Skolkovo Institute of Science and Technology
Center of Material Technologies

Report on the course

«Numerical Methods in Engineering and Science».

Executor:
PhD Student A.S. Kulikov

Lecturer:
Prof. O.V. Vasilyev

Skolkovo
2023

Содержание

Часть I

Lagrange and Hermite interpolation

Lagrange and Hermite interpolants are considered for a set of functions and grid distributions in the interval [0, 1]. Functions:

1. $\frac{1}{1+x^2}$.

2. $(x-\frac{1}{2})^2 sign(x-\frac{1}{2})$.

3. $|x-\frac{1}{2}|$.

4. $\sqrt{1-x^2}$.

Corresponding derivatives:

1. $\frac{-2}{(1+x^2)^2}$.

2. $2(x-\frac{1}{2})sign(x-\frac{1}{2})$.

3. $sign(x-\frac{1}{2})$.

4. $\frac{-x}{\sqrt{1-x^2}}$.

Grid distributions:

1. Equispaced: $x_i = \frac{i}{N}, \quad i = 0, ..., N$.

2. Chebyshev: $\frac{1}{2} - \frac{1}{2}cos(\frac{i}{N}\pi), \quad i = 0, ..., N$.

3. Asin: $\frac{1}{2} + \frac{1}{\pi}sin^{-1}\left(\frac{2i}{N} - 1\right), \quad i = 0, ..., N$.

where N is the number of data points.

1 $\frac{1}{1+x^2}$

1.1 Lagrange interpolant

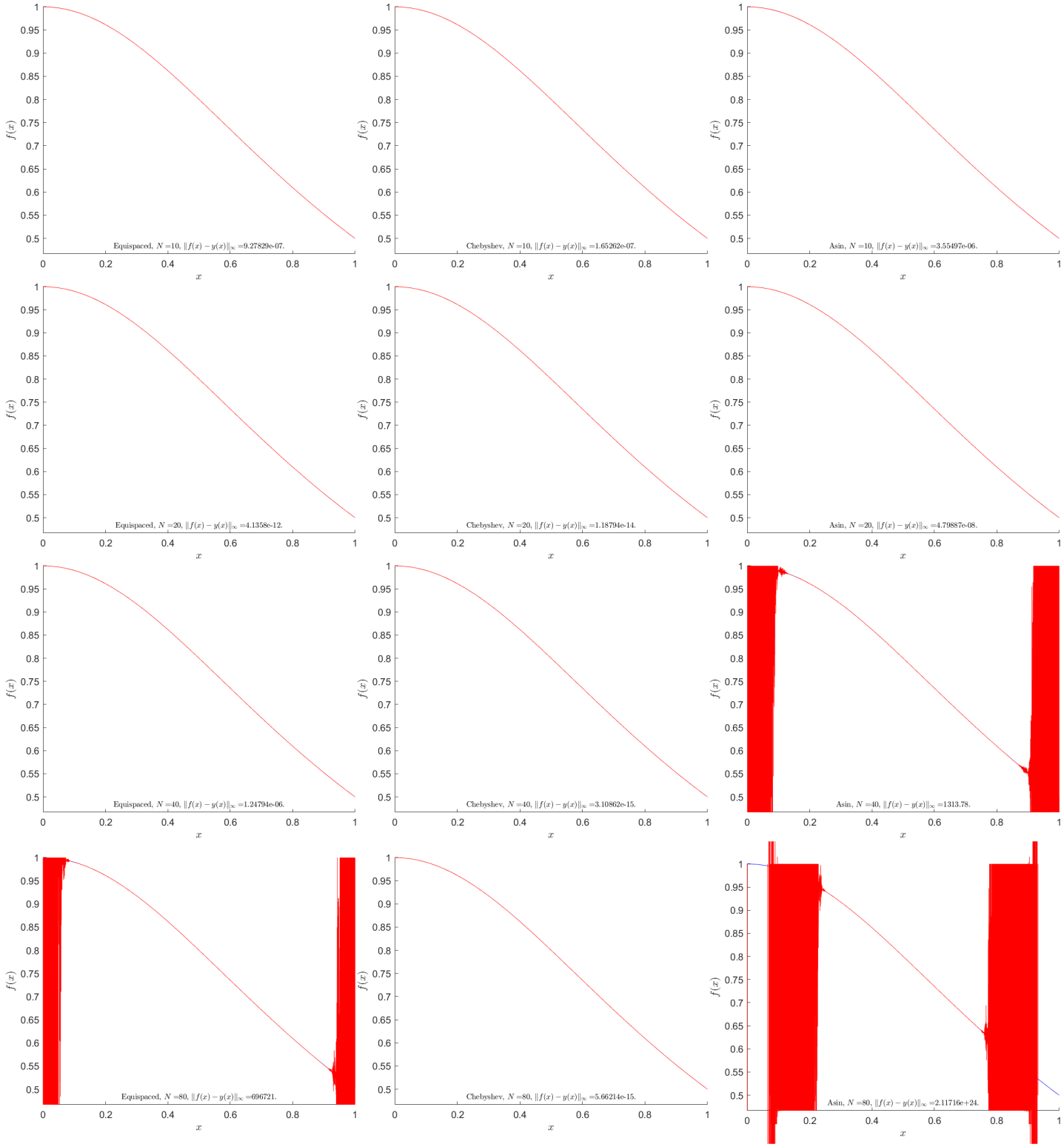

Рис. 1. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

1.2 Hermit interpolant

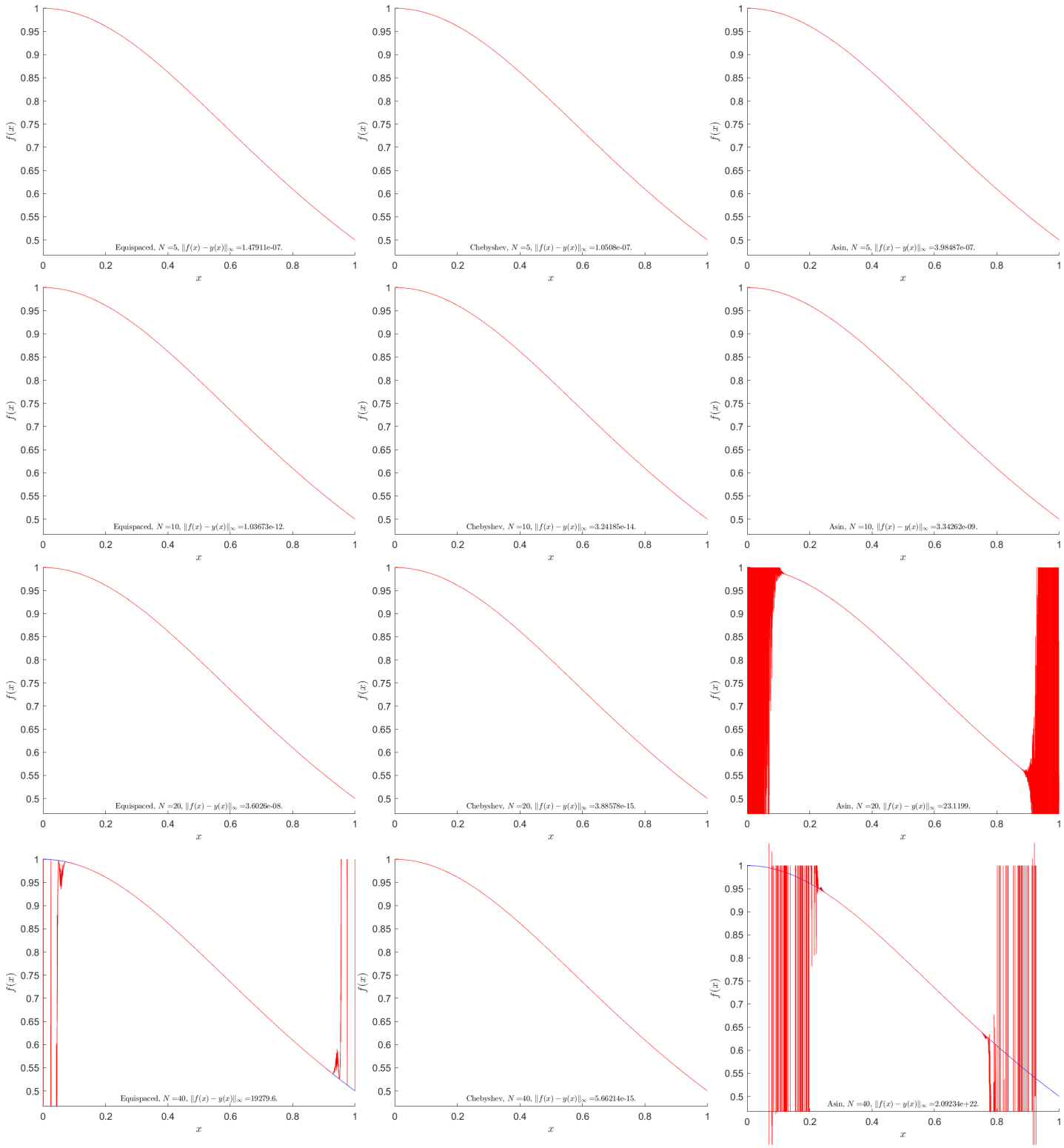

Рис. 2. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

1.3 Accuracy analysis

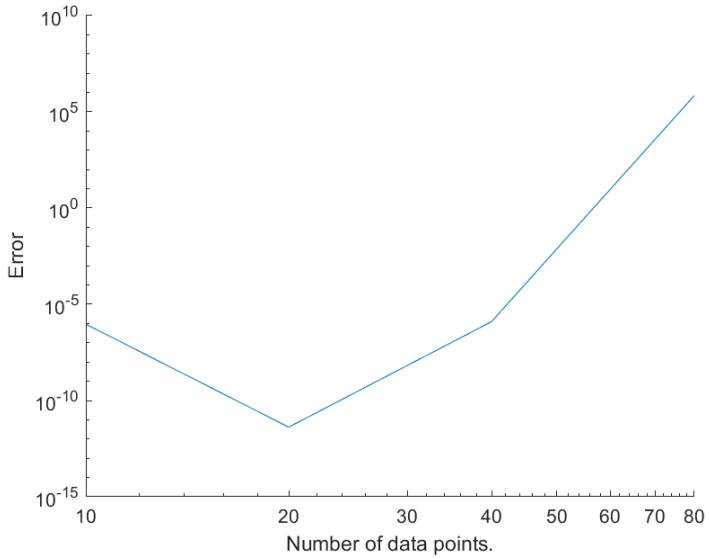

Рис. 3. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

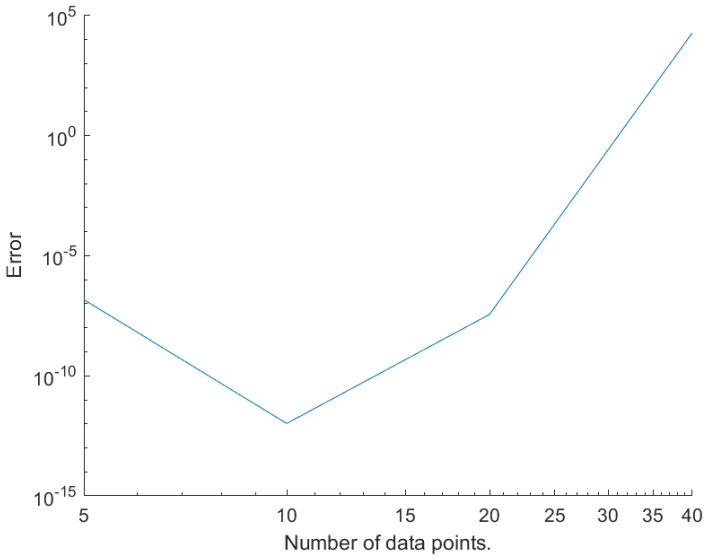

Рис. 4. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

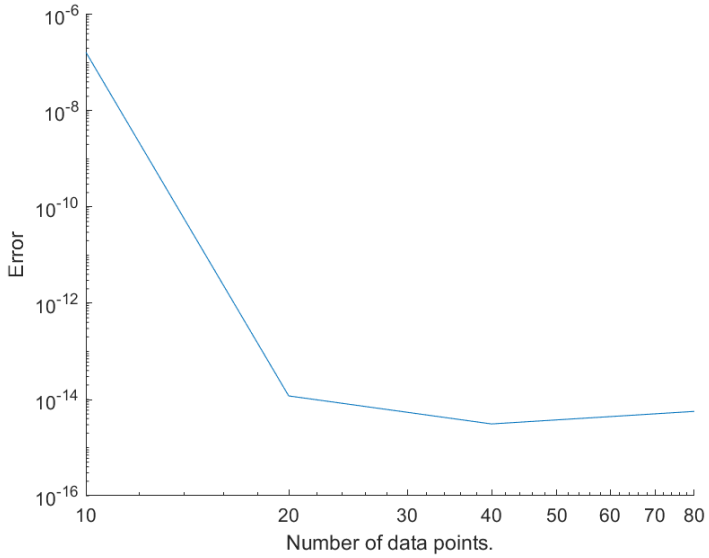

Рис. 5. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

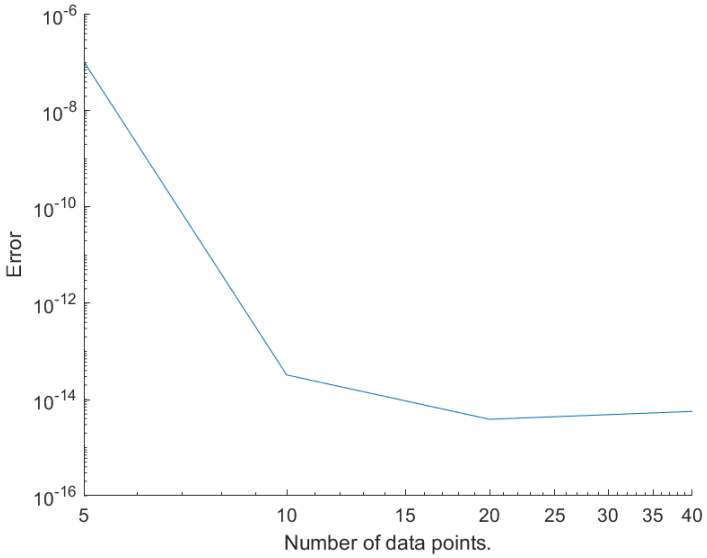

Рис. 6. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

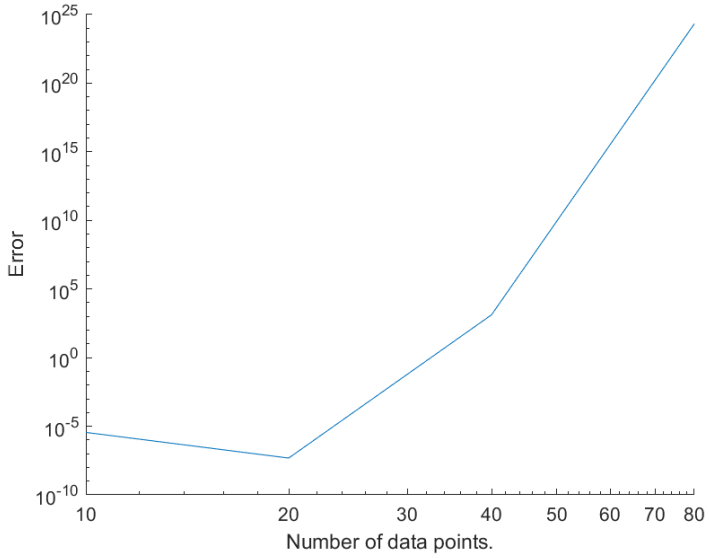

Рис. 7. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

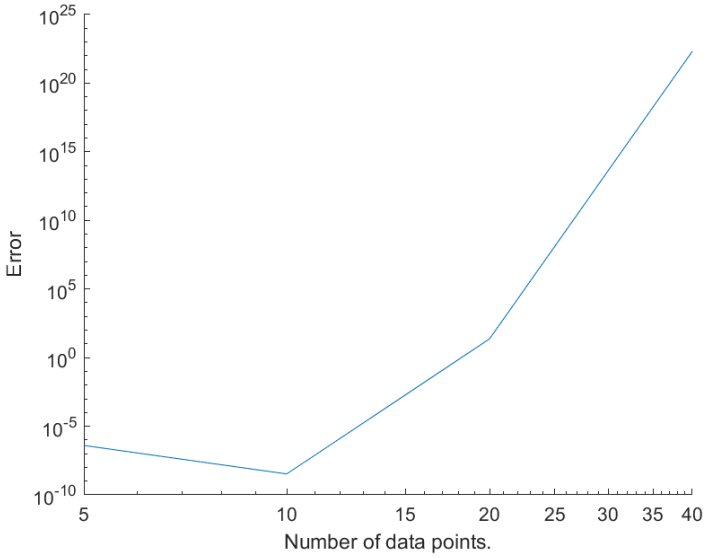

Рис. 8. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

2 $(x-\frac{1}{2})^2 sign(x-\frac{1}{2})$

2.1 Lagrange interpolant

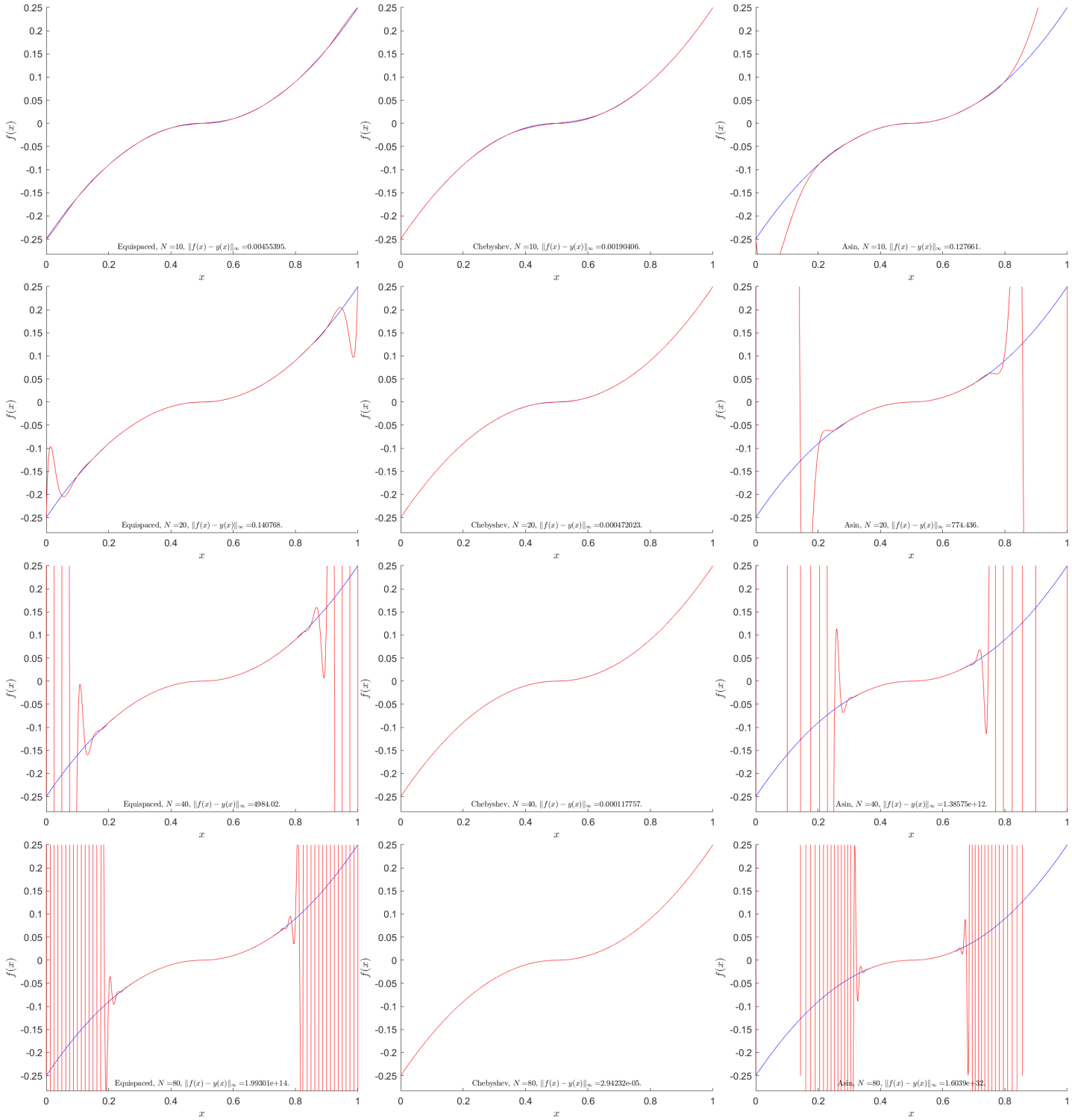

Рис. 9. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

2.2 Hermit interpolant

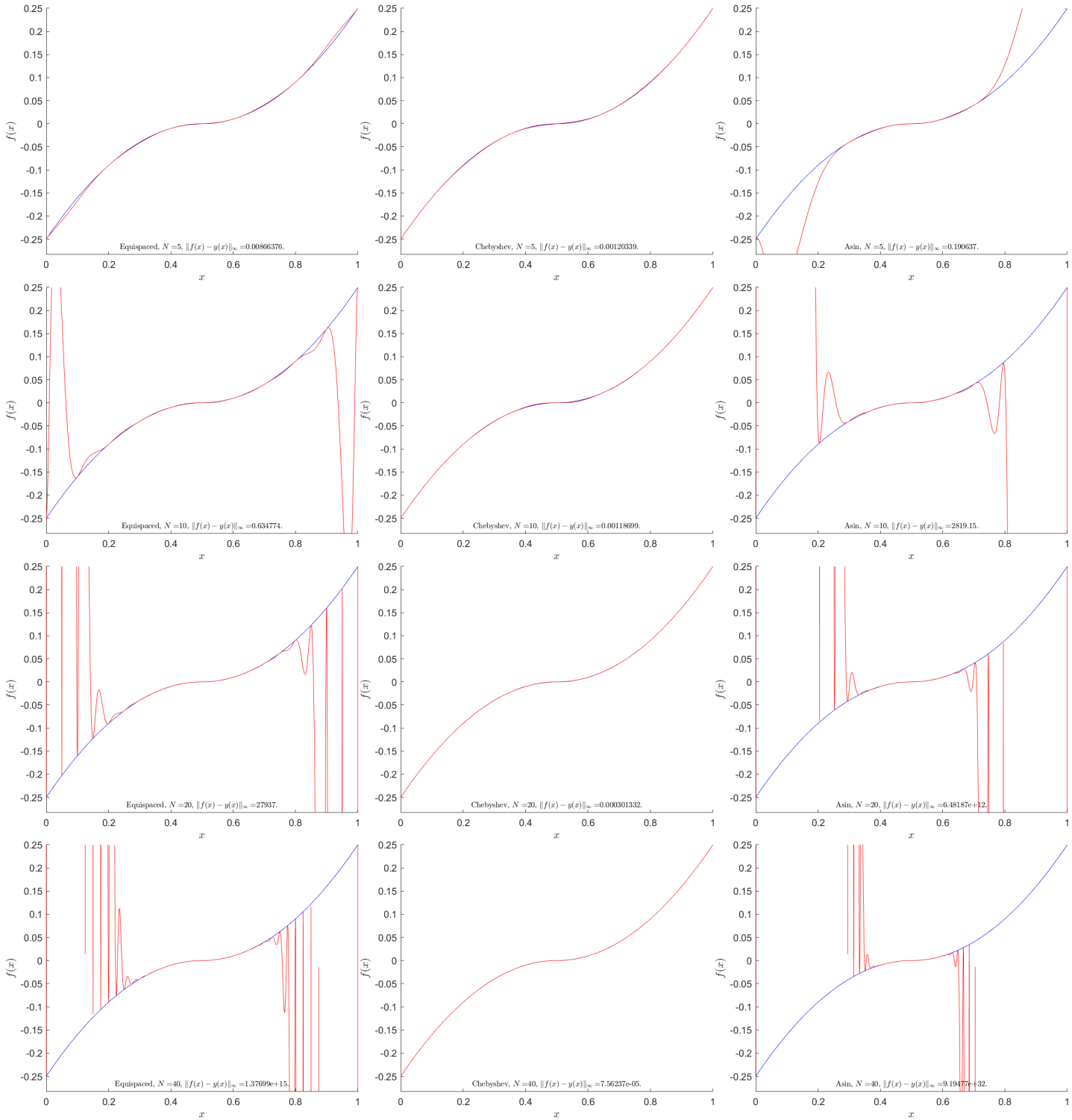

Рис. 10. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

2.3 Accuracy analysis

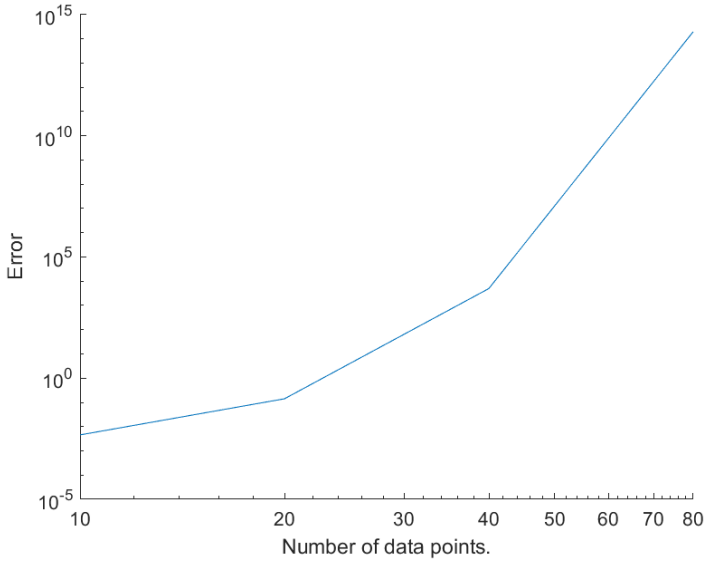

Рис. 11. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

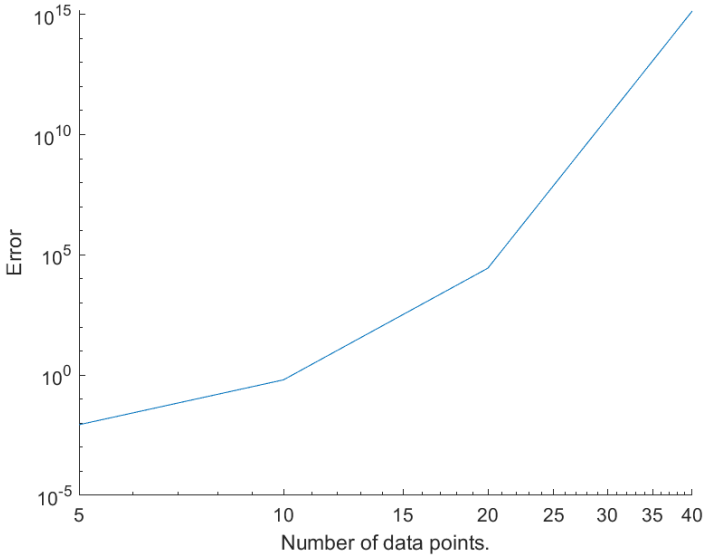

Рис. 12. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

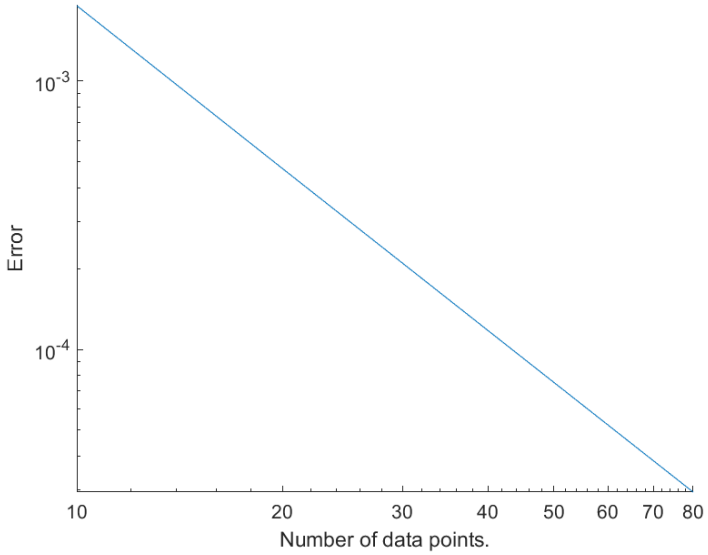

Рис. 13. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

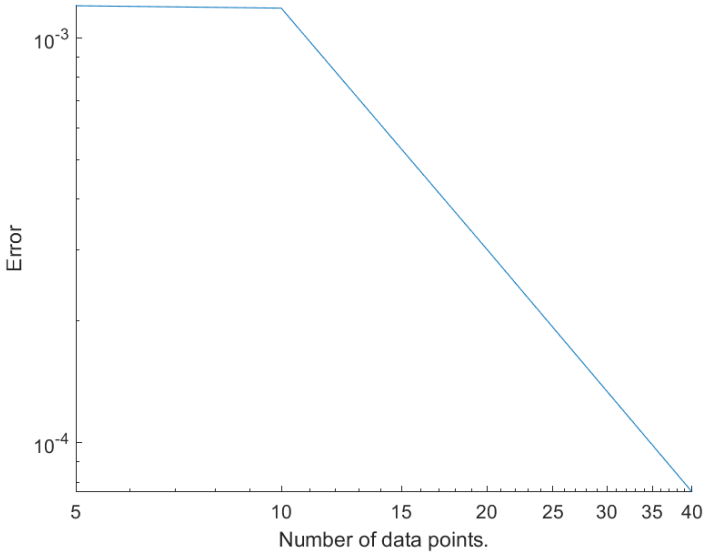

Рис. 14. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

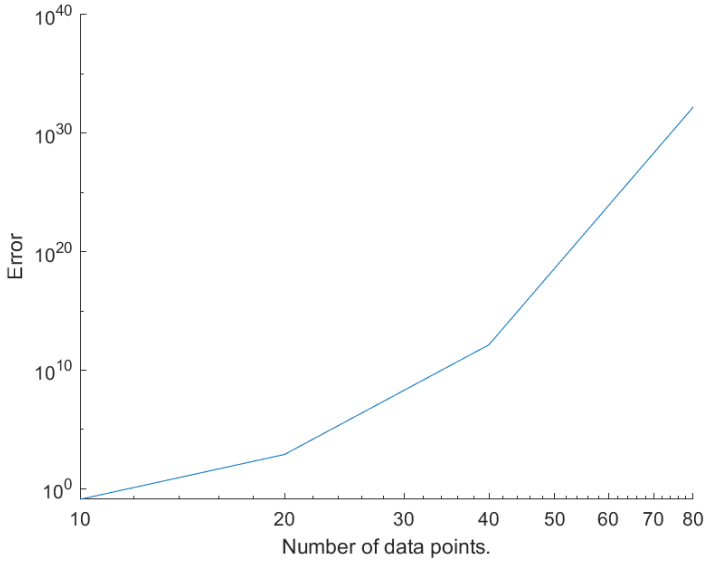

Рис. 15. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

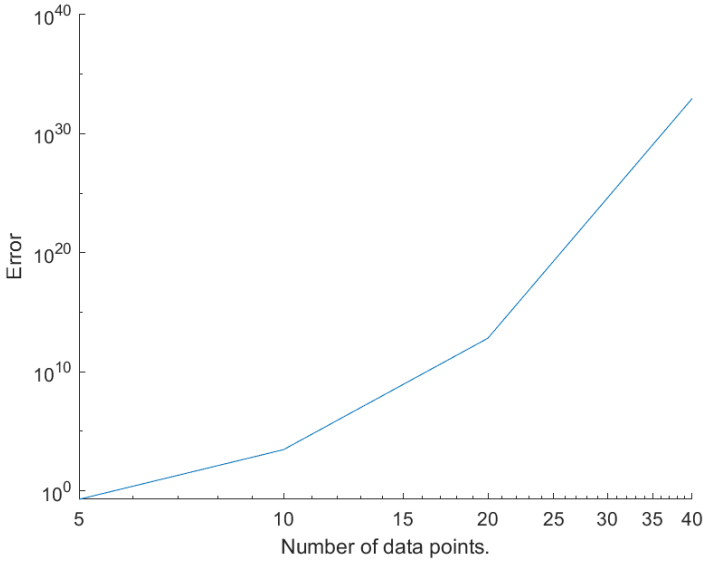

Рис. 16. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

3 $|x - \frac{1}{2}|$

3.1 Lagrange interpolant

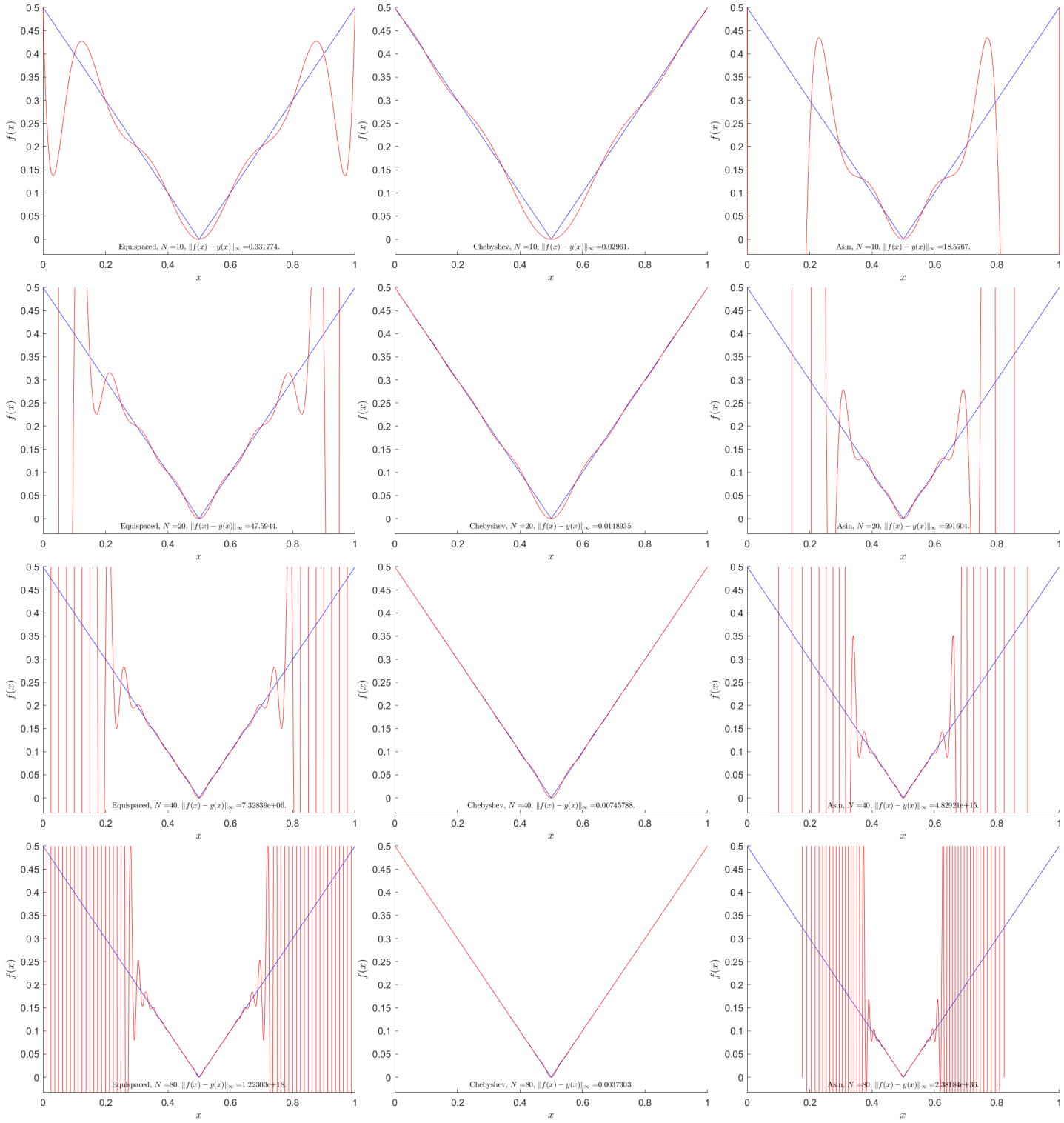

Рис. 17. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

3.2 Hermit interpolant

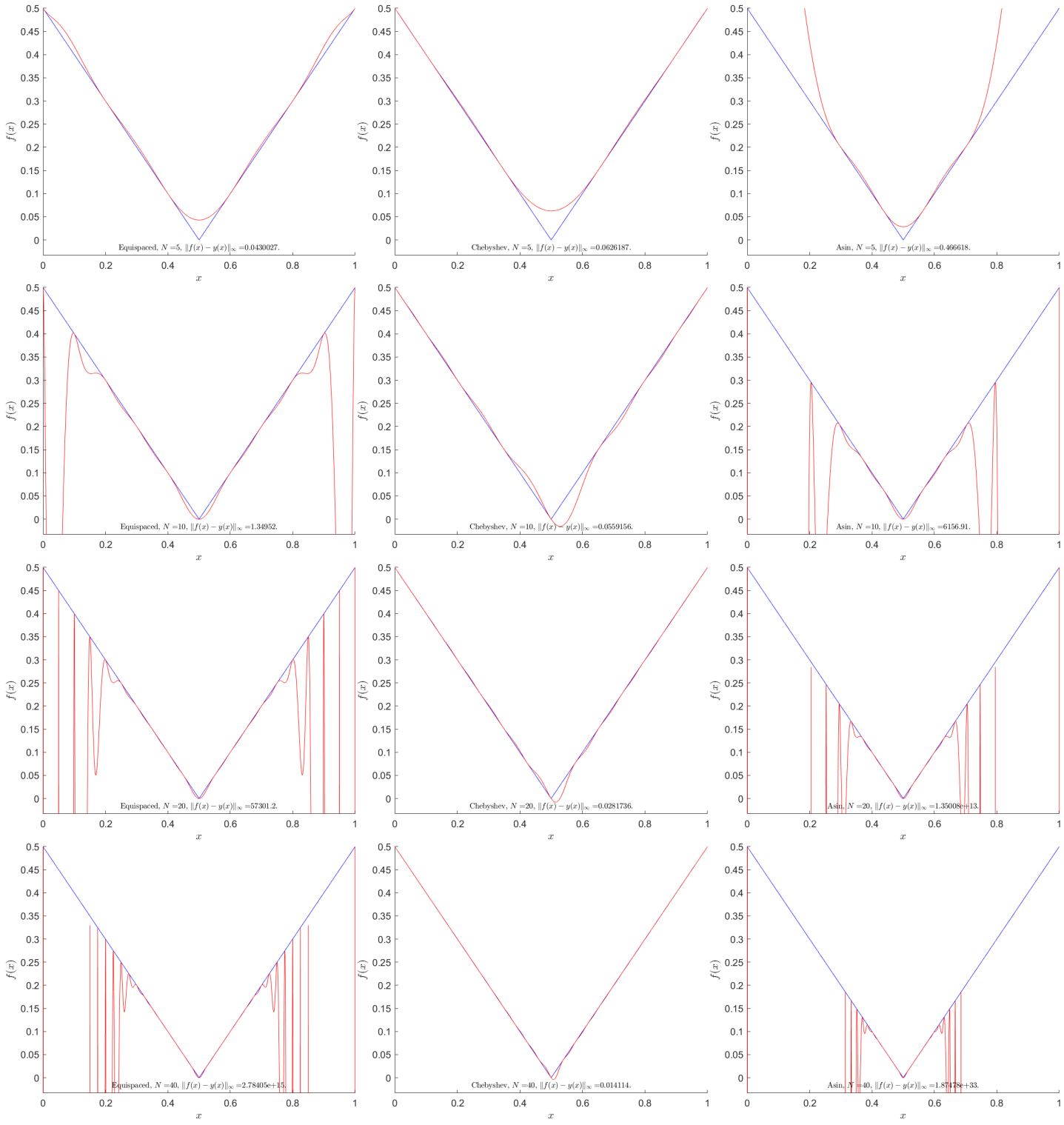

Рис. 18. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

3.3 Accuracy analysis

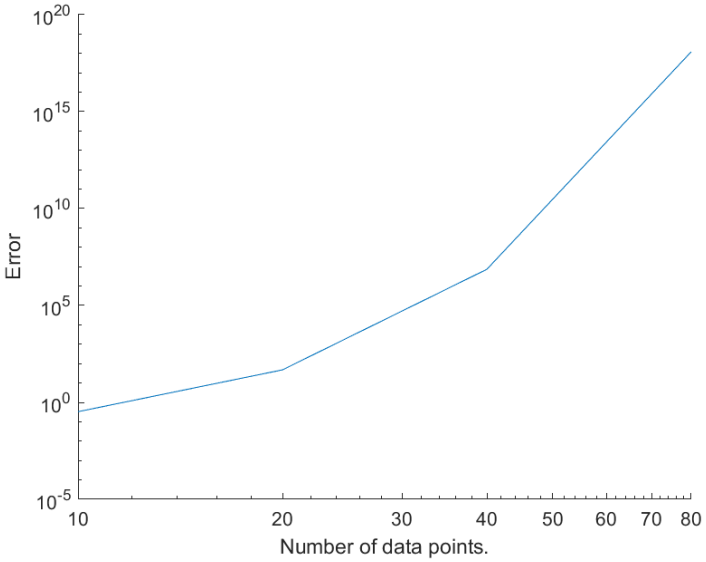

Рис. 19. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

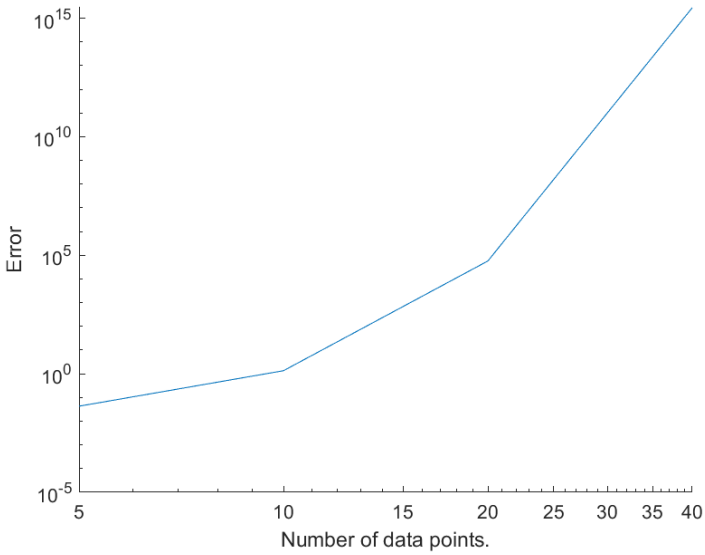

Рис. 20. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

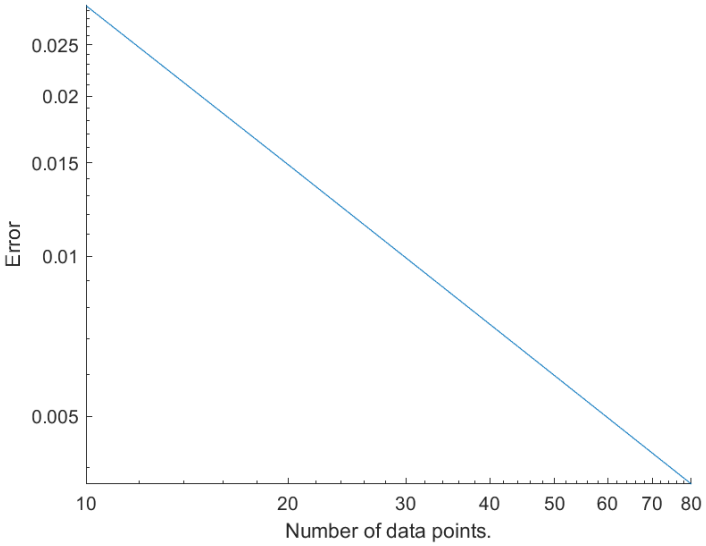

Рис. 21. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

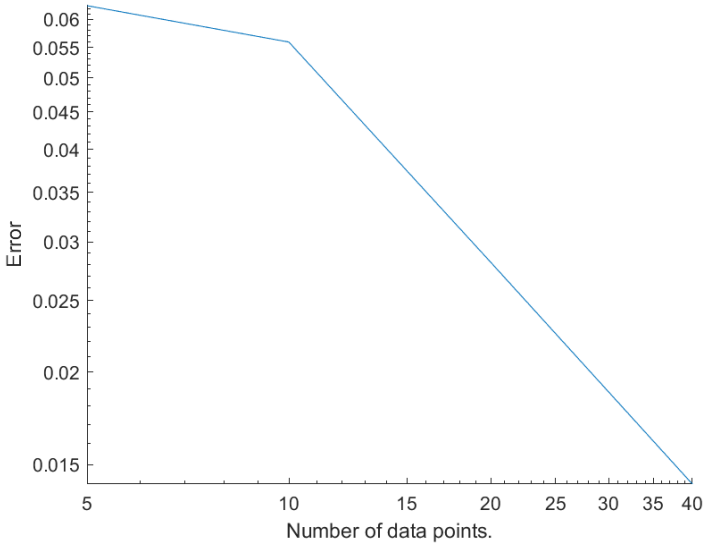

Рис. 22. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

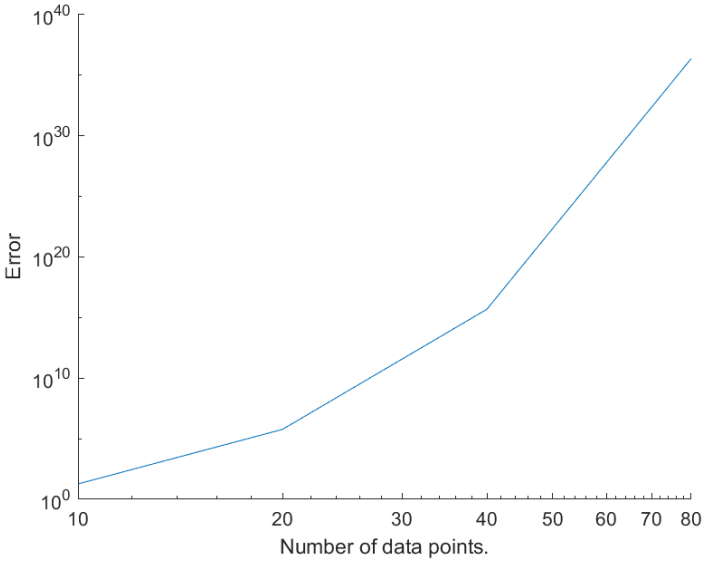

Рис. 23. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

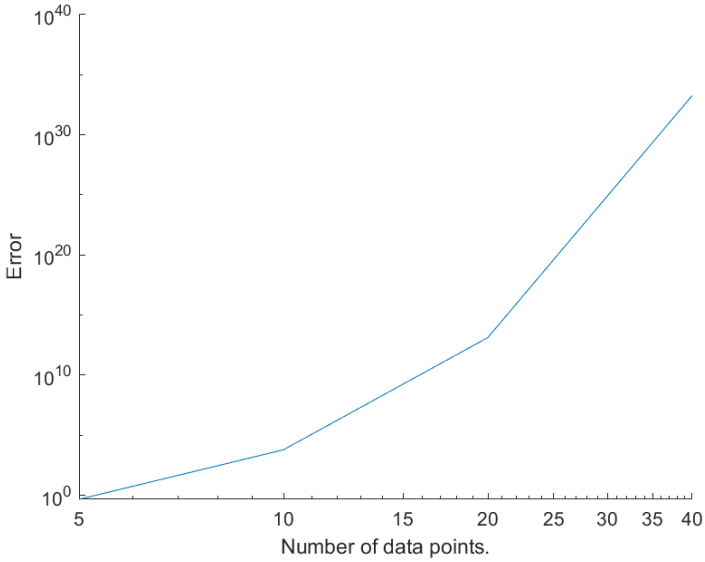

Рис. 24. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

4 $\sqrt{1-x^2}$

4.1 Lagrange interpolant

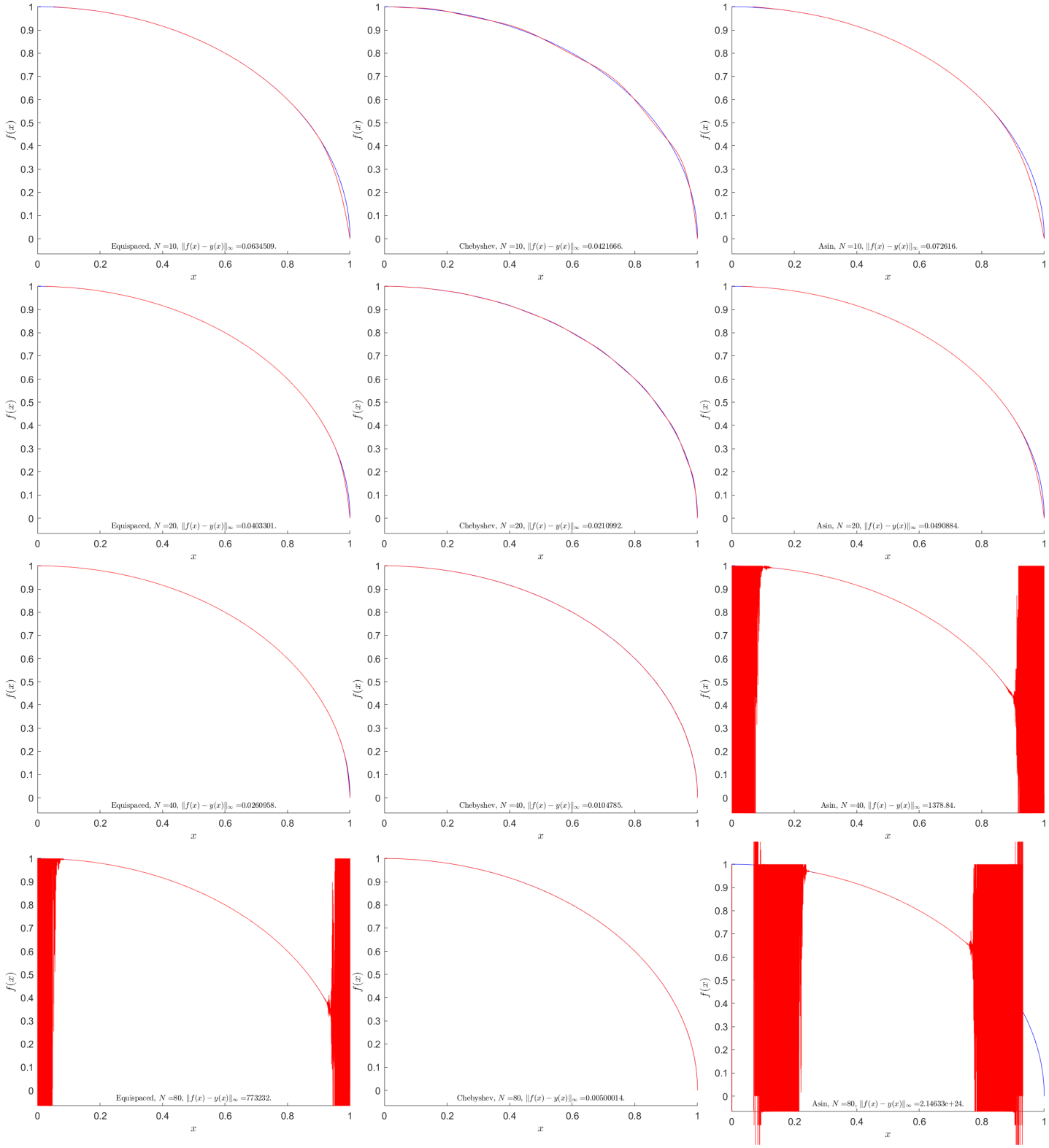

Рис. 25. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

4.2 Hermit interpolant

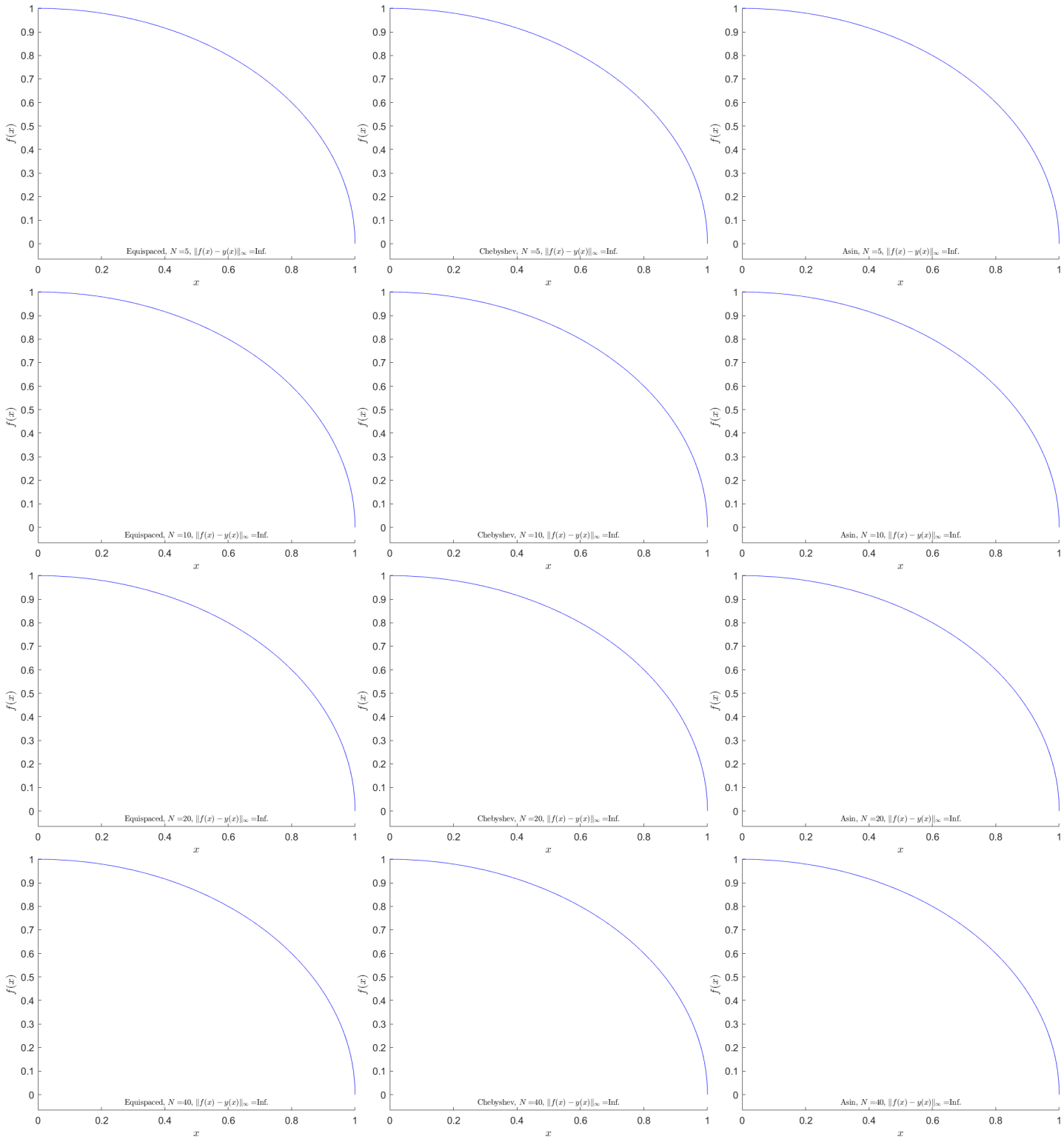

Рис. 26. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

4.3 Accuracy analysis

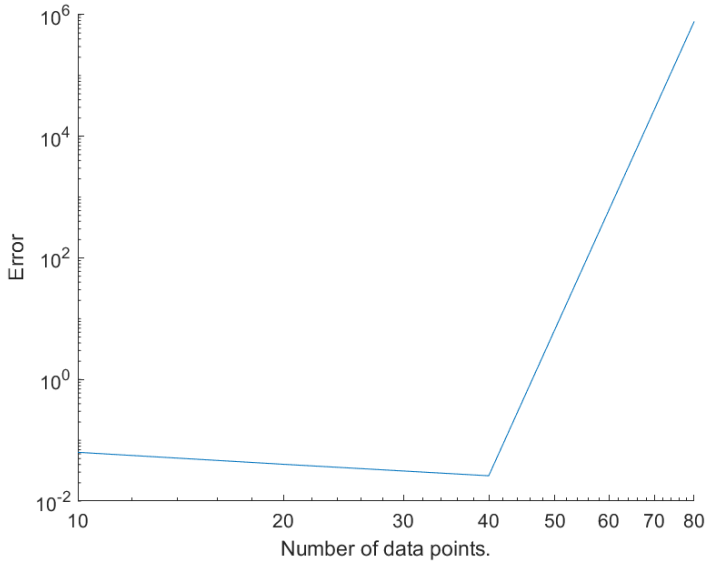

Рис. 27. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

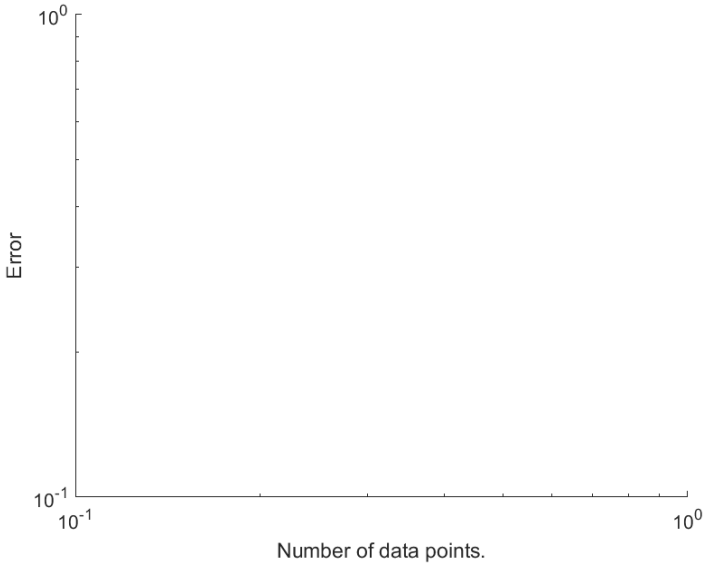

Рис. 28. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

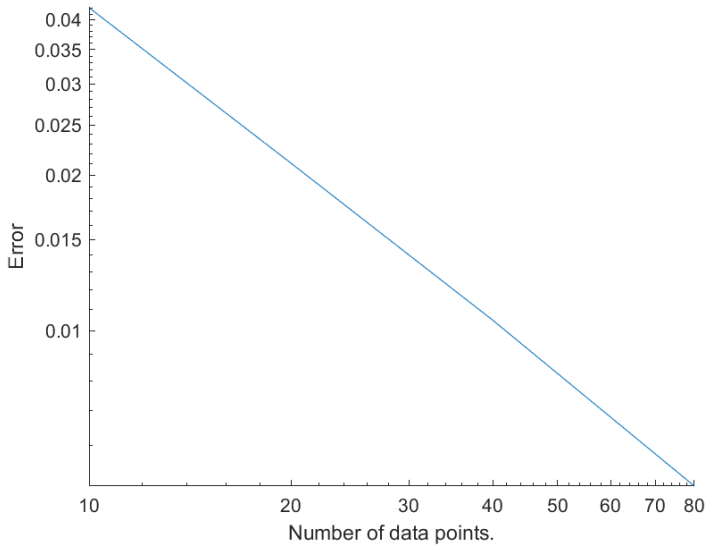

Рис. 29. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

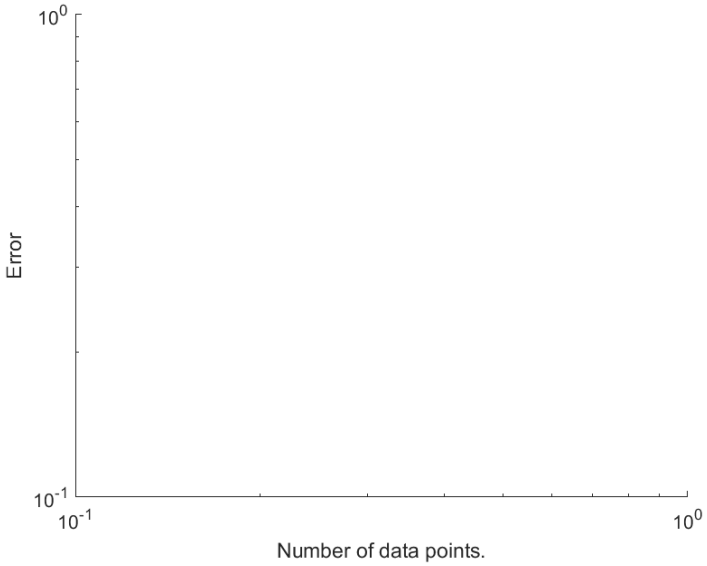

Рис. 30. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

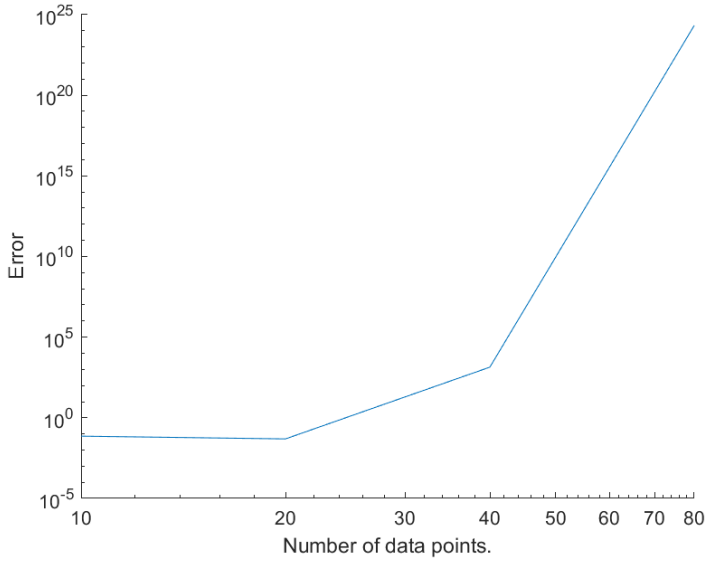

Рис. 31. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

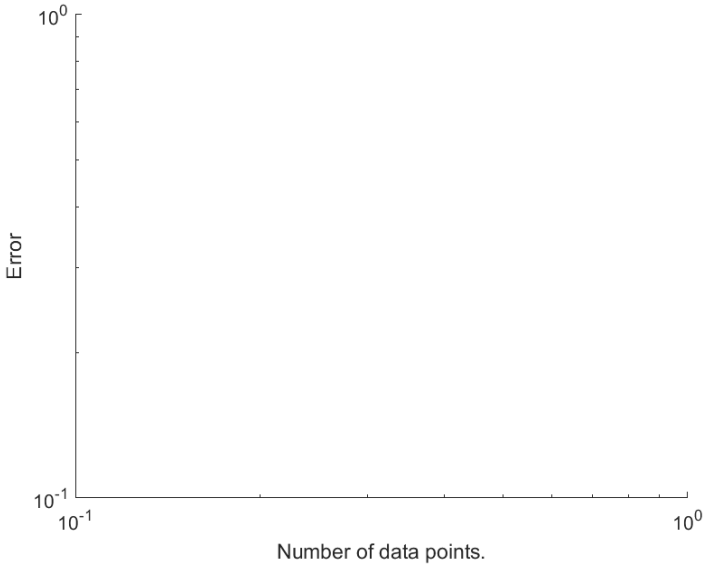

Рис. 32. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.