Set Theory Notes

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February 16, 2017

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1 Axioms of Zermelo-Fraenkel

Axiom 1 (Axiom of Extensionality). If X and Y have the same elements, then X = Y.

Axiom 2 (Axiom of Pairing). For any a and b there exists a set a, b that contains exactly a and b

Axiom 3 (Axiom Schema of Separation). if P is a property (with parameter p), then for any X and p there exists a set Y = {u ∈ : P(u,p)} that contains all those u ∈ X that have property P.

Axiom 4 (Axiom of Union). For any X there exists a set Y = ∪X, the union of all elements of X.

Axiom 5 (Axiom of Power Set). For any X there exists a set Y = P(X), the set of all subsets of X.

Axiom 6 (Axiom of Infinity). There exists an infinite set.

Axiom 7 (Axiom Schema of Replacement). If a class F is a function, the for any X there exists a set Y = F(X) = $\{F(x) : x \in X\}$