# Ideal city ( المدينة المثالية )

Leonardo, like many other Italian scientists and artists of his age, was extremely interested ( مهتم جدا ) in city planning ( تخطيط) and urban design ( تصميم حضاري ). He aimed (تهدف) to model (نموذج) an ideal city (المدينة المثالية): comfortable (مريحة), spacious (واسع) and rational (عقلانية) in its usage of resources(باستعمال- استخدام الموارد) , far away from the narrow (بعيده عن الضيق), claustrophobic cities (خنقه المدينة) of the Middle Ages.

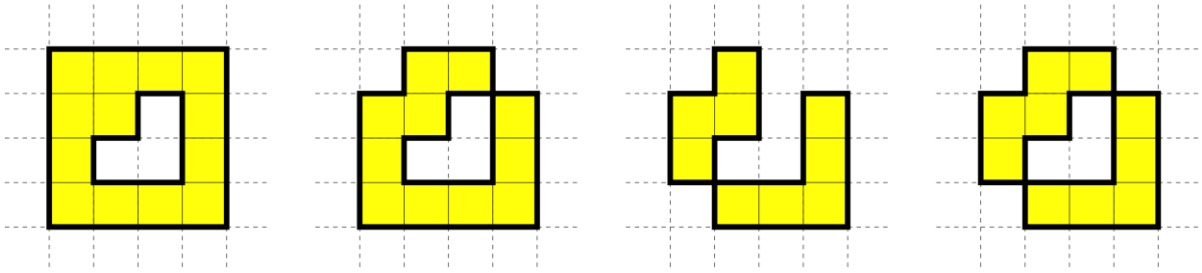
**The ideal city**

The city is made of N blocks placed on an infinite grid of cells (شبكة لانهائية من الخلايا). Each cell is identified (معرف- محدد) by a pair of coordinates (بزوج من الإحداثيات) (row, column). Given a cell (i, j), the adjacent cells (الخلايا المعدلة) are: (i - 1, j), (i + 1, j), (i, j - 1), and (i, j + 1). Each block, when placed onto the grid(شبكه), covers exactly one of the cells. A block can be placed onto the cell (i, j) if and only if $1 \leq i, j \leq 2^{31} - 2$. We will use the coordinates of the cells to also refer to the blocks on top of them. Two blocks are adjacent if they are placed in adjacent cells. In an ideal city, all of its blocks are connected in such a way that there are no “holes” inside its border, that is, the cells must satisfy both conditions below.

- For any two *empty* cells, there exists at least one sequence of adjacent *empty* cells connecting them.
- For any two *non-empty* cells, there exists at least one sequence of adjacent *non-empty* cells connecting them.

**Example 1**

None of the configurations (تعريفات تكوينات) of blocks below represent an ideal city: the first two on the left do not satisfy the first condition, the third one does not satisfy the second condition, and the fourth one does not satisfy either of the conditions.

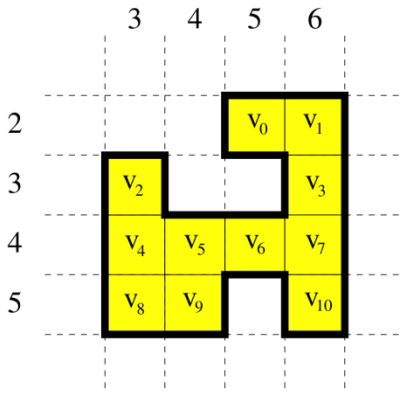


**Distance**

When traversing (تعبر) the city, a *hop* indicates going from one block to an adjacent one. Empty cells cannot be traversed (أجتاز). Let $v_0, v_1, \ldots, v_{N-1}$ be the coordinates of the N blocks placed on the grid. For any two distinct blocks at coordinates $v_i$ and $v_j$, their distance $d(v_i, v_j)$ is the smallest number of hops that are required to go from one of these blocks to the other one.

**Example 2**

The configuration below represents an ideal city made of N = 11 blocks at coordinates $v_0$ = (2, 5), $v_1$ = (2, 6), $v_2$ = (3, 3), $v_3$ = (3, 6), $v_4$ = (4, 3), $v_5$ = (4, 4), $v_6$ = (4, 5), $v_7$ = (4, 6), $v_8$ = (5, 3), $v_9$ = (5, 4), and $v_{10}$ = (5, 6). For example, $d(v_1, v_3) = 1$, $d(v_1, v_8) = 6$, $d(v_6, v_{10}) = 2$, and $d(v_9, v_{10}) = 4$.




# Statement

Your task is to, given an ideal city, write a program to compute the sum of all pairwise distances between blocks $v_i$ and $v_j$ for which i < j. Formally, your program should compute the value of the following sum:

$$\sum d(v_i, v_j), \text{ where } 0 \leq i < j \leq N - 1$$

Specifically, you have to implement a routine `DistanceSum`(N, X, Y) that, given N and two arrays X and Y that describe the city, calculates the formula above. Both X and Y are of size N; block i is at coordinates (X[i], Y[i]) for $0 \leq i \leq N - 1$, and $1 \leq X[i], Y[i] \leq 2^{31} - 2$. Since the result may be too big to be represented using 32 bits, you should report it modulo 1 000 000 000 (one billion).

In Example 2, there are 11 × 10 / 2 = 55 pairs of blocks. The sum of all the pairwise distances is 174.

# Subtask 1 [11 points]

You may assume that $N \leq 200$.

# Subtask 2 [21 points]

You may assume that $N \leq 2\,000$.

# Subtask 3 [23 points]

You may assume that $N \leq 100\,000$.

Additionally, the following two conditions hold: given any two non-empty cells i and j such that X[i] = X[j], every cell between them is non-empty too; given any two non-empty cells i and j such that Y[i] = Y[j], every cell between them is non-empty too.

# Subtask 4 [45 points]

You may assume that $N \leq 100\,000$.

# Implementation details

You have to submit exactly one file, called `city.c`, `city.cpp` or `city.pas`. This file must implement the subprogram described above using the following signatures.

### C/C++ programs

```
int DistanceSum(int N, int *X, int *Y);
```

### Pascal programs

```
function DistanceSum(N : LongInt; var X, Y : array of LongInt) : LongInt;
```

This subprogram must behave as described above. Of course you are free to implement other subprograms for its internal use. Your submissions must not interact in any way with standard input/output, nor with any other file.

### Sample grader

The sample grader provided with the task environment will expect input in the following format:

- line 1: N;
- lines 2, …, N + 1: X[i], Y[i].

# Time and Memory limits

- Time limit: 1 second.
- Memory limit: 256 MiB.