International Olympiad in Informatics 2016



12-19th August 2016 Kazan, Russia day1 2

railroad
Country: PHL

Roller Coaster Railroad

Anna is working in an amusement park and she is in charge of building the railroad for a new roller coaster. She has already designed (n) special sections (conveniently numbered from (0) to (n-1)) that affect the speed of a roller coaster train: lift hills, brake runs, and many others. She now has to put them together and propose a final design of the roller coaster.

For each (i) between (o) and (n-1), inclusive, the special section (i) has two properties:

- when entering the section, there is a speed limit: the speed of the train must be at most \(s_i\) km/h (kilometers per hour),
- when leaving the section, the speed of the train is **exactly** \(t_i\) km/h, regardless of the speed at which the train entered the section.

The finished roller coaster should contain each of the (n) sections exactly once. Moreover, there should be a track between every two consecutive sections. Anna should choose the order of the (n) sections and then decide the lengths of each track. The length of a track is measured in meters and may be equal to any nonnegative integer (possibly zero).

Each meter of the track between two special sections slows the train down by 1 km/h. At the beginning of the ride, the train enters the first special section going at 1 km/h.

The final design must satisfy the following requirements:

- the train does not violate any speed limit when entering the special sections;
- the speed of the train is positive at any moment until it reaches the end of the last special section.

In all subtasks except subtask 3, your task is to find the order of the (n) special sections and the lengths of tracks between consecutive sections, in such a way that the total length of tracks between sections is as small as possible. In subtask 3 you only need to check whether there exists a valid roller coaster design, such that each track has zero length.

Implementation details

You should implement the following function (method):

- int64 plan roller coaster(int[] s, int[] t).
 - \circ s: array of length \((n\), maximum allowed entry speeds.
 - t: array of length \(n\), exit speeds.
 - The function should return the minimum possible total length of all tracks between special sections (in subtask 3 you can output any positive integer if

answer is not zero, see details in the Subtasks section).

For the C language the function signature is slightly different:

- int64 plan roller coaster(int n, int[] s, int[] t)
 - n: the number of elements in s and t (i.e., the number of special sections),
 - the other parameters are the same as above.

Example

int64 plan_roller_coaster([1, 4, 5, 6], [7, 3, 8, 6])

It this example there are four special sections. The best solution is to build them in the order (0, 3, 1, 2), and to connect them by tracks of lengths (1, 2, 0) respectively. This is how a train travels along this railroad track:

- Initially the speed of the train is \(1\) km/h.
- The train starts the ride by entering special section \(o\).
- The train leaves section (0) going at (7) km/h.
- Then there is a track of length \(1\) m. When the train reaches the end of the track, its speed is \(6\) km/h.
- The train enters special section \(3\) going at \(6\) km/h and leaves it at the same speed.
- After leaving section \(3\), the train travels along a \(2\)m long track. Its speed decreases to \(4\) km/h.
- The train enters special section \(1\) going at \(4\) km/h and leaves it going at \(3\) km/h.
- Immediately after special section \(1\) the train enters special section \(2\).
- The train leaves section (2). Its final speed is (8) km/h.

The function should return the total length of track between special sections: (1+2+0=3).

Subtasks

In all subtasks $(1 \le i \le 10^9)$ and $(1 \le t_i \le 10^9)$.

- 1. (11 points): $(2 \le n \le 8)$,
- 2. (23 points): $(2 \le n \le 16)$,
- 3. (30 points): \(2\le n\le 200\,000\). In this subtask your program only needs to check whether the answer is zero or not. If the answer is not zero, any positive integer answer is considered correct.
- 4. $(36 \text{ points}): (2 \ln n \le 200,000).$

Sample grader

The sample grader reads the input in the following format:

- line 1: integer \((n\)).
- line 2 + i, for (i) between (o) and (n-1): integers (s_i) and (t_i) .