# AST1501 Notes

## 1. Coordinate Transformation

### a. Stream to Equatorial Coordinates

$$\begin{bmatrix} \cos(\phi_1)\cos(\phi_2) \\ \sin(\phi_1)\cos(\phi_2) \\ \sin(\phi_2) \end{bmatrix} = \begin{bmatrix} -0.4776303088 & -0.1738432154 & 0.8611897727 \\ 0.510844589 & -0.8524449229 & 0.111245042 \\ 0.7147776536 & 0.493068392 & 0.4959603976 \end{bmatrix} \times \begin{bmatrix} \cos(\alpha)\cos(\delta) \\ \sin(\alpha)\cos(\delta) \\ \sin(\delta) \end{bmatrix} \tag{1}$$

So if we assume the left hand side matric is A and the others are B and C, respectively, we can get C matrix by:

$$\begin{aligned} A &= BC \qquad (2) \\ B^{-1}A &= (B^{-1}B)C \\ B^{-1}A &= C, \end{aligned}$$

$$(3)$$

therefore, we get:

$$\begin{bmatrix} \cos(\alpha)\cos(\delta) \\ \sin(\alpha)\cos(\delta) \\ \sin(\delta) \end{bmatrix} = \begin{bmatrix} -0.4776303088 & 0.510844589 & 0.7147776536 \\ -0.1738432154 & -0.8524449229 & 0.493068392 \\ 0.8611897727 & 0.111245042 & 0.4959603976 \end{bmatrix} \times \begin{bmatrix} \cos(\phi_1)\cos(\phi_2) \\ \sin(\phi_1)\cos(\phi_2) \\ \sin(\phi_2), \end{bmatrix} \tag{4}$$

and we can get $\delta$ and $\alpha$ from $\phi_1$ and $\phi_2$ by:

$$\delta = \arcsin (B^{-1}A)[3] \tag{5}$$

$$\alpha = \arctan \frac{(B^{-1}A)[2]}{(B^{-1}A)[1]}$$

$$(6)$$

Velocity Transformations galpy uses positions of the stream and its velocity in cylindrical coordinate, but we have these values in the stream frame. So we can convert the velocity in the stream coordinate to the velocities in the equatorial coordinates by:

$$
\begin{aligned}
v_\delta &= \frac{d\delta}{dt} = \frac{d(\arcsin A_7 \cos(\phi_1)\cos(\phi_2) + A_8 \sin(\phi_1)\cos(\phi_2) + A_9 \sin(\phi_2))}{dt} \qquad (\\
&= \frac{-A_7\left[\sin(\phi_1)\cos(\phi_2)\mu_{\phi_1} + \cos(\phi_1)\sin(\phi_2)\mu_{\phi_2}\right] + A_8\left[\cos(\phi_1)\cos(\phi_2)\mu_{\phi_1} - \sin(\phi_1)\sin(\phi_2)\mu_{\phi_2}\right] + A_9\cos(\phi_2)\mu}{\sqrt{A_7\cos(\phi_1)\cos(\phi_2) + A_8\sin(\phi_1)\cos(\phi_2) + A_9\sin(\phi_2)) - 1}}
\end{aligned}
$$

$\mathrm{v}_\alpha = \frac{d\alpha}{dt} =$