

DISTORTION MEASURE OF WATERMARKING 2D VECTOR MAPS IN THE MESH-SPECTRAL DOMAIN

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ABSTRACT

The two main properties of digital watermarking algorithms for 2D vector data are robustness against attacks and minimization of the original data distortion. This paper proposes conformal energy as a measure of original data distortion to quantify the second property. We modify the watermarking algorithm proposed by Ohbuchi et al., which is, to the best of our knowledge, the most robust one against attacks. Experiments show that the modified algorithm is as resistant against random noise as the original one, but has smaller angle distortion.

Index Terms— Laplacian, Dirichlet energy, conformal map, quadratic form, Delaunay triangulation

1. INTRODUCTION

2D-vector data are typically used in Geographical Information System (GIS) related applications. For its high material value due to high efforts and costs in the acquisition of the point coordinates, GIS-data are sometimes watermarked to protect the copyrights and customers registration information [1] or to authenticate the original GIS-data. In some special cases such as military map watermarking, the precise point coordinates of the restored data are highly desired compared with its original version.

Research on digital watermarking has focused mainly on multimedia data, the research on vector data remaining relatively insufficient. Digital watermarking algorithms regarding vector data can be divided roughly into two categories. The first category is about spatial domain methods. These methods are more studied [1, 2, 3, 4], because they allow to control distortion easily. Unfortunately, the algorithms from the first category aren't resistant enough against such attacks as random noise or inserting/deleting vertices. Another category of algorithms of watermarking is about frequency domain method. These algorithms [5, 6, 7] are robust against attacks, but on the other hand it is hard to estimate distortion of the original data.

Another classification criterion for watermarking algorithms is using the original data during watermark extracting. Some watermarking algorithms need the original document to extract the watermark, they are called well-informed algorithms. By contrast, those not using the original document to extract the watermark are called blind. Blindness is an interesting property in case of huge and on the fly production of different documents as it doesn't oblige the producer to store every original document, but on the other hand it reduces a possibility of watermark extracting.

Among algorithms known to the authors, the most robust one against different attacks (global affine transformation, object order scrambling, vertex insertion or deletion, additive random noise, cropping) was proposed by Ohbuchi et al. in [5]. It is well-informed "frequency-domain" algorithm, its main idea is to apply techniques developed for 3D polygonal mesh [6] to 2D-vector map. Hereinafter this algorithm is referred as the base algorithm. It's described briefly in section 2. The goal of this paper is to develop quantitative measure of the original map distortion, and to modify the base algorithm accordingly. This is presented in section 3. We then describe the results of evaluation experiments in section 4, followed by a conclusion and future work in section 5.

2. BASE ALGORITHM

The base algorithm embeds message bits into a 2D vector digital map by modifying a "frequency" domain representation of the map. To compute the "frequency" domain representation, the algorithm provides the set of vertices of the map with a structure of planar graph, using Delaunay triangulation, creating a 2D mesh that covers every vertex in the map. The mesh is then transformed into a frequency domain representation using mesh spectral analysis proposed by Karny and others [8, 9]. Modification of the frequency domain coefficients according to the message bits embeds a watermark. Inverse transformation of the modified coefficients back into the coordinate domain produces a map with the watermark embedded. Modification of coefficients in the frequency domain therefore leads to a displacement of vertices in the spatial domain.

For computational efficiency and for robustness against cropping, a map is first divided into many rectangular subar-

The work of K. I. was supported by the Chebyshev Laboratory (Faculty of Mathematics and Mechanics, St Petersburg State University) under the grant 11.G34.31.2006 of the Government of the Russian Federation.

eas. Aforementioned mesh spectral analysis and watermark embedding is performed for each of the subareas. By embedding the same watermark repeatedly in multiple subareas, the watermark becomes resilient against cropping.

Watermarks are extracted by comparing the *reference map* (the map before watermarking) with the watermarked and possibly attacked *watermark map*. The two maps are first geometrically registered by using an iterative optimization process to minimize distance among a set of landmarks. This registration could remove an affine transformation applied to the watermarked map. Then the area subdivision equal to the one used for the embedding is recreated on the reference map, and the subdivision is transferred to the watermarked map. For each corresponding subarea, mesh spectral analysis and then comparison of spectral coefficients recovers the embedded watermark.

Let's consider the process of watermark embedding in detail. Let $V = \{v_1, v_2, \dots, v_n\}$ is set of vertices of the reference map, $n = |V|$. Let \mathfrak{T} denote some triangulation of the set V , $G(\mathfrak{T})$ denote graph of the triangulation \mathfrak{T} . The eigenvectors $\{\mathbf{e}_i\}_{i=1}^n$ of the graph $G(\mathfrak{T})$ are the eigenvectors of its Laplacian. There are several different Laplacian matrices, for example [10, 11, 12]. Bigg's definition of Laplacian R is employed in the base algorithm

$$R = I - D^{-1}A,$$

where I is the identity matrix, D is a diagonal matrix whose diagonal element $D_{ii} = \deg(v_i)$ is inverse degree of the vertex i of the graph $G(\mathfrak{T})$. A is the adjacency matrix of the graph $G(\mathfrak{T})$

$$D_{ij} = \begin{cases} \deg(v_i) & \text{if } i = j, \\ 0 & \text{otherwise;} \end{cases}$$

$$A_{ij} = \begin{cases} 1 & \text{if vertices } i \text{ and } j \text{ is adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

The eigenvectors $\{\mathbf{e}_i\}_{i=1}^n$ constitute basis of \mathbb{R}^n . We can assume that eigenvectors are normalized, $\|\mathbf{e}_i\| = 1$. Let $v_i = (x_i, y_i)$ then $\mathbf{r} = (r_1, r_2, \dots, r_n)$, $\mathbf{s} = (s_1, s_2, \dots, s_n)$ are decompositions of the vectors $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$, $\mathbf{y} = (y_1, y_2, \dots, y_n)^T$ correspondingly in the basis $\{\mathbf{e}_i\}_{i=1}^n$. In the other words

$$\mathbf{x} = r_1\mathbf{e}_1 + r_2\mathbf{e}_2 + \dots + r_n\mathbf{e}_n;$$

$$\mathbf{y} = s_1\mathbf{e}_1 + s_2\mathbf{e}_2 + \dots + s_n\mathbf{e}_n.$$

If the vectors $\{\mathbf{e}_i\}_{i=1}^n$ are orthogonal then $r_i = (\mathbf{x}, \mathbf{e}_i)$, $s_i = (\mathbf{y}, \mathbf{e}_i)$, but it is wrong in general case.

Assume the embedded message $\mathbf{m} = (m_1, m_2, \dots, m_k)$ consists of k bits, where $k \leq n$, $m_i \in \{0, 1\}$. Let $q_i = 2 * m_i - 1$, $q_i \in \{-1, 1\}$. Then change coefficients r_i, s_i , $1 \leq k \leq n$ in the following way

$$r'_i = r_i + \alpha p_i q_i;$$

$$s'_i = s_i + \alpha p_i q_i,$$

where the $\{p_i\}$ is pseudo-random number sequence generated from a private key, α is some positive constant. Increasing α increases resilience against attacks of the watermark, but simultaneously leads to a greater distortion of the reference map.

Let \mathbf{x}', \mathbf{y}' are coordinates of the vertices of the watermarked map.

$$\mathbf{x}' - \mathbf{x} = (r'_1 - r_1)\mathbf{e}_1 + (r'_2 - r_2)\mathbf{e}_2 + \dots + (r'_n - r_n)\mathbf{e}_n =$$

$$= \alpha [p_1 q_1 \mathbf{e}_1 + p_2 q_2 \mathbf{e}_2 + \dots + p_k q_k \mathbf{e}_k],$$

$$\mathbf{y}' - \mathbf{y} = \alpha [p_1 q_1 \mathbf{e}_1 + p_2 q_2 \mathbf{e}_2 + \dots + p_k q_k \mathbf{e}_k].$$

On the other words it's possible to express embedding of watermarking as constructing vector

$$\mathbf{g} = \alpha [p_1 \mathbf{w}_1 + p_2 \mathbf{w}_2 + \dots + p_k \mathbf{w}_k]; \quad (1)$$

$$\mathbf{w}_i = ((e_{i,1}, e_{i,1}), (e_{i,2}, e_{i,2}), \dots, (e_{i,n}, e_{i,n})) \text{ for } 1 \leq i \leq n.$$

If the vectors $\{\mathbf{e}_i\}_{i=1}^n$ are orthogonal and $|\mathbf{e}_i| = \frac{\sqrt{2}}{2}$ then it is possible to estimate mean length of displacement of an vertex Δv_{mean} .

$$|\mathbf{g}| = \alpha * \sqrt{k}; \quad \Delta v_{mean} = \alpha * \frac{\sqrt{k}}{n}. \quad (2)$$

To extract watermark from the watermarked map decompose vectors $\mathbf{x}' - \mathbf{x}, \mathbf{y}' - \mathbf{y}$ in the basis $\{\mathbf{e}_i\}_{i=1}^k$, or calculate coefficients $\mathbf{u} = (u_1, u_2, \dots, u_k)$, $\mathbf{w} = (w_1, w_2, \dots, w_k)$,

$$\mathbf{x}' - \mathbf{x} = u_1 \mathbf{e}_1 + u_2 \mathbf{e}_2 + \dots + u_k \mathbf{e}_k;$$

$$\mathbf{y}' - \mathbf{y} = w_1 \mathbf{e}_1 + w_2 \mathbf{e}_2 + \dots + w_k \mathbf{e}_k.$$

Then bits of the embedded message can be obtained in the following way

$$q_i = \text{sign}(p_i * (u_i + w_i)), m_i = (q_i + 1)/2.$$

3. MEASURE OF DISTORTION

3.1. Continuous case

We suppose that distortion of the reference map is defined by the change of the angles. Let's consider some domain $\Omega \subset \mathbb{R}^2$

and function $f : \Omega \rightarrow \mathbb{R}^2$, $f = \begin{pmatrix} f_x(x, y) \\ f_y(x, y) \end{pmatrix}$. We assume that f_x, f_y have partial derivatives with respect to x and y . We will use following notation:

$$\nabla f_x = \begin{pmatrix} \frac{\partial f_x}{\partial x} \\ \frac{\partial f_x}{\partial y} \end{pmatrix}, \nabla f_y = \begin{pmatrix} \frac{\partial f_y}{\partial x} \\ \frac{\partial f_y}{\partial y} \end{pmatrix}, \nabla f = \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix}.$$

Let's estimate change of a local angle after applying function f . Let $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ be an arbitrary point, \vec{r}_1 is the point displaced relative to \vec{r} by an arbitrary small vector \vec{dr} ,

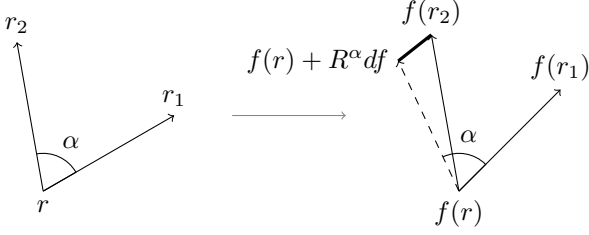


Fig. 1. Change of local angle.

$$\vec{r}_1 = \vec{r} + d\vec{r} = \begin{pmatrix} x + dx \\ y + dy \end{pmatrix}$$

and \vec{r}_2 is the point equal to \vec{r}_1 rotated relative to \vec{r} by an arbitrary angle α ,

$$\vec{r}_2 = \vec{r} + R^\alpha d\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

where R^α is the rotation by angle α (fig. 1). This points are mapped to

$$\begin{aligned} \overrightarrow{f(r), f(r_1)} &= \overrightarrow{f(r + d\vec{r})} = \overrightarrow{f(r)} + d\vec{f} \approx \overrightarrow{f(r)} + (\nabla f) d\vec{r}, \\ \overrightarrow{f(r), f(r_2)} &= \overrightarrow{f(r + R^\alpha d\vec{r})} \approx \overrightarrow{f(r)} + (\nabla f)(R^\alpha d\vec{r}). \end{aligned}$$

If f preserves local angles then $f(r_2)$ must be equal to the point $f(r_1)$ rotated relative to the point $f(r)$ by the angle α . Otherwise we can employ length of the vector $\overrightarrow{q(r, dr, \alpha)}$

$$\overrightarrow{q(r, dr, \alpha)} = \left(\overrightarrow{f(r)} + R^\alpha d\vec{f} \right) - \overrightarrow{f(r_2)}$$

as measure of the angle change.

$$\begin{aligned} \overrightarrow{q(r, dr, \alpha)} &= \left(\overrightarrow{f(r)} + R^\alpha d\vec{f} \right) - \overrightarrow{f(r_2)} \approx \\ &\approx \left(\overrightarrow{f(r)} + R^\alpha (\nabla f) d\vec{r} \right) - \left(\overrightarrow{f(r)} + (\nabla f)(R^\alpha d\vec{r}) \right) = \\ &= [R^\alpha (\nabla f) - (\nabla f) R^\alpha] d\vec{r}. \quad (3) \end{aligned}$$

$$\begin{aligned} R^\alpha (\nabla f) - (\nabla f) R^\alpha &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix} - \\ &- \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} = \\ &= \begin{pmatrix} \frac{\partial f_x}{\partial x} \cos \alpha - \frac{\partial f_y}{\partial x} \sin \alpha & \frac{\partial f_x}{\partial y} \cos \alpha - \frac{\partial f_y}{\partial y} \sin \alpha \\ \frac{\partial f_x}{\partial x} \sin \alpha + \frac{\partial f_y}{\partial x} \cos \alpha & \frac{\partial f_x}{\partial y} \sin \alpha + \frac{\partial f_y}{\partial y} \cos \alpha \end{pmatrix} - \\ &- \begin{pmatrix} \frac{\partial f_x}{\partial x} \cos \alpha + \frac{\partial f_y}{\partial x} \sin \alpha & -\frac{\partial f_x}{\partial x} \sin \alpha + \frac{\partial f_y}{\partial x} \cos \alpha \\ \frac{\partial f_x}{\partial y} \cos \alpha + \frac{\partial f_y}{\partial y} \sin \alpha & -\frac{\partial f_x}{\partial y} \sin \alpha + \frac{\partial f_y}{\partial y} \cos \alpha \end{pmatrix} = \\ &= \sin \alpha \begin{pmatrix} -\left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x}\right) & \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \\ \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} & \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \end{pmatrix}. \quad (4) \end{aligned}$$

Substituting (4) to the formula (3) obtain following formula

$$\begin{aligned} |\overrightarrow{q(r, dr, \alpha)}| &= |[R^\alpha (\nabla f) - (\nabla f) R^\alpha] d\vec{r}| = \\ &= \left| \sin \alpha \begin{pmatrix} -\left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x}\right) & \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \\ \frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} & \frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \end{pmatrix} d\vec{r} \right| = \\ &= |\sin \alpha| \sqrt{\left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x}\right)^2 + \left(\frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y}\right)^2} |d\vec{r}|. \end{aligned}$$

We can see that the vector $\overrightarrow{q(r, dr, \alpha)}$ vanishes when

$$\begin{cases} \frac{\partial f_x}{\partial x} = \frac{\partial f_y}{\partial y} \\ \frac{\partial f_x}{\partial y} = -\frac{\partial f_y}{\partial x} \end{cases}$$

i.e. f_x and f_y satisfy Cauchy-Riemann equations and the function f is conformal. If f is not conformal then length of the vector $\overrightarrow{q(r, dr, \alpha)}$ is maximal when $\alpha = \pm \frac{\pi}{2}$ and doesn't depend on direction of $d\vec{r}$.

$$\left(\frac{|\overrightarrow{q(r, dr, \pm \frac{\pi}{2})}|}{|d\vec{r}|} \right)^2 = \left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \right)^2 + \left(\frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \right)^2$$

We propose to use the functional

$$E_C(f) = \frac{1}{2} \int_{\Omega} \left[\left(\frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \right)^2 + \left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \right)^2 \right] d\Omega$$

called *conformal energy* ([13]) as measure of distortion of the domain Ω after applying to it function f .

3.2. Discrete case

Let's consider sets $V = \{v_1, v_2, \dots, v_n\}$, $V' = \{v'_1, v'_2, \dots, v'_n\}$, $v_i, v'_i \in \mathbb{R}^2$, $|V| = |V'| = n$. Let function $f : V \rightarrow V'$ maps set V to set V' , $f(v_i) = v'_i$ for $1 \leq i \leq n$. Let Ω denote convex hull of the set V . We can construct Piecewise Linear Interpolation Surface (PLIS) of function values $\{f(v_i)\}_{i=1}^n$ using some triangulation \mathfrak{T} of the set V , consequently f can be viewed as function from Ω to \mathbb{R}^2 . If a point v is in triangle $\triangle T = (v_i v_j v_k)$, $\triangle T \in \mathfrak{T}$; $v = \alpha v_i + \beta v_j + \gamma v_k$, $0 \leq \alpha, \beta, \gamma \leq 1$, $\alpha + \beta + \gamma = 1$, then $f(v) = \alpha f(v_i) + \beta f(v_j) + \gamma f(v_k)$.

$$\begin{aligned} E_C(f) &= \frac{1}{2} \int_{\Omega} \left[\left(\frac{\partial f_x}{\partial x} - \frac{\partial f_y}{\partial y} \right)^2 + \left(\frac{\partial f_x}{\partial y} + \frac{\partial f_y}{\partial x} \right)^2 \right] d\Omega = \\ &= \frac{1}{2} \int_{\Omega} \left[\left(\frac{\partial f_x}{\partial x} \right)^2 + \left(\frac{\partial f_y}{\partial y} \right)^2 + \left(\frac{\partial f_x}{\partial y} \right)^2 + \left(\frac{\partial f_y}{\partial x} \right)^2 \right] d\Omega - \\ &- \int_{\Omega} \left[\frac{\partial f_x}{\partial x} \frac{\partial f_y}{\partial y} - \frac{\partial f_x}{\partial y} \frac{\partial f_y}{\partial x} \right] d\Omega = \\ &= \frac{1}{2} \int_{\Omega} [|\nabla f_x|^2 + |\nabla f_y|^2] d\Omega - \int_{\Omega} \det \begin{pmatrix} \frac{\partial f_x}{\partial x} & \frac{\partial f_x}{\partial y} \\ \frac{\partial f_y}{\partial x} & \frac{\partial f_y}{\partial y} \end{pmatrix} d\Omega = \\ &= E_D(f) - \int_{\Omega} \det(\nabla f) d\Omega, \quad (5) \end{aligned}$$

where $E_D(f)$ is *Dirichlet energy*. It is well-known [14] that if f is PLIS of function values $\{f_i = f(v_i)\}_{i=1}^n$ over triangulation \mathfrak{T} , then

$$E_D(f) = \frac{1}{4} \sum_{\Delta(i,j,k) \in \mathfrak{T}} \cot \alpha_{ij} |f_i - f_j|^2 + \cot \alpha_{jk} |f_j - f_k|^2 + \cot \alpha_{ki} |f_k - f_i|^2, \quad (6)$$

where α_{ij} is the angle opposite the edge (v_i, v_j) in the triangle $\Delta T = (v_i, v_j, v_k)$. The formula (6) is equivalent to the following:

$$E_D(f) = \frac{1}{4} \sum_{(i,j) \in E(\mathfrak{T})} w_{ij} |f_i - f_j|^2, \quad (7)$$

where $E(\mathfrak{T})$ is the edges of \mathfrak{T} , $w_{ij} = \cot \alpha_{ij} + \cot \alpha_{ji}$, $\alpha_{\{ij,ji\}}$ are the angles opposite to the edge $v_i v_j$ in adjacent triangles $\Delta T_1 = v_i v_j v_{k_1}$, $\Delta T_2 = v_j v_i v_{k_2}$. If the edge $v_i v_j$ is contained in only triangle $\Delta T = v_i v_j v_k$ ($v_i v_j$ is contained in the bound of Ω) then $w_{ij} = \cot \alpha_{ij}$.

If we assume that f is complex-valued function and

$$f_x = \operatorname{Re} f = \frac{1}{2} (f + \bar{f}), f_y = \operatorname{Im} f = \frac{1}{2} (f - \bar{f}),$$

then

$$|f_i - f_j|^2 = \overline{(f_i - f_j)}(f_i - f_j) = (\bar{f}_i - \bar{f}_j)(f_i - f_j).$$

Let $G(\mathfrak{T})$ denote weighted graph of the triangulation \mathfrak{T} , $V(G) = V$, $E(G) = E(\mathfrak{T})$, $\text{weight}(u, v) = w_{uv}$. If we consider Laplacian L of the graph G defined as follows

$$L_{uv} = \begin{cases} \sum_{(v,x) \in E(G)} w_{vx} & \text{if } u = v, \\ -w_{uv} & \text{if } u \sim v, \\ 0 & \text{otherwise;} \end{cases}$$

then the formula (7) is equivalent to the following:

$$E_D(f) = \frac{1}{4} \bar{\mathbf{f}}^T L \mathbf{f}, \quad (8)$$

where $\mathbf{f} = (f_1, f_2, \dots, f_n)^T$, $\bar{\mathbf{f}}^T = (\bar{f}_1, \bar{f}_2, \dots, \bar{f}_n)$.

$\int_{\Omega} \det(\nabla f) d\Omega$ also can be viewed as value of some quadratic form applied to \mathbf{f} . Let's consider $\Delta T = v_i v_j v_k \in \mathfrak{T}$. f is linear on ΔT , consequently $\det(\nabla f)$ is constant, hence $\int_{\Delta T} \det(\nabla f) dT$ is oriented area of $\Delta T' = f_i f_j f_k$. Let's suppose that v_i, v_j, v_k constitute left turn then

$$\begin{aligned} \int_{\Delta T} \det(\nabla f) dT &= \frac{1}{2} (f_i - f_k) \times (f_j - f_k) = \\ &= \frac{1}{2} [(f_i - f_k)_x (f_j - f_k)_y - (f_i - f_k)_y (f_j - f_k)_x] = \\ &= \frac{i}{4} [(f_i - f_k) \overline{(f_j - f_k)} - (f_j - f_k) \overline{(f_i - f_k)}] = \\ &= \frac{i}{4} [(f_i \bar{f}_j - \bar{f}_i f_j) + (f_j \bar{f}_k - \bar{f}_j f_k) + (f_k \bar{f}_i - \bar{f}_k f_i)]. \end{aligned} \quad (9)$$

Substituting (9) to the formula

$$\int_{\Omega} \det(\nabla f) d\Omega = \sum_{\Delta T \in \mathfrak{T}} \int_{\Delta T} \det(\nabla f) dT, \quad (10)$$

we notice that for every edge $v_i v_j$ not contained in the bound of Ω there is the twin edge $v_j v_i$ and so expression $(f_i \bar{f}_j - \bar{f}_i f_j)$ is contained in the right side of the equation (10) together with the opposite expression $(f_j \bar{f}_i - \bar{f}_j f_i)$. Consequently formula (10) can be rewritten as

$$\int_{\Omega} \det(\nabla f) d\Omega = \frac{i}{4} \sum_{v_i v_j \in \partial E} (f_i \bar{f}_j - \bar{f}_i f_j), \quad (11)$$

where $\partial E = \partial\Omega \cap E(\mathfrak{T})$ is set of the edges of the triangulation \mathfrak{T} contained in the bound of Ω . If we consider matrix L' defined as follows

$$L'_{uv} = \begin{cases} i & \text{if edge } uv \in \partial E \text{ and } \Omega \text{ is left of the line } (u, v); \\ -i & \text{if edge } uv \in \partial E \text{ and } \Omega \text{ is right of the line } (u, v); \\ 0 & \text{otherwise} \end{cases}$$

then formula (11) is equivalent to the following:

$$\int_{\Omega} \det(\nabla f) d\Omega = \frac{1}{4} \bar{\mathbf{f}}^T L' \mathbf{f}. \quad (12)$$

Using formulas (5), (8), (12) we can express $E_C(f)$ as value of some quadratic form applied to \mathbf{f}

$$E_C(f) = \frac{1}{4} \bar{\mathbf{f}}^T L'' \mathbf{f},$$

where $L'' = L - L'$. It is easy to see that L'' is Hermitian, consequently eigenvalues of L'' are real and eigenvectors of L'' are orthogonal. If

$$\mathbf{f} = \alpha [p_1 \mathbf{e}_1 + p_2 \mathbf{e}_2 + \dots + p_k \mathbf{e}_k],$$

where α is some real positive constant, $p_i \in \{-1, 1\}$ and $\{\mathbf{e}_i\}_{i=1}^k$ is system of orthonormal vectors, then $E_C(f)$ will be minimized when $\{\mathbf{e}_i\}_{i=1}^k$ are the eigenvectors of L'' corresponding to k lowest eigenvalues.

The watermark embedding algorithm described in the section 2 constructs vector \mathbf{g} of differences between old and new coordinates of the vertices (formula (1)) and E_C is functional applicable to f , $f(v) = v + g(v)$. But

$$E_C(g) = E_C(f - v) = E_C(f),$$

because E_C is quadratic and the identity function v is conformal.

Thereby, if one wants to use algorithm described in the section 2 and to minimize the distortion measure defined as conformal energy, it's necessary to select the vectors $\{\mathbf{w}_i\}_{i=1}^k$ from formula (1) as the eigenvectors of L'' corresponding to the lowest eigenvalues where L'' is described above.

In the section 3.1 we argued that $E_C(f)$ is the measure of the change of local angles. Let's consider what $E_C(f)$ expresses in the sense of the angles of the triangulation \mathfrak{T}

$$\begin{aligned} E_C(f) &= E_D(f) - \int_{\Omega} \det(\nabla f) d\Omega = \\ &= \frac{1}{4} \sum_{v_i v_j v_k \in \mathfrak{T}} \cot \alpha_{ij} |f_i - f_j|^2 + \cot \alpha_{jk} |f_j - f_k|^2 + \\ &\quad + \cot \alpha_{ki} |f_k - f_i|^2 - 4S(f_i f_j f_k), \end{aligned}$$

where $S(f_i f_j f_k)$ is oriented area of the triangle $\triangle f_i f_j f_k$. If we assume that the function f distorts domain Ω a slightly, then orientation of $\triangle f_i f_j f_k$ is equal to the orientation of $\triangle v_i v_j v_k$ and so $\triangle f_i f_j f_k$ is oriented counterclockwise,

$$\begin{aligned} 4S(f_i f_j f_k) &= \cot \alpha'_{ij} |f_i - f_j|^2 + \\ &\quad + \cot \alpha'_{jk} |f_j - f_k|^2 + \cot \alpha'_{ki} |f_k - f_i|^2, \end{aligned}$$

where $\alpha'_{ij}, \alpha'_{jk}, \alpha'_{ki}$ are the angles opposite to the edges $f_i f_j, f_j f_k, f_k f_i$ correspondingly. And so under certain assumptions

$$\begin{aligned} E_C(f) &= \frac{1}{4} \sum_{v_i v_j v_k \in \mathfrak{T}} (\cot \alpha_{ij} - \cot \alpha'_{ij}) |f_i - f_j|^2 + \\ &\quad + (\cot \alpha_{jk} - \cot \alpha'_{jk}) |f_j - f_k|^2 + (\cot \alpha_{ki} - \cot \alpha'_{ki}) |f_k - f_i|^2. \end{aligned}$$

Up to this point we have assumed that the triangulation \mathfrak{T} is arbitrary, but it is obvious that E_C depends on \mathfrak{T} . We employ the triangulation \mathfrak{T} for passing from discrete function values to continuous function, consequently we have to select the triangulation which minimize the roughness of a PLIS. According to theorem of Rippa [15, 16] it is the Delaunay triangulation.

4. EXPERIMENT RESULTS

The base and modified algorithm have been tested on the two maps. The first reference map is map of rivers, it consists of polylines, 44482 vertices, 88548 edges. When magnitude $\alpha = 0.5\text{m}$, the watermark is 19 bits, it is embedded into 64 regions separately and duplicated 4 times, mean displacement of an vertex is 0.03m according to formula (2). This watermark can be restored after applying to map additive white noise with magnitude 1.5m. Mean change of an angle between adjacent edges after applying the base algorithm is 0.64° and after applying the proposed algorithm is 0.57° .

The second reference map is map of buildings of Saint Petersburg, it consists of polygons, 386257 vertices. When magnitude $\alpha = 0.1\text{m}$, the watermark is 119 bits, it is embedded into 1024 regions separately and duplicated 3 times, mean displacement of an vertex is 0.01m. This watermark can be restored after applying to map additive white noise with magnitude 1.5m. Mean change of an angle after applying the base algorithm is 1.72° and after applying the proposed algorithm is 0.97° .

5. CONCLUSION

In this paper we presented a measure of the original data distortion aroused by embedding watermark. We proposed to employ the conformal energy because minimization of this functional implies minimization of reference map angles change. Proposed measure was applied for modification of the known watermarking algorithm [5]. Modified algorithm is equivalent to the base algorithm in the sense of resilience against random noise attack and changes angles less.

Our future work includes improvement of the distortion measure to take into account distortion of topology of the reference map, what can happen in the case of intersection edges of the watermarked map.

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