

Introduction to Machine Learning

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Outline

1 Introduction

2 Preliminaries

3 Learning

4 Model Evaluation

5 Decision Trees

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Our Reality: We Are Drowning In Data

- Every click, every network packet and every online interaction is recorded and will outlive us.

2018 *This Is What Happens In An Internet Minute*



Data Science

- The umbrella field of data science aims to harness this massive amount of data to glean insights, drive industry and improve the human experience.



Scientific Literature Search - Finding The Needle in the Haystack

4

scientific papers published every minute



160

million documents indexed
on Google Scholar



24

million documents listed on PubMed

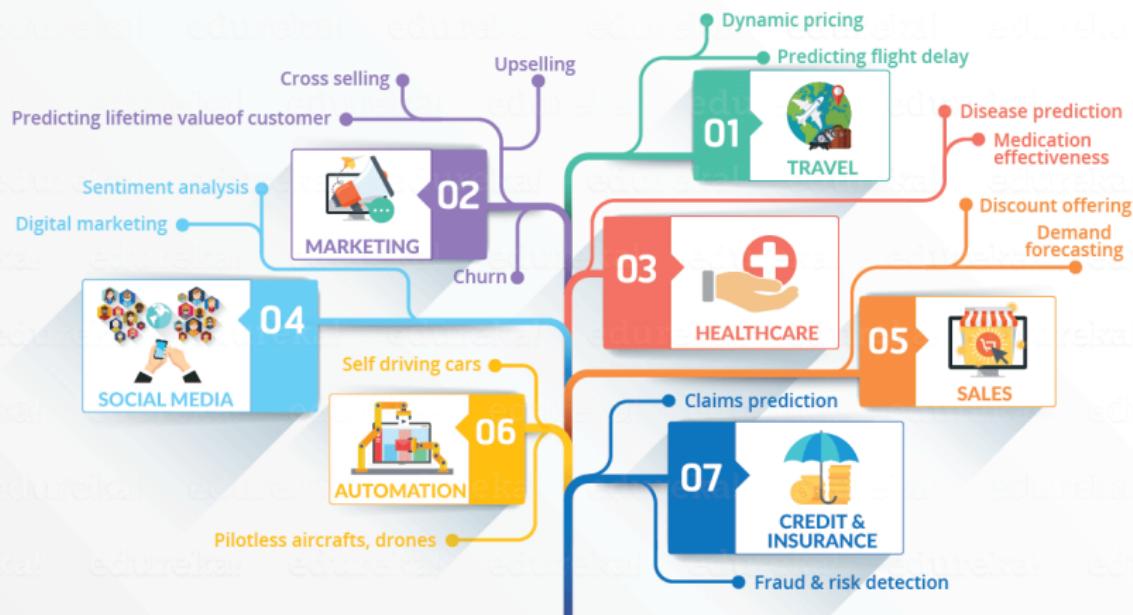


12

million documents listed on Science Direct

Societal Ramifications and Applications

- Health care (drug re-purposing, RNA sequencing)
- Automated knowledge discovery (scientific literature mining)
- Ease of mass surveillance (social credit systems, geographic tracking)
- Targeted advertising (political elections, big nudging)



Pattern Recognition and Machine Learning

- This talk focuses on gaining insight into the underlying algorithms and structures that produce models capable of tackling such problems.
- We will explore in-depth a heuristical algorithm - the supervised Decision Tree.

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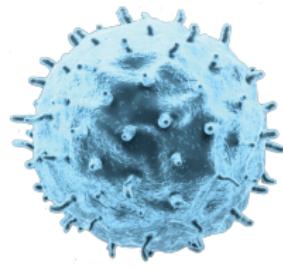
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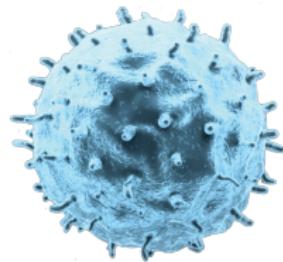
Entities

- The world around us can be succinctly described as comprised of entities.



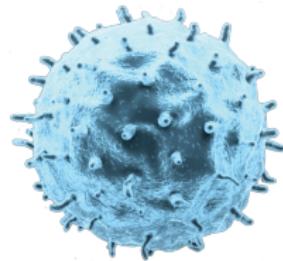
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- Entities interact (relate) with other entities and in the process possibly form new entities.



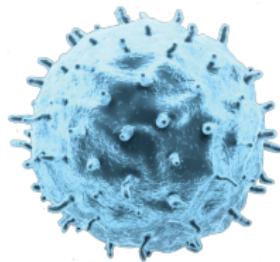
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- Entities are composed of **attributes**.
- An **instance** of an entity is a particular manifestation of the **attributes** that comprise the entity.



Instances of Entities

- A collection of instances $x_i \in \mathbb{D}$ from an entity domain \mathbb{D} forms a sample from the domain.



	account_balance	income	age	has_defaulted	credit_worthy
x_1	57103	50000	25	0	?
x_2	20343	40000		1	?
x_3	80000	150000	4	1	?
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- The **dimension** d of \mathbb{D} is the number of attributes describing its entity.
- We assume attributes (**observations, features**) of a given instance $x_{i,j}$ ($0 \leq j < d$) are real numbers.



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Paradigms

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Given a **representative** sample of **cardinality** n from a domain \mathbb{D} of dimension d :

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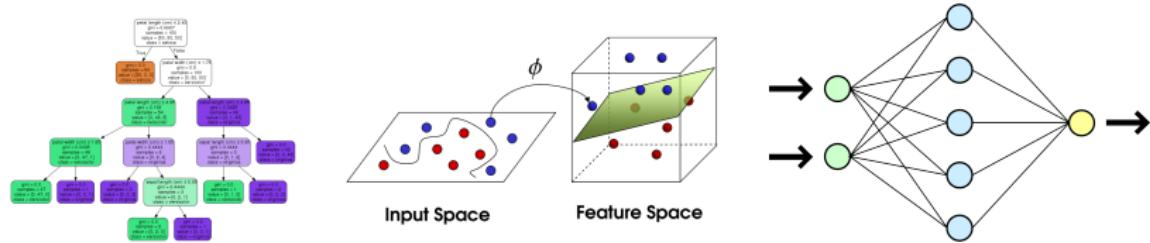
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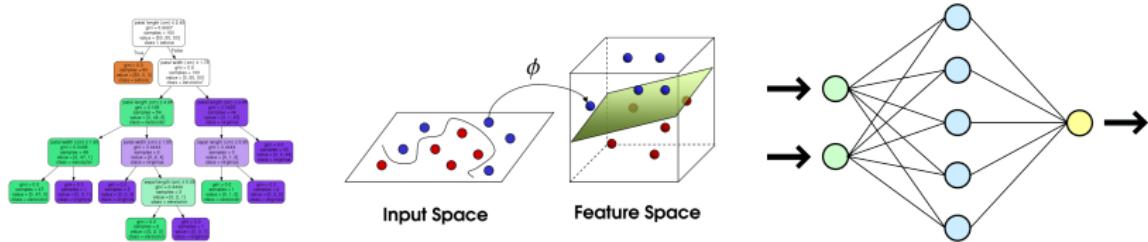
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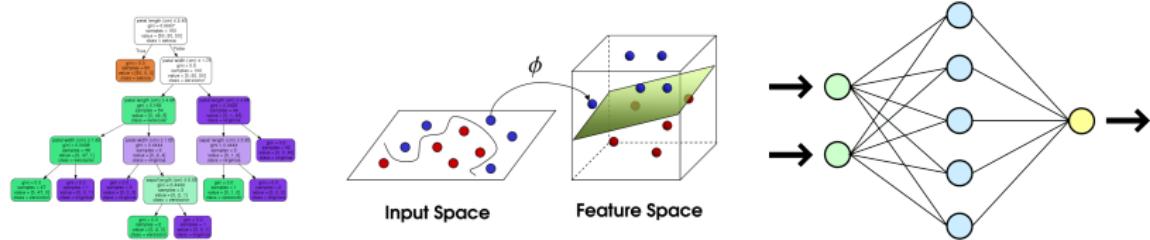
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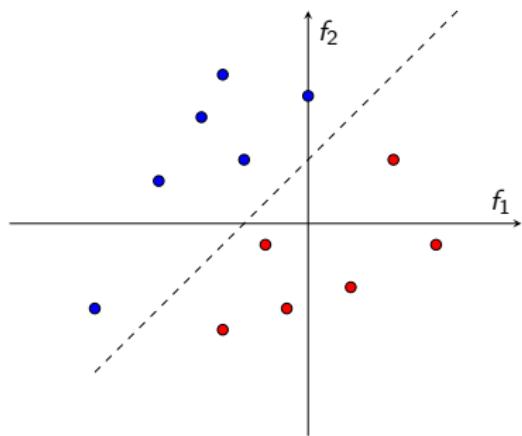
Supervised Learning

- Machine learning algorithms exploit X to estimate an h that has low predictive error.
- In the supervised setting, a machine learning algorithm finds (learns) a mapping from observations X to outputs y .
- The rest of this talk explores how such a mapping (**classifier, model**) can be learned.

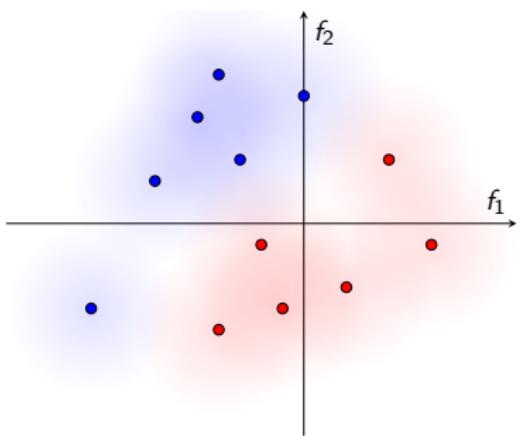


Learning a Supervised Model

- Supervised Learning algorithms differ by the space of hypothesis \mathbb{H} they search when selecting an $h \in \mathbb{H}$ with optimal predictive ability.



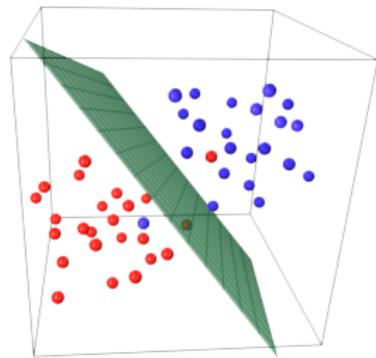
(a) Discriminative Classifier



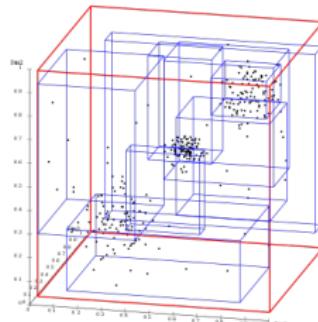
(b) Generative (probabilistic) Classifier

Discriminative Models

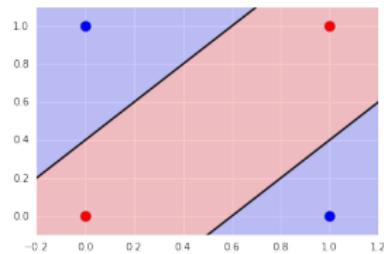
- Discriminative algorithms search the hypothesis space of geometric **decision boundaries** separating classes.



(a) Hyperplanes



(b) Hyper-rectangles



(c) Nested-Hyperplanes

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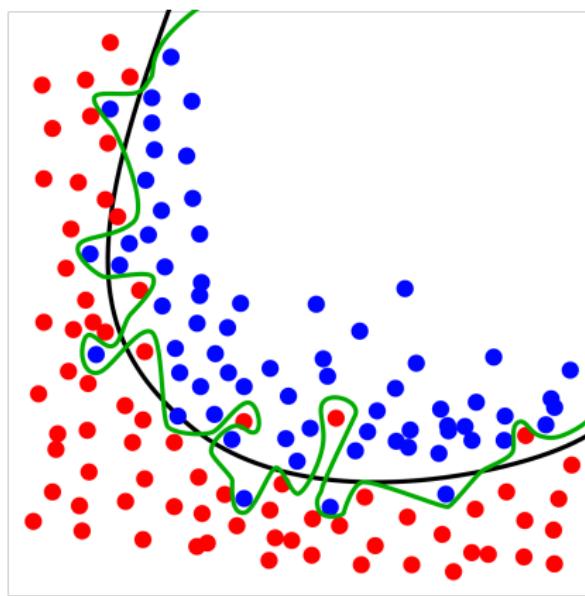
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Generalization Ability

- Recall that a hypothesis function (model) h is the output of our learning algorithm applied to our training sample X .

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- Recall that a hypothesis function (model) h is the output of our learning algorithm applied to our training sample X .
- How can we ensure that our selected h is consistent with \mathbb{D} (i.e. generalizes)?



Generalization Ability

- Recall our training set:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1d} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2d} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{d2} & x_{d3} & \dots & x_{nd} \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$



(a) Hold-out Subset



(b) Cross Validation (partitioning)

Hold Out Validation

Back to our loan example: a supervised algorithm trained on a training partition $T \subset X$ and evaluated on a test partition $X \setminus T$ building a model h :

$$h(T) = \begin{bmatrix} \text{give} \\ \text{deny} \\ \text{deny} \\ \text{give} \\ \text{give} \\ \text{deny} \end{bmatrix}$$

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True Positive (TP)

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	PP	PN
P	2	1
N	2	1

$$\text{Accuracy} = \frac{TP + TN}{N} = \frac{3}{6}$$

$$\text{Recall} = \frac{TP}{TP + FN} = \frac{2}{3}$$

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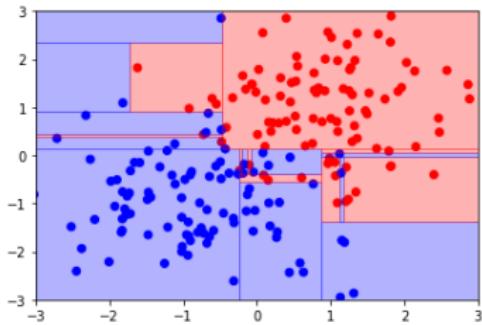
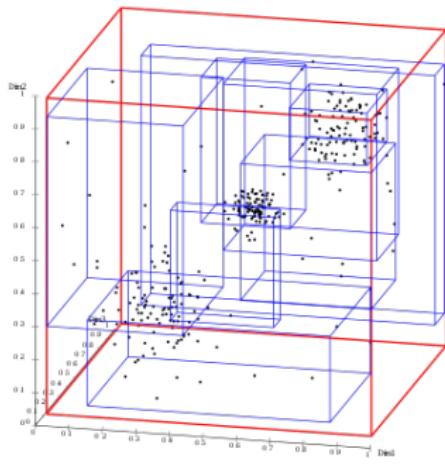
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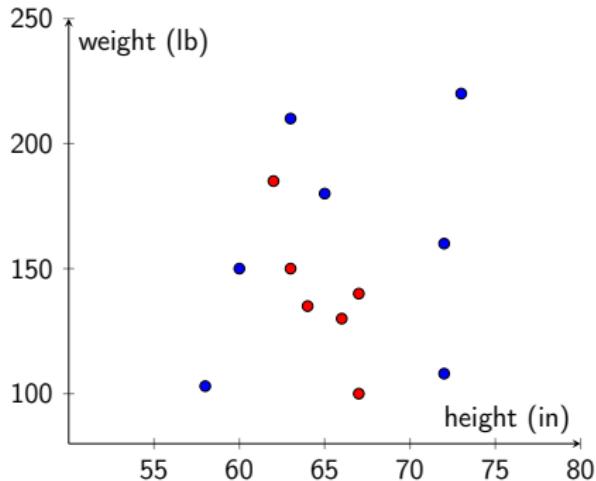
The Supervised Decision Tree

- Decision Trees are a class of discriminative learning algorithm searching the hypothesis space of hyper-rectangles.
- Given a labeled training sample X , a Decision Tree produces a model h mapping partitioned hyper-rectangles in \mathbb{R}^d to target classes.



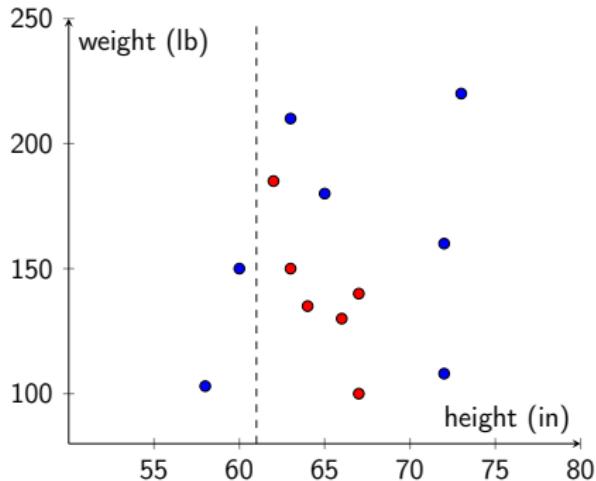
Decision Tree Induction

- Decision Trees build a model by recursively applying a simple heuristic: select a cut-point on the attribute that maximizes the class purity in the regions resulting from the cut-points partitioning.



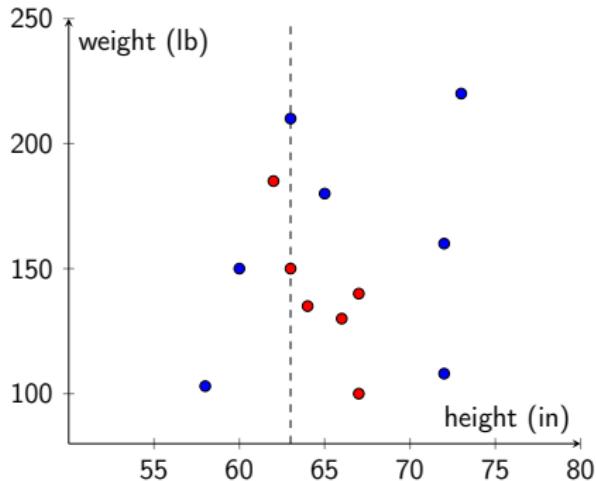
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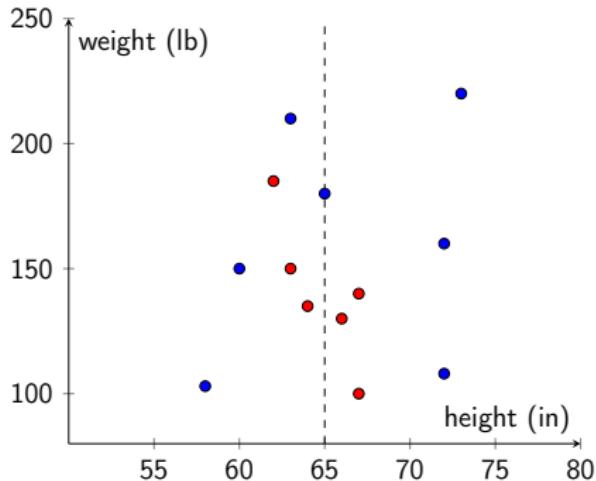
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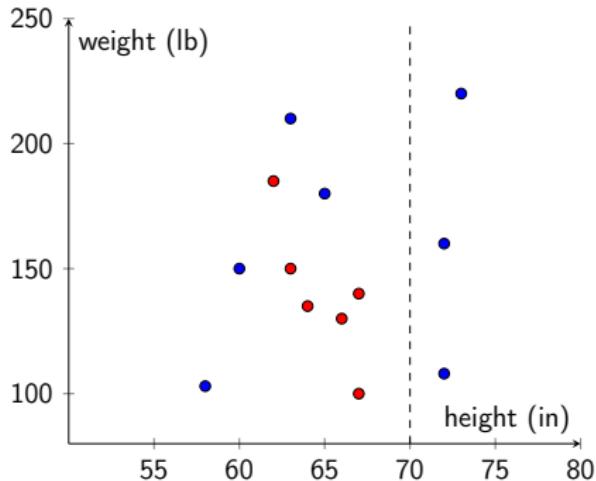
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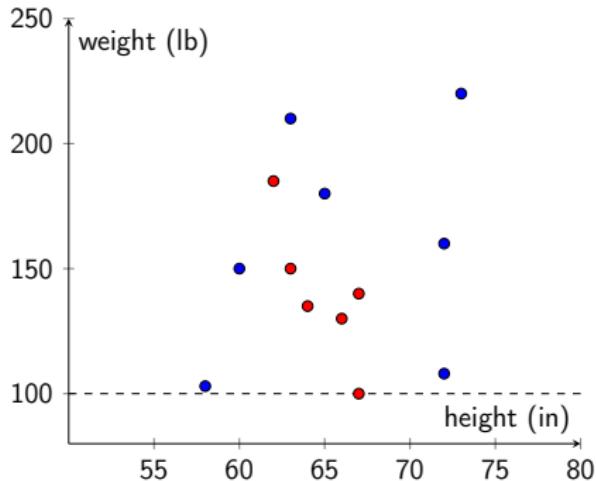
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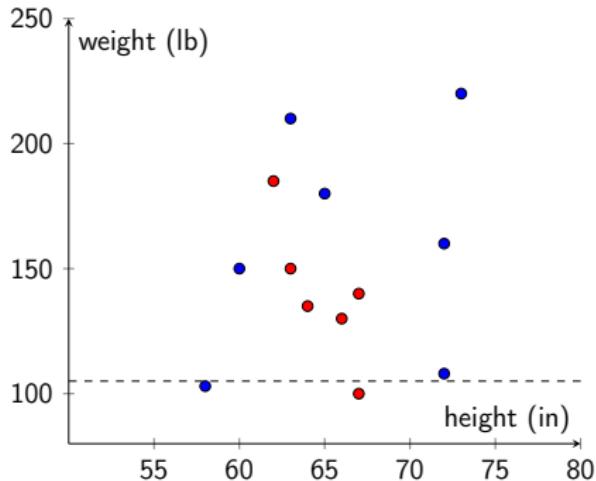
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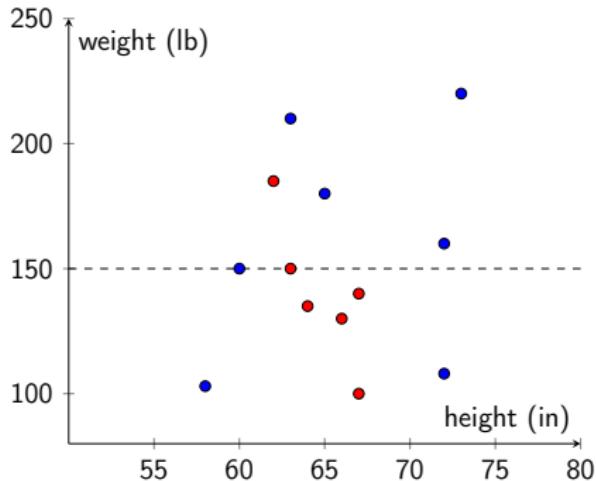
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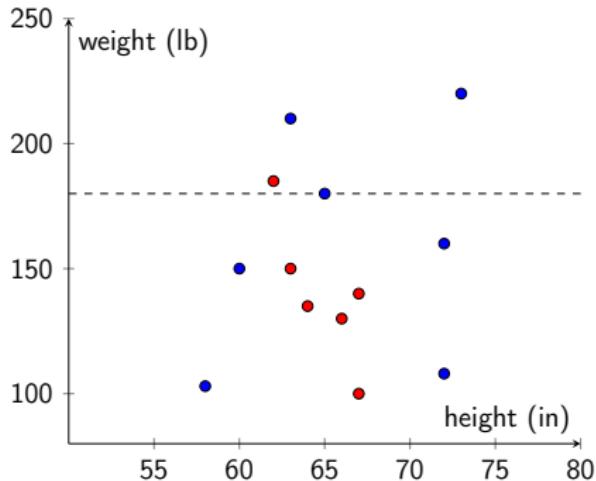
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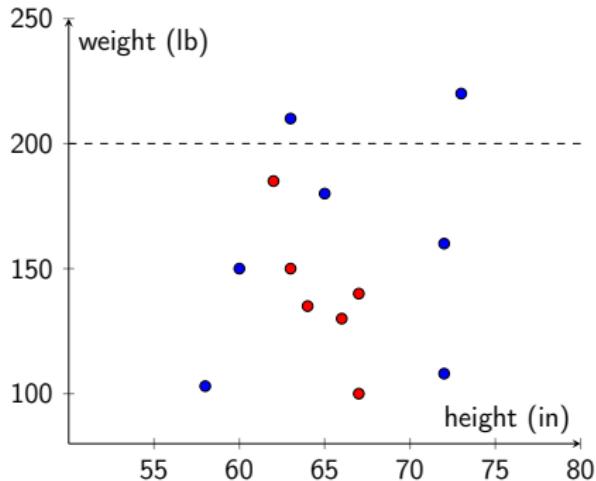
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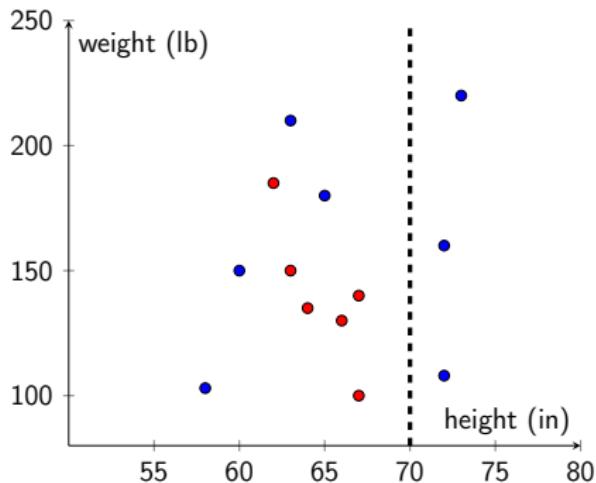
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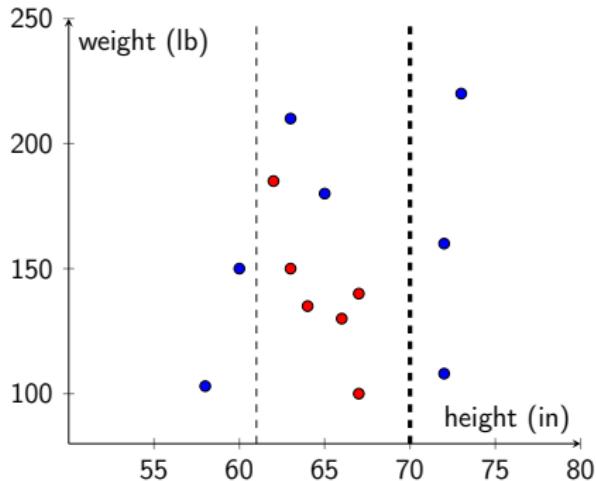
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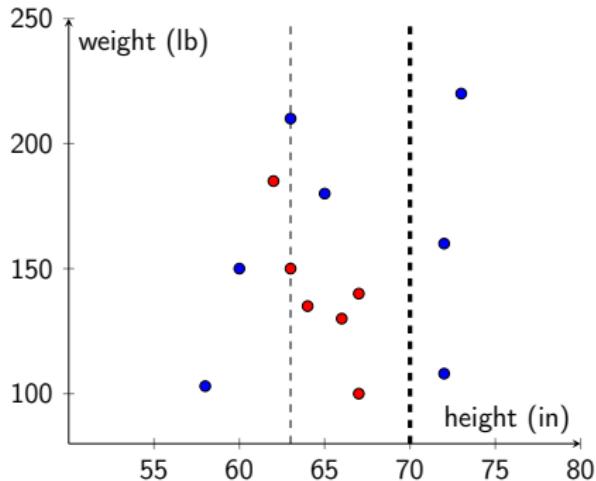
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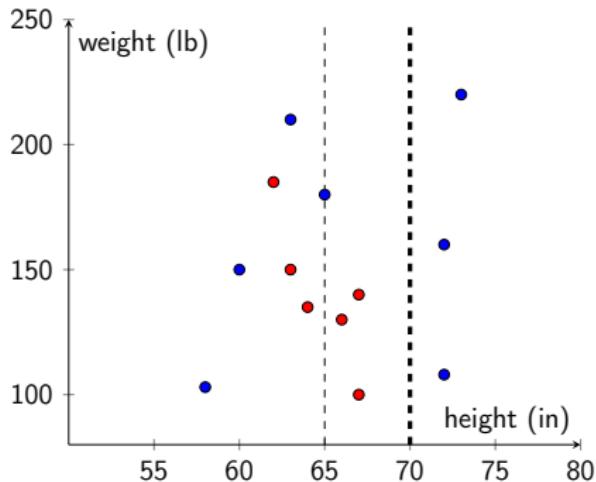
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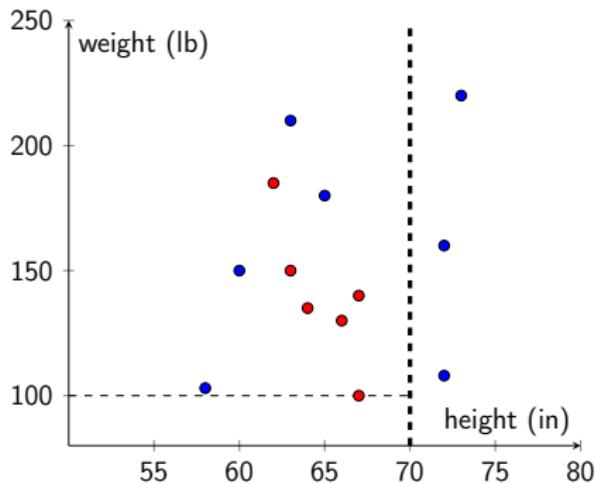
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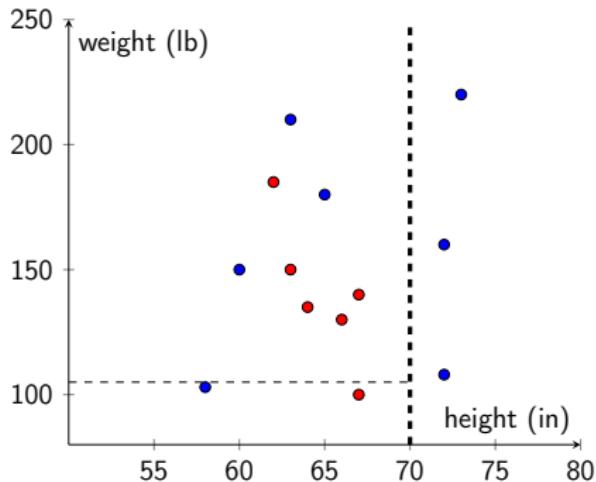
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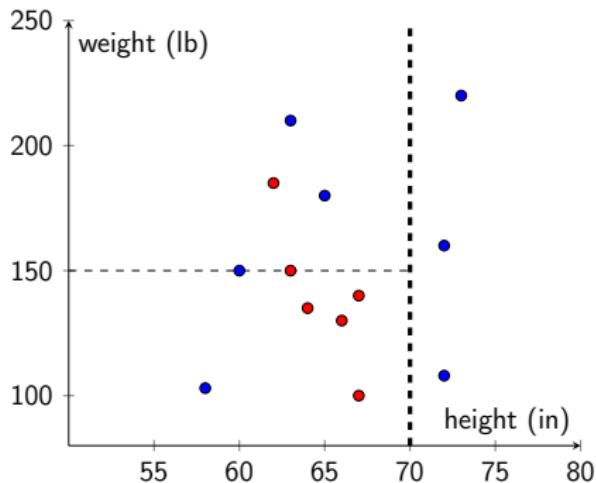
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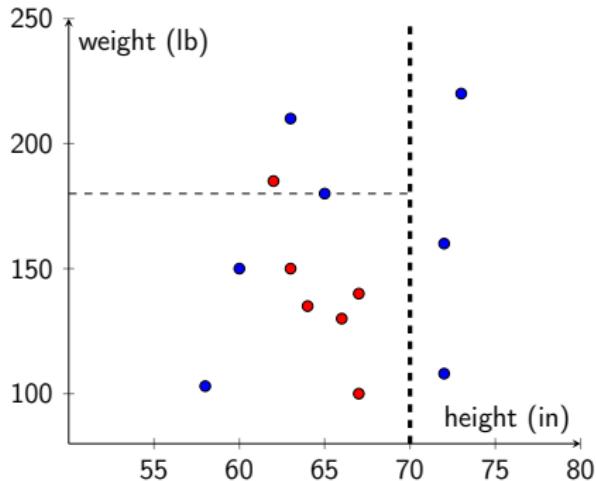
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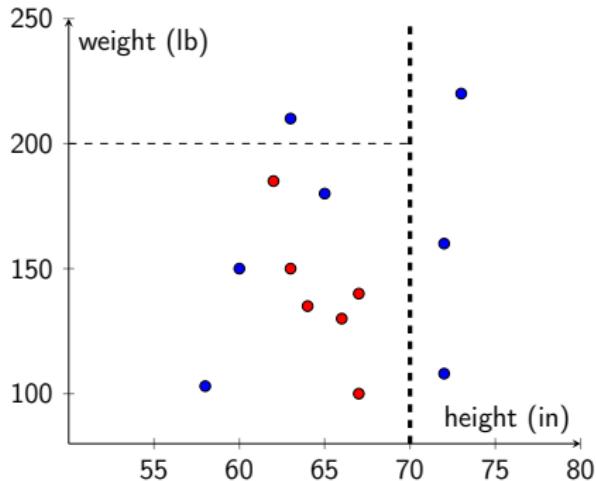
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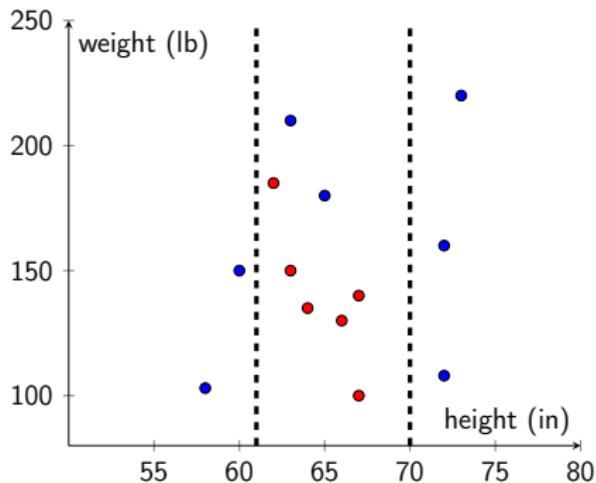
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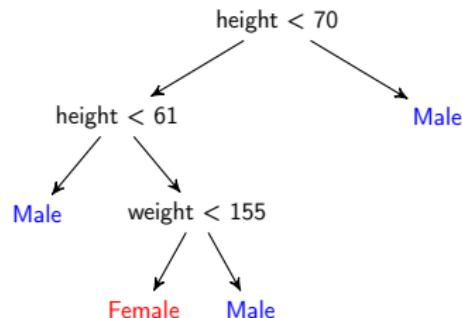
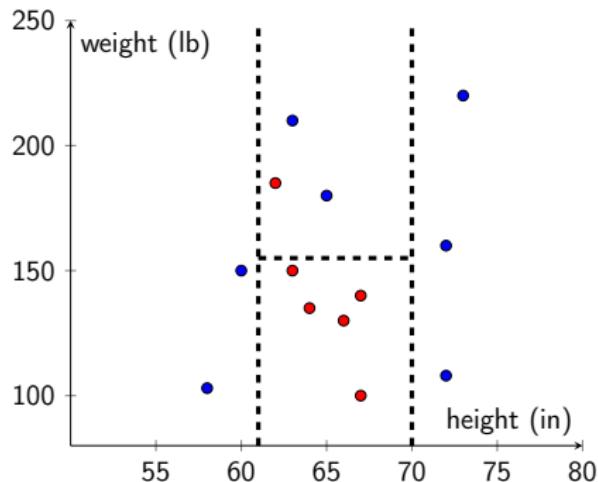
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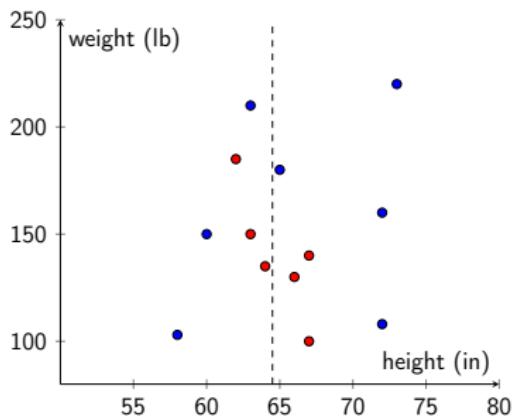
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The Impurity Criterion: A Heuristic

- A potential split (partition) induces two class distributions derived from the class distribution of the parent partition.



The Impurity Criterion: A Heuristic

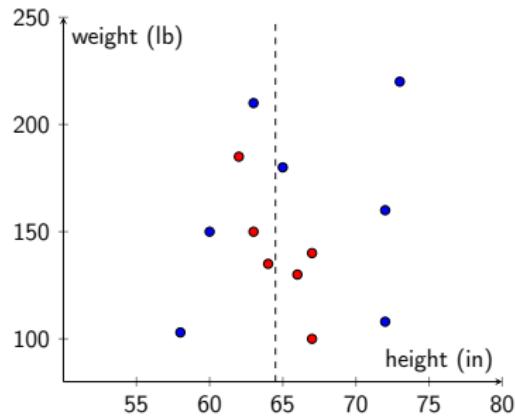
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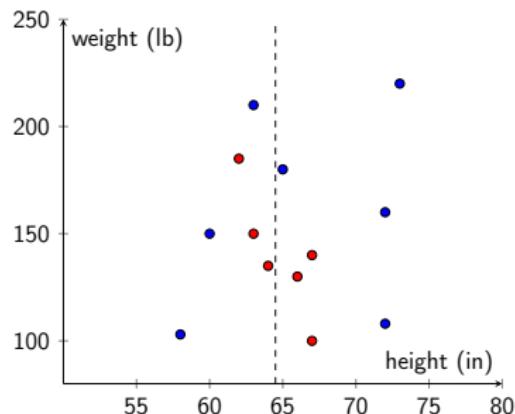
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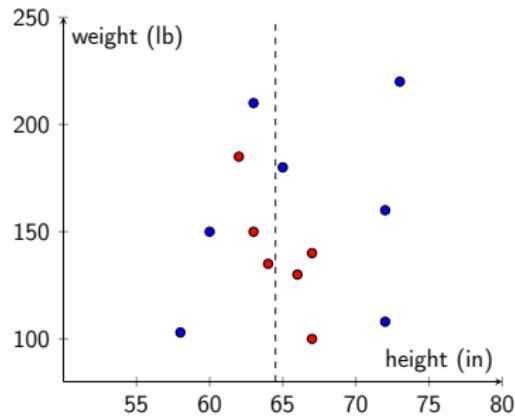
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Conclusion

**Thank you for your attention!
Questions?**

<https://bit.ly/2tPVPfv>

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