GLONASS Satellite Coordinates Computation

Fundamentals	
Title	GLONASS Satellite Coordinates Computation
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The GLONASS satellite coordinates shall be computed according to the specifications in the GLONASS-ICD document. An accuracy level of about three meters can be reached using the algorithm provided by this ICD.

In table 1 are listed the broadcast ephemeris parameters which are used to compute GLONASS satellites coordinates. Essentially, the ephemeris contain the initial conditions of position and velocity to perform the numerical integration of the GLONASS orbit within the interval of measurement $|t-t_e|<15$ minutes. The accelerations due solar and lunar gravitational perturbations are also given.

Parameter	Explanation
t_e	Ephemerides reference epoch
$x(t_e)$	Coordinate at t_e in PZ-90
$y(t_e)$	Coordinate at t_e in PZ-90
$z(t_e)$	Coordinate at t_e in PZ-90
$v_x(t_e)$	Velocity component at t_e in PZ-90
$v_y(t_e)$	Velocity component at t_e in PZ-90
$v_z(t_e)$	Velocity component at t_e in PZ-90
$X''(t_e)$	Moon and sun acceleration at t_e
$Y''(t_e)$	Moon and sun acceleration at t_e
$Z''(t_e)$	Moon and sun acceleration at t_e
$\tau_n(t_e)$	SV clock offset
$\gamma_n(t_e)$	SV relative frequency offset

Table 1: GLONASS broadcast ephemeris and clock message parameters.

In order to compute PZ-90 GLONASS satellite coordinates from the navigation message, the following algorithm must be used [GLONASS ICD, 1998] [1].

Computation equations and algorithm

- **1. Coordinates transformation to an inertial reference frame:**

  - The initial conditions $(x(t_e), y(t_e), z(t_e), v_x(t_e), v_y(t_e), v_z(t_e))$, as broadcast in the GLONASS navigation message, are in the ECEF Greenwich coordinate system PZ-90. Therefore, and previous to orbit integration, they must be transformed to an absolute (inertial) coordinate system using the following expressions [footnotes 1].

  Position:

$$\begin{aligned} x_a(t_e) &= x(t_e)\cos(\theta_{G_e}) - y(t_e)\sin(\theta_{G_e}) \\ y_a(t_e) &= x(t_e)\sin(\theta_{G_e}) + y(t_e)\cos(\theta_{G_e}) \\ z_a(t_e) &= z(t_e) \end{aligned} \qquad (1)$$

  Velocity:

$$\begin{aligned} v_{x_a}(t_e) &= v_x(t_e)\cos(\theta_{G_e}) - v_y(t_e)\sin(\theta_{G_e}) - \omega_E\ y_a(t_e) \\ v_{y_a}(t_e) &= v_x(t_e)\sin(\theta_{G_e}) + v_y(t_e)\cos(\theta_{G_e}) + \omega_E\ x_a(t_e) \\ v_{z_a}(t_e) &= v_z(t_e) \end{aligned} \qquad (2)$$

- The $\left(X''(t_e), Y''(t_e), Z''(t_e)\right)$ acceleration components broadcast in the navigation message are the projections of luni-solar accelerations to axes of the ECEF Greenwich coordinate system. Thence, these accelerations must be transformed to the inertial system by:

$$\begin{aligned}
(Jx_am + Jx_as) &= X''(t_e)\cos(\theta_{G_e}) - Y''(t_e)\sin(\theta_{G_e}) \\
(Jx_am + Jx_as) &= X''(t_e)\sin(\theta_{G_e}) + Y''(t_e)\cos(\theta_{G_e}) \qquad (3) \\
(Jx_am + Jx_as) &= Z''(t_e)
\end{aligned}$$

Where (θ_{G_e}) is the sidereal time at epoch t_e, to which are referred the initial conditions, in Greenwich meridian:

$$\theta_{G_e} = \theta_{G_0} + \omega_E(t_e - 3\ hours) \qquad (4)$$

being:

- ω_E: earth's rotation rate ($0.7292115\ 10^{-4}\ \ rad/s$)).
- θ_{G_0}: the sidereal time in Greenwich at midnight GMT of a date at which the epoch t_e is specified. (Notice: GLONASS_time = UTC(SU) + 3 hours).

- **2. Numerical integration of differential equations that describe the motion of the satellites.**

According to GLONASS-ICD, the re-calculation of ephemeris from epoch t_e to epoch t_i within the measurement interval ($|t_i - t_e| < 15min$) shall be performed by a numerical integration of the differential equations (5) describing the motion of the satellites. These equations shall be integrated in a direct absolute geocentric coordinate system OXa, OYa, OZa, connected with current equator and vernal equinox, using the 4th order Runge-Kutta technique:

$$\begin{cases}
\frac{dx_a}{dt} = v_{x_a}(t) \\
\frac{dy_a}{dt} = v_{y_a}(t) \\
\frac{dz_a}{dt} = v_{z_a}(t) \\
\frac{dv_{x_a}}{dt} = -\bar{\mu}\bar{x}_a + \frac{3}{2}C_{20}\bar{\mu}\bar{x}_a\rho^2(1 - 5\bar{z}_a^2) + Jx_am + Jx_as \\
\frac{dv_{y_a}}{dt} = -\bar{\mu}\bar{y}_a + \frac{3}{2}C_{20}\bar{\mu}\bar{y}_a\rho^2(1 - 5\bar{z}_a^2) + Jx_am + Jx_as \\
\frac{dv_{z_a}}{dt} = -\bar{\mu}\bar{z}_a + \frac{3}{2}C_{20}\bar{\mu}\bar{z}_a\rho^2(3 - 5\bar{z}_a^2) + Jx_am + Jx_as
\end{cases} \qquad (5)$$

where:

$$\bar{\mu} = \frac{\mu}{r^2}\ ,\bar{x}_a = \frac{x_a}{r}\ , \bar{y}_a = \frac{y_a}{r}\ , \bar{z}_a = \frac{x_a}{r}\ , \bar{\rho} = \frac{a_E}{r}\ , r = \sqrt{x_a^2 + y_a^2 + z_a^2}$$

$a_E = 6\,378.136\ km$ Equatorial radius of the Earth (PZ-90).

$\mu = 398\,600.44\ km^3/s^2$ Gravitational constant (PZ-90).

$C_{20} = -1\,082.63 \cdot 10^{-6}$ Second zonal coefficient of spherical harmonic expression.

Note: In the above differential equations system (5), the term $C_{20} = -J_2 = +\sqrt{5}\bar{C}_{20}$ is used instead of J_2 in equations $V(r,\phi,\lambda) = \frac{\mu}{r}\left[1 + \frac{1}{2}\left(\frac{a_e}{r}\right)^2 J_2\ \ (1 - 3\sin^2\phi)\right]$ and $\ddot{\mathbf{r}} = \nabla V + \mathbf{k}_{sun\_moon}$ to keep the same expressions as in the GLONASS-ICD (please refer to Perturbed Motion and GNSS Broadcast Orbits)

The right-hand side of the previous equation system (5) takes into account the accelerations determined by the central body gravitational constant μ, the second zonal coefficient C_{20} (that characterises polar flattening of the Earth), and the accelerations due to the luni-solar gravitational perturbation.

Runge-Kutta integration algorithm

- Given the following initial value problem:

$$\left\{\begin{array}{l} \frac{dy_1}{dt} = f_1(t, y_1, \cdots, y_n) \\ \qquad\vdots \\ \frac{dy_n}{dt} = f_1(t, y_1, \cdots, y_n) \end{array}\right. \iff \mathbf{Y}'(t) = \mathbf{F}(t, \mathbf{Y}(t)) \qquad (6)$$

$$\mathbf{Y}(t_0) = [y_1(t_0), \cdots, y_n(t_0)]^T, \mathbf{Y}'(t_0) = [y'_1(t_0), \cdots, y'_n(t_0)]^T$$

It is desired to find the $\mathbf{Y}(t_f)$ at some final time t_f, or $\mathbf{Y}(t_k)$ at some discrete list of points t_k (for example, at tabulated intervals).

- The Runge-Kutta method is based in the following algorithm:

$$\begin{array}{l} \mathbf{K}_1 = \mathbf{F}(t_n, \mathbf{Y}_n) \\ \mathbf{K}_2 = \mathbf{F}(t_n + h/2, \mathbf{Y}_n + h\mathbf{K}_1/2) \\ \mathbf{K}_3 = \mathbf{F}(t_n + h/2, \mathbf{Y}_n + h\mathbf{K}_2/2) \\ \mathbf{K}_4 = \mathbf{F}(t_n + h, \mathbf{Y}_n + h\mathbf{K}_3) \\ \mathbf{Y}_{n+1} = \mathbf{Y}_n + h/6(\mathbf{K}_1 + 2\mathbf{K}_2 + 2\mathbf{K}_3 + \mathbf{K}_4 + O(h^5) \end{array} \qquad (7)$$

The method is initialised with the initial conditions $\mathbf{Y}(t_0)$ and $\mathbf{Y}'(t_0)$. For the numerical integration of GLONASS satellite orbits, the function $\mathbf{F}(t, \mathbf{Y})$ is given by (7).

- **3. Coordinates transformation back to the PZ-90 reference system:**

The coordinates $(x(t), y(t), z(t))$, obtained from the motion equations numerical integration, shall be transformed back to the Earth fixed reference frame PZ-90 with the following equations:

$$\begin{array}{l} x(t) = x_a(t)cos(\theta_G) + y_a(t)sin(\theta_G) \\ y(t) = -x_a(t)sin(\theta_G) + y_a(t)cos(\theta_G) \\ z(t) = z_a(t) \end{array} \qquad (8)$$

where θ_G is the sidereal time at Greenwich meridian at time t, where t is in GLONASS time, see equation (4) :

$$\theta_G = \theta_{G_0} + \omega_E(t - 3\ hours) \qquad (9)$$

$$GLONASS\_time = UTC(SU) - 3\ hours \qquad (10)$$

Note that GLONASS satellite coordinates are computed in PZ-90 reference system, instead of WGS-84 where the GPS coordinates have been calculated. To bring the PZ-90 coordinate system in coincidence with WGS-84 the transformation given by equation (11) must be applied (see Reference Frames in GNSS):

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} -3\,ppb & -353\,mas & -4\,mas \\ 353\,mas & -3\,ppb & 19\,mas \\ 4\,mas & -19\,mas & -3\,ppb \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0.07\,m \\ -0.0\,m \\ -0.77\,m \end{bmatrix} \qquad (11)$$

The transformation from PZ-90.02 to WGS-84 (actually ITRF2000) is given by $\Delta x = -0.36\,m$, $\Delta y = +0.08\,m$, $\Delta z = +0.18\,m$, with no rotation, i.e., equation (12)[footnotes 2]:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ITRF2000} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{PZ-90.02} + \begin{bmatrix} -0.36\,m \\ 0.08\,m \\ 0.18\,m \end{bmatrix} \qquad (12)$$

References

1. ^ [GLONASS ICD, 1998] GLONASS ICD, 1998. Technical report. v.4.0.

Notes

1. ^ Note: Over a small integration intervals, a simple rotation of θ_{G_e} angle around Z-axis is enough to perform this transformation. Nutation and precession of the earth and polar motion are a very slow processes and will not introduce significant deviations on such short integration time intervals (see Transformation between Celestial and Terrestrial Frames).
2. ^ The PZ-90.02 was implemented in September 20th, 2007 at 18:00. (refer to Reference Frames in GNSS).