2D,

Skolkovo Institute of Science and Technology
Center of Material Technologies

Report on the course

«Numerical Methods in Engineering and Science».

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Part I

Lagrange and Hermite interpolation

Lagrange and Hermite interpolants are considered for a set of functions and grid distributions in the interval [0, 1]. Functions:

1. $\frac{1}{1+x^2}$.

2. $(x - \frac{1}{2})^2 sign(x - \frac{1}{2})$.

3. $|x - \frac{1}{2}|$.

4. $\sqrt{1 - x^2}$.

Corresponding derivatives:

1. $\frac{-2}{(1+x^2)^2}$.

2. $2(x - \frac{1}{2}) sign(x - \frac{1}{2})$.

3. $sign(x - \frac{1}{2})$.

4. $\frac{-x}{\sqrt{1-x^2}}$.

Grid distributions:

1. Equispaced: $x_i = \frac{i}{N}, \quad i = 0, ..., N$.

2. Chebyshev: $\frac{1}{2} - \frac{1}{2} cos(\frac{i}{N}\pi), \quad i = 0, ..., N$.

3. Asin: $\frac{1}{2} + \frac{1}{\pi} sin^{-1}\left(\frac{2i}{N} - 1\right), \quad i = 0, ..., N$.

where N is the number of data points.

1 $\frac{1}{1+x^2}$

1.1 Lagrange interpolant

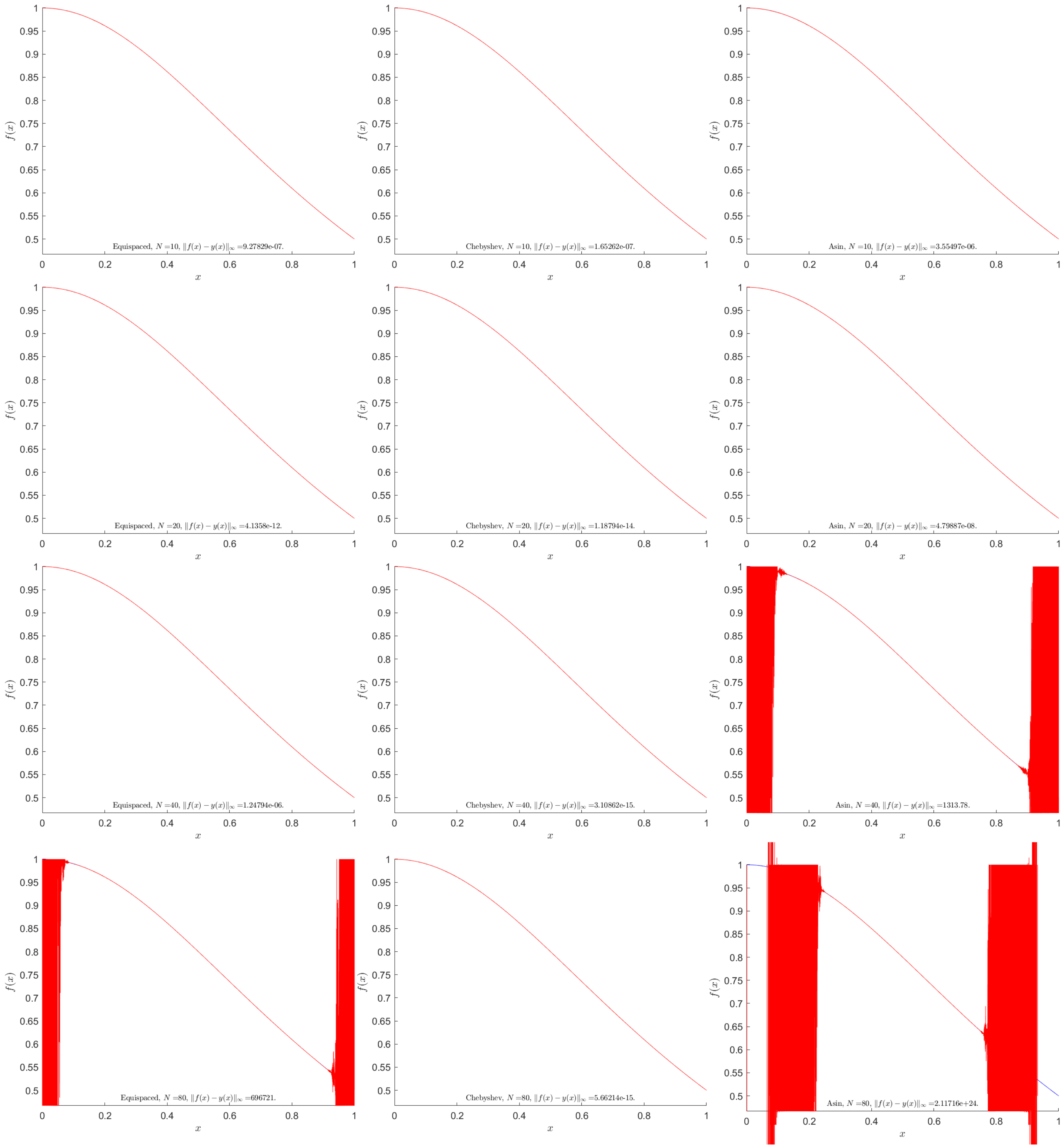

Figure 1. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

1.2 Hermit interpolant

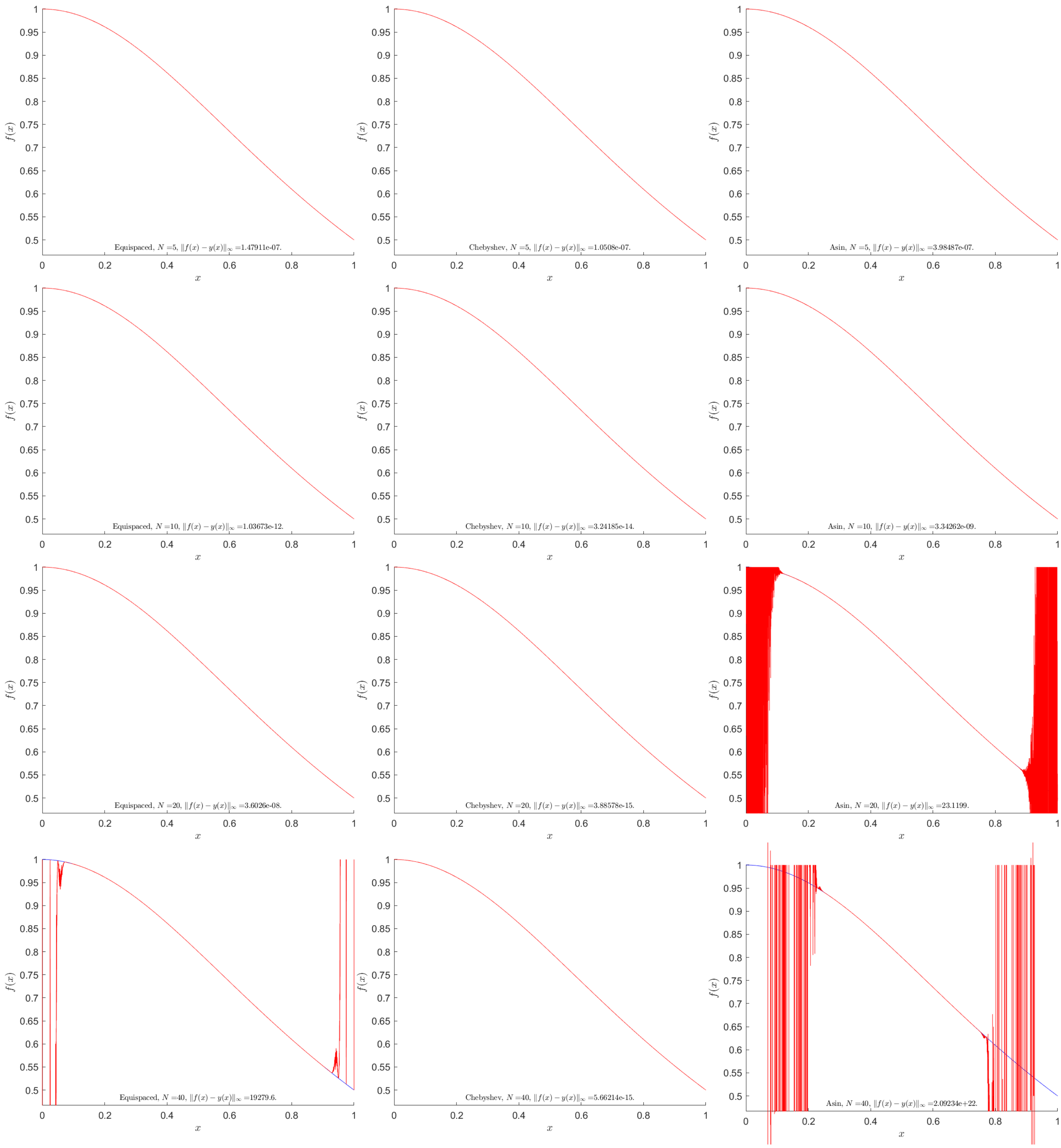

Figure 2. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

1.3 Accuracy analysis

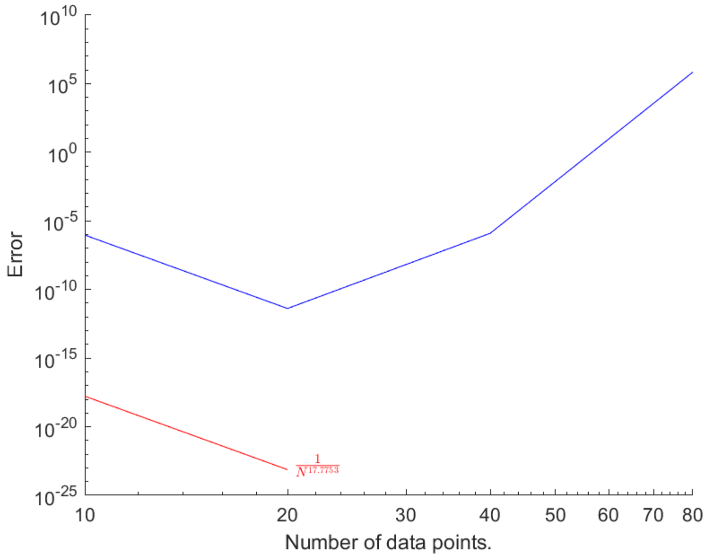

Figure 3. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

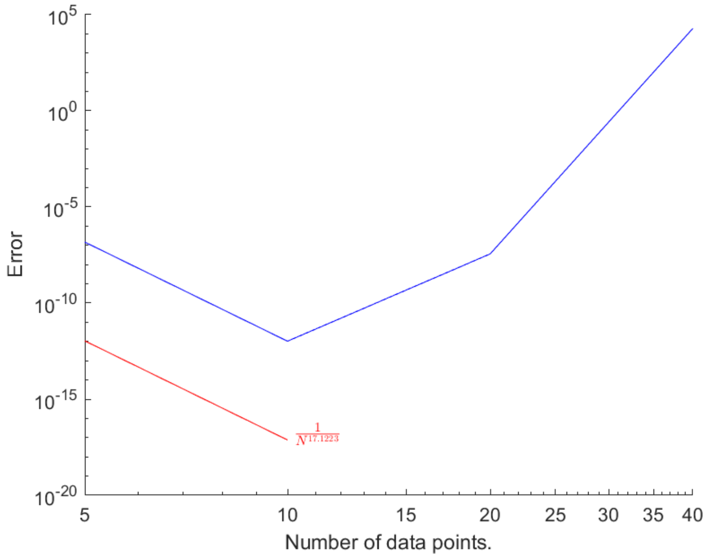

Figure 4. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

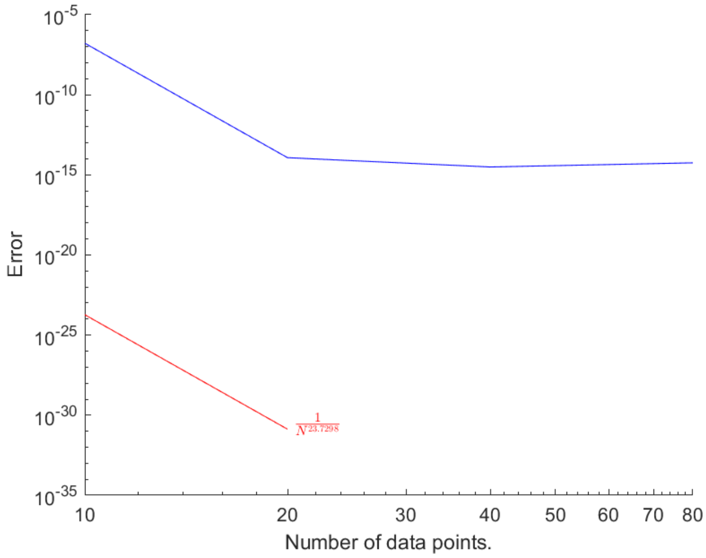

Figure 5. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

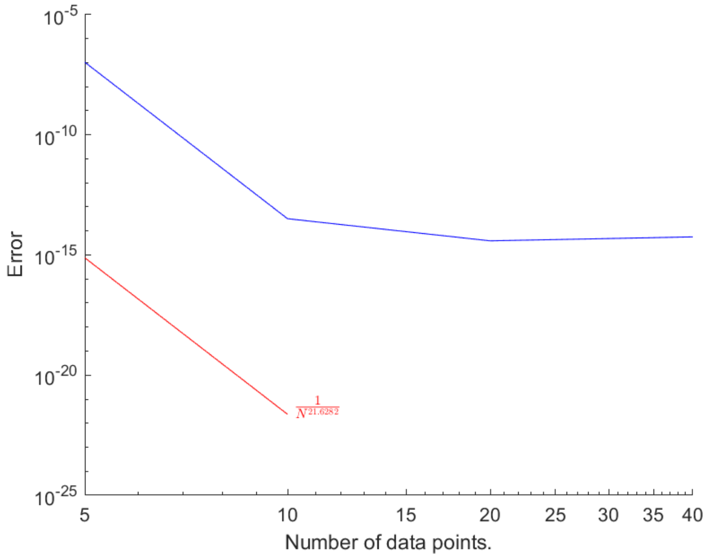

Figure 6. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

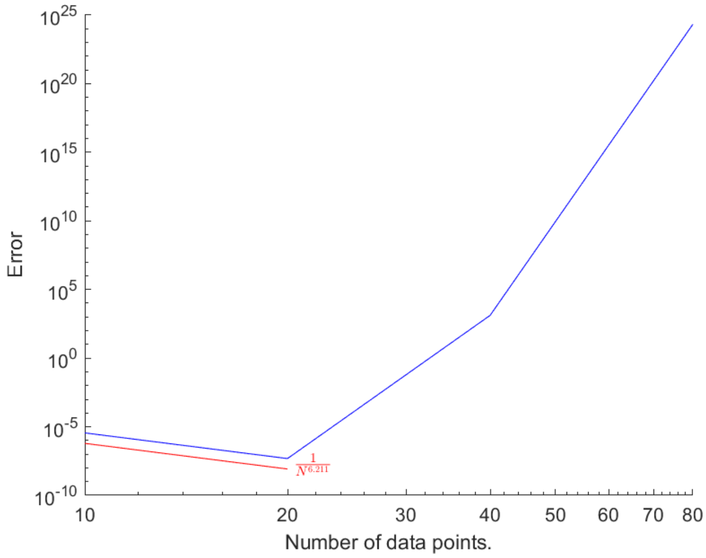

Figure 7. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

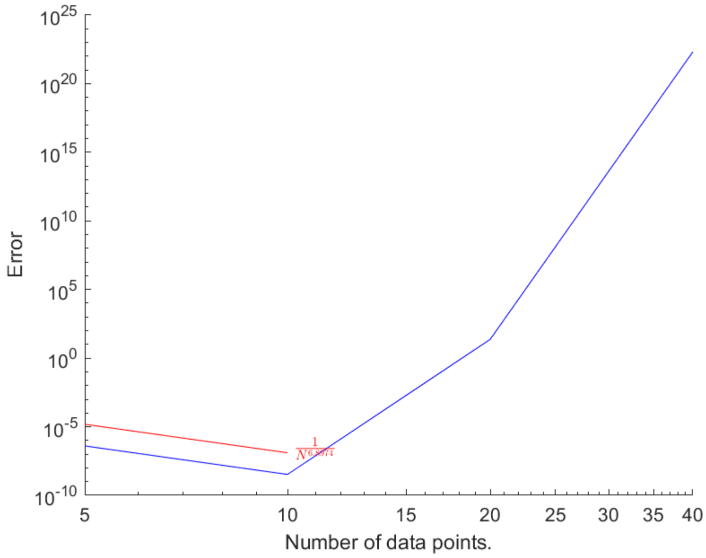

Figure 8. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

2 $(x - \frac{1}{2})^2 sign(x - \frac{1}{2})$

2.1 Lagrange interpolant

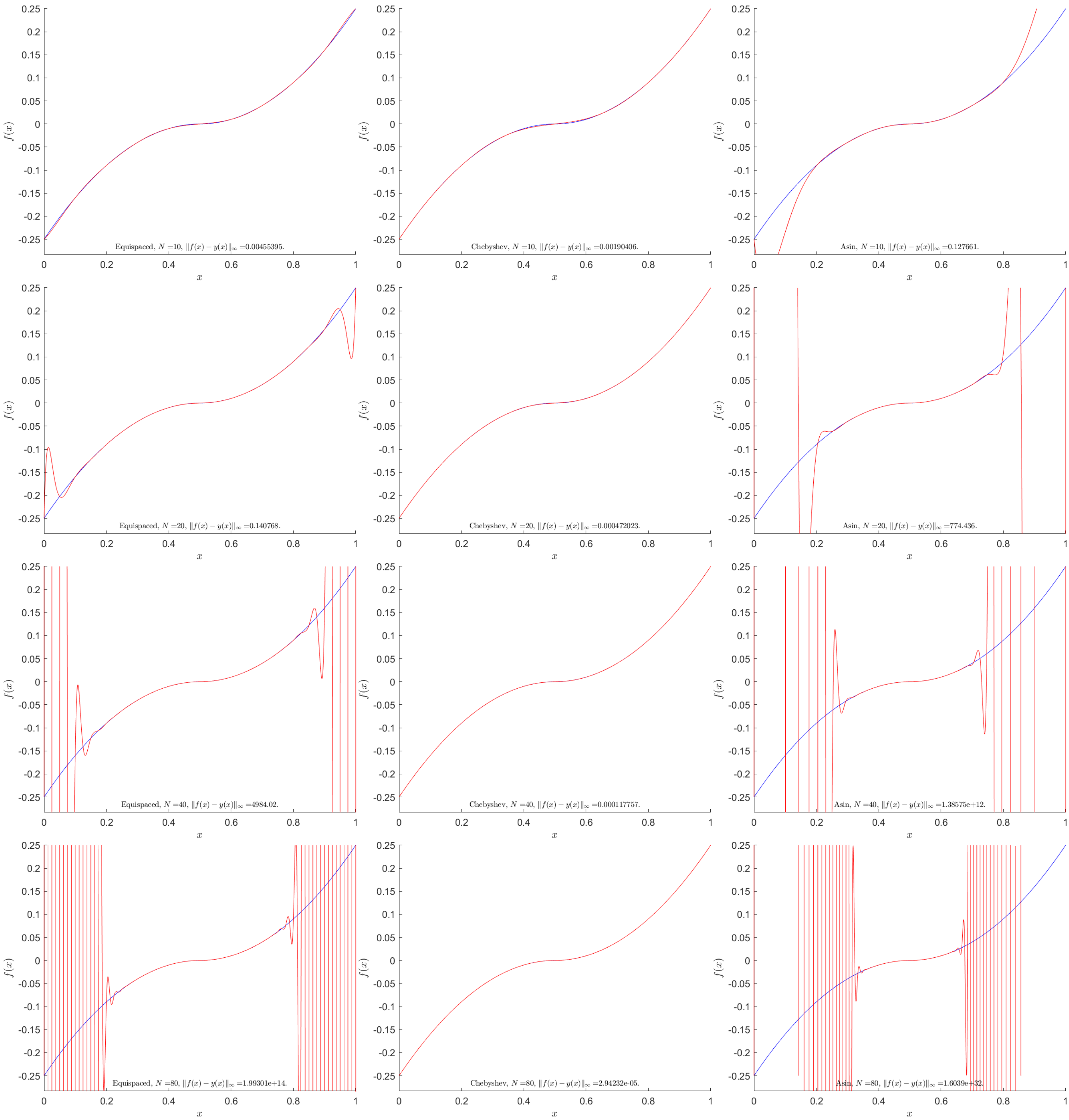

Figure 9. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

2.2 Hermit interpolant

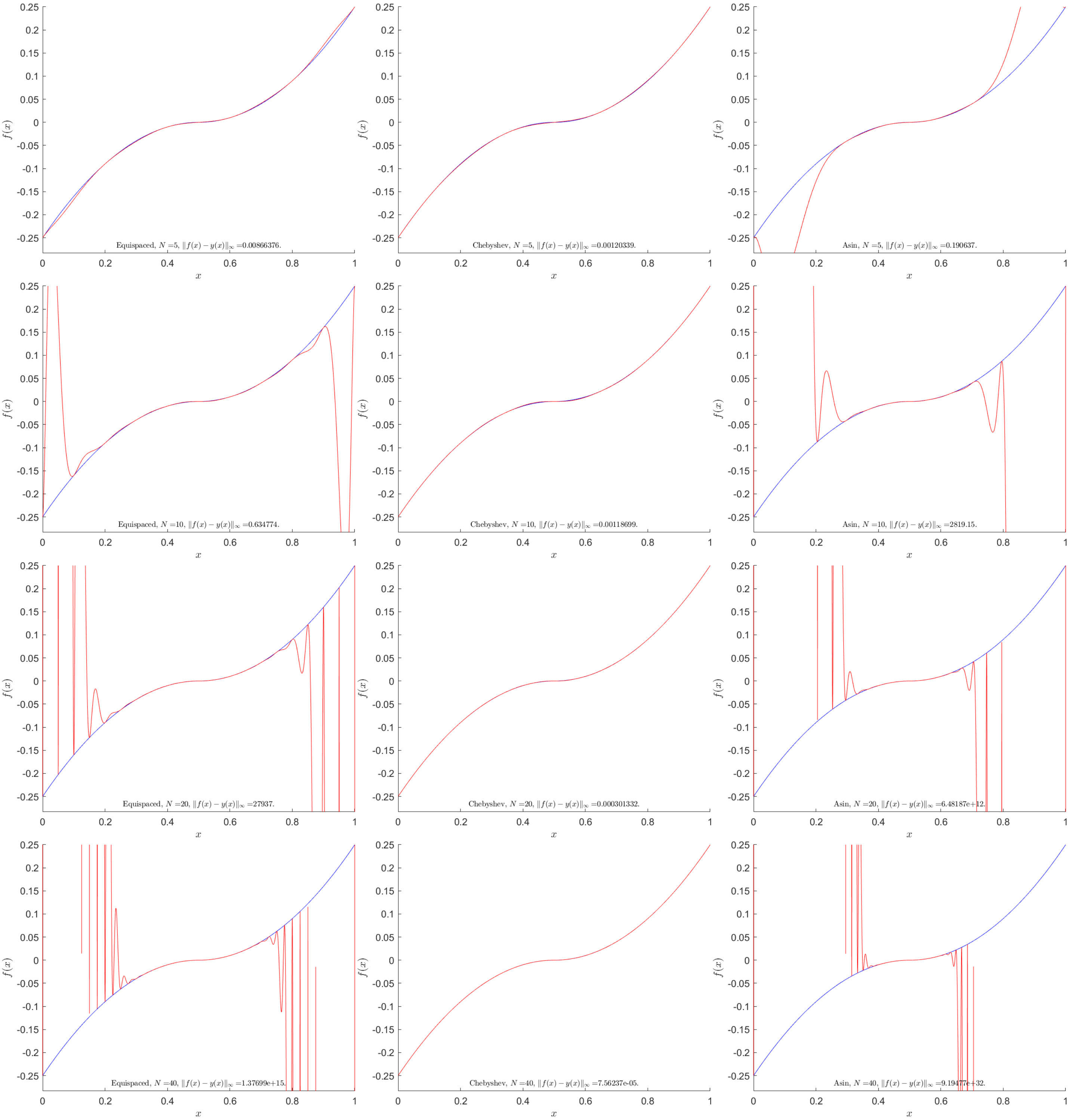

Figure 10. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

2.3 Accuracy analysis

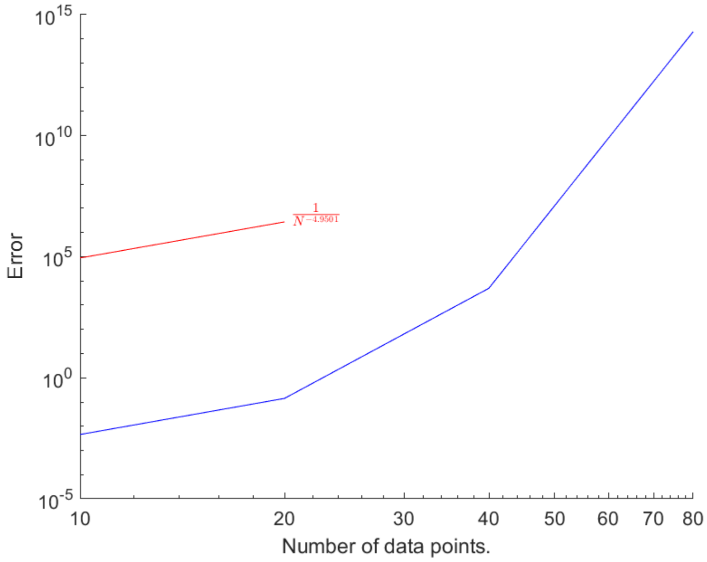

Figure 11. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

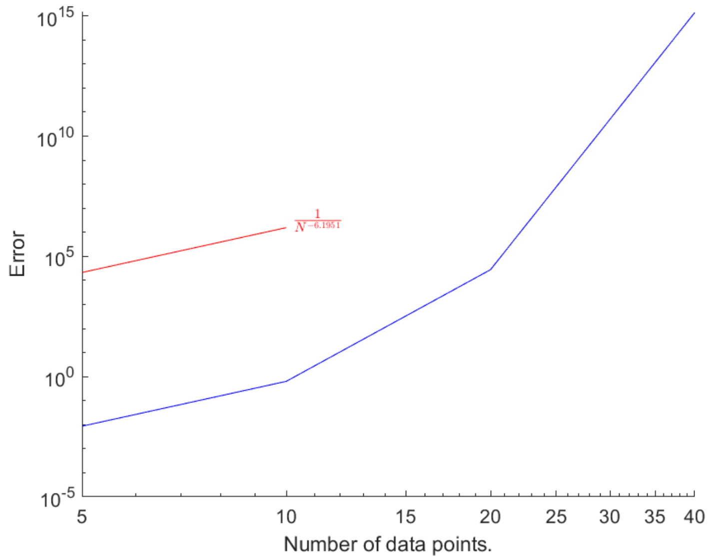

Figure 12. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

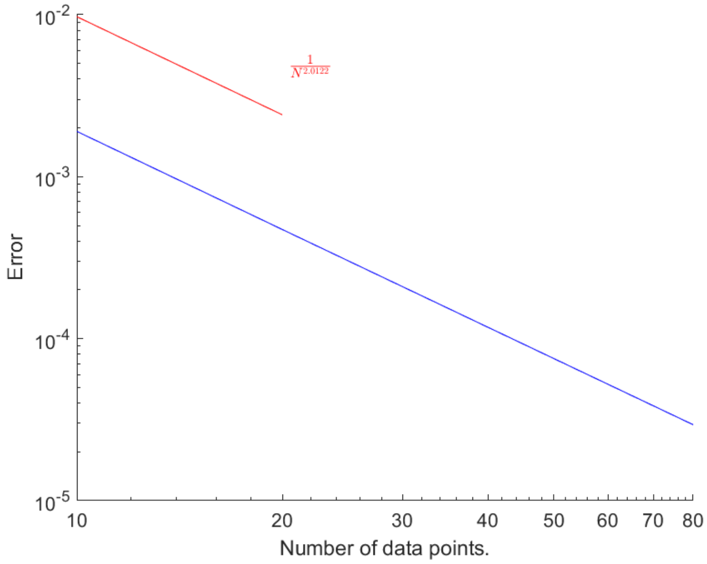

Figure 13. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

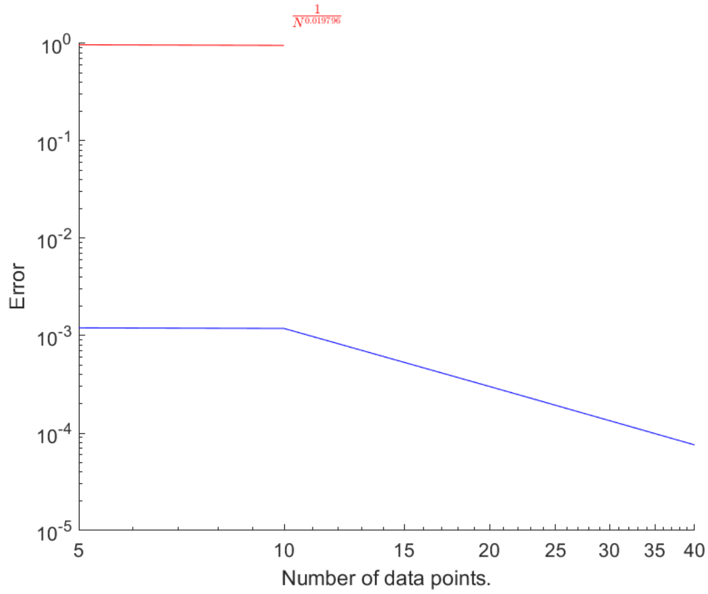

Figure 14. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

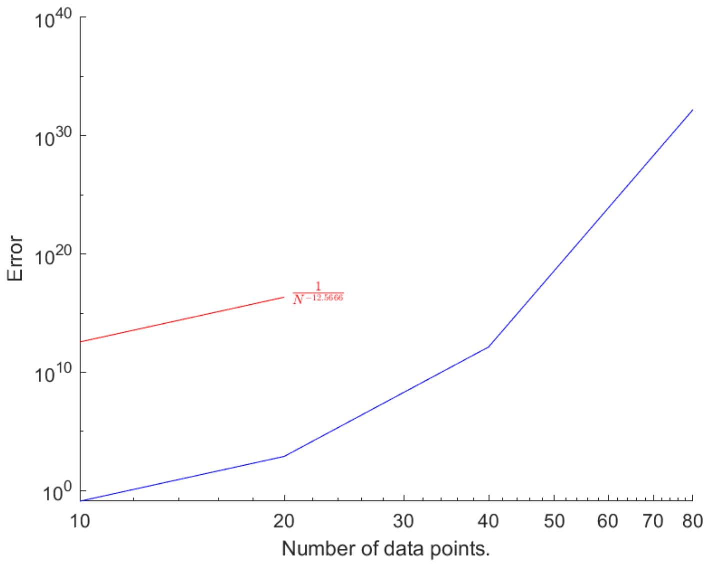

Figure 15. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

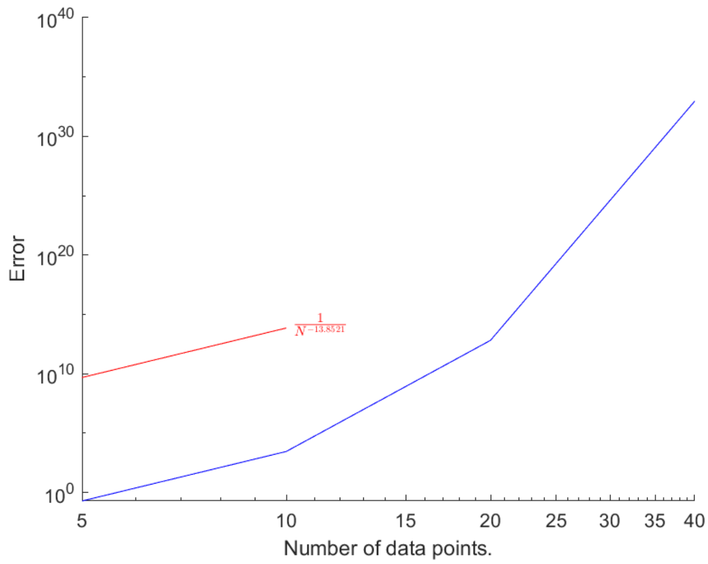

Figure 16. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

3 $|x - \frac{1}{2}|$

3.1 Lagrange interpolant

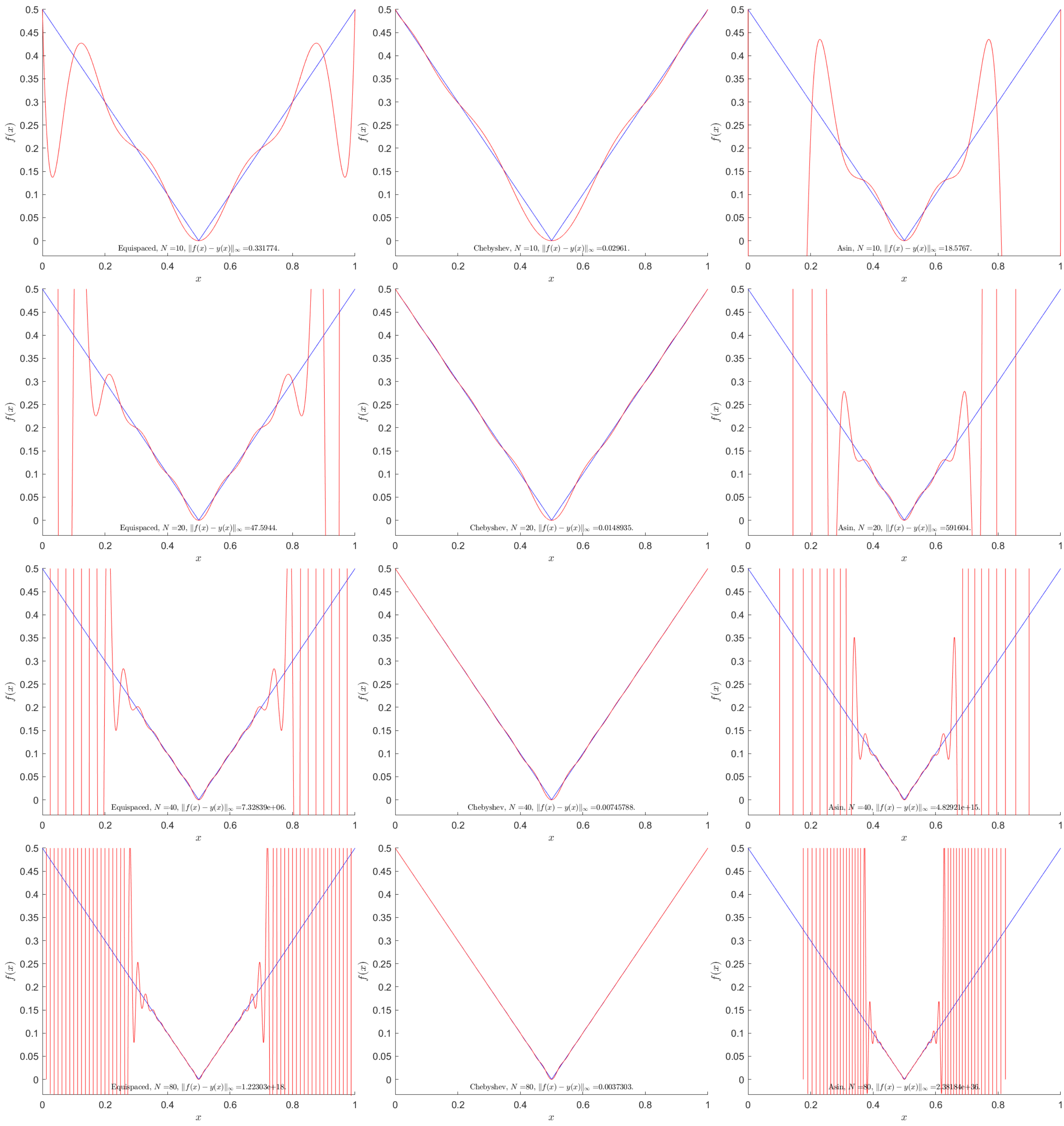

Figure 17. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

3.2 Hermit interpolant

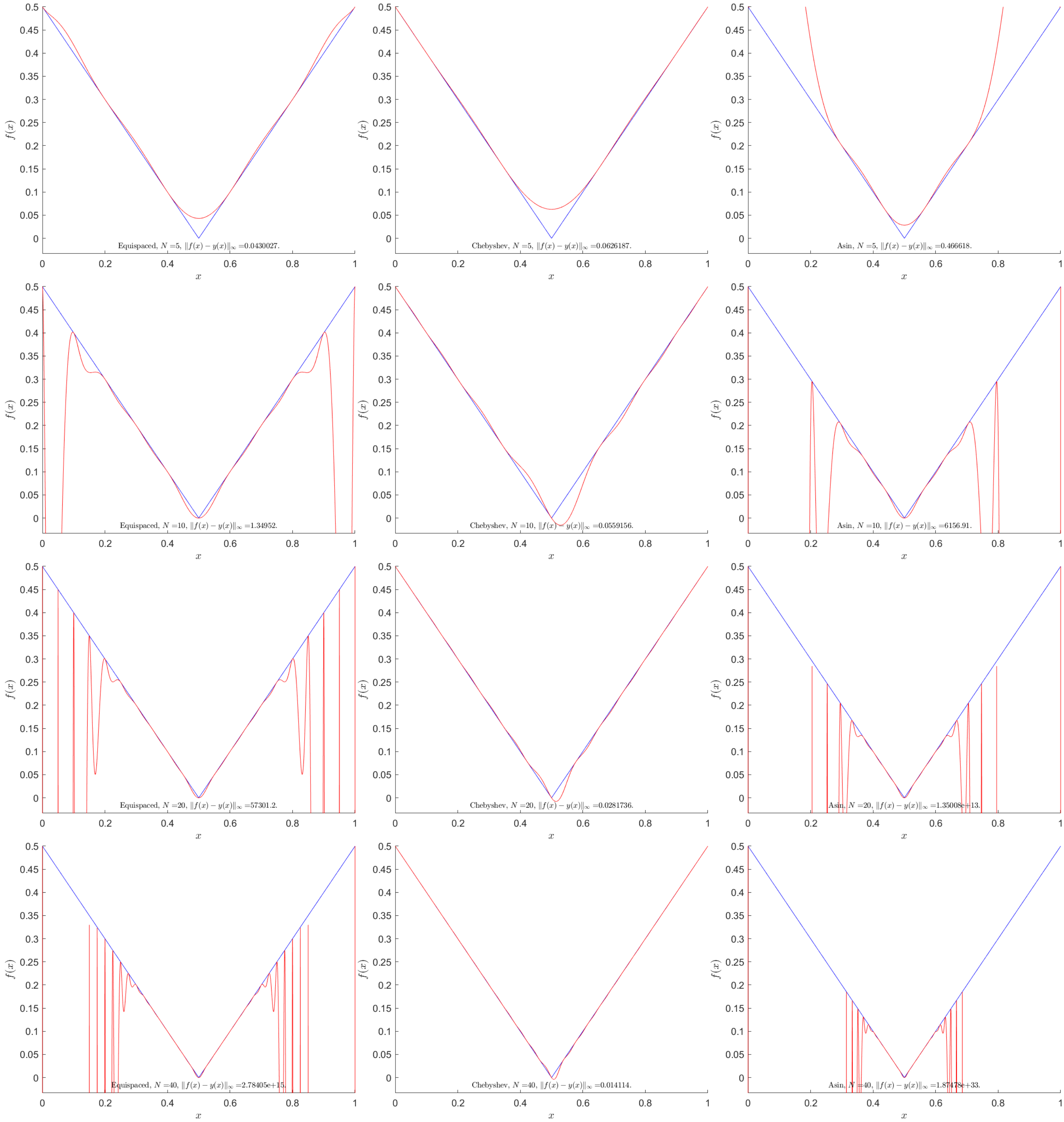

Figure 18. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

3.3 Accuracy analysis

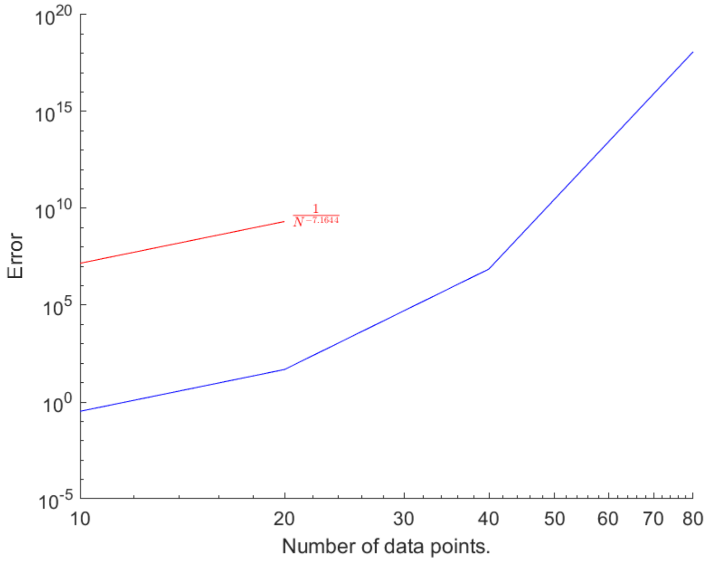

Figure 19. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

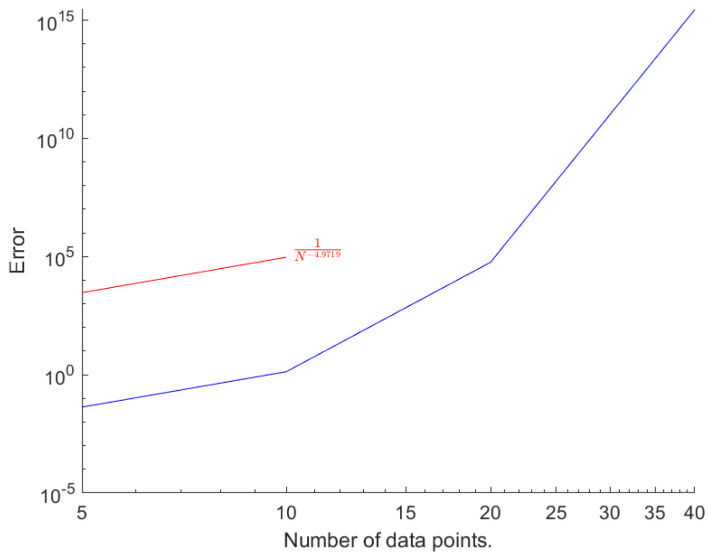

Figure 20. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

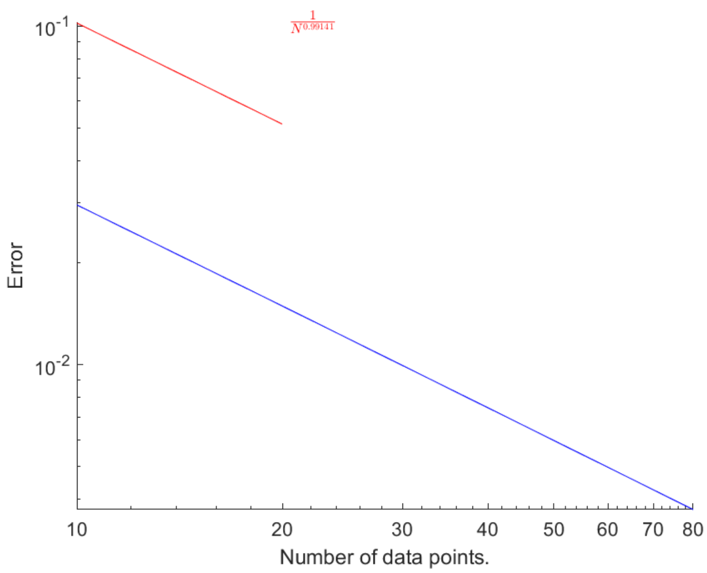

Figure 21. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

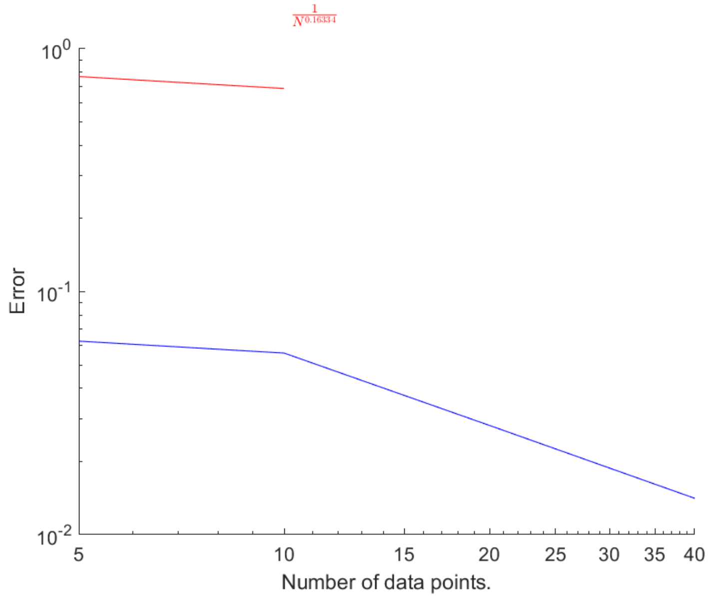

Figure 22. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

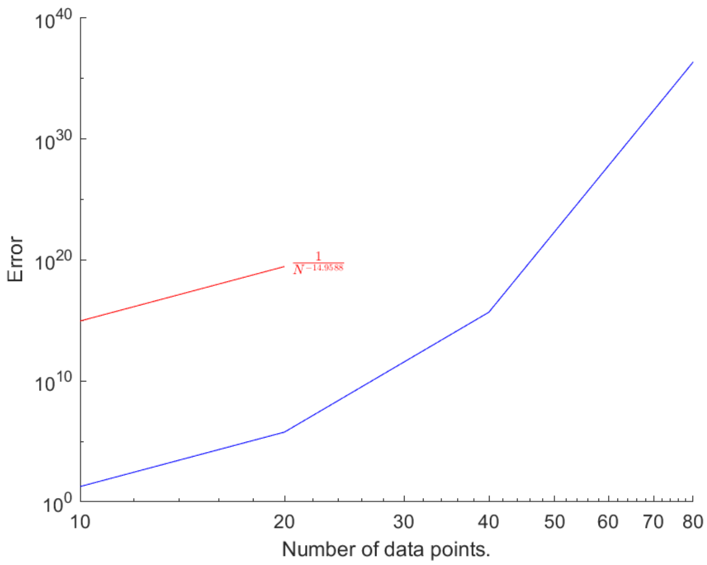

Figure 23. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

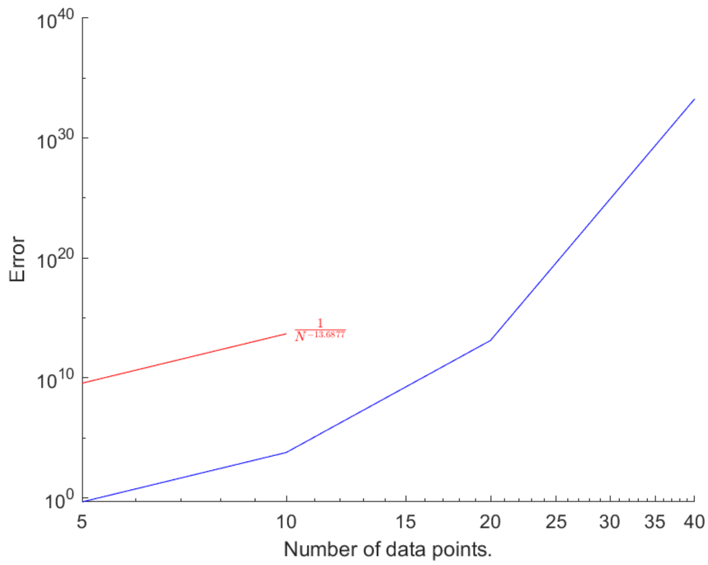

Figure 24. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

4 $\sqrt{1-x^2}$

4.1 Lagrange interpolant

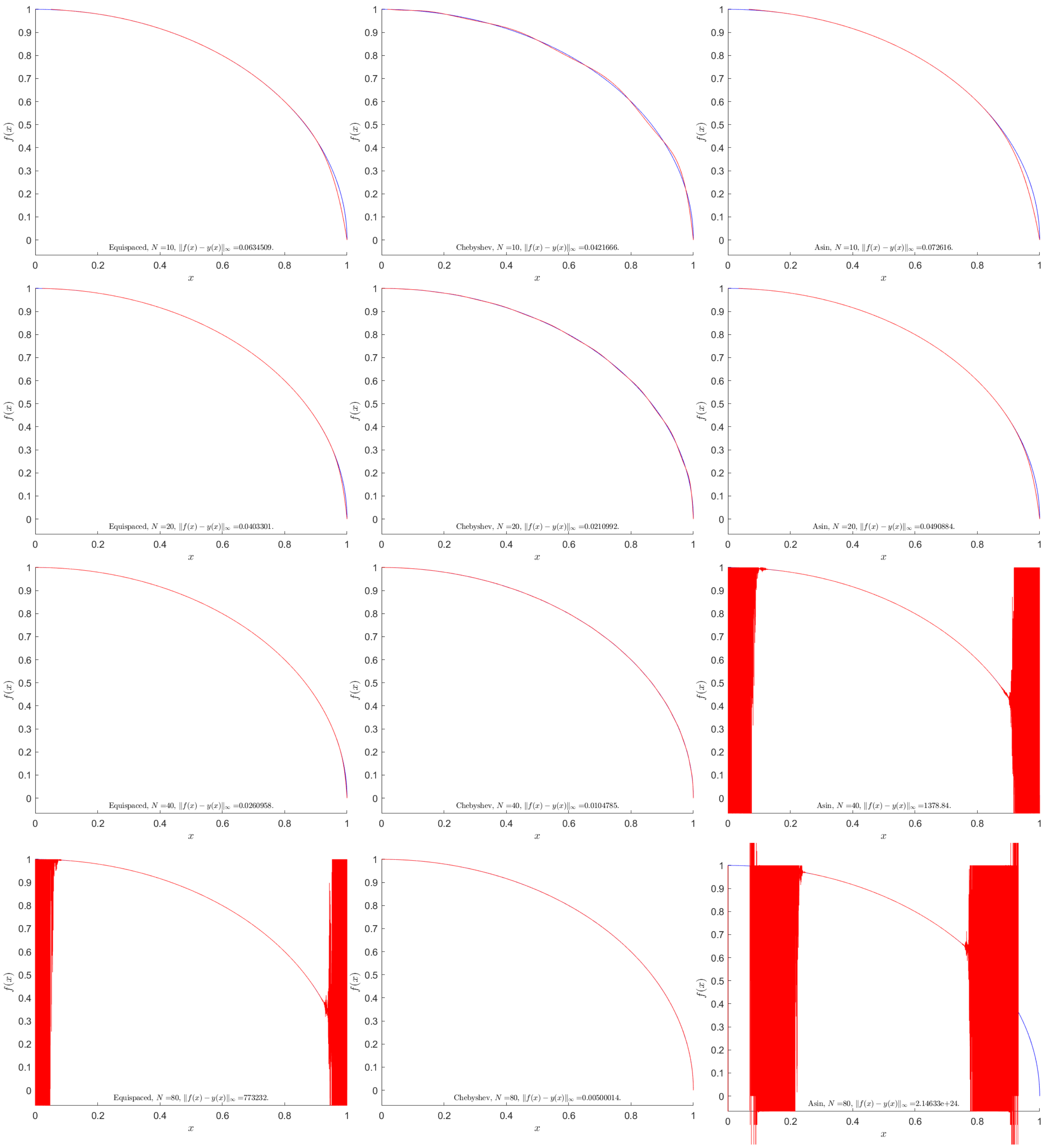

Figure 25. Results of Lagrange interpolation for 10, 20, 40 and 80 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

4.2 Hermit interpolant

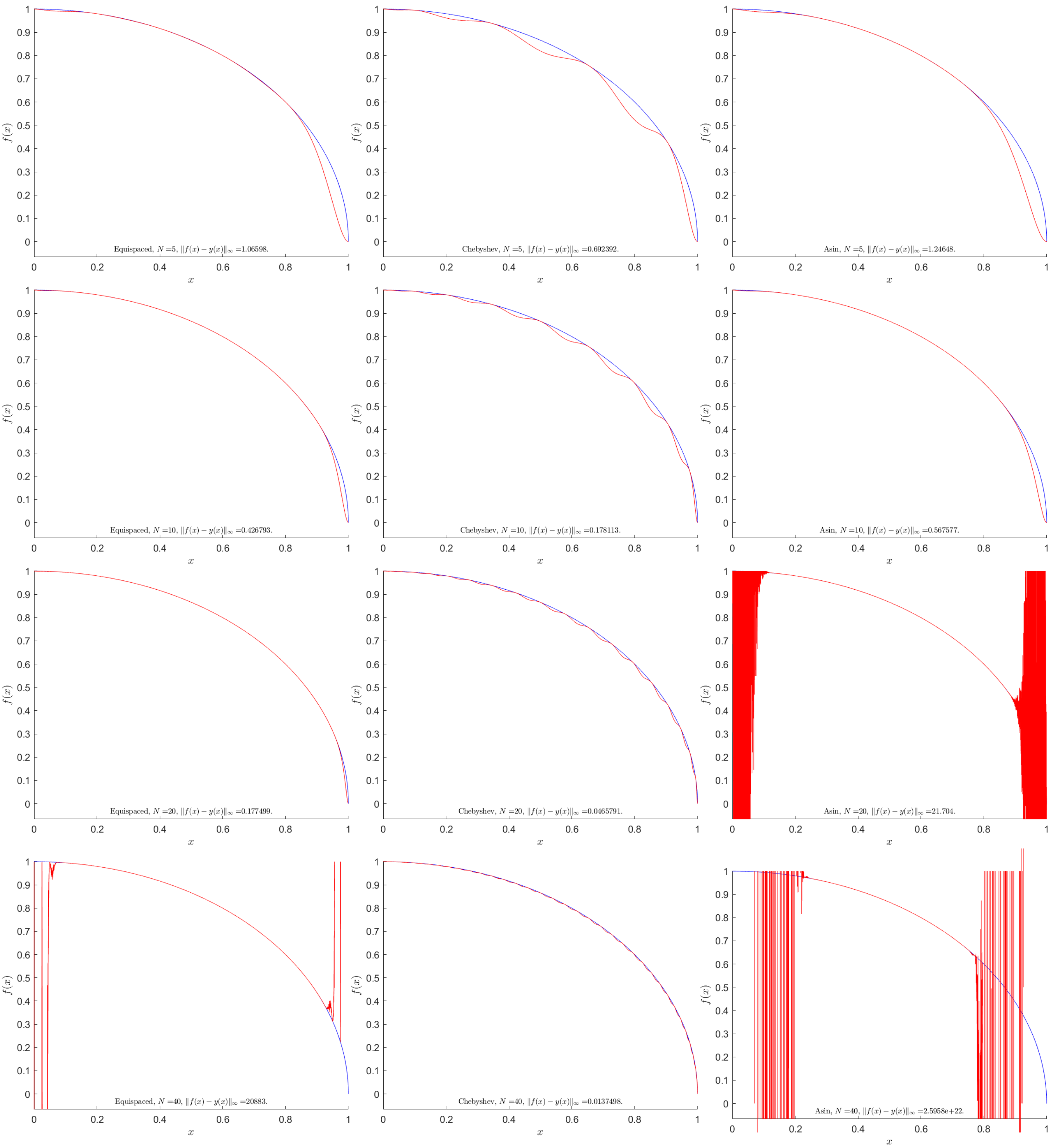

Figure 26. Results of Hermit interpolation for 5, 10, 20 and 40 data points. The function is pictured with blue, its interpolant with red. First colomn corresponds to Equispaced data point distribution, second to Chebyshev and third to Asin.

4.3 Accuracy analysis

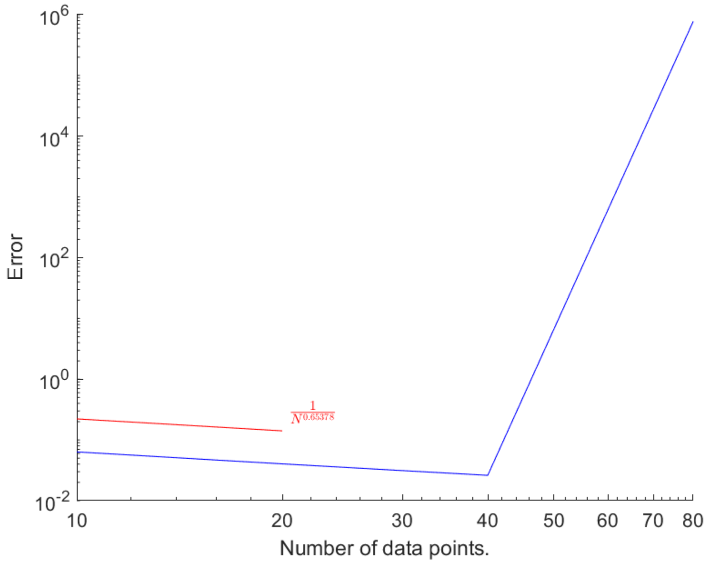

Figure 27. Dependence of error on the number of data points for Lagrange interpolant and Equispaced point distribution.

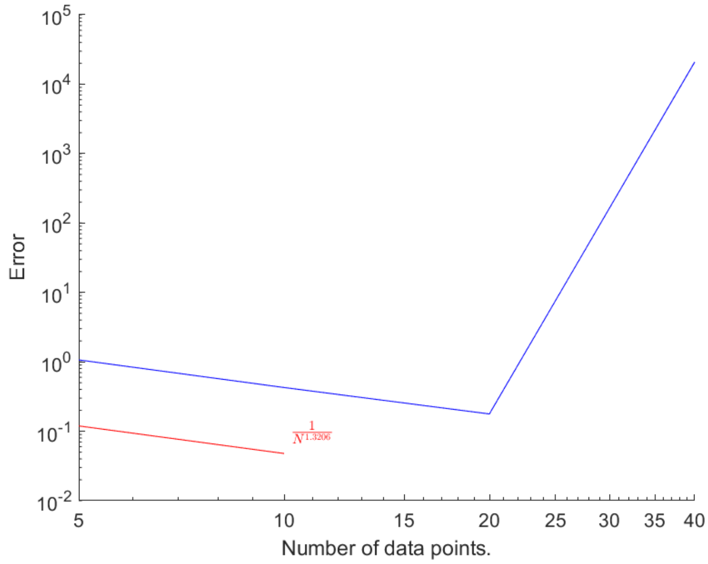

Figure 28. Dependence of error on the number of data points for Hermit interpolant and Equispaced point distribution.

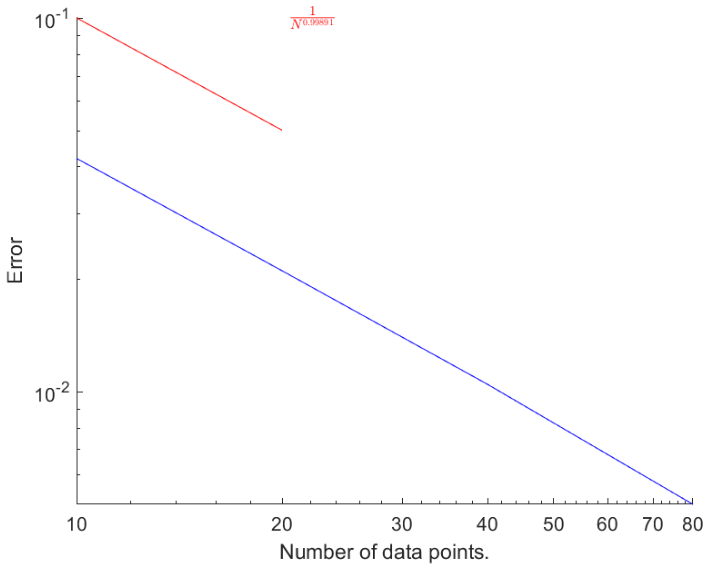

Figure 29. Dependence of error on the number of data points for Lagrange interpolant and Chebyshev point distribution.

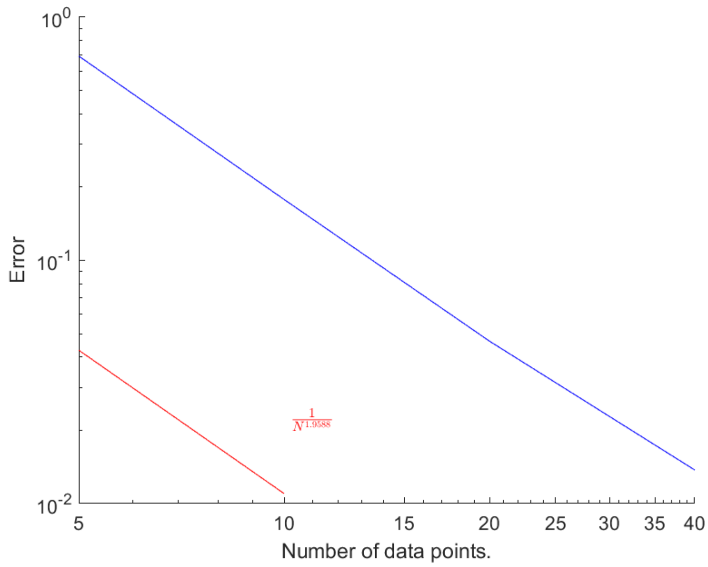

Figure 30. Dependence of error on the number of data points for Hermit interpolant and Chebyshev point distribution.

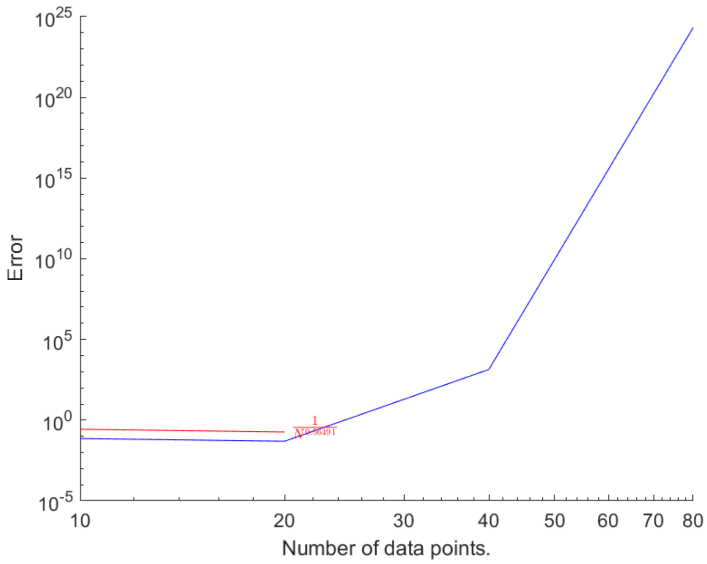

Figure 31. Dependence of error on the number of data points for Lagrange interpolant and Asin point distribution.

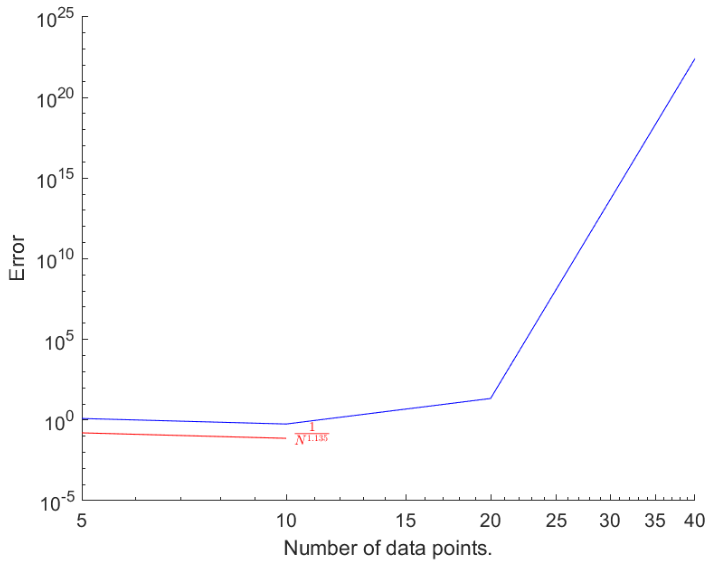

Figure 32. Dependence of error on the number of data points for Hermit interpolant and Asin point distribution.

Part II
Cubic spline interpolation

5 Parametrization

Cubic spline interpolation of an ellipse:

$$x^2 + \frac{y^2}{2} = 1, \tag{1}$$

is considered. Since the curve satisfying 1 can not be expressed in a form $y(x)$, we will work with its parametrization $(x(t), y(t))$. A set of data points is generated from:

$$\begin{cases} x = cos(t), \\ y = \sqrt{2}sin(t), \end{cases} \tag{2}$$

where $t \in [0, 2\pi + \delta]$. The interpolation was performed for N =9, 13, 17, and 21 data points, extension of the interval $\delta = \frac{2\pi}{N}$ is introduced to apply periodic boundary conditions: $f''(N-1) = f(0), \quad f''(N) = f(1)$.

6 Results

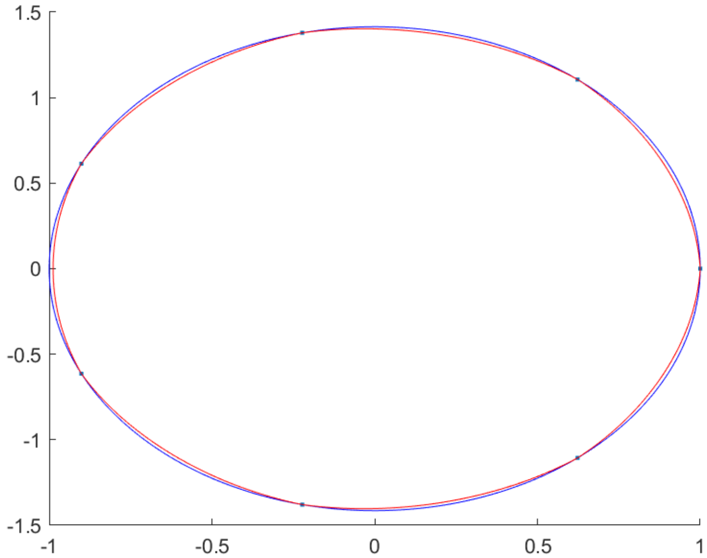

Figure 33. Interpolant for $N = 9$.

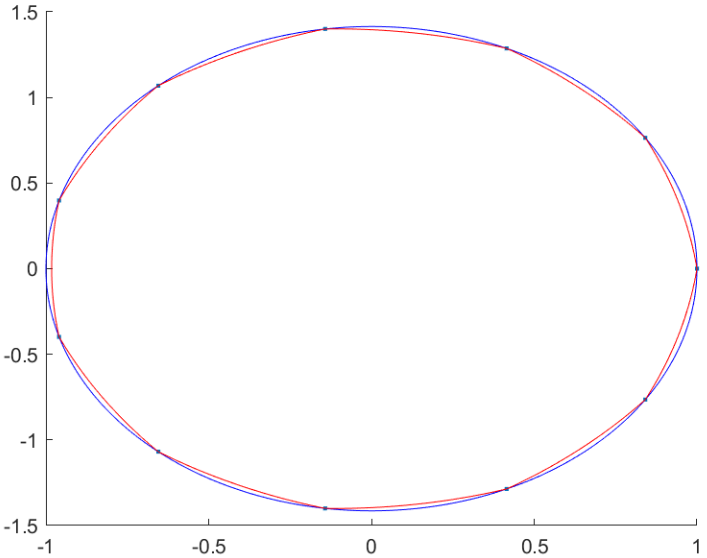

Figure 34. Interpolant for $N = 13$.

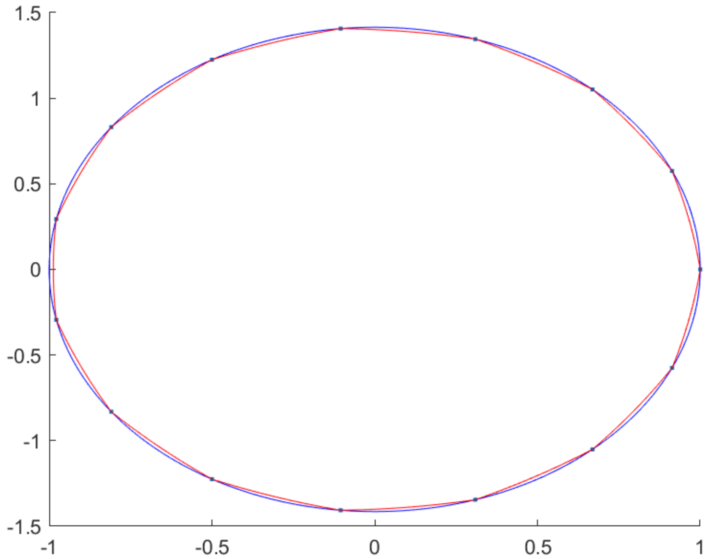

Figure 35. Interpolant for $N = 17$.

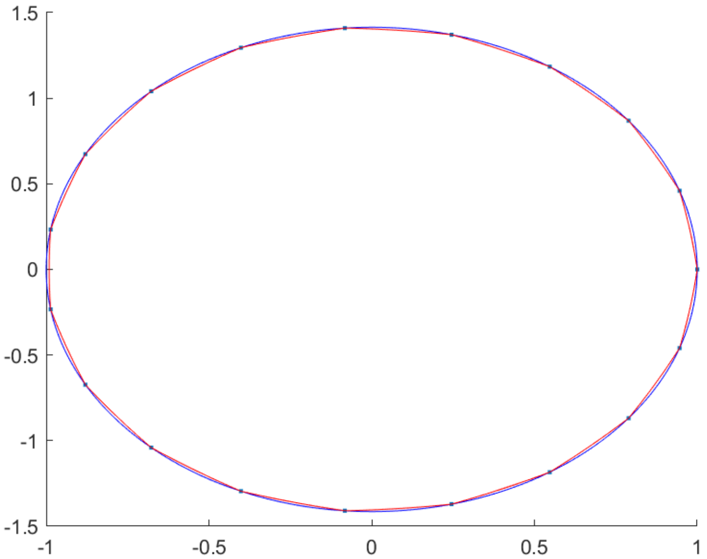

Figure 36. Interpolant for $N = 21$.

Figure 37. Cubic spline interpolant is pictured with red and the actual function with blue.

Part III
Finite difference and Padé approximation

7 Finite difference

8 Padé approximation

Part IV
Numeric integration

9 Trapezoidal Rule

10 Simpson's Rule

11 Trapezoidal Rule with End-Correction

12 Adaptive Quadrature

Part V
Numeric integration of improper integrals

13 Semi-Infinite intervals

14 Infinite intervals