



# Beech Tree

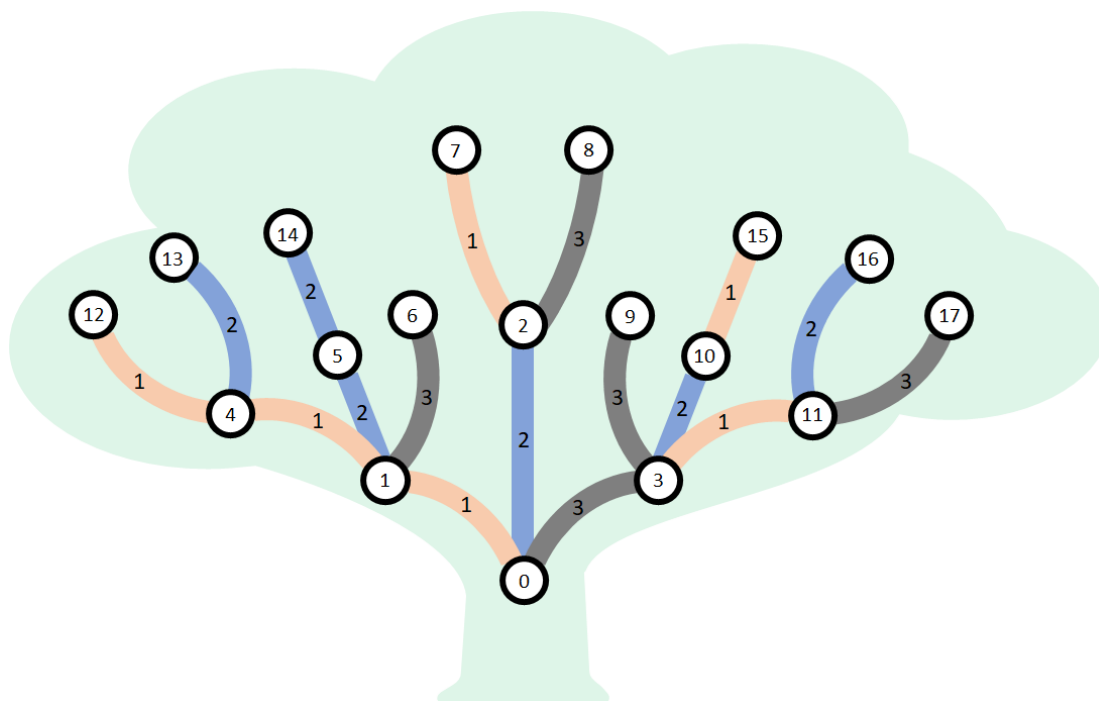
Vétyem Woods is a famous woodland with lots of colorful trees. One of the oldest and tallest beech trees is called Ős Vezér.

The tree Ős Vezér can be modeled as a set of  $N$  **nodes** and  $N - 1$  **edges**. Nodes are numbered from 0 to  $N - 1$  and edges are numbered from 1 to  $N - 1$ . Each edge connects two distinct nodes of the tree. Specifically, edge  $v$  ( $1 \leq v < N$ ) connects node  $v$  to node  $P[v]$ , where  $0 \leq P[v] < v$ .

Each edge has a color. There are  $M$  possible edge colors numbered from 1 to  $M$ . The color of edge  $v$  is  $C[v]$ . Different edges may have the same color.

Note that in the definitions above, the case  $v = 0$  does not correspond to an edge of the tree. For convenience, we let  $P[0] = -1$  and  $C[0] = 0$ .

For example, suppose that Ős Vezér has  $N = 18$  nodes and  $M = 3$  possible edge colors, with 17 edges described by connections  $P = [-1, 0, 0, 0, 1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 5, 10, 11, 11]$  and colors  $C = [0, 1, 2, 3, 1, 2, 3, 1, 3, 3, 2, 1, 1, 2, 2, 1, 2, 3]$ . The tree is displayed in the following figure:



Árpád is a talented forester who likes to study specific parts of the tree called **subtrees**. For each  $r$  such that  $0 \leq r < N$ , the subtree of node  $r$  (denoted by  $T(r)$ ) is a set of nodes. A node  $s$  is a member of  $T(r)$  if and only if:

- $s = r$ , or
- $s > r$  and node  $P[s]$  is a member of  $T(r)$ .

The size of the set  $T(r)$  is denoted as  $|T(r)|$ .

Árpád discovered an interesting subtree property. For each  $r$  such that  $0 \leq r < N$ , the subtree  $T(r)$  is **convoluted** if and only if there exists a sequence of *distinct* integers  $[v_0, v_1, \dots, v_{|T(r)|-1}]$  such that:

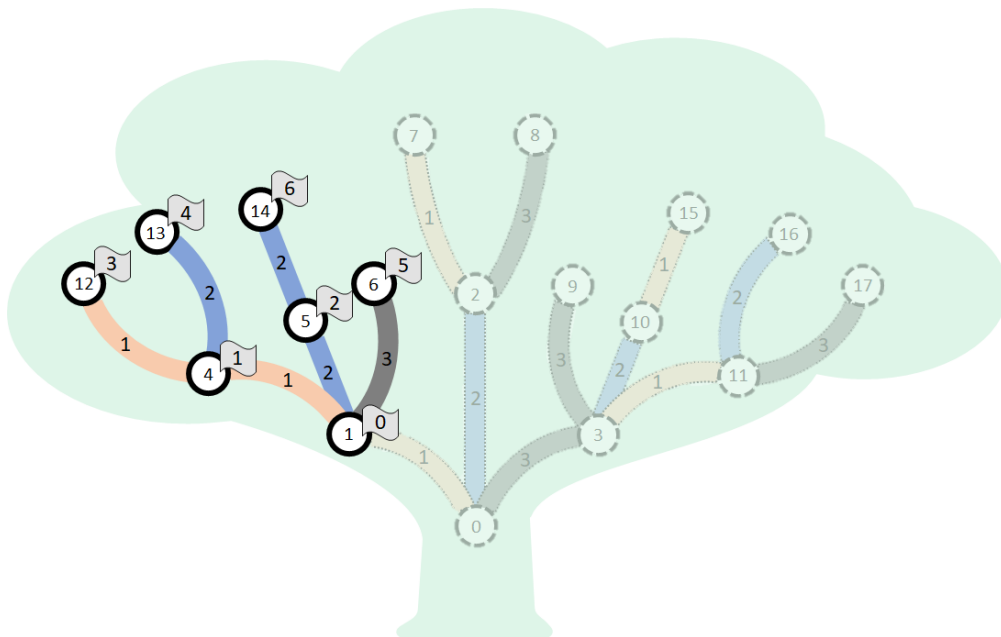
- For each  $i$  such that  $0 \leq i < |T(r)|$ , node  $v_i$  is a member of  $T(r)$ .
- $v_0 = r$ .
- For each  $i$  such that  $1 \leq i < |T(r)|$ ,  $P[v_i] = v_{f(i)}$ , where  $f(i)$  is defined as the number of times the color  $C[v_i]$  appears in the sequence  $[C[v_1], C[v_2], \dots, C[v_{i-1}]]$ .

Note that according to the definition:

- Every subtree which contains a single node is convoluted.
- For any subtree which contains two or more nodes,  $f(1) = 0$  because the sequence of colors in its definition is empty.

Consider the example tree above. The subtrees  $T(0)$  and  $T(3)$  of this tree are not convoluted. The subtree  $T(14)$  is convoluted, as it contains a single node. Below, we will show that the subtree  $T(1)$  is also convoluted.

Consider the sequence of distinct integers  $[v_0, v_1, v_2, v_3, v_4, v_5, v_6] = [1, 4, 5, 12, 13, 6, 14]$ . This sequence is represented in the following figure. The index of each node in this sequence is shown by the number on the label attached to the node.



The sequence of integers depicted above shows that  $T(1)$  is convoluted:

- $v_0 = r = 1$ .

- $f(1) = 0$  since  $C[v_1] = C[4] = 1$  appears 0 times in the sequence  $[],$  and  $P[v_1] = P[4] = 1 = v_0.$
- $f(2) = 0$  since  $C[v_2] = C[5] = 2$  appears 0 times in the sequence  $[1],$  and  $P[v_2] = P[5] = 1 = v_0.$
- $f(3) = 1$  since  $C[v_3] = C[12] = 1$  appears 1 time in the sequence  $[1,2],$  and  $P[v_3] = P[12] = 4 = v_1.$
- $f(4) = 1$  since  $C[v_4] = C[13] = 2$  appears 1 time in the sequence  $[1,2,1],$  and  $P[v_4] = P[13] = 4 = v_1.$
- $f(5) = 0$  since  $C[v_5] = C[6] = 3$  appears 0 times in the sequence  $[1,2,1,2],$  and  $P[v_5] = P[6] = 1 = v_0.$
- $f(6) = 2$  since  $C[v_6] = C[14] = 2$  appears 2 times in the sequence  $[1,2,1,2,3],$  and  $P[v_6] = P[14] = 5 = v_2.$

Your task is to help Árpád decide for every subtree of Ős Vezér whether it is convoluted.

## Implementation Details

You should implement the following procedure.

```
int[] beechtree(int N, int M, int[] P, int[] C)
```

- $N$ : the number of nodes in the tree.
- $M$ : the number of possible edge colors.
- $P, C$ : arrays of length  $N$  describing the edges of the tree.
- This procedure should return an array  $b$  of length  $N$ . For each  $r$  such that  $0 \leq r < N,$   $b[r]$  should be 1 if  $T(r)$  is convoluted, and 0 otherwise.
- This procedure is called exactly once for each test case.

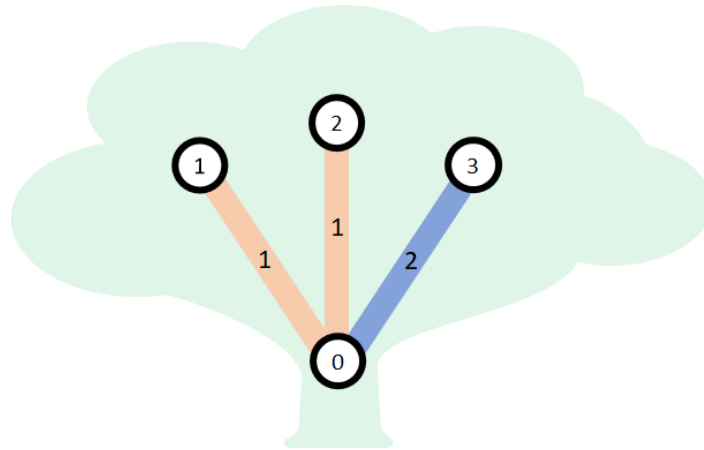
## Examples

### Example 1

Consider the following call:

```
beechtree(4, 2, [-1, 0, 0, 0], [0, 1, 1, 2])
```

The tree is displayed in the following figure:



$T(1)$ ,  $T(2)$ , and  $T(3)$  each contain a single node and are therefore convoluted.  $T(0)$  is not convoluted. Therefore, the procedure should return  $[0, 1, 1, 1]$ .

## Example 2

Consider the following call:

```
beechtree(18, 3,
          [-1, 0, 0, 0, 1, 1, 1, 2, 2, 3, 3, 3, 4, 4, 5, 10, 11, 11],
          [0, 1, 2, 3, 1, 2, 3, 1, 3, 3, 2, 1, 1, 2, 2, 1, 2, 3])
```

This example is illustrated in the task description above.

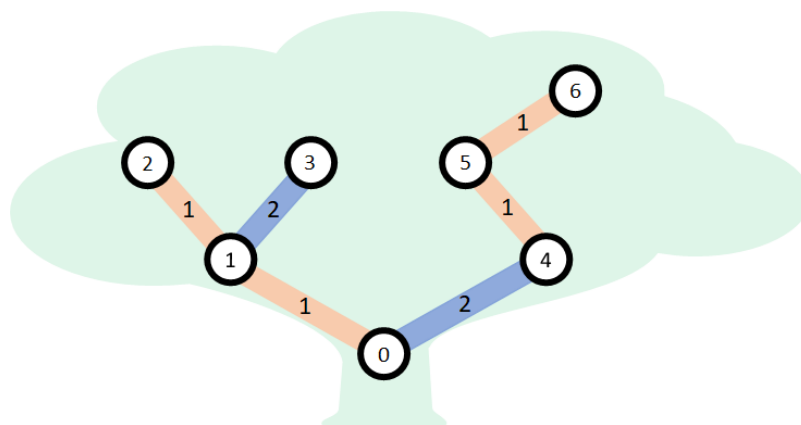
The procedure should return  $[0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$ .

## Example 3

Consider the following call:

```
beechtree(7, 2, [-1, 0, 1, 1, 0, 4, 5], [0, 1, 1, 2, 2, 1, 1])
```

This example is illustrated in the following figure.



$T(0)$  is the only subtree that is not convoluted. The procedure should return  $[0, 1, 1, 1, 1, 1, 1]$ .

## Constraints

- $3 \leq N \leq 200\,000$
- $2 \leq M \leq 200\,000$
- $0 \leq P[v] < v$  (for each  $v$  such that  $1 \leq v < N$ )
- $1 \leq C[v] \leq M$  (for each  $v$  such that  $1 \leq v < N$ )
- $P[0] = -1$  and  $C[0] = 0$

## Subtasks

1. (9 points)  $N \leq 8$  and  $M \leq 500$
2. (5 points) Edge  $v$  connects node  $v$  to node  $v - 1$ . That is, for each  $v$  such that  $1 \leq v < N$ ,  $P[v] = v - 1$ .
3. (9 points) Each node other than node 0 is either connected to node 0, or is connected to a node which is connected to node 0. That is, for each  $v$  such that  $1 \leq v < N$ , either  $P[v] = 0$  or  $P[P[v]] = 0$ .
4. (8 points) For each  $c$  such that  $1 \leq c \leq M$ , there are at most two edges of color  $c$ .
5. (14 points)  $N \leq 200$  and  $M \leq 500$
6. (14 points)  $N \leq 2\,000$  and  $M = 2$
7. (12 points)  $N \leq 2\,000$
8. (17 points)  $M = 2$
9. (12 points) No additional constraints.

## Sample Grader

The sample grader reads the input in the following format:

- line 1:  $N\ M$
- line 2:  $P[0]\ P[1]\ \dots\ P[N - 1]$
- line 3:  $C[0]\ C[1]\ \dots\ C[N - 1]$

Let  $b[0], b[1], \dots$  denote the elements of the array returned by `beechtree`. The sample grader prints your answer in a single line, in the following format:

- line 1:  $b[0]\ b[1]\ \dots$