Завдання 1. Продиференціювати задані функції.

1. a) $y = \sqrt{x^x + 2x} - \operatorname{ctg} 3x$, б) $y = \dfrac{4 + \ln x^2}{e^{3x}}$, в) $y = 2^x \sin\left(x + \dfrac{\pi}{3}\right) - \sqrt{x + 7}$

г) $y = x^{\sin x}$, д) $x \cdot \sin y + y^2 - x = 2$.

2. a) $y = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^{10}$, б) $y = e^{2x+3} \cdot \left(x^2 - x + \dfrac{1}{2}\right)$, в) $y = \dfrac{1 + \operatorname{tg} 3x}{\cos \dfrac{\pi}{2}(x - 4)}$,

г) $y = x^{e^x}$, д) $x \cdot \ln y + x^2 + y = 0$.

3. a) $y = \sin^2 \dfrac{x}{3} \cdot \operatorname{ctg} \dfrac{x}{2}$, б) $y = 2x - \sqrt[3]{x^6 - 8}$, в) $y = \dfrac{e^{-x^2} + 4}{\ln x}$,

г) $y = (\cos x)^x$, д) $e^{xy} - x^2 + y^2 = 0$.

4. a) $y = x \cdot \operatorname{arctg}\left(5x + \dfrac{\pi}{4}\right)$, б) $y = \sqrt{3 + 2 \cdot \operatorname{tg}^2\left(x - \dfrac{\pi}{4}\right)}$, в) $y = \dfrac{1 + \sin 3x}{1 + \cos x}$,

г) $y = (\sin 3x)^{\sqrt{x}}$, д) $x \cdot y^2 - 2y + \ln x^2 = 7$.

5. a) $y = \left(\sin x + x^2\right)^4$, б) $y = 3^{4-x^2} \cdot \arcsin x - 2x$, в) $y = \dfrac{\cos^2 3x}{x^3 - 1}$,

г) $y = (\operatorname{tg} x)^{x^2}$, д) $x^3 - \operatorname{arctg} y + x \cdot y = 1$.

6. a) $y = \sqrt[4]{x - \arcsin x}$, б) $y = \ln\left(x^2 + 4\right) - x \cdot \sin^2 2x$, в) $y = \dfrac{\cos x^2 - 3x}{6^{x-1}}$,

г) $y = (x)^{2\operatorname{tg} x}$, д) $\dfrac{y}{x} - \operatorname{arctg} x + e^y = 0$.

7. a) $y = x^3 \cdot \arccos x - 7x$, б) $y = 5^x \cdot \operatorname{tg} \dfrac{x}{4} - 2$, в) $y = \dfrac{\ln(3x - 1)}{\sin 2x}$,

г) $y = (\cos x)^{x^2}$, д) $x^2 \cdot y^2 - \sin y + \sqrt{x} = 3$.

8. a) $y = \sqrt[3]{\dfrac{1 - x}{1 + x}}$, б) $y = \sin^2 x \cdot \cos x^2$, в) $y = 2^{\frac{x}{\ln x}} + 1$,

г) $y = (\arcsin x)^{\sqrt{x}}$, д) $(x + y)^2 - \operatorname{arctg} y = 7$.

9. a) $y = \dfrac{\arcsin 4x}{1 - 4x}$, б) $y = x \cdot 10^{\sqrt{x}} - 7x^2$, в) $y = \sqrt{\sin^2 x - \ln(x + 2)}$,

г) $y = \left(1 + x^2\right)^x$, д) $3\ln\left(x^2 + y^2\right) - e^{2x} = 0$.

10.

а) $y = \dfrac{1}{3}\operatorname{tg}^3 x - \operatorname{ctg} x + x$,   б) $y = e^{ax} \cdot \cos(bx + c)$,

в) $y = \dfrac{\arccos x}{\sqrt{1 - x^2}}$,   г) $y = (1 - x)^{\operatorname{arctg} x}$,   д) $6^{x+y} - \sin y^2 + \sqrt{x} = 0$.

11.

а) $y = \dfrac{\sqrt{2}}{3}\operatorname{arctg}\dfrac{x}{\sqrt{2}} + (1 - x)^3$,   б) $y = \dfrac{2x}{\operatorname{tg}^2 3x}$,

в) $y = \sin^3 x \cdot \ln x + 7x$,   г) $y = (\operatorname{ctg} x)^{\sqrt{x}}$,   д) $y^2 - x \cdot y + \ln(x + 1) = 1$.

12.

а) $y = \dfrac{m}{2}\ln\!\left(x^2 - a^2\right) + \dfrac{n}{2a}\ln\dfrac{x - a}{x + a}$,   б) $y = \sqrt{\dfrac{\cos x}{x^3 - 1}}$,

в) $y = (x + 1)\cdot \sin^2 x^2$,   г) $y = (\operatorname{tg} x)^{e^x}$,   д) $5^{xy} - x^2 + \operatorname{arctg} x = 1$.

13.

а) $y = x^n \cdot a^{-x}$,   б) $y = \arcsin\dfrac{x^2 - 4}{4} + 7x$,

в) $y = \dfrac{x + \cos^3 2x}{\operatorname{tg}(x + 1)}$,   г) $y = (x - 2)^{\cos x}$,   д) $x^2 - x \cdot y + 3^{x+y} = 2$.

14.

а) $y = \sin^2 5x \cdot \cos\dfrac{x}{3}$,   б) $y = \dfrac{4}{x - 2} + \arcsin\sqrt{x}$,   в) $y = \dfrac{3^{\operatorname{tg} x}}{1 - x^2}$,

г) $y = (\operatorname{arctg} x)^{x^3}$,   д) $\operatorname{tg}\dfrac{y}{x} - y^2 - x = 1$.

15.

а) $y = x^4 \cdot \left(a - 2x^3\right)^2$,   б) $y = e^{-2x} \cdot (3\sin x - x)$,

в) $y = \dfrac{\operatorname{ctg}^3 4x}{\cos x}$,   г) $y = (\arcsin x)^{x+3}$,   д) $y^3 - x^2 \cdot \cos y + \dfrac{4}{x} = 2^x$.

16.

а) $y = 5\arccos x + \sin^3 2x$,   б) $y = 2x - \sqrt[3]{\left(x^2 + 1\right)\cdot \operatorname{tg}\dfrac{x}{3}}$,

в) $y = \dfrac{1 - 2^x}{3 + 2^x}$,   г) $y = (\ln x)^{x^2}$,   д) $\sin(x \cdot y) - \operatorname{arctg} x^2 + 4 = 0$.

17.

в) $y = \dfrac{\operatorname{tg}^2(x + 7)}{5 + \sin 2x}$,   г) $y = x^{\ln x}$,   д) $6^{x^2+y} - \dfrac{y}{x} + x^2 = 0$.

18.

а) $y = \arcsin e^x$,   б) $y = \operatorname{tg}^4\!\left(x^2 + 1\right) - x\sqrt{1 - 2x^2}$,

в) $y = \dfrac{6^{\sin x}}{\ln x + 2}$,   г) $y = (\cos 2x)^x$,   д) $y^2 - x^3 + \operatorname{arctg} y = 2$.

19.

а) $y = \left(x^2 + 10\right)^7 + \ln 2$,   б) $y = \sqrt{x} \cdot \cos^2(3x - 1)$,

в) $y = \dfrac{\ln\!\left(x^2 + 4x - 1\right)}{x - 2}$,   г) $y = (\sin 5x)^{\operatorname{arctg} x^2}$,   д) $x^2 - 7^{x-y} + x \cdot y = 0$.

20.

а) $y = \operatorname{tg} 5x \cdot \sin^3 2x$,   б) $y = \sqrt{1 + \arcsin x}$,   в) $y = \dfrac{5\ln\!\left(x - 4x^2\right)}{3^{2x} - 4}$,

г) $y = (\cos(x + 2))^{x-1}$,   д) $\operatorname{arctg}\dfrac{y}{x} - x^2 + 2y^2 = 1$.

21.

а) $y = 2e^{-x}\left(3\sin 3x + \cos^2 x\right)$,   б) $y = \sqrt{\cos x} \cdot \ln\dfrac{x}{4}$,

в) $y = \dfrac{1}{6}\ln\dfrac{x-1}{x+1} + 4x$, г) $y = \left(x^2 + 3\right)^{\sqrt{x}}$, д) $\operatorname{arctg}\dfrac{y}{4} + \sin^3 2x + x\cdot y^2 = 0$.

22.

а) $y = \dfrac{\sqrt{2}}{3}\operatorname{arctg}\dfrac{x}{\sqrt{2}} + \dfrac{1}{6}\ln\left(x^3 + 4\right)$,   б) $y = \dfrac{\sin^3 2x}{\cos^2 5x}$,

в) $y = \dfrac{1}{2}\cdot 5^{x^2+6}\cdot \operatorname{ctg}\dfrac{x}{4}$,   г) $y = \left(\arcsin x\right)^{\ln x}$,   д) $\sqrt{x^2 + y^2} = \operatorname{tg}\dfrac{y}{x}$.

23.

а) $y = x^n \cdot a^{-x^2}$,   б) $y = \dfrac{\arccos(1-x)}{\cos x + \operatorname{tg} x}$,   в) $y = \dfrac{\arccos x^2}{\sqrt{2x - x^2}}$,

г) $y = \sqrt[x]{\operatorname{arctg} x}$,   д) $\ln y + \dfrac{x}{y} = c$.

24.

а) $y = \cos^3\dfrac{x}{4}\cdot \operatorname{tg}\left(5x + \dfrac{\pi}{3}\right)$,   б) $y = \ln(\arcsin 5x)$, в) $y = \dfrac{\sqrt[3]{1 + x^5}}{\sin 2x}$,

г) $y = (x-1)^{\operatorname{arctg} x}$,   д) $\sqrt{x} + \sqrt{y} = 2^{xy}$.

25.

а) $y = \dfrac{1}{2}\ln\left(\operatorname{tg}\dfrac{x}{2}\right) + \dfrac{x+1}{4}$,   б) $y = 6^{\sin x}\cdot \operatorname{arctg}\dfrac{x}{3}$,

в) $y = \dfrac{\cos^3(2x-1)}{x^4 - 7}$,   г) $y = (\sin 2x)^{x^2}$,   д) $x^y = y^x$.

а) $y = 0{,}4\left(\cos(2x+1) - \sin\dfrac{x}{2}\right)^2$,   б) $y = 10^{x\cdot \arcsin x}$,

26.

а) $y = 0{,}4\left(\cos(2x+1) - \sin\dfrac{x}{2}\right)^2$,   б) $y = 10^{x\cdot \arcsin x}$,

в) $y = \dfrac{\ln \operatorname{tg} x}{\ln \operatorname{ctg} x}$,   г) $y = \left(\operatorname{arctg}(x+3)\right)^{\frac{1}{x}}$,   д) $\sin(x\cdot y) + 6^y - x^4 = 0$.

27.

а) $y = \dfrac{\sqrt[9]{4x^5 + 2}}{3x^4}$,   б) $y = \dfrac{1}{18}\arcsin\dfrac{x}{4} - \ln\dfrac{x+2}{x-1}$,

в) $y = 7^{\frac{x}{3}}\cdot \cos^3\left(2x - \dfrac{\pi}{4}\right)$,   г) $y = (\operatorname{arctg} x)^{x^2+4}$,   д) $y\cdot \sin x - \cos(x - y) = 0$.

а) $y = \operatorname{ctg}\dfrac{x}{2}\cdot \sin^2\dfrac{x}{2}$,   б) $y = \dfrac{1}{4}\ln\dfrac{1-x}{1+x} + \dfrac{1}{\sqrt{2}}\operatorname{arctg}\sqrt{x}$,

28.

а) $y = \operatorname{ctg}\dfrac{x}{2}\cdot \sin^2\dfrac{x}{2}$,   б) $y = \dfrac{1}{4}\ln\dfrac{1-x}{1+x} + \dfrac{1}{\sqrt{2}}\operatorname{arctg}\sqrt{x}$,

в) $y = \dfrac{10^x}{\cos\left(3x - \dfrac{\pi}{4}\right)}$,   г) $y = \left(\operatorname{tg} x^2\right)^{\sqrt{x}}$,   д) $2y\cdot \ln y = x^2 - 1$.

29.

в)$y = \dfrac{1 - e^{2x}}{\cos\dfrac{x}{3}}$,   г)$y = (\text{ctg}\,x)^{x-2}$,   д)$3x^2 \cdot \arcsin y + 6^y - 7x^2 = 0$.

30.

а)$y = \dfrac{\text{arctg}x}{\ln^5 x}$,   б)$y = \dfrac{\sqrt{1-x^2}}{4} \cdot e^{3x-1} + \cos\left(\dfrac{x}{2} - 3\right)$,

в)$y = 4^{x^2-3} - x \cdot \sin\dfrac{x+2}{3}$,   г)$y = (\text{tg}x)^{\text{ctg}x}$, д)$\sin^3 4x - x \cdot y + \sqrt{y-1} = 0$.

Завдання 2. Знайти u', u'' при заданому значенні x чи t.

1.   а) $y = e^{x^2}x$, $x = 0$.   б)$\begin{cases} x = t + \ln\cos t \\ y = t - \ln\sin t \end{cases}$, $t = -\dfrac{\pi}{4}$.

2.   а) $y = \ln\sqrt[3]{1+x^2}$, $x = 1$.   б)$\begin{cases} x = \ln t \\ y = t^3 \end{cases}$, $t = -\dfrac{\pi}{4}$.

3.   а) $y = x^6 - 4x^3 + 4$, $x = 1$.   б)$\begin{cases} x = \text{arctg}t \\ y = \ln(1+t^2) \end{cases}$ $t = 1$.

4.   а) $y = 1 - x^2 - x^4$, $x = 2$.   б)$\begin{cases} x = \arcsin t \\ y = \sqrt{1-t^2} \end{cases}$, $t = 0$.

5.   а) $y = (x^2 + 3)^3$, $x = 1$.   б)$\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$, $t = \dfrac{\pi}{2}$.

6.   а) $y = x^3 \ln x$, $x = 1$.   б)$\begin{cases} x = a \cdot \cos^3 t \\ y = a \cdot \sin^3 t \end{cases}$, $t = \dfrac{\pi}{4}$.

7.   а) $y = \dfrac{1}{1-x}$, $x = -1$.   б)$\begin{cases} x = a(\sin t - t\cos t) \\ y = a(\cos t + t\sin t) \end{cases}$, $t = \dfrac{\pi}{2}$.

8.   а)$y = \text{arctg}\,x$, $x = 2$. б)$\begin{cases} x = \cos 2t \\ y = \sin^2 t \end{cases}$, $t = \dfrac{\pi}{8}$.

9.   а) $y = e^{\sqrt{x}}x$, $x = 2$.   б)$\begin{cases} x = \text{arctg}t \\ y = \dfrac{t^2}{2} \end{cases}$, $t = 1$.

10.   а) $y = \ln(\ln x)$, $x = e$.   б)$\begin{cases} x = e^{-at} \\ y = e^{at} \end{cases}$, $t = 0$.

11.   а) $y = (1 - x^2)\cos x$, $x = 0$.   б)$\begin{cases} x = \ln t \\ y = \dfrac{1}{1-t} \end{cases}$, $t = \dfrac{1}{2}$.

12.   а) $y = \dfrac{1+x}{\sqrt{x}}$, $x = 1$.   б)$\begin{cases} x = e^t \cdot \cos t \\ y = e^t \cdot \sin t \end{cases}$, $t = \dfrac{\pi}{2}$.

13.   а) $y = \dfrac{x^2+1}{x-1}$, $x = \dfrac{1}{2}$.   б)$\begin{cases} x = \arcsin(t^2 - 1) \\ y = \arccos 2t \end{cases}$, $t = \dfrac{1}{2}$.

14. a) $y = x^3 + 2x + 6$, $x = 3$. б) $\begin{cases} x = \operatorname{ctg} t \\ y = \dfrac{1}{\cos^3 t} \end{cases}$, $t = \dfrac{\pi}{4}$.

15. a) $y = \sqrt[3]{(1-x)^2}$, $x = -1$. б) $\begin{cases} x = 2t - \sin 2t \\ y = \sin^3 t \end{cases}$, $t = \dfrac{\pi}{4}$.

16. a) $y = \arcsin x$, $x = \dfrac{1}{4}$. б) $\begin{cases} x = 2\cos^3 2t \\ y = \sin^3 2t \end{cases}$, $t = \dfrac{\pi}{8}$.

17. a) $y = \sin ax + \cos bx$, $x = 0$. б) $\begin{cases} x = t^5 \\ y = t^3 + 8t - 1 \end{cases}$, $t = 1$.

18. a) $y = \ln \operatorname{ctg} 4x$, $x = \dfrac{\pi}{8}$. б) $\begin{cases} x = 2t - \dfrac{t^3}{3} \\ y = t^4 + 6t - 1 \end{cases}$, $t = 2$.

19. a) $y = x^2 \cdot \sin 2x$, $x = \dfrac{\pi}{4}$. б) $\begin{cases} x = \dfrac{1}{\cos t} \\ y = \operatorname{tg} t \end{cases}$, $t = \dfrac{\pi}{6}$.

20. a) $y = x \cdot e^{x^2}$, $x = -1$. б) $\begin{cases} x = a\cos^2 t \\ y = a\sin^2 t \end{cases}$, $t = \dfrac{\pi}{4}$.

21. a) $y = \sqrt{a^2 - x^2}$, $x = \dfrac{a}{2}$. б) $\begin{cases} x = \ln(1 + t^2) \\ y = t^2 \end{cases}$, $t = 0$.

22. a) $y = \dfrac{1}{a + \sqrt{x}}$, $x = a$. б) $\begin{cases} x = \operatorname{arctg} t \\ y = \cos^2 t \end{cases}$, $t = \dfrac{\pi}{4}$.

23. a) $y = \ln\left(x + \sqrt{1 + x^2}\right)$, $x = 1$. б) $\begin{cases} x = t^3 + 4t - 1 \\ y = t^6 - 2t \end{cases}$, $t = 0$.

24. a) $y = \dfrac{1}{x^2 - 3x + 2}$, $x = 0$. б) $\begin{cases} x = \ln(t^3 + 1) \\ y = e^{2t} \end{cases}$, $t = 1$.

25. a) $y = x^3 \sin^2 x$, $x = \dfrac{\pi}{2}$. б) $\begin{cases} x = t \cdot 2^t \\ y = \ln(1 + t^2) \end{cases}$, $t = 0$.

26. a) $y = 5x^7 - 4x^2 + 7x$, $x = 1$. б) $\begin{cases} x = \sin^3 2t \\ y = \ln(1 + t) \end{cases}$, $t = \dfrac{\pi}{8}$.

27. a) $y = \sqrt{2x^2 + 4x}$, $x = 1$. б) $\begin{cases} x = e^{-2t} \\ y = t^3 + 1 \end{cases}$, $t = 0$.

28. a) $y = x \cdot 3^{x+1}$, $x = 1$. б) $\begin{cases} x = \dfrac{2 - t}{2 + t^2} \\ y = \dfrac{t^2}{2 + t^2} \end{cases}$, $t = 0$.

29. а) $y = \ln \operatorname{tg} \dfrac{x}{3},\ x = \pi.$ б) $\begin{cases} x = t^2 + t + 1 \\ y = t^3 + t \end{cases},\ t = 1.$

30. а) $y = 6x^5 - 2x^3 - 3x + 4,\ x = 0.$ б) $\begin{cases} x = e^{-3t} \\ y = \sin t \ln\left(1 + t^2\right) \end{cases} t = -\dfrac{\pi}{4}.$

Завдання 3. Знайти границі функцій, використовуючи правило Лопіталя:

1. 1) $\displaystyle\lim_{x\to 0} \frac{\sqrt[3]{1-6x}-1+2x}{\sin^2 x}$;   2) $\displaystyle\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{x}{\ln x}\right)$;   3) $\displaystyle\lim_{x\to 0}\left(2 - 4^{\sin^2 x}\right)^{\frac{1}{\ln\cos x}}$.

2. 1) $\displaystyle\lim_{x\to 0} \frac{x\cdot \operatorname{arctg} x}{\ln\cos x}$;   2) $\displaystyle\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$;   3) $\displaystyle\lim_{x\to 0}\left(2e^x - 1\right)^{\frac{x-2}{x}}$.

3. 1) $\displaystyle\lim_{x\to 0} \frac{\cos 3x - e^{x^2}}{\operatorname{arctg}^2 5x}$;   2) $\displaystyle\lim_{x\to 0}\left(\operatorname{ctg} x - \frac{1}{x}\right)$;   3) $\displaystyle\lim_{x\to 0+0}\left(\cos\sqrt{x}\right)^{\frac{2}{x}}$.

4. 1) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x^2}-\cos 4x}{\ln^2(1+x)}$;   2) $\displaystyle\lim_{x\to 0+0} \frac{\operatorname{ctg} 2x}{\ln x}$;   3) $\displaystyle\lim_{x\to 0}\left(\frac{1}{1+\sin x}\right)^{\frac{3}{x}}$.

5. 1) $\displaystyle\lim_{x\to 0} \frac{e^x - x - \cos 2x}{\sin\left(x^2\right)}$;   2) $\displaystyle\lim_{x\to +\infty} \frac{e^{2x}}{x^4}$;   3) $\displaystyle\lim_{x\to 0}\left(\cos x\right)^{\frac{5}{\operatorname{tg}^2 5x}}$.

6. 1) $\displaystyle\lim_{x\to 0} \frac{\ln(1-3x)+3x}{\arcsin^2 5x}$;   2) $\displaystyle\lim_{x\to \frac{\pi}{2}}(\pi - 2x)\operatorname{tg} x$;   3) $\displaystyle\lim_{x\to 0}\left(2 - \cos x\right)^{\frac{-2}{x^2}}$.

7. 1) $\displaystyle\lim_{x\to 0} \frac{x^2}{e^{2x}-\sqrt{1+2x}}$;   2) $\displaystyle\lim_{x\to 0+0} x\ln x$;   3) $\displaystyle\lim_{x\to 0}\left(\cos\left(x^2\right)\right)^{\frac{1}{\sin^4 x}}$.

8. 1) $\displaystyle\lim_{x\to 0} \frac{\sqrt[3]{1+6x}-e^{2x}}{\sin^2 2x}$;   2) $\displaystyle\lim_{x\to 0+0} \frac{\ln \operatorname{tg} x}{\ln \operatorname{tg} 2x}$;   3) $\displaystyle\lim_{x\to 0}\left(5 - \frac{4}{\cos x}\right)^{\operatorname{ctg}^2 x}$.

9. 1) $\displaystyle\lim_{x\to 0} \frac{e^{-x} - \cos 2x}{x^2}$;   2) $\displaystyle\lim_{x\to 0} \frac{x - \operatorname{arctg} 2x}{x^3}$;   3) $\displaystyle\lim_{x\to 0}\left(1 - 5x\right)^{\frac{1}{\ln(1+2x)}}$.

10. 1) $\displaystyle\lim_{x\to 0} \frac{x^2}{\ln(1+5x)-5x}$;   2) $\displaystyle\lim_{x\to 1}(1-x)\operatorname{tg} \frac{\pi x}{2}$;   3) $\displaystyle\lim_{x\to 0}\left(1 + \sin 2x\right)^{\frac{-1}{\operatorname{tg} x}}$.

11. 1) $\displaystyle\lim_{x\to +\infty} \frac{\pi - 2\operatorname{arctg} 2x}{e^{\frac{3}{x}}-1}$;   2) $\displaystyle\lim_{x\to 4+0} \frac{\ln(x-4)}{\ln(e^x - e^4)}$;   3) $\displaystyle\lim_{x\to 0}(e^x + x)^{\frac{2}{x}}$.

12. 1) $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{\cos x-1}$;   2) $\lim\limits_{x\to 0}\arcsin 2x\cdot ctg\,x$;   3) $\lim\limits_{x\to 1}(3-2x)^{tg\frac{\pi x}{2}}$.

13. 1) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\dfrac{1}{\cos^2 x}-2tgx}{1+\cos 4x}$;   2) $\lim\limits_{x\to 1}\dfrac{\ln(x-1)}{ctg\pi x}$;   3) $\lim\limits_{x\to\frac{\pi}{2}}(\pi-2x)^{\cos x}$.

14. 1) $\lim\limits_{x\to 0}\dfrac{x\cos x-\sin x}{x^3}$;   2) $\lim\limits_{x\to 0+0}\dfrac{\ln x}{1+2\ln\sin x}$;   3) $\lim\limits_{x\to 0}(\cos 2x)^{\frac{3}{x^2}}$.

15. 1) $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$;   2) $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2}-\dfrac{1}{\sin x}\right)$;   3) $\lim\limits_{x\to\infty}(x+2^x)^{\frac{1}{x}}$.

16. 1) $\lim\limits_{x\to 0}\dfrac{e^x-2^x}{x\sqrt{1-x^2}}$;   2) $\lim\limits_{x\to 1-0}\dfrac{e^{\frac{1}{1-x}}}{\ln(1-x)}$;   3) $\lim\limits_{x\to 0}(tg\,2x)^{\sin x}$.

17. 1) $\lim\limits_{x\to 0}\dfrac{\ln\left(1+x^2\right)}{\cos 3x-e^{-x}}$;   2) $\lim\limits_{x\to 3}\dfrac{\ln(2x-5)}{e^{\sin\pi x}-1}$;   3) $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{6tg3x}$.

18. 1) $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{1-2\sin x}{\cos 3x}$;   2) $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{x}{\ln x}\right)$;   3) $\lim\limits_{x\to\frac{\pi}{2}}(1+\cos 3x)^{\frac{1}{\cos x}}$.

19. 1) $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{\cos x-1}$;   2) $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x}-\dfrac{1}{x\ln x}\right)$;   3) $\lim\limits_{x\to 2}\left(2-\dfrac{x}{2}\right)^{tg\frac{\pi x}{4}}$.

20. 1) $\lim\limits_{x\to 0}\dfrac{e^x-e^{\sin x}}{x^3}$;   2) $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[4]{x}-1}$;   3) $\lim\limits_{x\to 0}(ctg\,2x)^{\frac{1}{\ln x}}$.

21. 1) $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{2-2e^{-4x}}$;   2) $\lim\limits_{x\to+\infty}\dfrac{\ln(x+7)}{\sqrt[7]{x-3}}$;   3) $\lim\limits_{x\to\infty}x^{\frac{6}{1+2\ln x}}$.

22. 1) $\lim\limits_{x\to+\infty}\dfrac{e^{\frac{2}{x^2}}-1}{\pi-2arctg\,x}$;   2) $\lim\limits_{x\to 0}\left(\dfrac{1}{tg\,2x}-\dfrac{1}{x}\right)$;   3) $\lim\limits_{x\to 0}(\cos 4x)^{\frac{2}{x^2}}$.

23. 1) $\lim\limits_{x\to 0+0}\dfrac{e^{\sqrt{x}}-1}{\sqrt{\sin x}}$;   2) $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\ln\sin 3x}{(6x-\pi)^2}$;   3) $\lim\limits_{x\to 0}(\cos\pi x)^{\frac{1}{\sin^2\pi x}}$.

24. 1) $\lim\limits_{x\to\infty} \dfrac{e^{\frac{4}{x^2}}-1}{2\,arctg\left(x^2\right)-\pi}$; 2) $\lim\limits_{x\to3}\left(\dfrac{1}{x-3}-\dfrac{6}{x^2-9}\right)$; 3) $\lim\limits_{x\to0+0}\left(-\ln x\right)^x$.

25. 1) $\lim\limits_{x\to1}\dfrac{\ln x}{1-x^3}$; 2) $\lim\limits_{x\to\frac{\pi}{4}}\left[tg\,2x\cdot ctg\left(\dfrac{\pi}{4}+x\right)\right]$; 3) $\lim\limits_{x\to\frac{\pi}{2}}\left(\cos x\right)^{\frac{\pi}{2}-x}$.

26. 1) $\lim\limits_{x\to0}\dfrac{e^{2x}-e^{-x}}{\sin 2x}$; 2) $\lim\limits_{x\to0}\left(1-\cos 2x\right)\cdot ctg\,4x$; 3) $\lim\limits_{x\to1}\left(1-x\right)^{\ln x}$.

27. 1) $\lim\limits_{x\to0}\dfrac{\ln\cos x}{x}$; 2) $\lim\limits_{x\to0}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$; 3) $\lim\limits_{x\to\infty}\left(x^2+3\right)^{\frac{1}{x}}$.

28. 1) $\lim\limits_{x\to0}\dfrac{e^{2x}-\cos x}{e^{-x}-\cos 2x}$; 2) $\lim\limits_{x\to0,5}\ln 2x\cdot\ln\left(2x-1\right)$; 3) $\lim\limits_{x\to0+0} x^{\frac{1}{\ln\left(e^x-1\right)}}$.

29. 1) $\lim\limits_{x\to0}\dfrac{\pi\cdot tg\left(2x\right)}{\ln\left(1-\pi x\right)}$; 2) $\lim\limits_{x\to+\infty}\dfrac{\ln 3x}{\sqrt[3]{x}}$; 3) $\lim\limits_{x\to1}\left(tg\dfrac{\pi x}{4}\right)^{tg\frac{\pi x}{2}}$.

30. 1) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sqrt[3]{tg\,x}-1}{2\sin^2 x-1}$; 2) $\lim\limits_{x\to0+0}\dfrac{\ln\left(\sin 4x\right)}{\ln\left(\sin x\right)}$; 3) $\lim\limits_{x\to+\infty} x^{\frac{2}{1+2\ln x}}$.

Завдання 4. Провести повне дослідження функцій та їх побудувати графіки

1. 1) $y=\dfrac{(x+1)^2}{x-2}$; 2) $y=\ln(x^2+2x+2)$.

2. 1) $y=\dfrac{x^3+4}{x^2}$; 2) $y=\dfrac{e^{-x-1}}{x+1}$.

3. 1) $y=\dfrac{x-1}{x^2-4}$; 2) $y=xe^{-\frac{x^2}{2}}$.

4. 1) $y=x+\dfrac{2x}{x^2-1}$; 2) $y=x-\ln(1+x^2)$.

5. 1) $y=\dfrac{2}{x^2+2x}$; 2) $y=(2-x)\cdot e^{x-1}$.

6. 1) $y=\left(\dfrac{x-1}{x}\right)^2$; 2) $y=x^2 e^{\frac{1}{x}}$.

7. 1) $y=\dfrac{1-x^3}{x^2}$; 2) $y=\ln\dfrac{x-1}{x+2}$.

8. 1) $y = \dfrac{x}{(x-1)^2}$; 2) $y = 2\ln\dfrac{x+2}{x-1} - 1$.

9. 1) $y = \dfrac{x-1}{x^2+2x}$; 2) $y = \ln(x^2 - 4x + 8)$.

10. 1) $y = \dfrac{x^4}{x^3-1}$; 2) $y = x^2 e^x$.

11. 1) $y = -\left(\dfrac{x}{x+2}\right)^2$; 2) $y = \dfrac{e^{2x}}{2x}$.

12. 1) $y = \left(\dfrac{x+2}{x-1}\right)^2$; 2) $y = x^2 - 2\ln x$.

13. 1) $y = \dfrac{2x-1}{(x-1)^2}$; 2) $y = \dfrac{2\ln x}{x}$.

14. 1) $y = \dfrac{4(x+1)^2}{x^2-4x+4}$; 2) $y = 2 - 2\ln\dfrac{x}{x+4}$.

15. 1) $y = \dfrac{x^3+1}{x^2}$; 2) $y = e^{6x-x^2}$.

16. 1) $y = \left(\dfrac{x+1}{x-1}\right)^2$; 2) $y = x^{\frac{3}{2}} e^{-x}$.

17. 1) $y = \dfrac{1}{1-x^4}$; 2) $y = (x-1)\cdot e^{2-x}$.

18. 1) $y = \dfrac{x^3}{x^2-x+1}$; 2) $y = \ln\dfrac{x+6}{x}$.

19. 1) $y = \dfrac{x^3-1}{4x^2}$; 2) $y = e^{3x-x^2}$.

20. 1) $y = \dfrac{x^4}{x^3-1}$; 2) $y = \dfrac{x}{e^{x^2}}$.

21. 1) $y = \dfrac{x^2+6}{x^2+1}$; 2) $y = \dfrac{1}{3}\sqrt[3]{x^2}\,(x-5)$.

22. 1) $y = \dfrac{(1-x)^3}{(x-2)^2}$; 2) $y = \sqrt{x^2 - 4x + 5}$

23. 1) $y = \dfrac{x^2-1}{x^2+1}$; 2) $y = -(x+2)\cdot e^{x+3}$.

24. 1) $y = \dfrac{4x}{1+x^2}$; 2) $y = x\ln|x|$.

25. 1) $y = \dfrac{x^3}{(x-2)(x+3)}$; 2) $y = \dfrac{2^x}{x}$.

26. 1) $y = \dfrac{x-1}{x^2}$; 2) $y = \ln\left(x^2 + 1\right)$.

27. 1) $y = \dfrac{x^3 - 1}{4x^2}$; 2) $y = x\ln^2 x$.

28. 1) $y = 2x + 3\sqrt[3]{(2-x)^2}$; 2) $y = \dfrac{e^x}{x+3}$.

29. 1) $y = \dfrac{x^3}{x^2 - 1}$; 2) $y = -\ln\dfrac{1+x}{1-x}$.

30. 1) $y = \dfrac{4 - 2x}{x^2 - 1}$; 2) $y = x^{\frac{2}{3}}e^{-\frac{x^2}{3}}$.

Завдання 5. Знайти найменше та найбільше значення функції на відрізку:

1. $y = x^2 + \dfrac{16}{x} - 16$, $[1,4]$.   2. $y = x - 4\sqrt{x} + 5$, $[0,4]$.

3. $y = 3 - x - \dfrac{4}{(x+2)^2}$, $[-1,2]$.   4. $y = \ln(x^2 - 2x + 2)$, $[0,3]$.

5. $y = x\ln x$, $[-1,2]$.   6. $y = x^3 e^{x+1}$, $[-4,0]$.

7. $y = \dfrac{3x}{x^2 + 1}$, $[0,5]$.   8. $y = \dfrac{2x - 1}{(x-1)^2}$, $[-0,5;0]$.

9. $y = \dfrac{x^5 - 8}{x^4}$, $[-3,-1]$.   10. $y = x^5 - 5x^4 + 5x^3 + 1$, $[-1,2]$.

11. $y = (3 - x)e^{-x}$, $[0,5]$.   12. $y = 108x - x^4$, $[-1,4]$.

13. $y = \dfrac{4x}{4 + x^2}$, $[-4,2]$.   14. $y = -\dfrac{2(x^2 + 3)}{x^2 + 2x + 5}$, $[-5,1]$.

15. $y = \dfrac{4}{x^2} - 8x - 15$, $[-2;-0,5]$.   16. $y = 8x + \dfrac{4}{x^2} - 15$, $[0,5;2]$.

17. $y = \dfrac{2(x^2 + 3)}{x^2 - 2x + 5}$, $[-3,3]$.   18. $y = \dfrac{10(x + 1)}{x^2 + 2x + 2}$, $[-1,2]$.

19. $y = 2x^2 + \dfrac{108}{x} - 59$, $[2,4]$.   20. $y = \dfrac{10x}{1 + x^2}$, $[0,3]$.

21. $y = 4 - x - \dfrac{4}{x^2}$, $[1,4]$.   22. $y = 3 - x - \dfrac{4}{(x+2)^2}$, $[-1,2]$.

23. $y = (x+2)e^{1-x}$, $[-2,2]$.   24. $y = \ln(x^2 - 2x + 4)$, $[-1;1,5]$.

25. $y = \dfrac{x^3}{x^2 - x + 1}$, $[-1,1]$.   26. $y = \sqrt{x - x^3}$, $[-2,2]$.

27. $y = \dfrac{x^3 + 4}{x^2}$, $[1,2]$.   28. $y = \dfrac{x}{9 - x^2}$, $[-2,2]$.

29. $y = e^{4x - x^2}$, $[1,3]$.   30. $y = \dfrac{x^5 - 8}{x^4}$, $[-3,-1]$.