Question 1)

One of the first option models developed was the Black-Scholes model. By making some key assumptions, the authors were able to derive a pricing model as follows:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

$$\text{Where } d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

N(x) is the cumulative distribution function, S_0 is the current spot price, K is the strike price of the option, r is the continuously compounding risk-free rate, σ is the stock price volatility, and T is the time to maturity.

A) What are the fundamental assumptions made to get to this result?
   - Do any of these assumptions cause problems in employing the BS model in the current trading environment? Explain why.

B) How do the above equations change if our asset is based on a future instead of a stock? Write out the four equations using futures without using the current stock price.

C) Assuming a fixed future price, what happens to the price of the option if the risk-free rate increases? Explain the price change in each of c, p, d_1, and d_2 above.

Please familiarize yourself with the following Greeks for both calls and puts

- Gamma
- Vega
- Theta

D) What Greek/s do you think are most important to consider when holding an option that expires
   a. In a week?
   b. In a month?
   c. In a year?

Question 2)

You were provided a set of Dec 20, 2019 ES Mini Future European Option Market Data. The data contains a strip of strikes along with the theoretical values for calls and puts. Assume today is August 1, 2019. There are 99 trading days until Dec 20, 2019 and 252 trading days throughout the year. The December ES Mini Future is trading 2900. The ES Mini Future has no dividend. The current risk-free rate is 250 basis points. Each contract ES Mini Future contract is for 100 options.

A) Using the data provided, generate an implied volatility curve **with no changes in convexity** from the Out-of-the-Money options. If an initial vol is needed, use the ATM vol as the initial guess. Explain the methodology and if possible, write the equation as a function of the strike.

B) Please describe your volatility curve. Why do you think that the option market yields this curve?

C) Based on your generated implied volatility curve, which options are the best buys? What are the best sells? Please explain your thought process in determining these best buys and best sells.

D) Looking at the data, are there any arbitrage opportunities? If so, please explain how you would capture them. If not, explain what tests you conducted to check for arbitrages.

E) If you want to create a long Vega portfolio with limited Gamma risk, how would you construct your portfolio? Choose one strike in the Dec 20, 2019 expiration to put on this position.
F) Building off of your portfolio from part E, eliminate your exposure to directional underlying moves. Use only Dec 20, 2019 calls, Dec 20, 2019 puts and Dec 20, 2019 ES futures.

Assume the trading date is now, December 1, 2019. There are 15 trading days until expiration. The vol curve for the Dec 20, 2019 expiration is identical to what you solved in part A. The ES Future is still 2900.

G) You have decided that you want to be short volatility. How do you hope that you will profit? What are your portfolio risks?
H) If you enter into a short vol position at one strike (using calls, puts or both), which strike would you choose? How much does it cost per contract to enter into your position?
I) Assuming you don't exit your position before expiration, what is your PNL if the ES Futures settles at 2500? 2800? 2900? 3000? 3300?

Question 3)

The S&P 500 is a market weighted collection of the largest 500 companies in the United States. It is used as a benchmark for investor portfolios and the US economy overall. Two assets that track the S&P 500 are SPY, a passive ETF that is approximately one-tenth the size of the S&P 500 and ticks in $0.01 increments, and ES Minis, a CME listed future that ticks in $0.25 increments. ES Minis have a 50-contract multiplier. Assume both ES Minis and SPY are extremely liquid at all price levels. The current risk-free rate is 260 basis points. The yield curve is flat.

A) As an investor with a long-time horizon, which asset do you think would be more appropriate to use in gaining exposure to large cap US stocks for your portfolio? Justify your response.
B) You are dealing with a trader who has just bought 100 30 Delta SPX Call Options expiring next month, to become delta neutral, how many shares of SPY need to be traded? How many ES Minis need to be traded? SPX options have a 100 multiplier.
C) How would you suggest that the trader hedge her deltas? Please explain your decision.
D) Currently it is October 1st, the Dec 20, 2019 ES Mini is trading 2920, what is the fair value of SPY? Where would you quote your market? Detail the steps you took to get to this fair value.
E) How much would it cost to execute your hedging trade? Assume you must execute immediately and cannot wait for markets to tick.
F) What about hedging with the other asset?

The yield curve has changed. The Dec 20, 2019 spot rate is still 260 basis points, but the Nov 15, 2019, Dec 20, 2019 forward rate is now 300 basis points. It is still October 1st, 2019.

G) What is the current Nov 15, 2019 spot rate?
H) If the ES Future remains at 2920, what is the new SPY price?
I) How much does the market expect SPY to change in value over the next day? What about the ES Mini?
J) Does this change in interest rate have any impact on which asset you use to hedge? Explain any cash flow changes in hedging with SPY or ES.
K) Which interest rate scenario is cheaper to hedge your delta assuming you enter into your one-month option contract on October 1st 2019? What about November 15st, 2019?