Tests of Modified Gravity with Simulations and Numerical Methods

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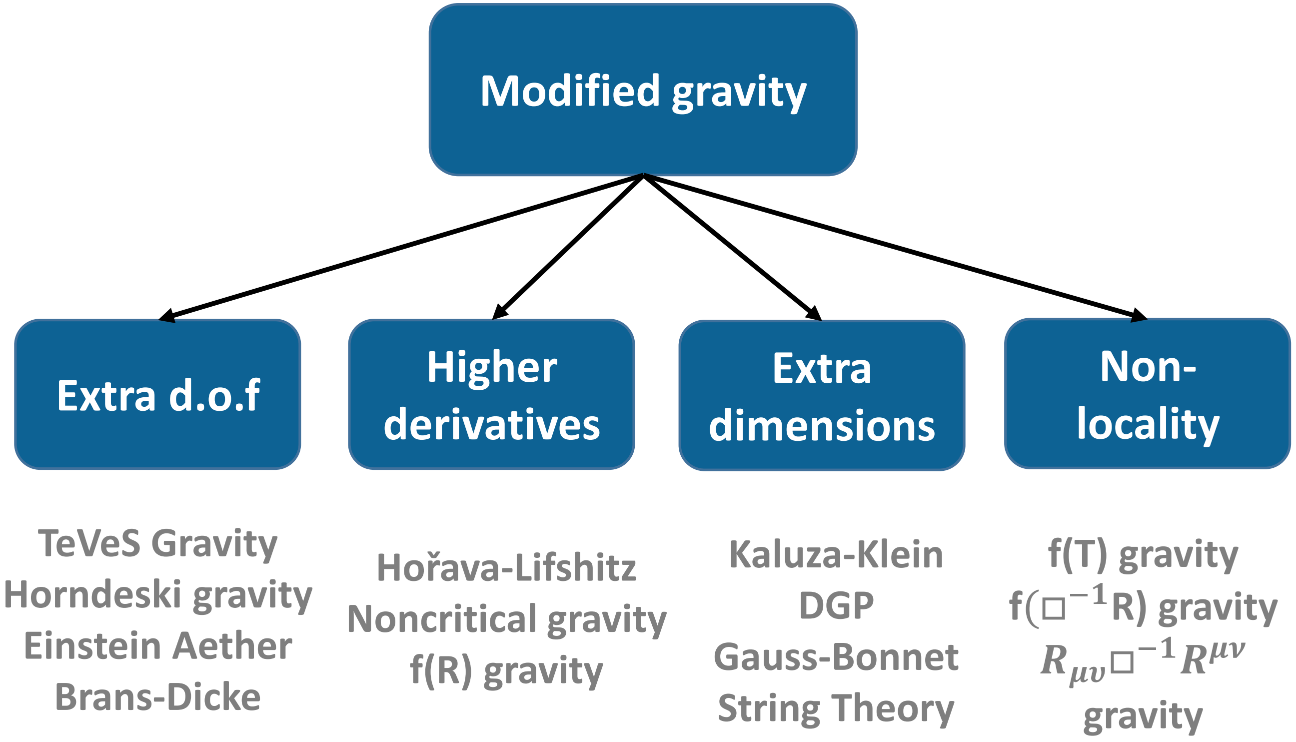
Veiled Chameleon
(Chamaeleo calyptratus)

Modified Gravity

- The case for modified gravity:
  - Dark matter,
  - Dark energy,
  - Quantum gravity,
  - “What I cannot create, I do not understand”. Richard P. Feynman
- A wide variety of models:
  - Different model families break different **assumptions** in GR,
  - Interesting phenomenology on **different scales**.



Chameleon Gravity

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- Chameleon gravity: **scalar-tensor** theory with non-minimal coupling:
- Varying the action gives the **non–linear** equation of motion:
- Solving the EOM gives the **fifth force**:
- The challenge: solving the EOM for **astrophysical** density distributions.

$$S = \int dx^4 \sqrt{-g}\left(\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right) - \int dx^4 \mathcal{L}_m\left(\varphi_m^{(i)}, \tilde{g}_{\mu\nu}^{(i)}\right)$$

General relativity Scalar field Matter Lagrangian

$$\Box\phi = V_{,\phi} - \sum_i \frac{\beta_i}{M_{pl}} T_{\mu\nu}^{(i)} \tilde{g}_{(i)}^{\mu\nu}$$

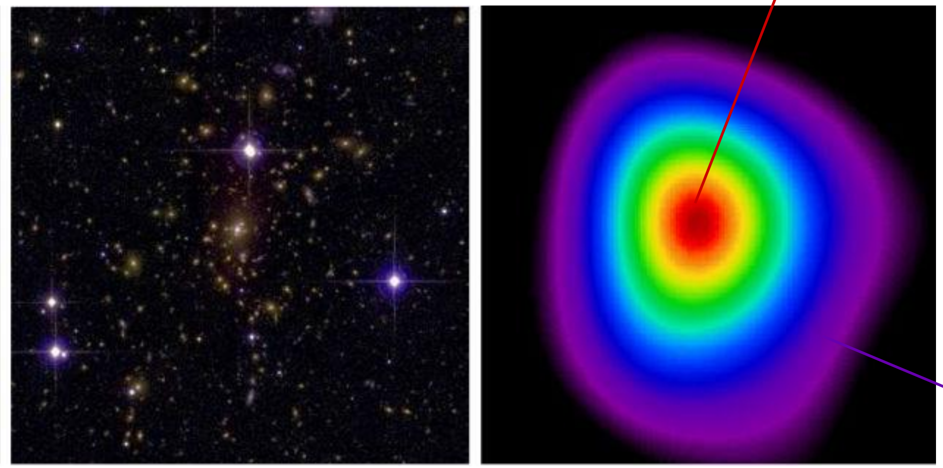
$$\boxed{\nabla^2\phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{\rm Pl}}}$$

$$F_\phi = -\frac{m\beta}{M_{pl}}\nabla\phi$$

Chameleon Screening

Effective potential: $V_{\text{eff}}(\phi) = V(\phi) + \sum_i \frac{\beta_i \rho_i}{M_{\text{Pl}}} \phi$

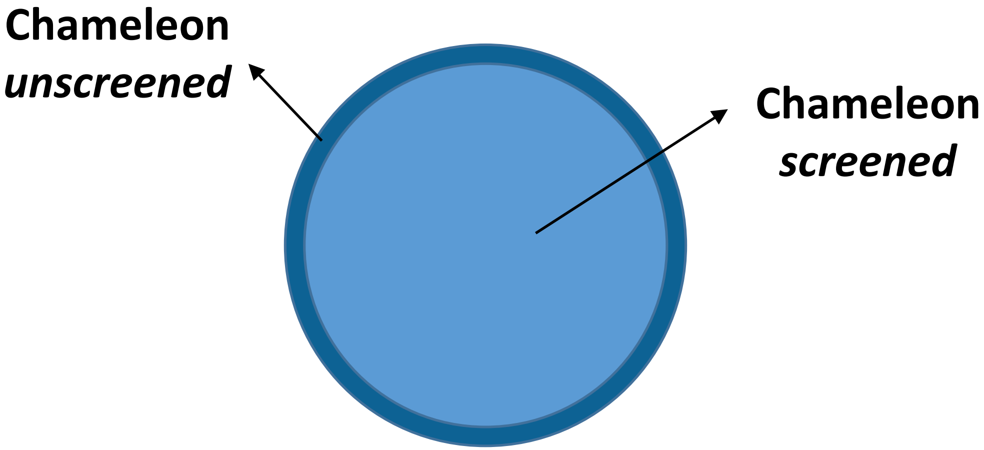
Chameleon *screened*
(behaves like GR)



Chameleon *unscreened*
(extra *fifth* force)

Abell 1835 ([Mantz et al. 2010a]())

The thin-shell effect:



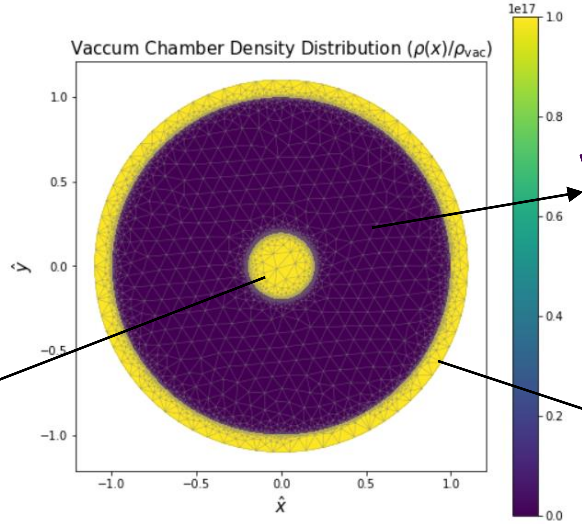
Part 2: The Finite-Element Approach

Panther Chameleon
(Furcifer pardalis)

The Finite Element Approach

- A numerical method to solve **linear** and **non-linear** equations in 1D, 2D and 3D.
- The problem domain is discretize into ***finite elements***.
- The equations are rewritten into an integral form via Green's theorem.
- Widely used in engineering, fluid dynamics, structural analysis etc.

An example of applying the FEM approach:

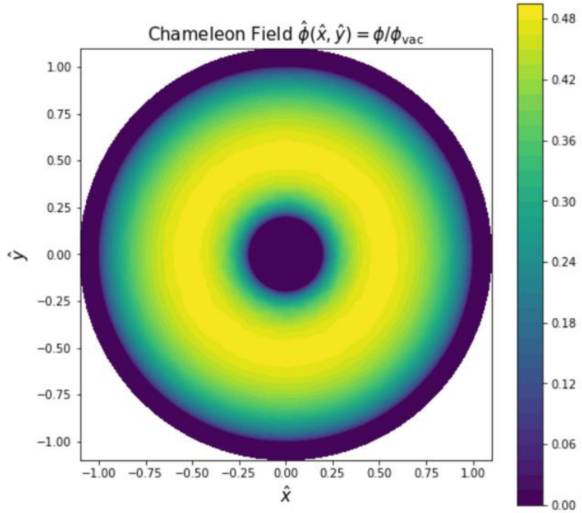


Vacuum

Source

Wall

The resulting field:



Credit: Chad Briddon

The Finite Element Approach

- Rewriting the Chameleon EOM:
  - Use **Green's function** (with v_j as the *test* function),
  - Rescale the EOM by the field **vacuum value** and the domain size,
  - **Taylor expand** the non-linear term,
  - Solve the resulting equation using the **Picard/Newton** iteration methods.

$$\int_\Omega (\nabla^2\phi) v_j dx + \int_\Omega \nabla\phi \cdot \nabla v_j dx = \int_{\partial\Omega} (\partial_n \phi) v_j dx$$

$$\nabla^2\phi = -\frac{n\Lambda^{n+4}}{\phi^{n+1}} + \frac{\beta\rho}{M_{pl}} \rightarrow \alpha\hat{\nabla}^2\hat{\phi} = -\hat{\phi}^{-(n+1)} + \hat{\rho}$$

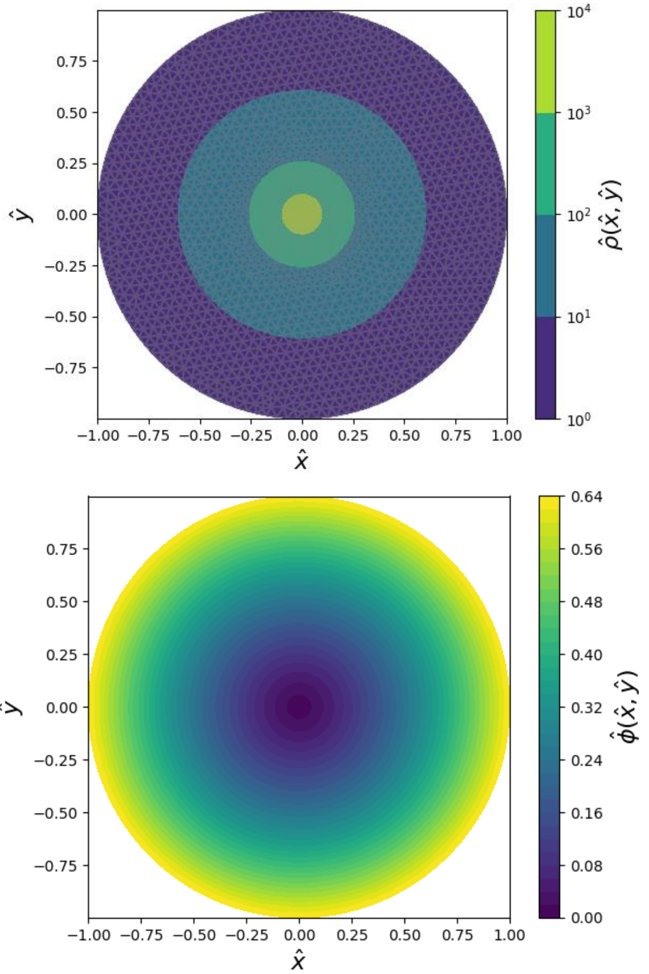
$$\alpha \equiv \left(\frac{M\Lambda}{L^2\rho_\infty}\right)\left(\frac{nM\Lambda^3}{\rho_\infty}\right)^{\frac{1}{n+1}}$$

$$\hat{\phi}^{-(n+1)} \approx (n+2)\hat{\phi}_k^{-(n+1)} - (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} + O\left(\hat{\phi} - \hat{\phi}_k\right)^2$$

$$\alpha\int_\Omega \hat{\nabla}\hat{\phi} \cdot \hat{\nabla} v_j dx + \int_\Omega (n+1)\hat{\phi}_k^{-(n+2)}\hat{\phi} v_j dx = \int_\Omega (n+2)\hat{\phi}_k^{-(n+1)} v_j dx - \int_\Omega \hat{\rho} v_j dx$$

The Finite Element Approach: FEniCS

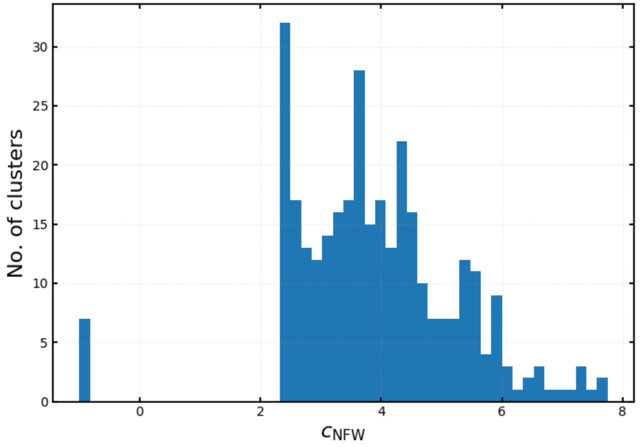
- *Selkie* (**to be released soon**) – a Python code based on the *FEniCS* software library.
- *Selkie* allows solving non-linear equations in 1D, 2D and 3D.
- Chameleon equations can be solved for various systems:
  - ➢ Vacuum chamber,
  - ➢ Galaxy clusters,
  - ➢ Cosmic voids.

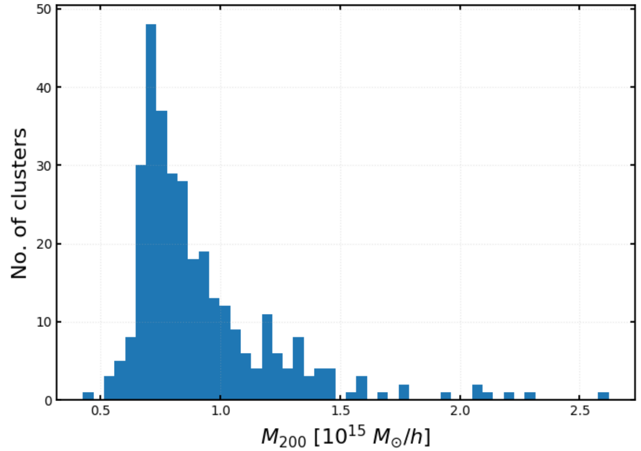


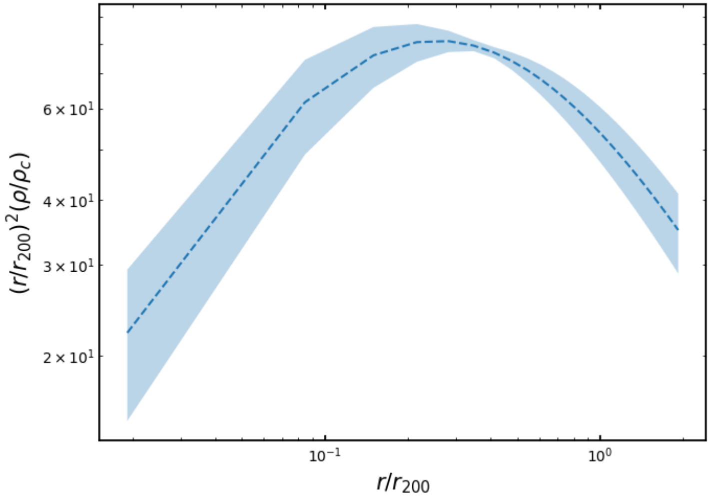
Chameleon field in an NFW halo

The Simulation Data

- For the density data we used **The Project 300** cluster data.
- We used only the **central** 324 cluster density data (GadgetX, z=0).
- Tested both direct density fit and AHF NFW parameters (Prada et al. 2012).
- Considered both spherical and triaxial NFW density distributions.







The 300 Project Data ([Cui et al.,2018](#))

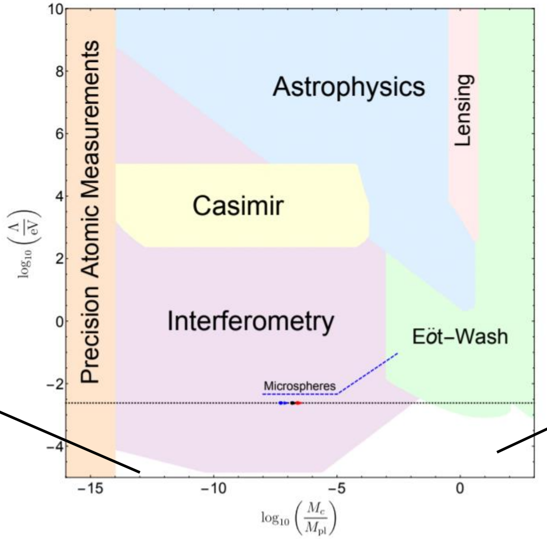
Key Questions

- We tackled the following questions:
  - ➢ What is the **chameleon-to-NFW** acceleration/force ratio at $\sim r_{200}$?
  - ➢ How does this ratio depend on the c_{NFW} and M_{200}?
  - ➢ Is it potentially **measurable**?
  - ➢ What are the effects of **triaxiality**?

$$R^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = \frac{x^2}{q_a^2} + \frac{y^2}{q_b^2} + \frac{z^2}{q_c^2} \; ; (q_c \equiv 1)$$

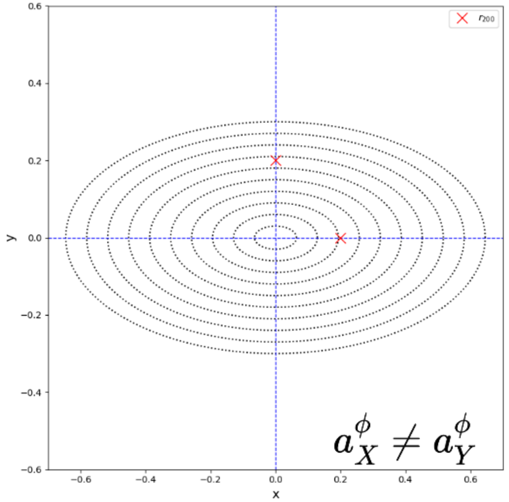
$$\rho(r) \to \rho(R)$$

Large coupling



Small coupling

Current chameleon constraints
([Burrage and Sakstein (2018)](#))

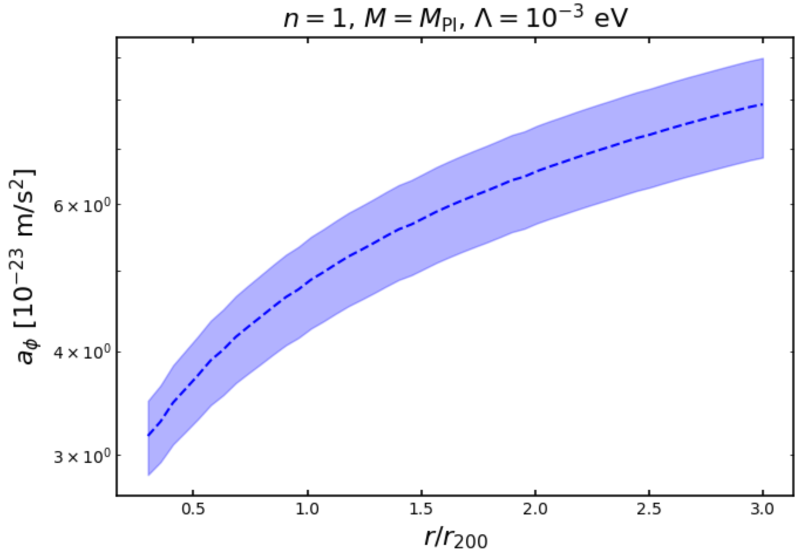


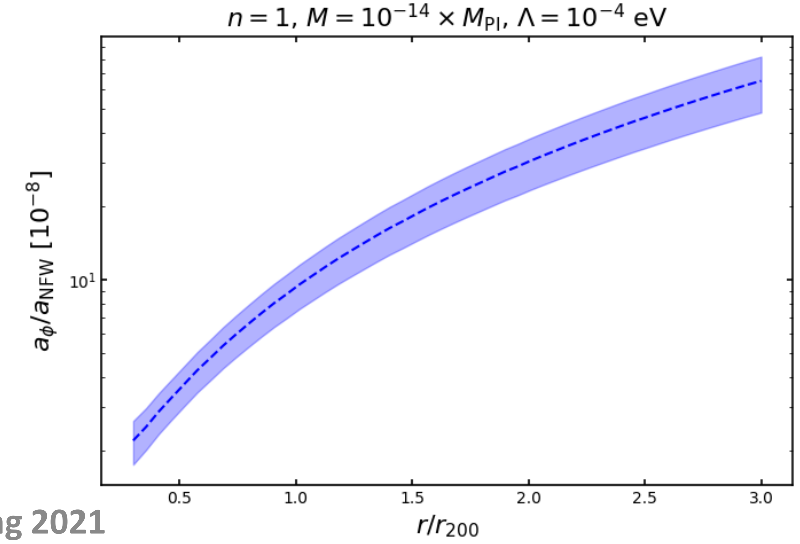
Fischer's Chameleon
(Kinyongia fischeri)

Part 3: Results

Galaxy Cluster Results

- The chameleon field profile **increases** monotonically.
- The NFW/GR acceleration is **significantly higher** throughout the cluster.
- Such small deviations would likely **not be measurable** on cluster scales. Potentially measurable in vacuum experiments.





Individual Cluster Results

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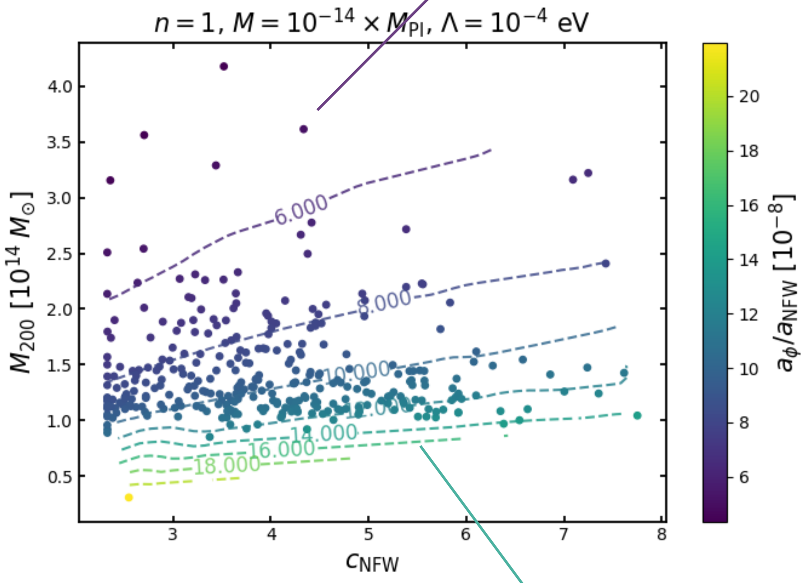
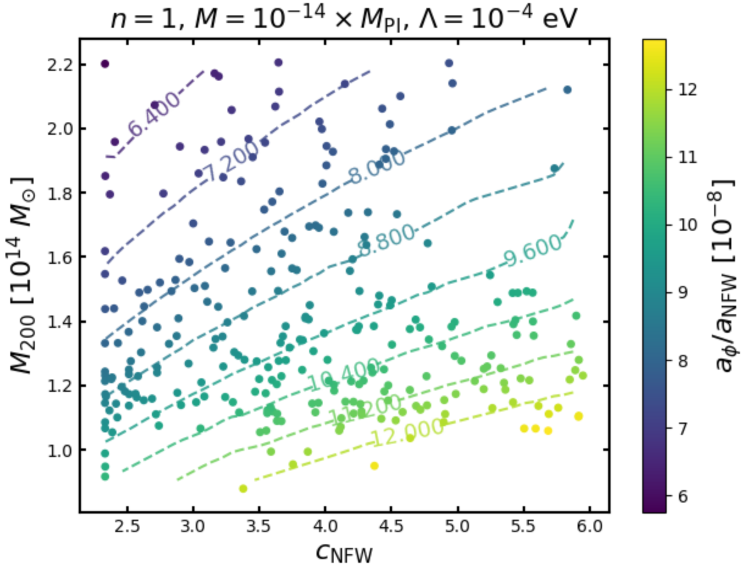
Cluster (Project 300 dataset)

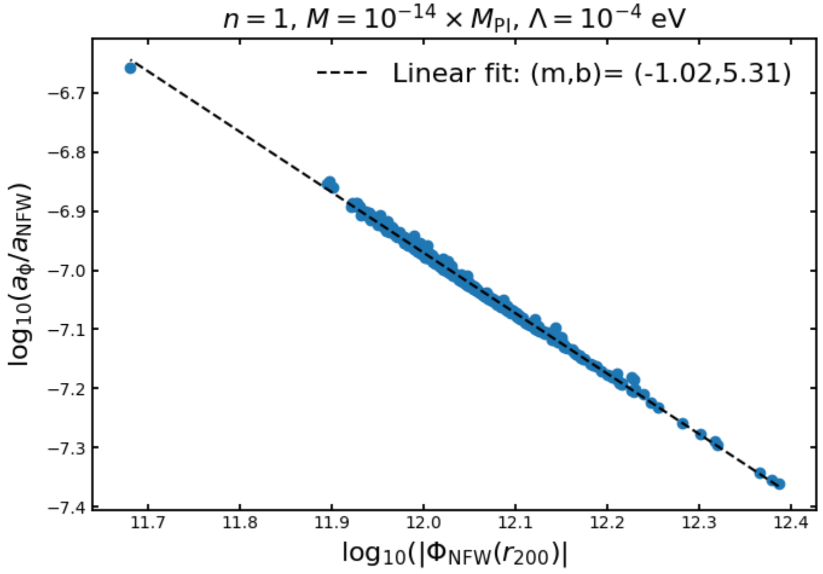


Lines of equal value

Outliers removed:

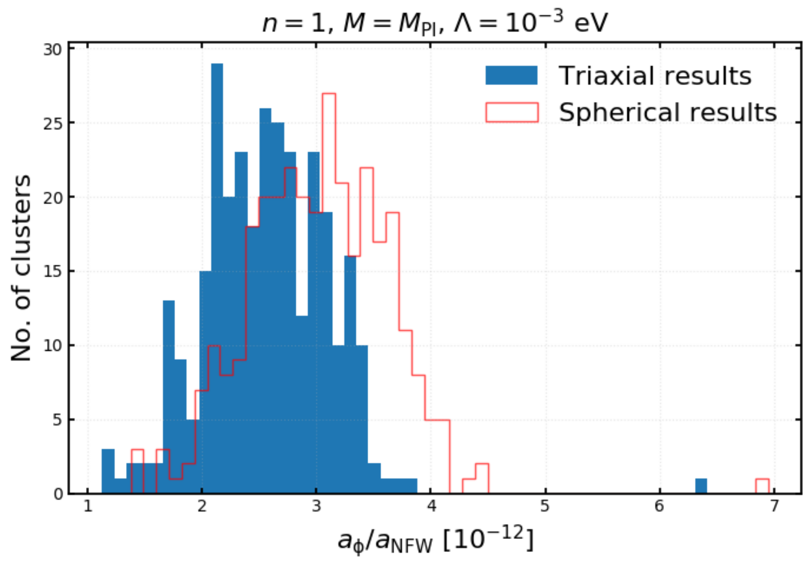


NFW potential vs. acc. ratio

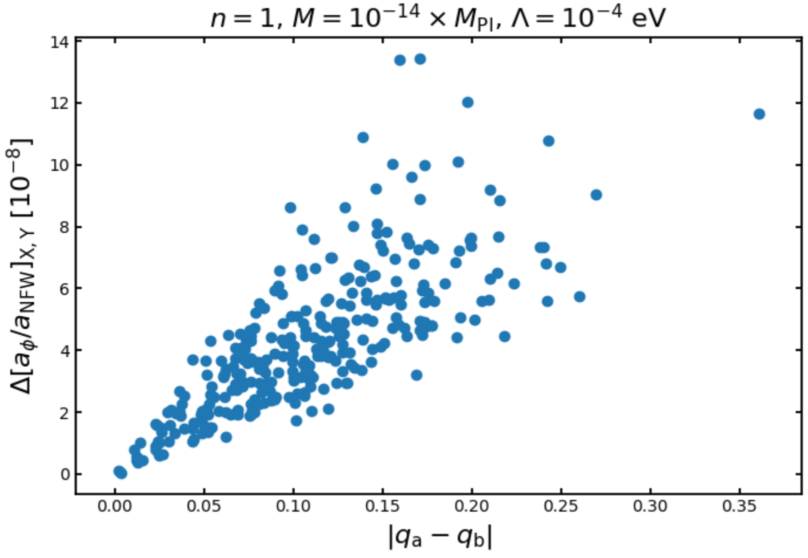


NFW Triaxiality effects

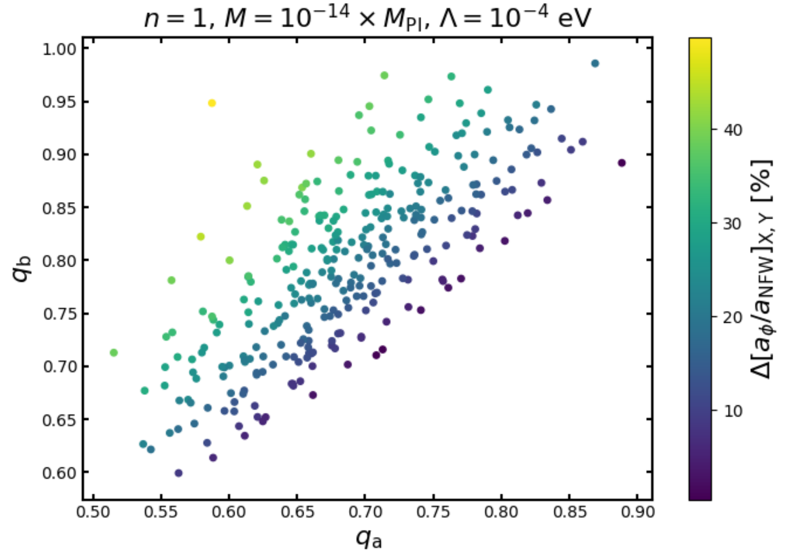
Spherical vs Triaxial Results



Axes diff. vs $\Delta\, a_\phi/a_{\rm NFW}$



Shape vs $\Delta\, a_\phi/a_{\rm NFW}$



Chameleon Screening in Voids

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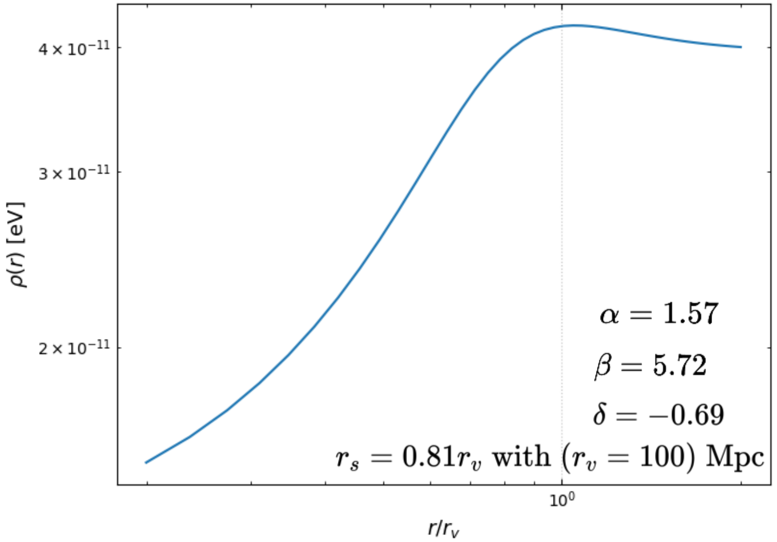
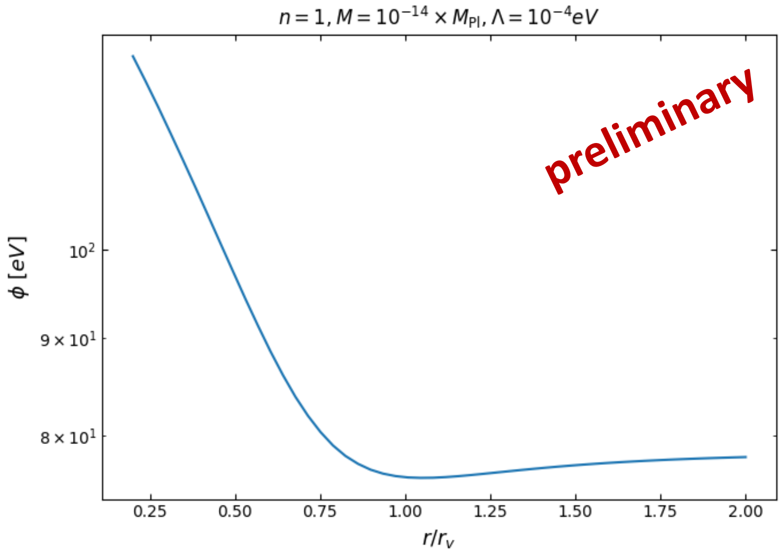
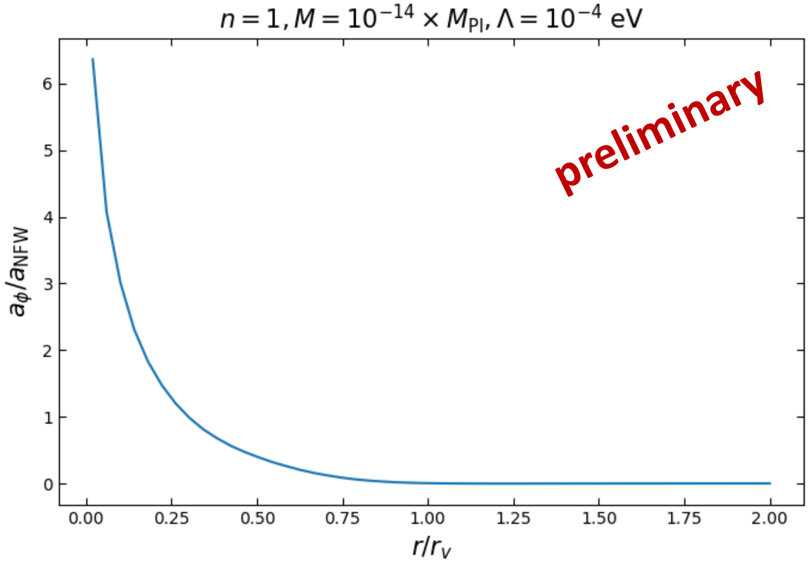
Void profile (Nadathur, 2014):
$$\frac{\rho(r)}{\bar{\rho}} = 1 + \delta\left(\frac{1-(r/r_s)^{\alpha}}{1+(r/r_s)^{\beta}}\right)$$







Conclusions and Future Work

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Summary:

- There is a clear relationship between the NFW parameters and $a_\phi / a_{\mathrm{NFW}}$.
- **Insignificant** effects on galaxy cluster scales.
- Triaxiality introduces **extra directional effects.** This is (mostly) model-independent.

Future work:

- Explore triaxiality effects in **3D**.
- Explore arbitrary mass distributions.
- Explore the Chameleon effects in **cosmic voids**.
- To be published **soon**!