MA06129, Bayesian Machine Learning
Homework 2: Theoretical Problems (20 points)

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1 Problem 1, 10 points

Here we consider the model of the probabilistic PCA. There are some references: video about Probabilistic PCA model and derivation of the EM steps for it. We consider an extension of the model making it more flexible and robust to the noise. Consider a K-mixture model of Robust PCA. For each point from the dataset $y_n \in \mathbb{R}^D$, we consider the model with the local hidden variables.

$$p(z_n) = \prod_{k=1}^{K} \pi_k^{z_{nk}}$$

$$p(u_n|z_n) = \prod_{k=1}^{K} \text{Gamma}(u_{nk}|\frac{\nu_k}{2}, \frac{\nu_k}{2})^{z_{nk}}$$

$$p(x_n|u_n, z_n) = \prod_{k=1}^{K} \mathcal{N}(x_{nk}|0, u_{nk}^{-1} I_d)^{z_{nk}}$$

$$p(y_n|x_n, u_n, z_n) = \prod_{k=1}^{K} \mathcal{N}(y_n|W_k x_{nk} + \mu_k, u_{nk}^{-1} I_D)^{z_{nk}}$$

Derive E and M steps for the dataset $\{y_n\}_{n=1}^N$.

2 Problem 2, 10 point

Consider log-concave function $\phi(x)$. Make a statement about concavity over (μ, Σ) of the function:

$$\langle \log \phi(w^T x) \rangle_{\mathcal{N}(x|\mu,\Sigma)}$$