

Looking for the Fifth Force Using Galaxy Clusters

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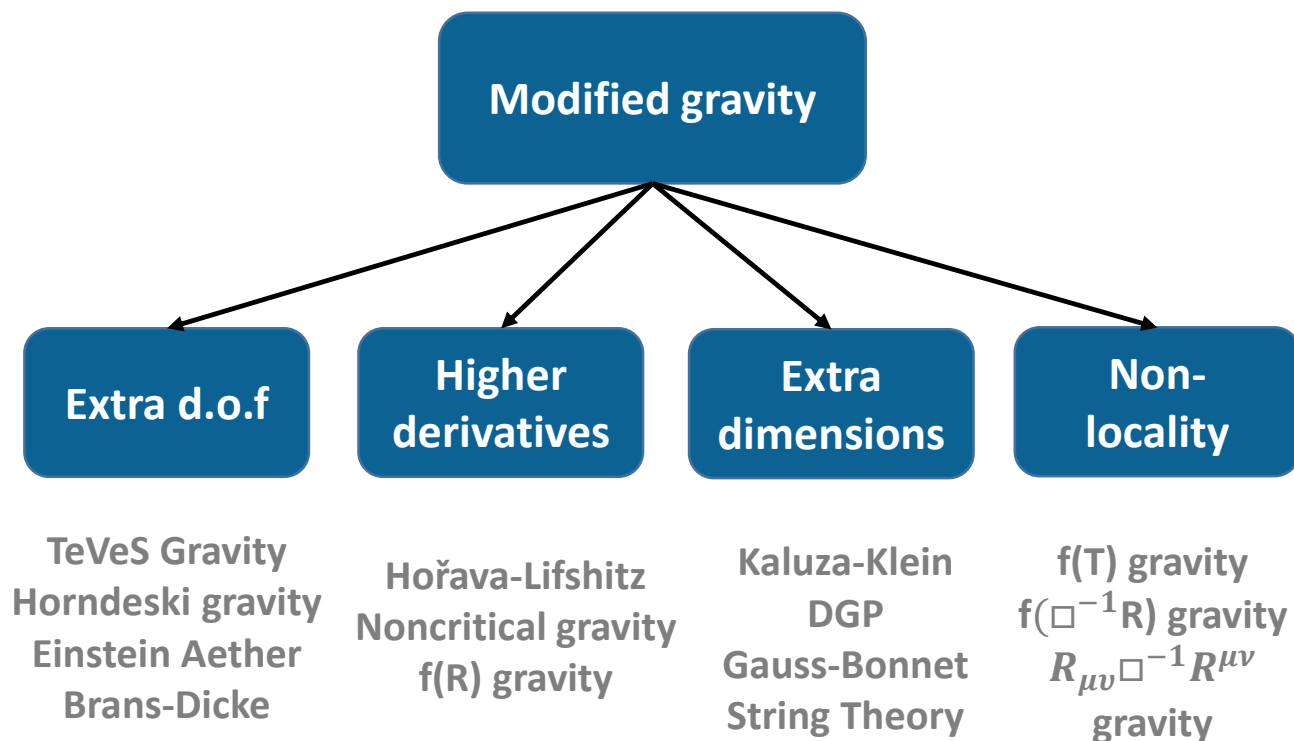
*Veiled Chameleon
(Chamaeleo calyptratus)*





Modified Gravity

- The case for modified gravity:
 - Dark matter,
 - Dark energy,
 - Quantum gravity,
 - “What I cannot create, I cannot understand”.
- A wide variety of models:
 - Different model families break different **assumptions** in GR,
 - Interesting phenomenology on **different scales**.





Chameleon Gravity

- Chameleon gravity: **scalar-tensor** theory with non-minimal coupling:
- Varying the action gives the **non-linear** equations of motion:
- Solving the EOM gives the **fifth force**:
- The challenge: solving the EOM for **astrophysical** density distributions.

$$S = \int dx^4 \sqrt{-g} \left(\underbrace{\frac{M_{\text{Pl}}^2}{2} R}_{\text{General relativity}} - \underbrace{\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi)}_{\text{Scalar field}} \right) - \int dx^4 \underbrace{\mathcal{L}_m(\varphi_m^{(i)}, \tilde{g}_{\mu\nu}^{(i)})}_{\text{Matter Lagrangian}}$$

$$\square \phi = V_{,\phi} - \sum_i \frac{\beta_i}{M_{\text{pl}}} T_{\mu\nu}^{(i)} \tilde{g}_{(i)}^{\mu\nu}$$

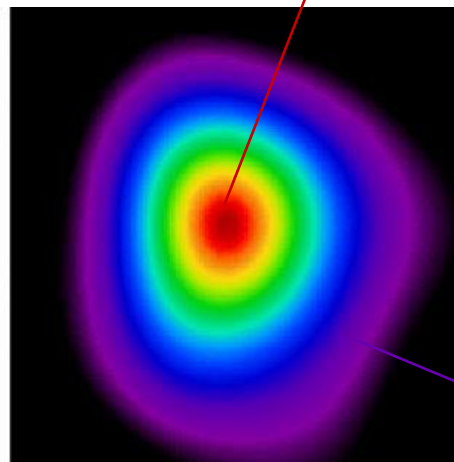
$$\nabla^2 \phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{\text{Pl}}}$$

$$F_\phi = -\frac{m\beta}{M_{\text{pl}}} \nabla \phi$$

Chameleon Screening

Effective potential:
$$V_{\text{eff}}(\phi) = V(\phi) + \sum_i \frac{\beta_i \rho_i}{M_{\text{Pl}}} \phi$$

Chameleon screened
(behaves like GR)

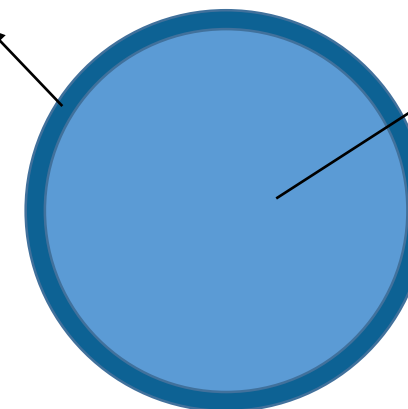


Abell 1835 ([Mantz et al. 2010a](#))

Chameleon unscreened
(extra fifth force)

The thin-shell effect:

Chameleon
unscreened



Chameleon
screened

Part 2: The Finite-Element Approach

Panther Chameleon
(*Furcifer pardalis*)



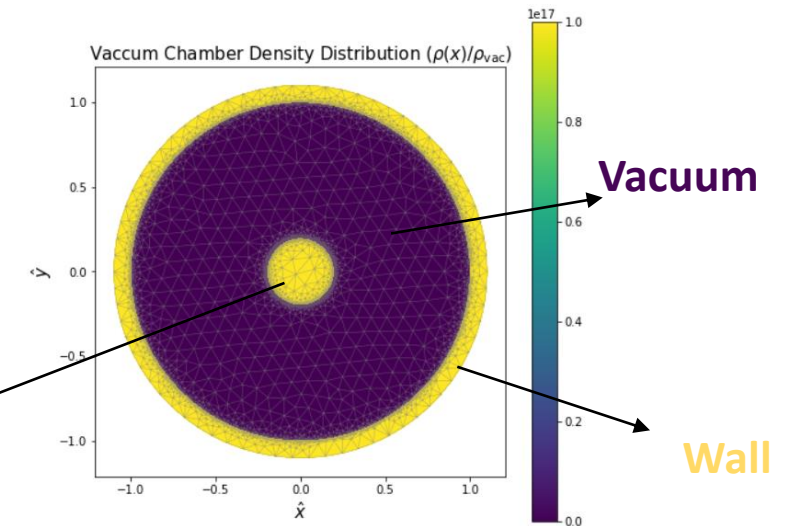


The Finite Element Approach

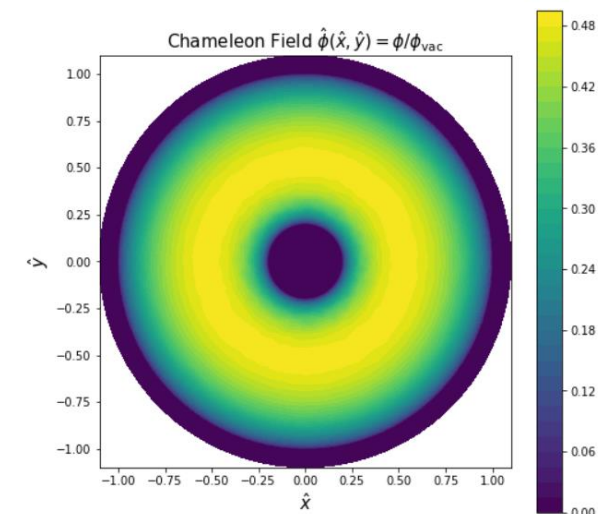
- A numerical method to solve **linear** and **non-linear** equations in 1D, 2D and 3D.
- The problem domain is discretize into *finite elements*.
- The equations are rewritten into an integral form via Green's theorem.
- Widely used in engineering, fluid dynamics, structural analysis etc.

An example of applying the FEM approach:

Source



The resulting field:





The Finite Element Approach

- Rewriting the Chameleon EOM:
 - Use **Green's function** (with v_j as the *test* function,
 - Rescale the EOM by the field **vacuum value** and the domain size,
 - **Taylor expand** the non-linear term,
 - Solve the resulting equation using the **Picard/Newton** iteration methods.

$$\int_{\Omega} (\nabla^2 \phi) v_j dx + \int_{\Omega} \nabla \phi \cdot \nabla v_j dx = \int_{\partial\Omega} (\partial_n \phi) v_j dx$$

$$\nabla^2 \phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{pl}} \rightarrow \alpha \hat{\nabla}^2 \hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho}$$

$$\alpha \equiv \left(\frac{M\Lambda}{L^2 \rho_{\infty}} \right) \left(\frac{nM\Lambda^3}{\rho_{\infty}} \right)^{\frac{1}{n+1}}$$

$$\hat{\phi}^{-(n+1)} \approx (n+2)\hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} + O\left(\hat{\phi} - \hat{\phi}_k\right)^2$$

$$\alpha \int_{\Omega} \hat{\nabla} \hat{\phi} \cdot \hat{\nabla} v_j dx + \int_{\Omega} (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} v_j dx = \int_{\Omega} (n+2)\hat{\phi}_k^{-(n+1)} v_j dx - \int_{\Omega} \hat{\rho} v_j dx$$

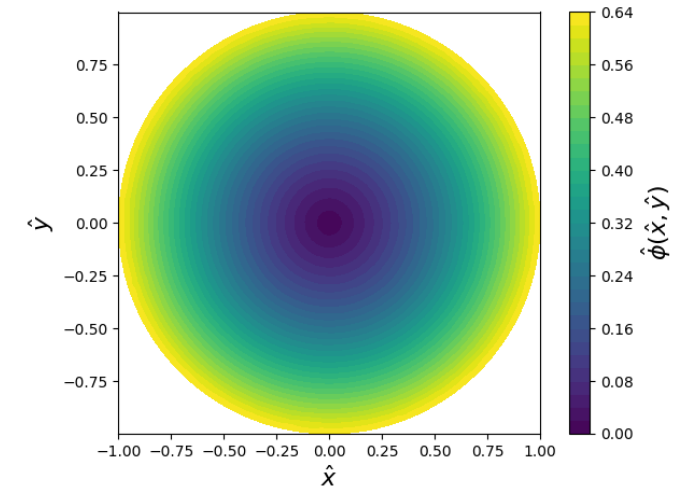
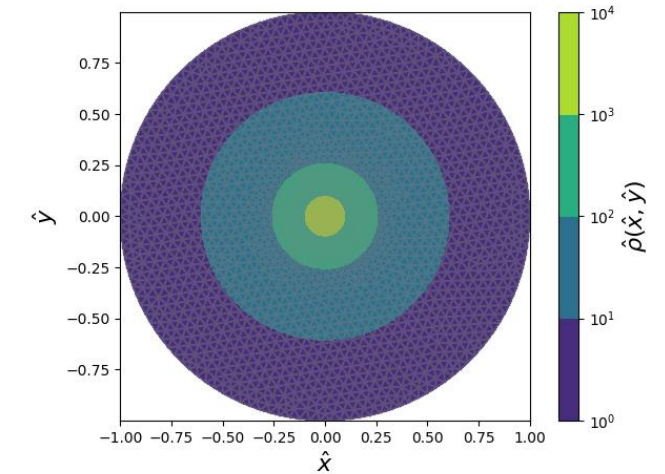
The Finite Element Approach: FEniCS



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- *Selkie* (to be released soon) – a Python code based on the *FEniCS* software library.
- *Selkie* allows solving non-linear equations in 1D, 2D and 3D.
- Chameleon equations can be solved for various systems:
 - Vacuum chamber,
 - Galaxy clusters,
 - Cosmic voids.

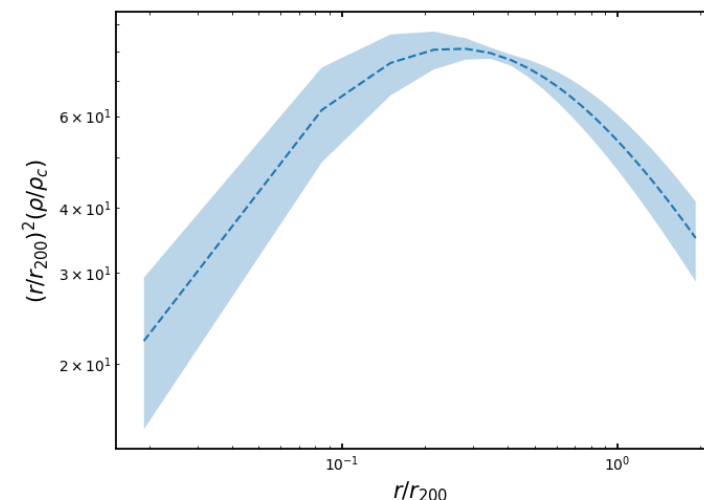
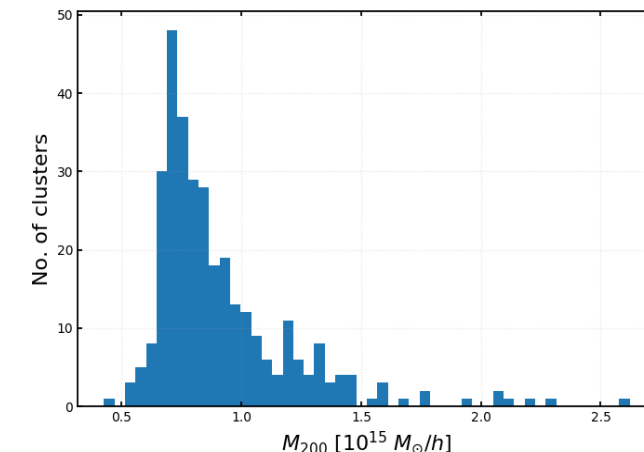
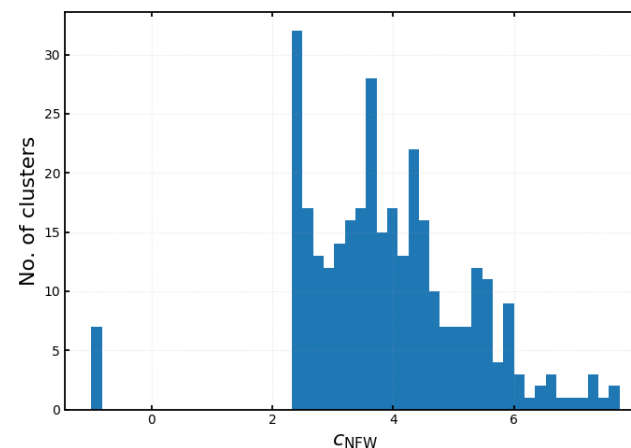


Chameleon field in an NFW
halo



The Simulation Data

- For the density data we used **The 300 Project** cluster data.
- We used only the **central 324** cluster density data (GadgetX, $z=0$).
- Tested both direct density fit and AHF NFW parameters (Prada et al. 2012).
- Considered both spherical and triaxial NFW density distributions.

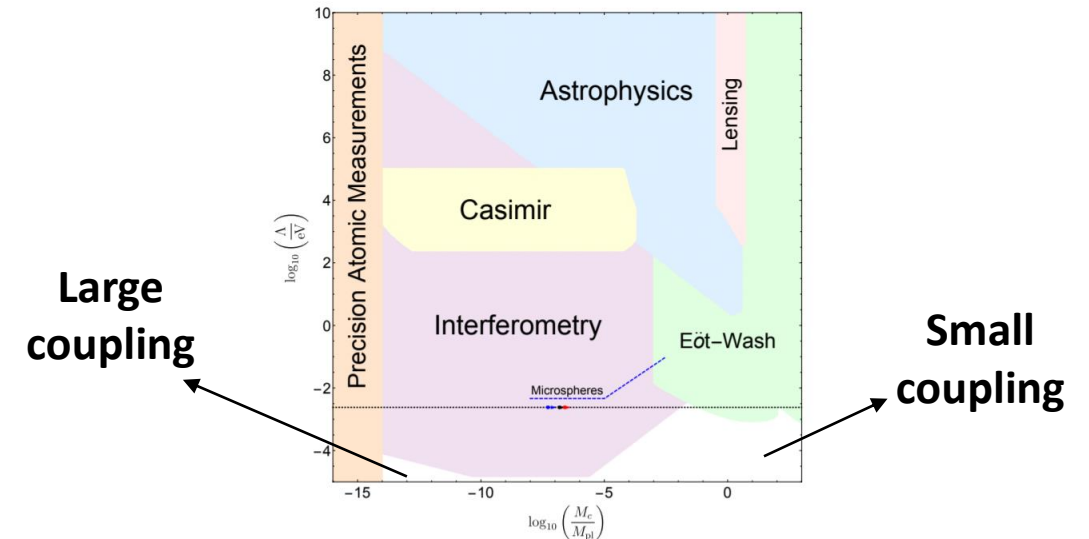




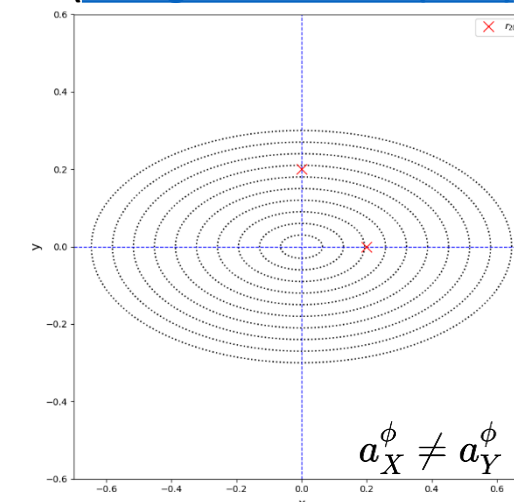
Key Questions

- We tackled the following questions:
 - What is the **chameleon-to-NFW** acceleration/force ratio at $\sim r_{200}$?
 - How does this ratio depend on the c_{NFW} and M_{200} ?
 - Is it potentially **measurable**?
 - What are the effects of **triaxiality**?

$$R^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x^2}{q_a^2} + \frac{y^2}{q_b^2} + \frac{z^2}{q_c^2} ; (q_c \equiv 1)$$
$$\rho(r) \rightarrow \rho(R)$$



Current chameleon constraints
(Burrage and Sakstein (2018))



Fischer's Chameleon
(*Kinyongia fischeri*)

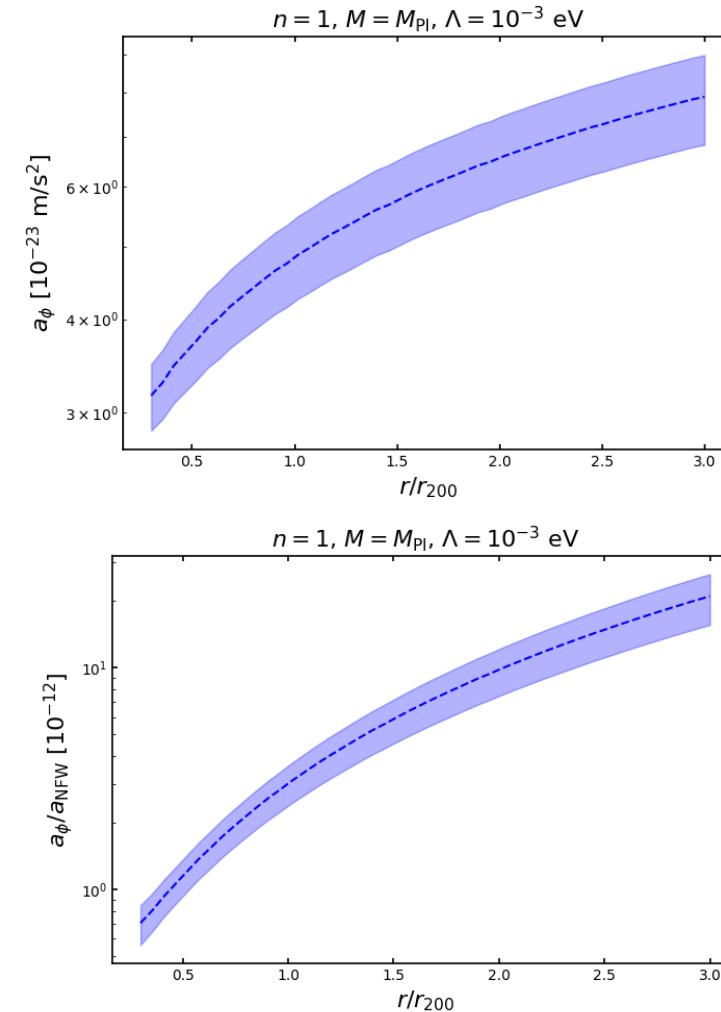
Part 3: Results



Results ($n = 1, M = M_{Pl}, \Lambda = 1 \times 10^{-3} \text{ eV}$)



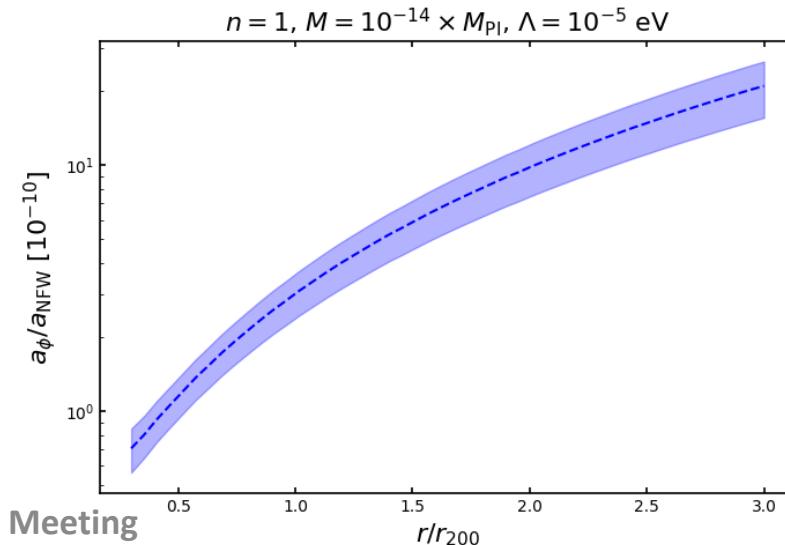
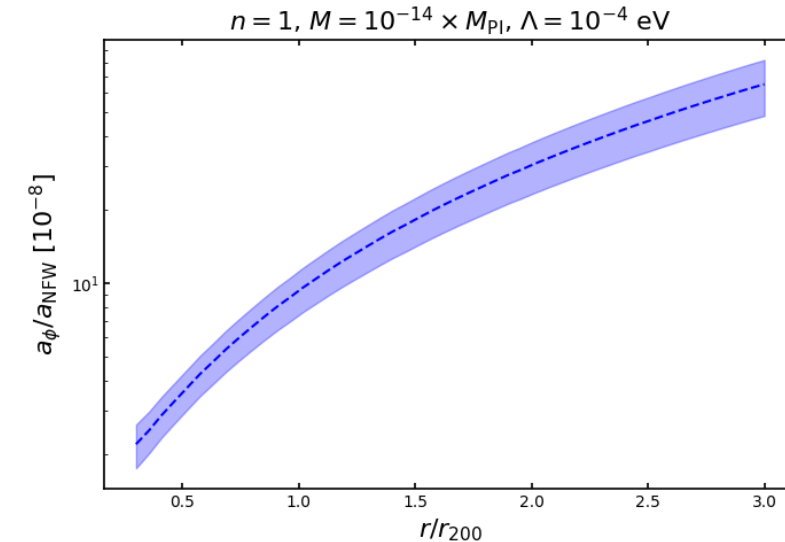
- The chameleon field profile **increases** monotonically throughout the cluster.
- The NFW/GR acceleration is **significantly higher** throughout the cluster.
- Such small deviations would likely **not be measurable** on cluster scales.



Results ($n = 1, M = 10^{-14} \times M_{Pl}, \Lambda = 1 \times 10^{-4} \text{ eV}$)

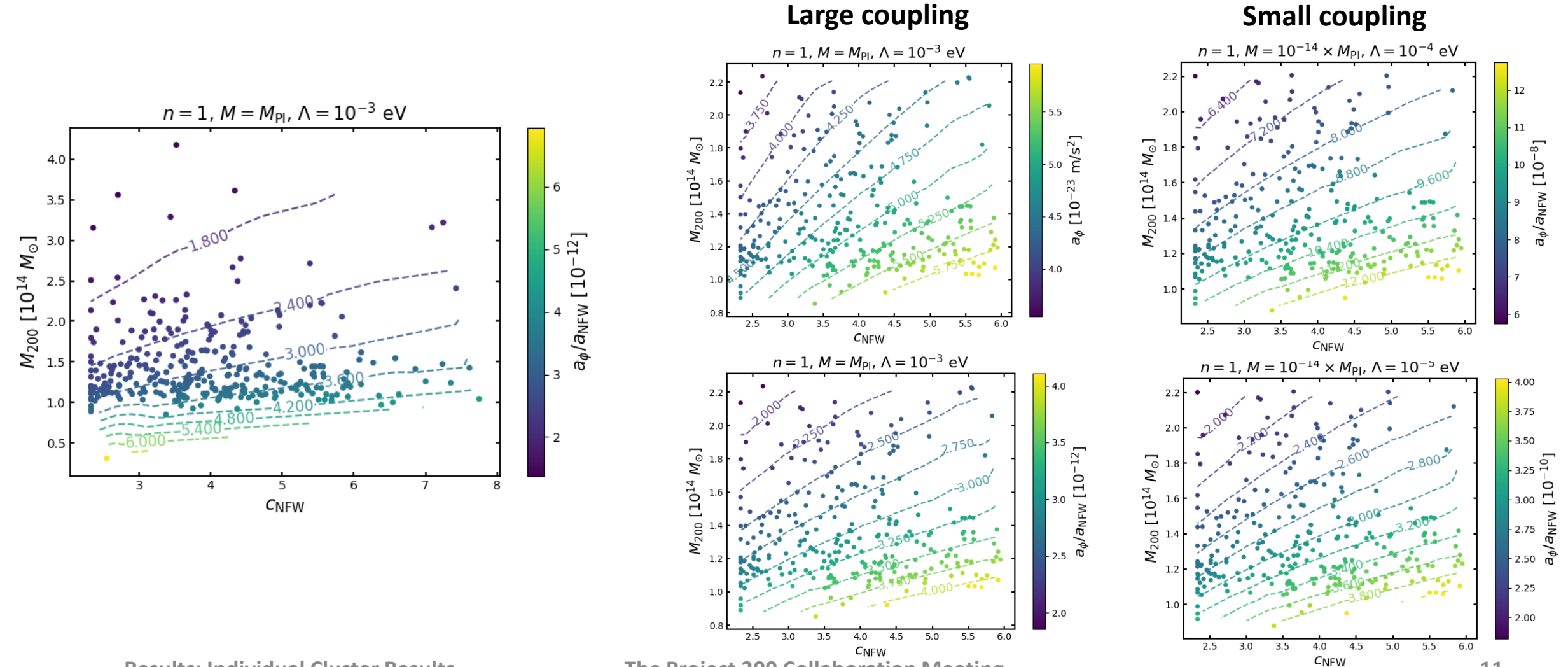


- Significantly increased the **coupling** and reduced the **energy scale**.
- The acceleration ratio also corresponds to the (effective) **mass ratio**.
- Likely **not measurable** on galaxy cluster scales. However, certain models could be detected in vacuum chamber tests.





Individual Cluster Results



Results: Individual Cluster Results

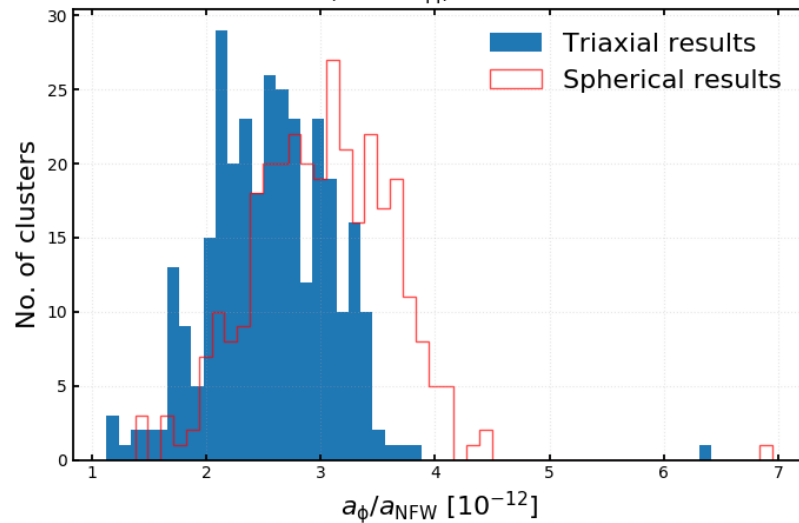
NFW Triaxiality effects



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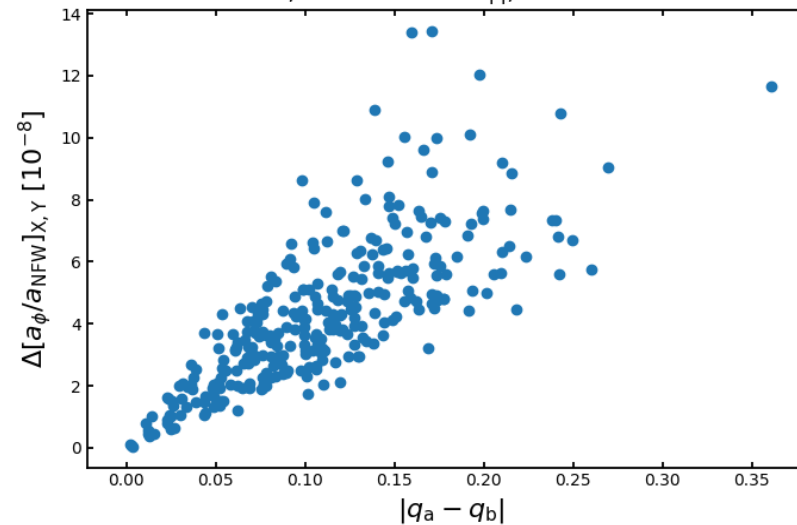
Spherical vs Triaxial Results

$n = 1, M = M_{\text{Pl}}, \Lambda = 10^{-3} \text{ eV}$



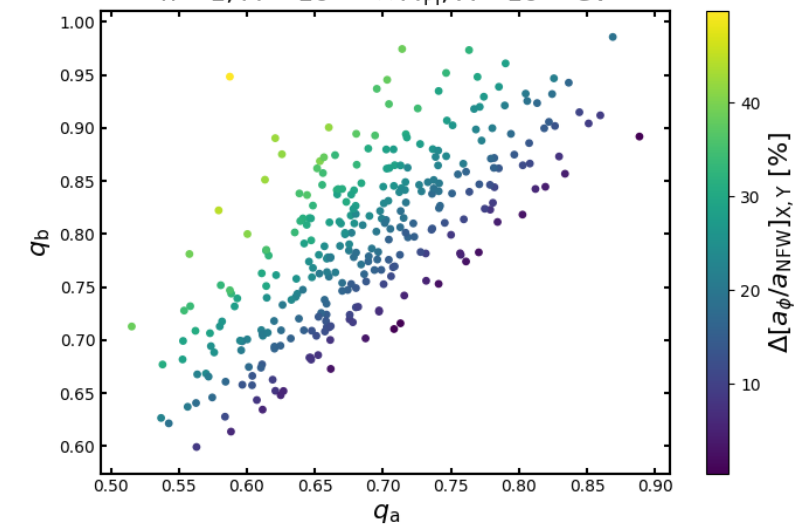
Axes diff. vs $\Delta a_\phi/a_{\text{NFW}}$

$n = 1, M = 10^{-14} \times M_{\text{Pl}}, \Lambda = 10^{-4} \text{ eV}$



Shape vs $\Delta a_\phi/a_{\text{NFW}}$

$n = 1, M = 10^{-14} \times M_{\text{Pl}}, \Lambda = 10^{-4} \text{ eV}$





Conclusions and Future Work

Summary:

- There is a clear relationship between the NFW parameters and a_ϕ/a_{NFW} .
- **Insignificant** effects on galaxy cluster scales.
- Triaxiality introduces **extra directional effects**. This is (mostly) model-independent.

Future work:

- Explore triaxiality effects in **3D**.
- Explore arbitrary mass distributions.
- Explore the Chameleon effects in **cosmic voids**.
- To be published **soon!**