TODO TITLE

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TODO ABSTRACT

TODO ITALICIZE DEFINITIONS? TODO DEFINITIONS? TODO THE AUTHORS INSTEAD OF WE?

I Introduction

A ribbon knot is a knot that contains only ribbon singularities. A ribbon singularity occurs when the disk that is bounded by the knot only intersects itself in pairs, e.g. when the boundary of the disk passes through said disk, there will be another intersection "nearby" with the opposite sign. A "ribbon presentation" for a knot depicts the knot as the image of a series of disks, each connected to the next by an arc, the ribbon, which is "split" into two strands after applying the map.

A symmetric union is a knot that is symmetric about a central axis, save the crossings that lie on the axis itself. It is well-known that any symmetric union is a ribbon knot. However, at this time of this writing TODO SOURCE it is not yet known whether every ribbon knot can be expressed as a symmetric union. For all 21 knots with 10 or fewer crossings it has been shown that they can be expressed as symmetric unions TODO SOURCE.

II Tangles

A tangle is a collection of joined crossings with a certain number of "loose ends" sticking out. A tangle with n strands will have $2n$ such ends sticking out. For convenience, a tangle is drawn as a braid, with n ends along the top and n ends along the bottom where the order of the strands along the top is the same as along the bottom. In other words, if one were to follow each strand in a tangle, they would end up exactly below the point at which they started. Let the tops of the strands be denoted by $1_t,\ 2_t$, ..., n_t, and the bottoms by $1_b,\ 2_b$, ..., n_b. TODO DIAGRAM

A closure of a tangle is a manner of stitching the ends of the tangle together to reduce the resulting number of strands. For a tangle with $2n$ strands, we define the top-closure of the tangle as the closure that stitches together the pairs $(1_t,\ 2_t),\ (3_t,\ 4_t),\ \ldots,\ ((2n-1)_t,\ (2n)_t)$ to create n strands. For a tangle with $2n$ strands, we define the full-closure of the tangle as the closure that stitches together the pairs $(1_b,\ 2_b),\ (2_t,\ 3_t),\ (3_b,\ 4_b),\ \ldots,\ ((2n-2)_t,\ (2n-1)_t),\ ((2n-1)_b,\ (2n)_b)$ to create 1 strand. TODO DIAGRAM

We point out that in both scenarios, one is effectively closing the strands along the bottom in the same manner. In the top-closure, the strands create an untangle which can be turned into the unlink by closing the strands along the bottom as it is done in the full-closure.

Theorem 2.1. *A knot K is ribbon $\iff \exists$ a tangle T with $2n$ strands where the top-closure of T creates the trivial tangle with n strands, while full-closure of T creates a tangle with 1 strand, identical to the knot K (obtained by stitching $(1_t,\ (2n)_t)$).*

Proof. We s If the top-closure of a tangle T results in n trivial strands that can be untangled, that means among the top-closure of T, there are only ribbon singularities, as any clasp singularities would not make the top-closure of T trivial. We can now consider closing those n strands along the bottom to create n unknots. Then each unknot is connected to the next using a single strand. That strand may pass through the plane of the original knot, but if it were to be unzipped into strands, it would result in only ribbon singularities. This has effectively connected every adjacent pair of links as well as the first and last links together, creating the original knot by our assumption. Since the resultant knot only contains ribbon singularities, K is ribbon.

Conversely, consider a tangle T with n strands and $2n$ ends along the top numbered 1_t to $(2n)_t$.

Lemma 2.2. *If a tangle T with n strands with $2n$ ends along the top can be top-closed (with n stitchings) to create the unlink consisting of n unknots, then $\exists$ a tangle T' consisting of $2n$ strands with $2n$ ends along the top and $2n$ ends along the bottom such that applying the stitchings $(1_b,\ 2_b)$, $(2_t,\ 3_t)$, $(3_b\ 4_b)$, ..., $((2n-2)_t,\ (2n-1)_t)$, $((2n-1)_b,\ (2n)_b)$ (just the bottom part of the full-closure) results in T. TODO DIAGRAM*

Proof of Lemma. Once T is closed, a continuous deformation of three-dimensional space turns it into the unlink. Here, we extend the n segments from the unlink to points outside the tangle, making sure that they do not intersect. This can be done trivially as T is untangled. If we were to reverse the earlier spatial deformation, we will end up a tangle with $2n$ ends along both the top and the bottom, where the bottom ends extend to be stitched pairwise as described earlier and the tangle can be top-closed to produce the untangle. This tangle is therefore T'. TODO DIAGRAM □

For any knot in its ribbon presentation, we can enclose the lower half of each component in one large tangle. The knot can be deformed until the ribbon connects the tops of the components, making sure the ends of the ribbons are above the tangle. TODO DIAGRAM It is important to note that everything above the tangle appears exactly as stated while everything inside the tangle can have any number of crossings. Since the space in the tangle but above the ribbon is connected to the outside of the knot, any strands that are there can be moved □

III Symmetric Unions

We now provide a list of all 21 ribbon knots with 10 crossings or fewer. Each was originally a symmetric union provided by TODO SOURCE and was deformed to appear as the full-closure of a tangle (with the extra stitching of 4_t and 1_t). TODO DIAGRAM

The locations of the cuts that result in the ends 1_b and 2_b as well as 3_b and 4_b are, in general, not very interesting. It is merely a location on the knot that can be kept on the outside while untangling the top-closure of the tangle. We do note that in the symmetric union, the locations of the two important cuts can always be obtained by deforming the knot (but preserving its symmetry) until one of the bridges appears above any crossings on the axis of symmetry. Then the knot is cut just above this bridge along the first strands that cross it, moving out from the middle of the bridge. TODO DIAGRAM TODO QUESTION

IV Database

Here we attempt to publish a database of the planar diagram crossing information of the 21 knots. They are 6_1, 8_8, 8_9, 8_{20}, 9_{27}, 9_{41}, 9_{46}, 10_3, 10_{22}, 10_{35}, 10_{42}, 10_{48}, 10_{75}, 10_{87}, 10_{99}, 10_{123}, 10_{129}, 10_{137}, 10_{140}, 10_{153}, and 10_{155}. This notation consists of a list of crossings of the form $X_{a,b,c,d}$ which labels the four strands in a crossing from the lower incoming string and continues counterclockwise. TODO DIAGRAM

The knots are encoded as they appear in TODO FIGURE NUMBER. The tangle information consists of the list of segments of the knot (before any stitching between crossings is done) of each strand of the tangle. A strand in a crossing will carry the number of the knot segment coming into it. Here we present TODO KNOT as an example. TODO DIAGRAM

The full database may be accessed at the following URL. TODO URL

V A Correction

We would like to use this opportunity to point out some slight errors in TODO BOOK TITLE. In Appendix TODO APPENDIX AND PAGE NUMBER, the ribbon presentations of the knots TODO LIST KNOTS are incorrect, on account of the Jones polynomial of the presentations not matching that of the intended knots. Interestingly, all 4 knots evaluate to have the correct Alexander polynomial. For this reason, we suspect the mistake was caused by one or more $\Delta\Delta$ moves while deforming the knot into the intended form.

VI Acknowledgments

VII References