Astrophysical Tests of Gravity Using FEniCS

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The Context: The ΛCDM Model

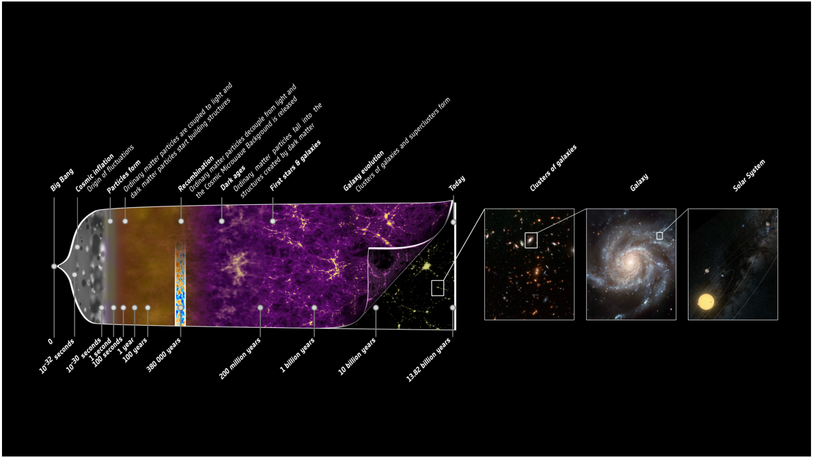


Fig.: The Universe according to the standard model of cosmology (NASA).

The ΛCDM Model

1. Gravity is described by the theory of **general relativity** (GR):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (1)$$

2. The Universe is dominated by **dark matter** and **dark energy**.

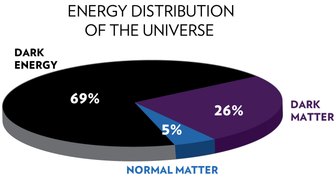


Fig.: The mass-energy content of the Universe (Chandra).

Key Questions:

1. Dark matter?
2. Dark energy?
3. Galaxy-scale problems?
4. The Hubble constant problem?
5. The theory of quantum gravity?

Possible Solution: Modified Gravity

- It is possible that some of these problems could be tackled by modifying Einstein's general relativity.

- General relativity can be modified in a variety ways:

  1. Some of the assumptions can be relaxed ($f(R)$ gravity);
  2. Extra spacetime dimensions (Kaluza Klein);
  3. Non-local theories (Infinite derivative gravity);
  4. Extra (scalar, vector, tensor) fields.

- Modifying GR results in an extra fundamental force ("fifth" force).

An Example: Chameleon Gravity

- An example of a scalar-tensor theory: Chameleon gravity;
- An extra scalar field interacting with matter in a special way is introduced;
- A "fifth" force arises from the interactions between the field and matter in a **density-dependent** way.

Key Equation to Solve:

$$\nabla^2\phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{\rm pl}}, \quad (2)$$

where: ϕ- Chameleon field, ρ-density, n, Λ, β, $M_{\rm pl}$-constants.

Fifth Force in Cosmology

- A fifth force could potentially explain dark energy;
- A fifth would not be relevant in the Solar System;
- Modifying gravity would change the properties of galaxy clusters:

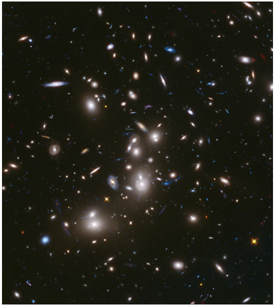
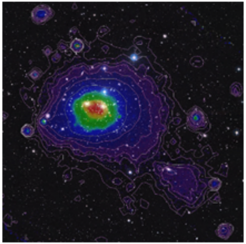


Fig.: Galaxy cluster Abel 2744 (NASA).



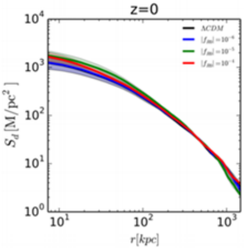


Fig.: Clusters in X-ray (ESA).

Fifth Force in the Lab

Atom Interferometry Tests:

- Fifth forces could be potentially detected in laboratory tests;
- Recent tests have not found evidence for a fifth force;
- Such tests allow putting strong constraints on the Chameleon model parameters.

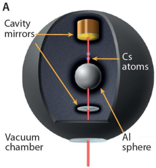


Fig.: A vacuum chamber experiment (Hamilton et al. 2015).

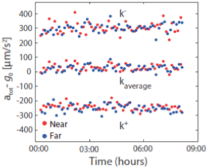


Fig.: Key results from the experiment (Hamilton et al. 2015).

Solving the Chameleon Eq. in FEniCS

- Normalizing the eq.: $\hat{\phi} = \phi/\phi_\infty$, $\hat{\rho} = \rho/\rho_\infty$,

$$\nabla^2\phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{pl}} \rightarrow \alpha\hat{\nabla}^2\hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho}, \tag{3}$$

- Rewriting the eq. in the variational form:

$$\alpha \int_\Omega \hat{\nabla}\hat{\phi} \cdot \hat{\nabla} v_j dx = \int_\Omega \left(\hat{\phi}^{-(n+1)} - \hat{\rho}\right) v_j dx, \tag{4}$$

- Rewriting using Taylor expansion:

$$\hat{\phi}^{-(n+1)} \approx (n+2)\hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} + \mathcal{O}\left(\hat{\phi} - \hat{\phi}_k\right)^2 \tag{5}$$

- The eq. is solved using the Newton/Picard methods:

$$\alpha \int_\Omega \hat{\nabla}\hat{\phi}\cdot\hat{\nabla} v_j dx + \int_\Omega (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} v_j dx = \int_\Omega (n+2)\hat{\phi}_k^{-(n+1)} v_j dx - \int_\Omega \hat{\rho} v_j dx \tag{6}$$

Numerical Solutions Using FEniCS

- Using FEniCS allows to simulate the mentioned vacuum chamber experiment;
- The mesh is generated and refined using Gmsh;
- Complex source shapes can be easily handled:

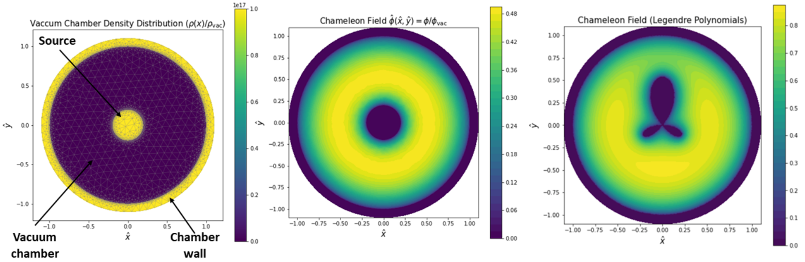


Fig.: A vacuum chamber simulation using FEniCS (Chad Briddon).

- Galaxy and galaxy cluster density distribution:

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}; \tag{7}$$

- Galaxy cluster Chameleon field simulation:

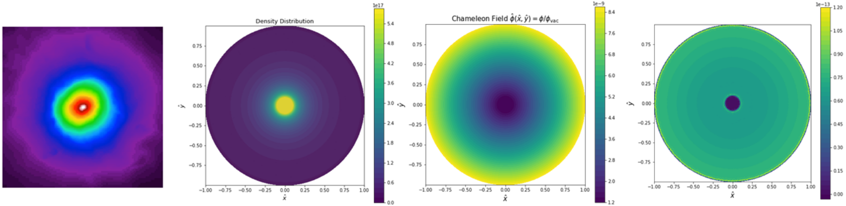


Fig.: Left: a typical galaxy cluster (X-ray); **center** the density distribution and the Chameleon field **right:** the field gradient.

3D Solutions

- An analogous method allows finding the 3D solutions;
- The typical size of the residuals: $\sim 0.1 - 1\ \%$;
- Finding the solution typically takes 10-40 min depending on the model parameters.

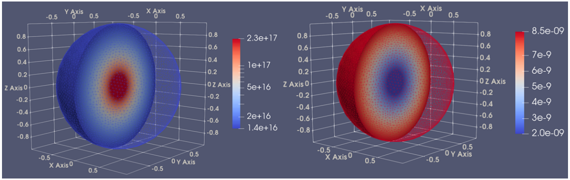


Fig.: Left 3D NFW distribution; **right:** Chameleon field solution.

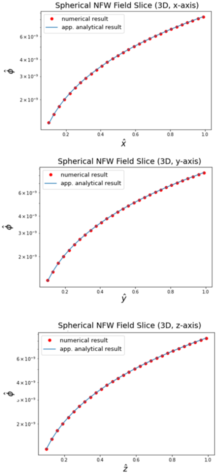
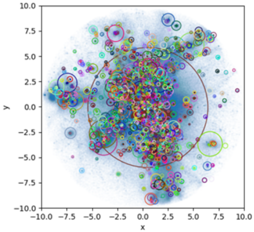


Fig.: Comparing the numerical and analytic solutions.

Realistic Galaxy Clusters

- Realistic galaxy clusters based on simulations:



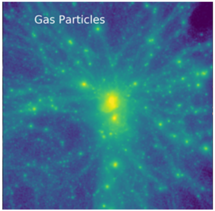


Fig.: Cluster density distribution (Project 300 simulation).

Key Questions

1. Optimal cluster form?
2. Optimal galaxy form?
3. Time-dependent density distributions?

Summary and Future Plans

The long-term goal:

An open-source FEniCS code for modelling experimental and observational tests for a variety of gravity models.

Key problems:

- Other gravity theories: Symmetron Gravity, $f(R)$, Vainshtein screening models;
- Other cosmology/astrophysics questions: stars, galaxies, cosmic voids;
- Other laboratory setups: spinning sources, multiple sources, more complicated vacuum chambers.