

Самостійна робота. Виконав студент групи ВТ-21-1(1) – Бабушко Андрій.

Тема: Невизначений інтеграл.

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Варіант. 1.

№1.1. $\int \frac{(\sqrt{x}-1)^3}{x} dx \quad \textcircled{=}$

$\textcircled{=}$ $\begin{cases} t = \sqrt{x} - 1 \\ dt = \frac{dx}{2\sqrt{x}} \end{cases} = \int t^3 dt = \frac{t^4}{4} + C = \frac{(\sqrt{x}-1)^4}{4} + C$

№2.1. $\int \frac{dx}{x(1+\ln x)} = \int \frac{dx}{x + \ln x^2} = \int \frac{dx}{x} + \int \frac{dx}{\ln x^2}$

$= \ln|x| + \int \frac{t = \ln x}{dt = \frac{dx}{x}} + C = \ln|x| + \int \frac{dt}{t^2} \quad \textcircled{=}$

$\int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-1}}{-1} + C$

$\textcircled{=} \ln|x| - \frac{1}{\ln x} + C$

№3.1. $\int \frac{dx}{\sqrt{1-2x-x^2}} = \int \frac{dx}{\sqrt{(x+1)^2-2}} \quad \textcircled{=}$

$\sqrt{x^2+2x-1} = \sqrt{(x^2+2 \cdot x \cdot 1+1)-2} = \sqrt{[(x+1)^2-2]}$

$\textcircled{=} \sqrt{2-(x+1)^2}$

$\textcircled{=} \int \frac{dx}{\sqrt{2-(x+1)^2}} = \left[\begin{matrix} dt = \frac{dx}{x} \\ t = x+1 \end{matrix} \right] = \int \frac{dt}{\sqrt{2-t^2}} =$

$= \arcsin \frac{t}{\sqrt{2}} + C = \arcsin \frac{x+1}{\sqrt{2}} + C =$

$= \arcsin \frac{\sqrt{2}x + \sqrt{2}}{2} + C.$

$$N.4.1. \int x^2 \cos x dx = \begin{cases} u = x^2 & du = 2x dx \\ dv = \cos x dx & v = \sin x \end{cases}$$

$$= x^2 \cdot \sin x - \int \sin x \cdot 2x dx = x^2 \sin x - 2 \int x \sin x dx = x^2 \sin x - 2(x \cdot (-\cos x) - \int -\cos x dx) = x^2 \sin x - 2(x \cdot (-\cos x) + \int \cos x dx) = x^2 \sin x + 2x \cdot \cos x - 2 \sin x + C$$

$$N.5.1. \int \cos^3 x \sin^2 x dx = \begin{cases} t = \sin x & \textcircled{2} \\ dt = \cos x dx \end{cases}$$

$$\textcircled{2} \int t^2 - t^4 dt = \int t^2 dt - \int t^4 dt = \frac{t^3}{3} - \frac{t^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$N.6.1. \int \frac{3x^2 + 2x + 3}{x^3 - x} dx = \int \frac{3}{x} + \frac{1}{x-1} - \frac{1}{x+1}$$

$$dx = \int \frac{3}{x} dx + \int \frac{1}{x-1} dx - \int \frac{1}{x+1} dx =$$

$$= 3 \ln |x| + \ln |x-1| - \ln |x+1| + C$$

$$\frac{3x^2 + 2x - 3}{x^3 - x} = \frac{3x^2 + 2x - 3}{x(x^2 - 1)} = \frac{3x^2 + 2x - 3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$$

$$3x^2 + 2x - 3 = Ax^2 - \underline{A} + Bx^2 + Bx + Cx^2 - \underline{Cx} =$$

$$3x^2 = x^2(A+B+C) + x(B-C) - A$$

$$\begin{cases} -3 = -A \\ 2 = B - C \\ 3 = A + B + C \end{cases} \Rightarrow \begin{cases} A + B + C = 3 \cdot (-1) \\ B - C = 2 \\ A = 3 \end{cases} \Rightarrow \begin{cases} A + B + C = 3 \\ B - C = 2 \\ A = 3 \end{cases}$$

$$\Rightarrow \begin{cases} A + B + C = 3 \\ C + C = 2 \\ A = 3 \end{cases}$$

$$\begin{cases} A + B + C = 3 \\ C = -1 \Rightarrow \\ A = 3 \end{cases}$$

$$\Rightarrow 3 + B + (-1) = 3 \quad B = 1$$

$$A = 3 \quad B = 1 \quad C = -1$$

$$\frac{3}{x} + \frac{1}{x-1} - \frac{1}{x+1}$$