Check a(x)

Andre YI

$Y_t = h(X_t),\ h'(x) = \dfrac{1}{\sigma(x)}$、ここで $\sigma(x) = \sqrt{x^2+1}$ である。
よって、

$$h'(x) = \frac{1}{\sqrt{x^2+1}} \implies h(x) = \operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

伊藤公式より、

$$dY_t = h'(X_t)dX_t + \frac{1}{2}h''(X_t)\sigma^2(X_t)dt \cdots\cdots (1)$$

ここで、

$$h'(X) = \frac{1}{\sqrt{X^2+1}},\ h''(X) = -\frac{X}{(X^2+1)^{3/2}}$$

次に、 $dX_t = a(X_t)dt + \sigma(X_t)dW_t$ と $\sigma^2(X_t) = X_t^2 + 1$ を (1) 式に代入すると、

$$\begin{aligned} dY_t &= \frac{1}{\sqrt{X_t^2+1}}\left[a(X_t)dt + \sigma(X_t)dW_t\right] + \frac{1}{2}\left(-\frac{X_t}{(X_t^2+1)^{3/2}}\right)(X_t^2+1)dt \\ &= \left[\frac{a(X_t)}{\sqrt{X_t^2+1}} - \frac{X_t}{2\sqrt{X_t^2+1}}\right]dt + \frac{\sigma(X_t)}{\sqrt{X_t^2+1}}dW_t. \end{aligned}$$

ここで、$\sigma(X_t) = \sqrt{X_t^2+1}$, $\sigma'(X_t) = \frac{X_t}{\sqrt{X_t^2+1}}$, $a(X_t) = \dfrac{X_t}{2} + \dfrac{\sqrt{X_t^2+1}}{2}$ である。
したがって、

$$dY_t = \left[\frac{X_t + \sqrt{X_t^2+1}}{2\sqrt{X_t^2+1}} - \frac{X_t}{2\sqrt{X_t^2+1}}\right]dt + dW_t = \frac{1}{2}dt + dW_t$$