try to align the three images of it produced by the beams at the viewing position by tilting and rotating the solid. Moreover, since the transition of the interference pattern is sudden, there is no way to count the fringes “as they pass”, like you did to get a calibration curve. And if you use a monochromatic light source, all the fringes are virtually indistinguishable — they have different radii, but without a precise scale it would be impossible to find the displacement of the interference pattern. Instead, we shall do the following: as in the previous part of this experiment, we locate the white light fringes and note the reading of the micrometer. The fact that they only occur over a narrow range of mirror positions will be to our advantage this time. As soon as you insert the solid in the optical path of the beam headed toward mirror A , the interference pattern will be displaced and you will no longer see the white light fringes. However, if you then slowly move the carriage (toward the observer, as you have to account for increased path difference by decreasing the geometric distance between the mirror and the beam splitter), you can locate the fringes once again. The difference in the reading of the micrometer between these two positions can be related, via your

¹⁰The beam traverses a distance t through the solid. Before the solid is inserted into place, the optical path length across a stretch of air of length t with index of refraction $\mu_{air} = 1$ is simply $r_0 = 2t\mu_{air} = 2t$ (the factor of two accounts for the fact that the beam traverses this stretch of air twice: on the way to the mirror, and on the way back). After the solid is put into place, the new optical path length is $r = 2t\mu$. Hence the path difference δ , introduced by the solid, is simply $\delta = r - r_0 = 2t(\mu - 1)$.

calibration curve, to the amount the mirror actually moved, which can be tied back to equation (6). Using equation (3) in the form $2d = m\lambda$ and noting that $d = Mf$, where M is the micrometer distance reading and f is the conversion factor (slope of calibration curve obtained earlier), we obtain

$$\mu = \frac{Mf}{t} + 1 \quad (7)$$

So all we really need is to measure the thickness of the plate and, having found the fringe pattern again, to note the distance traversed by the micrometer. Nevertheless, even finding the displaced fringes is difficult: you can estimate the index of refraction of your microscope slide (they are usually made out of soda lime or borosilicate glass, with indices of refraction around 1.5) and from that you can outline a range of distances you have to explore with the micrometer to find the fringes. Be aware that it might take quite some time to find them even still. Once you found the region, it is advisable to mark its approximate location on the micrometer. Having done this several times, calculate the index of refraction using equation (7).

Refractive index of gas (1 wt): experiment

Another useful application of the Michelson interferometer is the measurement of the index of refraction of air (or any gas) by exploiting the relationship between the index of refraction n and pressure P in the gas chamber.

Consider an evacuated cylindrical gas cell, positioned on the viewing axis of mirror A , of length l . Suppose gas is admitted into it, with index of refraction n . The change in the optical path length will be simply $2l(n - 1)$ ¹¹, hence we obtain the exact same relationship as in the previous section, for a light of wavelength λ

$$2l(n - 1) = N\lambda$$

where N is the number of fringes counted. Take the derivative with respect to pressure:

$$\frac{dN}{dP} = \frac{2l}{\lambda} \frac{dn}{dP} \quad (8)$$

It is an experimental fact that, for gases, the number $n - 1$ is proportional to the density of the gas ρ , as long as the chemical composition of the gas does not change, i.e. $n - 1 = c\rho$ for some constant c [4]. Assume the gas obeys the ideal gas law,

$$PV = \frac{m}{M}RT$$

where V is the volume, m and M the total and the molar masses, respectively, R is the

¹¹Derivation is similar to that presented in footnote (11).

universal gas constant and T is the absolute temperature. We will rewrite it in the form

$$P = \frac{\rho}{M} RT \quad (9)$$

where, of course $\rho \equiv m/V$. Let us consider the gas in two states: one will be defined by the variables P, V, ρ, T, n , and the other will be the gas in some other (reference) state, with the corresponding variables $P_0, V_0, \rho_0, T_0, n_0$. We may write

$$\frac{n-1}{n_0-1} = \frac{\rho}{\rho_0} \quad (10)$$

From the ideal gas law in the form (9) for each of the states we see that

$$\frac{\rho}{\rho_0} = \frac{PT_0}{P_0T}$$

and thus, combining this with (10)

$$n-1 = (n_0-1) \frac{PT_0}{P_0T} \quad (11)$$

Let us take the reference state to be at normal temperature and pressure, i.e. let $T_0 = 273$ K, $P_0 = 760$ mm Hg¹². Take the derivative with respect to pressure on both sides (we assume the process is isothermal, so that temperature remains constant throughout the measurements and does not vary with pressure) to obtain

$$\frac{dn}{dP} = \frac{n_0-1}{T} \frac{T_0}{P_0}$$

Substituting into equation (8) yields

$$n_0 = 1 + \frac{dN}{dP} \frac{\lambda}{2l} \frac{760T}{273} \quad (12)$$

From this equation we see that if we measure the rate of change of passage of fringes through the field of view with pressure (i.e. take several measurements of the number of fringes passed, counting from zero, and the corresponding pressure, do a linear fit on the data and take the slope), while noting the temperature at which the measurements are taking place, we can find the index of refraction of a given gas n_0 at normal temperature and pressure.

We now describe the procedure of the experiment.

¹²The reason for the choice of such strange pressure units makes sense if you recall the description of the vacuum pump apparatus used in this experiment — those are the units its gauge readings are in.

Refractive index of gas: procedure

Begin by taking the gas cell (it should be connected via three reinforced plastic tubes to the vacuum pump) and measuring its length. Since you do not want to take into account the thickness of the glass walls of the cell, it is best to measure the overall length of the gas cell, then measure the thickness of one of the walls and perform the subtraction. Once finished with the measurements, attach the cell to the stand between mirror A and the beam splitter by inserting the two screws into the holes in the optical bench and tightening the nuts. Since we only want to take into account the path difference created by the gas and not the glass walls of the glass cell, we need to compensate for them by inserting two thick circular glass plates into a holder in the path of the other beam.

Plug in the vacuum pump, and open the valve so that the gauge reads 760 mm Hg. By slowly turning the release valve (the small attachment with a knob on it) you can lower and raise the pressure by an incremental amount. Now set up the interferometer to view circular fringes with a monochromatic light source (using a sodium lamp is advisable). By setting up the pressure as above, take measurements of the change in pressure, and how many fringes cross the field of view, all the while keeping track of ambient temperature. One person should slowly vary the pressure, and another should count the fringes and announce the number of them that passed by at specific pressure increments. Try to conduct measurements quickly, so that the temperature does not have time to change.

Using equation (12) to find the value for the index of refraction you determined, calculate the ratio of your value vs. the accepted value of $n_0^{\text{accepted}} = 1.000277$.

With the Fabry-Perót interferometer (1 wt)

The chief advantage of the Fabry-Perót interferometer is its higher resolving power compared to that of the Michelson interferometer: as such, it is often used to investigate the fine structure of spectral lines. In this experiment we will look at the spectral line of sodium, which is actually a doublet of two lines separated by a very small wavelength difference: we will be able to discern them and quantify the separation.

Set up the interferometer as described in the “Apparatus” section, with the mirror A set all the way back. The first task is to make the adjustable mirror of unit E almost exactly parallel to mirror A : we begin by employing an incandescent light bulb and, without the telescope, adjusting the calibration screws of mirror E to make the light bulb electrical arc aligned with its multiple reflections. Once this is accomplished, the two mirrors are approximately parallel: now finer adjustments are needed. Replace the light bulb with the sodium lamp and observe the interference fringes (again without the telescope): the pattern is hard to discern, as the mirrors are likely still not perfectly aligned — there might be several underlying interference patterns. Focus on the ones in the background, and try to align them with each other by bringing the centre of the circular fringes to the middle of the field of view. Finally, when you think you’ve brought

the pattern to the centre, insert the telescope tube, first without the magnifying piece, and make more adjustments to bring the pattern to the middle. Once you have done that, insert the magnifying piece and focus on the pattern by adjusting the depth of insertion of the magnifying piece into the telescope tube; once you are in focus, complete the final adjustments to bring the pattern to the middle of the field of view of the telescope. If at any point of this procedure you feel you completely lost the pattern, go back to the light bulb. In the end, you should observe perfectly focused circular fringes, with the doublet clearly visible (to check if you got it right, try turning the micrometer screw - the fringes should come in and out of coincidence). The view should resemble fig. 10.

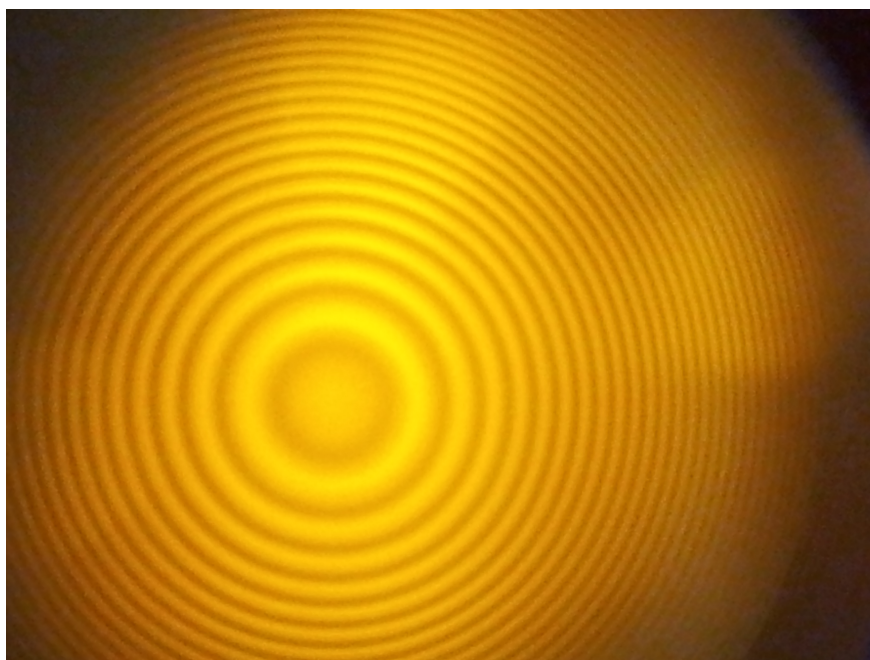


Figure 10: Circular fringes from a sodium light source as seen in the Fabry-Perót interferometer.

If that is not the case, try starting from scratch: reset the adjustable mirror E to the very back, then start from the beginning with the light bulb.

Once you have located the circular fringes, proceed to calibrate the device just as you have done for the Michelson interferometer: count fringes passing the field of view and record the micrometer readings every 50 fringes (have the mirror carriage move toward you, as before).

Now that you have the calibration curve, you can use the interference pattern to determine the wavelength separation of the doublet. We proceed as follows.

Let the two wavelengths of the spectral lines in the doublet be λ_1 and λ_2 , with $\lambda_2 < \lambda_1$. Now for certain path differences the two interference patterns produced by the spectral lines may be interfering with each other (on top of with interfering with themselves to produce the fringes in the first place), moving in and out of complete coincidence with

each other. The coincidence condition is

$$f\lambda_1 = g\lambda_2$$

where f, g are some integers. The next time a coincidence will occur, as we increase the fringe order, is when the condition

$$(f + h)\lambda_1 = (g + h + 1)\lambda_2$$

is satisfied, where h is another integer. Subtracting the two, we obtain

$$h\lambda_1 = (h + 1)\lambda_2$$

or, rearranging for the difference

$$\frac{\lambda_2}{h} = \lambda_1 - \lambda_2 \equiv \Delta\lambda$$

where $\Delta\lambda$ is the sought wavelength separation. Thus, knowing the number of fringes h between positions of coincidence of the two wavelengths, along with the wavelength value of one of the lines in the doublet, λ_2 , we can find the separation we seek. Now let us make use of our calibration curve: instead of patiently counting the fringes, let us note the micrometer readings of displacements — call it M — and convert it to actual mirror displacement d via the conversion factor, $d = fM$. Knowing the mirror displacement d , we use the basic result that $h\lambda_1 = 2d$ and write

$$h = \frac{2d}{\lambda_1} \Rightarrow \Delta\lambda = \frac{\lambda_1\lambda_2}{2d}$$

We will approximate the above by treating the product of the two wavelengths as their geometric mean¹³ squared, i.e. $\lambda_1\lambda_2 \approx \bar{\lambda}^2$ — where we take the value of the average sodium spectral line wavelength to be $\bar{\lambda} = 589.3$ nm. Hence our final expression for the wavelength difference is

$$\Delta\lambda = \frac{\bar{\lambda}^2}{2fM} \tag{13}$$

¹³Recall that the geometric mean of numbers $n_1, n_2, n_3, \dots, n_k$ is defined by $\bar{n} = \left(\prod_{i=1}^k n_i\right)^{\frac{1}{k}}$.

Questions

Michelson interferometer

Initial adjustments, observation of fringes, and calibration

1. Produce a calibration curve, as discussed in the “Experiment and Procedure” section, using Python (and use it for all subsequent data analysis and plotting).
2. Be sure to include a χ^2 value for the fit, and, in its light, discuss the uniformity of the screw thread.

Refractive index of a transparent solid

1. Find the index of refraction of a microscope slide, following the procedure outlined in the “Refractive index of a transparent solid: procedure” section.
2. Compare with known values of index of refraction of material employed in manufacturing microscope slides and comment on the accuracy of the measurement.

Refractive index of gas

1. Build a curve of pressure versus the number of fringes having passed the field of view, and from its slope calculate the index of refraction of air at normal temperature and pressure, as described in section “Refractive index of gas”.
2. In your error estimates and calculations, evaluate the significance and impact of the following (possible) sources of error: (a) change in cell length when the cell is partially evacuated, (b) influence of relative humidity of air.

Fabry-Perót interferometer

1. Build a calibration curve for the micrometer.
2. Using this calibration curve, find the wavelength separation of the doublet of sodium spectral lines.

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Additional reference for general optics inquiries

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The lab manual was revised and greatly expanded in 2014 by P. Albanelli and S. Fomichev.