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ALFVÉN SURFACE WAVES ALONG CYLINDRICAL PLASMA COLUMNS

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Abstract—The properties of Alfvén surface waves along a cylindrical plasma column surrounded by vacuum or by another plasma medium are discussed. Both symmetric ($m = 0$) and asymmetric ($m = \pm 1$) modes are found to be dispersive in nature. The interfacial symmetric modes propagate in a certain frequency window (ω_{A1}, ω_{As}), where ω_{As} is the Alvén surface wave frequency along the interface of two semi-infinite media; when $v_{A1} > v_{A2}$ these modes propagate as backward waves and when $v_{A1} < v_{A2}$ as forward waves. The asymmetric modes change from backward to forward waves at a critical wave number $k_{Tr} \simeq 1.59/a$ when $v_{A1} < v_{A2}$ or vice versa when $v_{A1} > v_{A2}$.

1. INTRODUCTION

IT IS NOW well known that an interface between two plasma media can support low frequency Alfvén surface waves propagating along the magnetic field, in the plane of the interface (KRUSKAL and SCWARZSCHILD, 1954; GERWIN, 1967; CHEN and HASEGAWA, 1974). For a semi-infinite plane geometry these waves are non-dispersive and propagate along a plasma–plasma interface with constant frequency

$$\omega_{As} = \frac{k_{11}}{\sqrt{4\pi}}\left[\frac{B_{01}^{\,2} + B_{02}^{\,2}}{\rho_{01} + \rho_{02}}\right]^{1/2},$$

where k_{11} is the wave number along the magnetic field.

Recently, the study of Alfvén surface waves has acquired a new interest as the resonant absorption of these waves provides a mechanism for irreversible heating of laboratory plasma (UBEROI, 1972; HASEGAWA and CHEN, 1976) and Coronal plasma (IONSON, 1978).

Surface waves constitute an entirely two-dimensional phenomenon and, therefore, their propagation characteristics can, in general, be affected by the surface geometry. Since, many discontinuous structures in the laboratory and in astrophysical context like coronal loops are cylindrical rather than planar, in this short note we study the properties of Alfvén surface waves along cylindrical plasma columns. We discuss both symmetric ($m = 0$) and asymmetric ($m = \pm 1$) surface modes both for plasma–vacuum and plasma–plasma interfaces. As our aim is to find the curvature effects on 'pure' Alfvén surface waves we consider the ideal situation as in earlier works with semi-infinite plane geometry. We find that, just as in the case of high frequency electrostatic surface waves (TRIVELPIECE and GOULD, 1959), the curvature introduces dispersion which gives rise to many interesting features in the characteristic propagation of Alfvén surface waves.

2. DISPERSION RELATION

In magnetohydrodynamic approximation the linearised equations governing the electromagnetic properties of an incompressible, conducting plasma fluid of equilibrium mass density ρ_0, embedded in an external magnetic field $\bar{B}_0$ are:

$$\nabla \bar{v} = 0 \tag{2.1}$$

$$\rho_0 \frac{\partial \bar{v}}{\partial t} = -\nabla p + \frac{1}{4\pi}(\nabla \times \bar{b}) \times \bar{B}_0 \tag{2.2}$$

$$\frac{\partial \bar{b}}{\partial t} = \nabla \times (\bar{v} \times \bar{B}_0) \tag{2.3}$$

$$\nabla \cdot \bar{b} = 0 \tag{2.4}$$

where $\bar{v}$, p and $\bar{b}$ are the perturbed fluid velocity, pressure and magnetic field respectively.

Taking divergence of equation (2.2) and defining the total pressure $\tilde{p} = p + \frac{\bar{B}_0 \cdot \bar{b}}{4\pi}$, we get

$$\nabla^2 \tilde{p} = (B_0 \cdot \nabla)(\nabla \cdot \bar{b})/4\pi - \rho_0 \frac{\partial}{\partial t}(\nabla \cdot \bar{v}) \tag{2.5}$$

which, on using equations (2.1) and (2.4), reduced to the Laplace's equation

$$\nabla^2 \tilde{p} = 0. \tag{2.6}$$

Using equations (2.2) and (2.3), the fluid velocity can be written as:

$$\left[\frac{(\bar{B}_0 \cdot \nabla)^2}{4\pi} - \rho_0 \frac{\partial^2}{\partial t^2}\right] \bar{v} = \frac{\partial}{\partial t} \nabla \tilde{p}. \tag{2.7}$$

Equations (2.6) and (2.7) form the complete set of equations for the study of Alfvén surface waves.

The dispersion relation for Alfvén surface waves is obtained by applying the boundary conditions, that the total pressure $\tilde{p}$ and the normal component of the fluid velocity across the boundary are continuous. From equation (2.7) we note that the normal component v_n of the fluid velocity is proportional to $\nabla_n \tilde{p}$. Hence, the discussion of Alfvén surface waves is reduced to solving Laplace's equation (2.6) for $\tilde{p}$, with the boundary conditions on $\tilde{p}$ and its normal derivative $\nabla_n \tilde{p}$, thus making the problem similar to the study of high frequency electrostatic surface waves in cold plasmas (see for example, TALMADGE *et al.*, 1973).

Consider a cylindrical plasma column of radius 'a' (medium 1) surrounded by another plasma medium (medium 2) with mass density ρ_{01} and ρ_{02} respectively, immersed in an external magnetic field $\bar{B}_{01,2}$ oriented along the interface. For perturbations of the form $f(r, \varphi, z, t) = f(r) \exp i(kz + m\varphi - \omega t)$ the solution of equation (2.6) in media 1 and 2 gives:

$$\tilde{p}_1 = A_1 I_m(kr) \qquad r < a \tag{2.8}$$

$$\tilde{p}_2 = A_2 K_m(kr) \qquad r > a \tag{2.9}$$

where $I_m(kr)$ and $K_m(kr)$ are the modified Bessel's function of order m.

Matching the boundary conditions:

(i) $\tilde{p} = \tilde{p}_2$

(ii) $\frac{1}{\varepsilon_1(k, \omega)} \frac{\partial \tilde{p}_1}{\partial r} = \frac{1}{\varepsilon_2(k, \omega)} \frac{\partial \tilde{p}_2}{\partial r}$

where $\varepsilon_{1,2}(k, \omega) = B_{01,2}^2 k^2/4\pi - \rho_{01,2}\omega^2$; at $r = a$ we get the dispersion relation as:

$$\varepsilon_1(k, \omega) + \varepsilon_2(k, \omega)F_m(ka) = 0 \tag{2.10}$$

where

$$F_m(ka) = -\frac{I_m'(ka)K_m(ka)}{K_m'(ka)I_m(ka)}. \tag{2.11}$$

Equation (2.10) can be written as

$$\rho_{01}(v_s^2 - v_{A1}^2) + \rho_{02}(v_s^2 - v_{A2}^2)F_m(ka) = 0 \tag{2.12}$$

where $v_s = \omega/k$ is the Alfvén surface wave velocity and $v_{A1,2}$ are the bulk Alfvén velocities in the two media. The phase velocity of the Alfvén surface wave for cylindrical geometry is then given by

$$v_s = v_{A1}\left[\frac{1+\beta^2 F_m(ka)}{1+\eta F_m(ka)}\right]^{1/2} \tag{2.13}$$

where $\beta = B_{02}/B_{01}$ and $\eta = \rho_{02}/\rho_{01}$ are the interface parameters.

3. DISCUSSION

From equation (2.12) we note that the curvature effect on the propagation characteristics of Alfvén surface waves in cylindrical geometry appears through the function $F_m(ka)$, independent of the interface parameters β and η. The function $F_m(ka)$ is always a positive quantity for all positive values of ka. Figure 1 shows the variation of $F_m(ka)$ with ka for $m = 0$ and $m = \pm 1$. $F_0(ka)$ increases

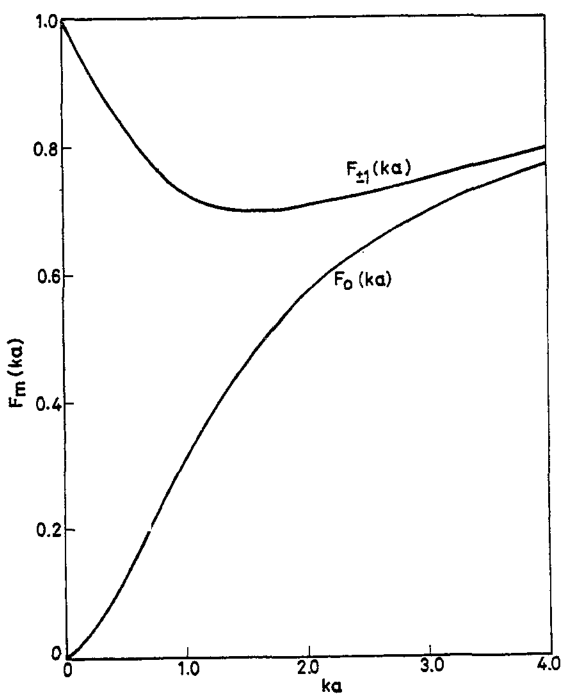

FIG. 1.—Variation of the function $F_m(ka)$ with ka for $m = 0$ and $m = \pm 1$.

with ka and tends to one as $ka \to \infty$, whereas $F_{\pm 1}(ka)$ shows a minimum at $k = k_{Tr}$ given by $F_{\pm 1}'(k_{Tr}a) = 0$ before reaching the maximum value of one.

When $ka \to \infty$, equation (2.13) gives

$$v_s = v_{A1}\left(\frac{1+\beta^2}{1+\eta}\right)^{1/2}, \tag{3.1}$$

the Alfvén surface wave velocity for the semi-infinite plane geometry. It is interesting to note that equation (3.1) for plasma–vacuum interface i.e. when $\eta = 0$ gives

$$\omega_{A2} = k_{11}\left[\frac{B_{01}{}^2 + B_{02}{}^2}{4\pi\rho_{01}}\right]^{1/2}$$

which is the Alfvén surface wave frequency obtained by Kruskal and Scwarzschild (1954). In the particular case with $B_{01} = B_{02}$, this frequency becomes (Chen and Hasegawa, 1974)

$$\omega_{As} = k_{11}\sqrt{2}v_{A1}.$$

It is to be noted that the surface wave frequency in this case is higher than the bulk Alfvén wave frequency, whereas in the case $\rho_{01} \approx \rho_{02}$, met with in coronal loops, for example.

$$\omega As = \frac{1}{\sqrt{2}} v_{A1} k_{11}$$

on taking $B_{02} = 0$. This frequency is less than the bulk Alfvén frequency, similar to the case of high frequency electrostatic plasma surface wave (Trivelpiece and Gould, 1959).

For cylindrical geometry, considering first the case of a plasma column surrounded by vacuum, we have

$$v_s = v_{A1}[1 + \beta^2 F_m(ka)]^{1/2}. \tag{3.2}$$

The phase velocity is always greater than the bulk Alfvén velocity. In Fig. 2 the dispersion characteristics of symmetric (solid lines) and asymmetric (broken lines) modes are shown for different values of the interface parameter β. The phase velocity of the symmetric mode is seen to increase with increase in β. The asymmetric modes $m = \pm 1$ have the same phase velocity. Due to the nature of the function $F_{\pm 1}(ka)$ the asymmetric modes exhibit an interesting property that at $k = k_{Tr} (\simeq 1.59/a)$ the waves change from backward to forward waves.

Considering the equation (2.12) for $\eta \neq 0$, it can be easily seen that real roots of this equation will exist only if v_s satisfies the following inequality:

$$\min(v_{A2}, v_{A1}) < v_s < \max(v_{A1}, v_{A2}).$$

In Fig. 3, the dispersion characteristics for symmetric (solid lines) and asymmetric (broken lines) modes are shown for different values of the interface parameter β for $\eta = 0.5$. The symmetric modes change from backward to forward waves at $\beta^2 = \eta$. The asymmetric modes at the wave number k_{Tr} transform from forward to backward waves when $\beta^2 < \eta$ and backward to forward when $\beta^2 > \eta$. The most interesting result as seen from Fig. 3 is that the symmetric modes propagate in a

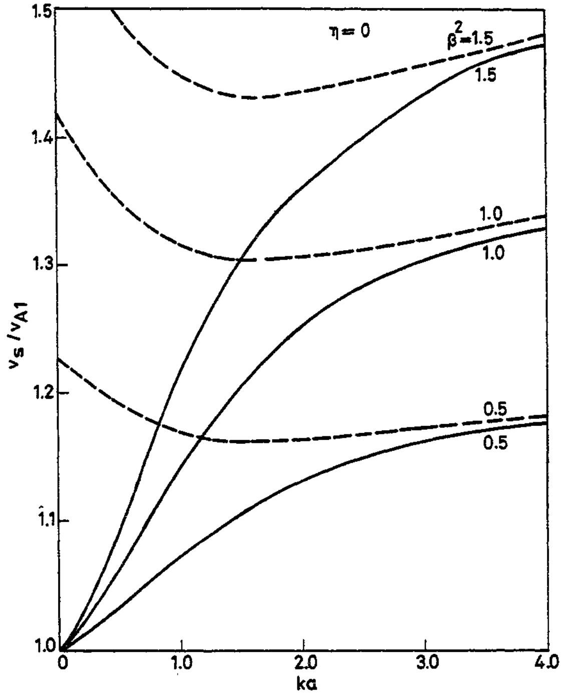

FIG. 2.—Dispersion characteristics of Alfvén surface waves along plasma–vacuum ($\eta = 0$) interface for various values of β. Solid lines represent the symmetric mode ($m = 0$) and broken lines the asymmetric mode ($m = \pm 1$).

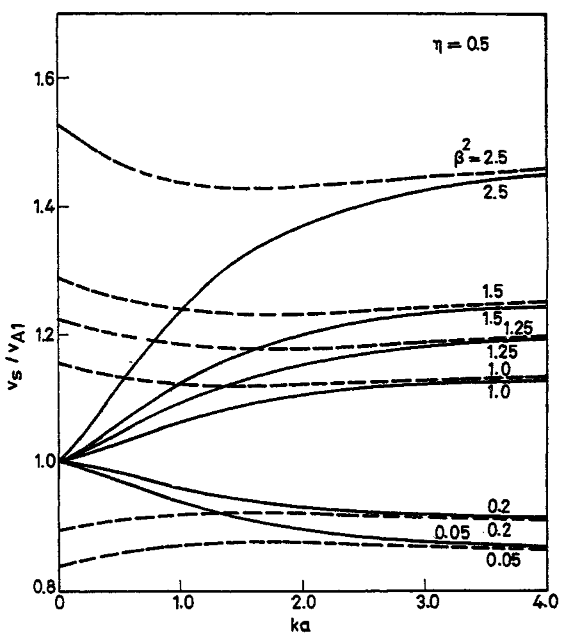

FIG. 3.—Dispersion characteristics of Alfvén surface wave along plasma–plasma interface ($\eta = 0.5$) for various values of β. Solid lines represent symmetric modes ($m = 0$) and broken lines the asymmetric modes ($m = \pm 1$).

certain frequency 'window', $(\omega_{A1}, \omega_{As})$. This window narrows down as β^2/η is close to one i.e., when $v_{A1} \simeq v_{A2}$ resulting in very little dispersion (ω_s = constant). Interestingly, this result is similar to that obtained in the case of high frequency electromagnetic surface waves propagating along the plane interface of two plasma media (UBEROI and RAO, 1975) or two semi-conductors (HALEVI, 1975).

The difference between the Alfvén speed inside and outside a cylindrical column is given as $\Delta v_A = |v_{A1} - v_{A2}| = v_{A1}|1-\sqrt{\beta^2/\eta}|$. From Fig. 3, we note that for $\beta^2 < \eta$ for a fixed ka the surface wave phase velocity v_s increases with decrease in the value of Δv_A, whereas for $\beta^2 > \eta$ it increases with increasing value of Δv_A.

4. CONCLUSION

A cylindrical plasma column supports Alfvén surface waves dispersive in nature. When the plasma column is surrounded by vacuum the phase velocity of the surface waves is greater than the bulk Alfvén velocity. The symmetric modes propagate as forward waves, but the asymmetric modes change from backward to forward waves at a critical wave number $k_{Tr}(\simeq 1.59/a)$, This feature of the asymmetric mode is also noted when the cylinder is surrounded by another plasma medium. In the case of plasma–plasma interface the symmetric modes propagate in the frequency range $(\omega_{A1}, \omega_{As})$ which can be made very narrow when Δv_A is small thus resulting in very little dispersion. Another interesting feature in the case of plasma–plasma interface is that both the symmetric and asymmetric modes show variation in the direction of the phase and group velocities at the critical value of the interface parameters given by $\beta^2 = \eta$.

It will be of interest to observe the Alfvén surface wave properties discussed in this note in the laboratory. The study of surface waves in a real plasma with resistivity and Hall current effects is under consideration and will form the subject matter of a separate paper.

REFERENCES

CHEN L. and HASEGAWA A. (1974) *J. geophys. Res.* **79,** 1033.
GERWIN R. (1967) *Phys. Fluids* **10,** 2164.
HASEGAWA A. and CHEN L. (1976) *Phys. Fluids* **19,** 1924.
HALEVI P. (1975) *Phys. Rev. B,* **12,** 4032.
IONSON J. A. (1978) *Astrophys. J.* **226,** 650.
KRUSKAL M. and SCHWARZSCHILD M. (1954) *Proc. R. Soc.* (Lond.) **A223,** 348.
TALMADGE S. MARSHALL T. C. and SCHLESINGER S. T. (1973) *Phys. Fluids* **16,** 1171.
TRIVELPIECE A. W. and GOULD R. W. (1959) *J. appl. Phys.* **30,** 1784.
UBEROI C. (1972) *Phys. Fluids* **15,** 1673.
UBEROI C. and RAO U. J. (1975) *Plasma Phys.* **17,** 659.