

MAGNETOACOUSTIC SURFACE WAVES AT A SINGLE INTERFACE

REKHA JAIN and B. ROBERTS

*Department of Mathematical and Computational Sciences, University of St Andrews, St Andrews,
Fife KY16 9SS, Scotland*

(Received 27 September, 1990)

Abstract. The occurrence of magnetoacoustic surface waves at a single magnetic interface one side of which is field-free is explored for the case of non-parallel propagation. Phase-speeds and penetration depths of the waves are investigated for various Alfvén speeds, sound speeds and angles of propagation to the applied field. Both slow and fast magnetoacoustic surface waves can exist depending on the values of sound speeds and propagation angle. The fast waves penetrate more than the slow waves.

The parallel propagation of fast and slow magnetoacoustic surface waves on a magnetic-magnetic interface is investigated. The slow surface wave is unable to propagate below a critical sound speed. In a low β -plasma, only the fast mode exists ($\theta \neq 0$).

1. Introduction

The increasing interest in the study of wave propagation in a magnetically structured atmosphere, motivated by the observed inhomogeneous nature of the solar atmosphere, has high-lighted many important properties of magnetoacoustic surface waves. Magnetoacoustic surface waves can exist on discontinuities in the magnetic field, plasma density, pressure or temperature. In the solar atmosphere, for example, discontinuities (or at least rapid changes) in the magnetic field arise on the edges of sunspots or intense flux tubes.

A discussion of magnetohydrodynamic surface waves at a single interface, in an incompressible medium, may be found in Chandrasekhar (1961); see also Kruskal and Schwarzschild (1954) and Gerwin (1967). The phase-speed of such a wave is given by

$$c_k = \left(\frac{\rho_0 v_A^2 + \rho_e v_{Ae}^2}{\rho_0 + \rho_e} \right)^{1/2}, \quad (1)$$

where ρ_0 and v_A are the density and Alfvén speed on one side of the interface and ρ_e and v_{Ae} are their respective values on the other side. The investigation of waves in flux tubes also gives rise to the speed c_k (see, for example, the reviews by Spruit, 1983; Spruit and Roberts, 1983; Thomas, 1985; Roberts, 1986; Ryutova, 1990).

Surface waves in a compressible medium have been discussed by various authors. Wentzel (1979) and Roberts (1981) formulated the magnetoacoustic surface wave problem for a magnetic field which is uniform in the direction of the field but varies in a direction perpendicular to itself, considering in particular the case of a single magnetic interface. The detailed properties of magnetoacoustic surface waves on such an interface have been investigated by Uberoi (1982) and Somasundaram and Uberoi (1982) for

non-parallel propagation and by Miles and Roberts (1989) for the case of parallel propagation. Somasundaram and Uberoi (1982) have investigated the effects of compressibility on the propagation of surface waves by considering different magnetic field strengths on either side of the interface. They were concerned with the longitudinal phase-speeds of the surface waves at various propagation angles. Miles and Roberts (1989) discussed longitudinal phase-speeds and penetration depths of surface waves on a magnetic interface, one side of which is field-free. They also investigated the associated pressure perturbations and motions for a variety of field strengths and sound speeds. In another context, Cadez and Okretic (1989) discuss the possibility of surface wave energy leakage, which may occur in the case of a two-step configuration. A study of phase-speed diagrams, which can yield information on the spectrum of modes that a magnetic structure can support, has been carried out by Rae and Roberts (1983).

An application of magnetoacoustic surface waves to running penumbral waves was suggested by Small and Roberts (1984). Running penumbral waves were first observed by Giovanelli (1972) and Zirin and Stein (1972), and have been modelled by Nye and Thomas (1974, 1976) and Cally and Adam (1983) as magnetoacoustic-gravity modes (see also Zhugzhda and Dzhililov (1984)).

Magnetoacoustic surface waves clearly play a prominent role in any discussion of waves in a structured atmosphere. They may gain an additional importance in view of their possible role in the heating of the corona (Ionson, 1978, 1985; Wentzel, 1979; Rae and Roberts, 1981; Lee and Roberts, 1986; Hollweg, 1986, 1987a, b, 1990; Davila, 1990). It is of interest, then, to consider the properties of surface waves, and to assess the dependence of phase-speeds on various field strengths.

In this paper we consider the parallel propagation of magnetoacoustic surface waves on a magnetic interface. We then go on to consider the non-parallel propagation of magnetoacoustic surface waves at a single interface one side of which is field-free.

2. The Wave Equations

We consider magnetohydrodynamic wave propagation in an ideal, perfectly conducting and compressible plasma. The effect of gravity is ignored. The basic equations of ideal magnetohydrodynamics are taken in the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (2)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (3)$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = \frac{\gamma p}{\rho} \left(\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho \right), \quad (4)$$

$$p = \frac{k_B}{\hat{m}} \rho T, \quad (5)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (6)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (7)$$

where p , ρ , and T are the pressure, density, and temperature of the gas with velocity \mathbf{v} and magnetic field \mathbf{B} ; γ is the ratio of specific heats, μ is the magnetic permeability, k_B is Boltzmann's constant, and \bar{m} is the mean particle mass.

We consider an equilibrium field $\mathbf{B}_0 = B_0(x)\hat{z}$, lying parallel to the z -axis and varying in strength in the x -direction. The equilibrium gas pressure $p_0(x)$ is related to the magnetic pressure $B_0^2(x)/2\mu$ by

$$\frac{d}{dx} \left(p_0 + \frac{B_0^2}{2\mu} \right) = 0. \quad (8)$$

To investigate the propagation of small amplitude plane waves we write

$$\mathbf{v} = \hat{\mathbf{v}}(x) e^{i(\omega t - ly - kz)}, \quad p = p_0(x) + \hat{p}(x) e^{i(\omega t - ly - kz)}, \quad (9)$$

etc., for perturbations in flow, pressure, etc. Here ω is the angular frequency and l and k are the wave numbers in the y - and z -directions, defining a wave vector $\mathbf{K} = (0, l, k)$ in the yz -plane. We write $k = K \cos \theta$ and $l = K \sin \theta$, for angle of propagation θ to the applied field; $K = (l^2 + k^2)^{1/2}$. The sound speed in the basic state of density $\rho_0(x)$ is $c_s(x) = (\gamma p_0(x)/\rho_0(x))^{1/2}$ and the Alfvén speed is $v_A(x) = B_0(x)/(\mu \rho_0(x))^{1/2}$. It is also useful to define the total pressure $\hat{p}_T = \hat{p} + B_0 \hat{B}_z/\mu$, the sum of the perturbed gas pressure and the perturbed magnetic pressure (for field perturbation \hat{B}_z in the direction of the applied magnetic field).

The linearised form of Equations (2)–(7), about the equilibrium (8), give rise to a second order ordinary differential equation for the x -component of velocity (Goedbloed, 1971; Roberts, 1981):

$$\frac{d}{dx} \left\{ \frac{\rho_0(x) (k^2 v_A^2(x) - \omega^2)}{m^2 + l^2} \frac{d\hat{v}_x}{dx} \right\} - \rho_0(x) (k^2 v_A^2(x) - \omega^2) \hat{v}_x = 0, \quad (10)$$

where

$$m^2(x) = \frac{(k^2 c_s^2(x) - \omega^2) (k^2 v_A^2(x) - \omega^2)}{(c_s^2(x) + v_A^2(x)) (k^2 c_s^2(x) - \omega^2)}, \quad c_T^2(x) = \frac{c_s^2(x) v_A^2(x)}{c_s^2(x) + v_A^2(x)}. \quad (11)$$

In a uniform medium (for which ρ_0 , c_s , v_A , and m^2 are constants), Equation (10) becomes

$$(k^2 v_A^2 - \omega^2) \left(\frac{d^2 \hat{v}_x}{dx^2} - (m^2 + l^2) \hat{v}_x \right) = 0, \quad (12)$$

showing that there are two possibilities: either $\omega^2 = k^2 v_A^2$, with $\hat{v}_x(x)$ arbitrary; or $\omega^2 \neq k^2 v_A^2$ and $\hat{v}_x(x)$ satisfies

$$\frac{d^2 \hat{v}_x}{dx^2} - (m^2 + l^2) \hat{v}_x = 0. \quad (13)$$

3. Surface Waves on a Magnetic-Magnetic Interface

3.1. THE DISPERSION RELATION

We are interested in investigating the behaviour of plane waves on an interface which has a uniform field (of different magnitude) on either side of it. Consider a basic state in which the magnetic field is of the form (see Figure 1)

$$B_0(x) = \begin{cases} B_e, & x > 0, \\ B_0, & x < 0, \end{cases} \quad (14)$$

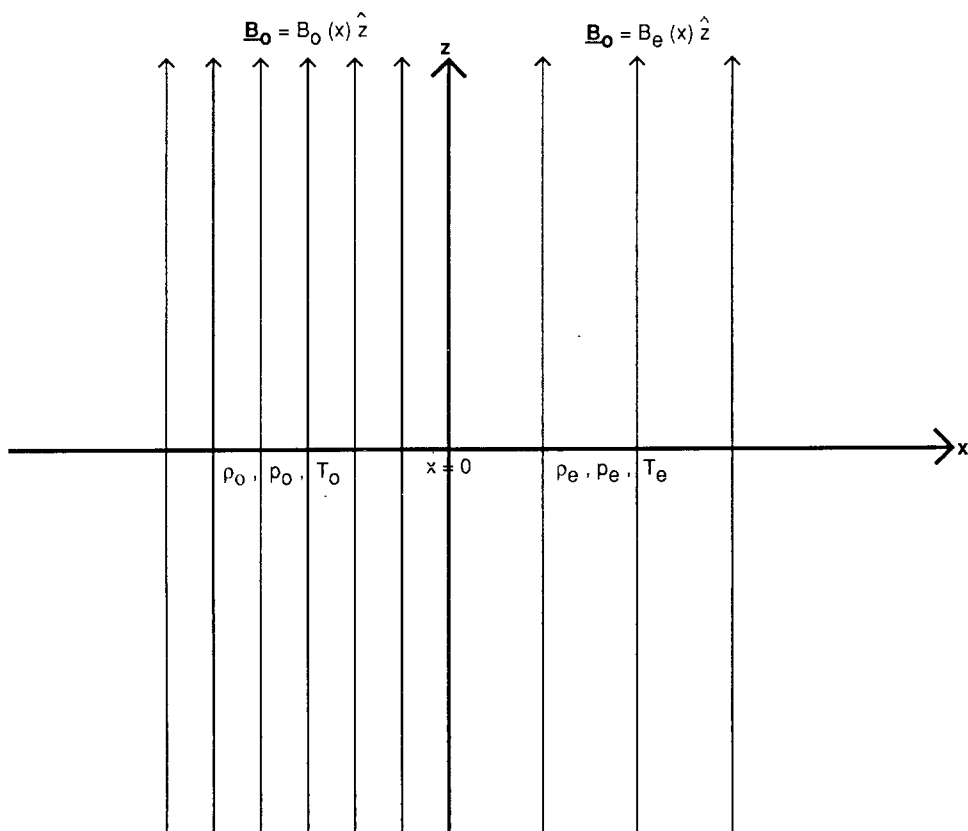


Fig. 1. The equilibrium atmosphere of a single magnetic interface.

with B_0 and B_e constants. The pressure balance condition (Equation (8)) yields

$$p_e + \frac{B_e^2}{2\mu} = p_0 + \frac{B_0^2}{2\mu}, \quad (15)$$

which, together with the ideal gas law (5), implies that

$$\frac{\rho_e}{\rho_0} = \frac{c_0^2 + \frac{\gamma}{2} v_A^2}{c_e^2 + \frac{\gamma}{2} v_{Ae}^2}, \quad (16)$$

where $c_e = (\gamma p_e / \rho_e)^{1/2}$ and $v_{Ae} = B_e / (\mu \rho_e)^{1/2}$ are the sound and Alfvén speeds in $x > 0$, and $c_0 = (\gamma p_0 / \rho_0)^{1/2}$ and $v_A = B_0 / (\mu \rho_0)^{1/2}$ are the sound and Alfvén speeds in $x < 0$.

Since the media on either side of the discontinuity at $x = 0$ are uniform, Equation (12) applies with the coefficients being constants. Thus, in addition to Alfvén waves, given by the first factor in Equation (12), there are magnetoacoustic modes given by Equation (13) applied to the regions $x > 0$ and $x < 0$. Hence, in $x < 0$ we have

$$\frac{d^2 \hat{v}_x}{dx^2} - (m_0^2 + l^2) \hat{v}_x = 0, \quad x < 0, \quad (17)$$

and in $x > 0$ we have

$$\frac{d^2 \hat{v}_x}{dx^2} - (m_e^2 + l^2) \hat{v}_x = 0, \quad x > 0. \quad (18)$$

Here

$$m_0^2 = \frac{(k^2 v_A^2 - \omega^2)(k^2 c_0^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}, \quad m_e^2 = \frac{(k^2 v_{Ae}^2 - \omega^2)(k^2 c_e^2 - \omega^2)}{(c_e^2 + v_{Ae}^2)(k^2 c_{Te}^2 - \omega^2)}; \quad (19)$$

note that m_0^2 and m_e^2 may be positive or negative (for ω^2 and k^2 real).

Solving Equations (17) and (18) subject to the requirement of boundedness at $\pm \infty$ yields

$$\hat{v}_x(x) = \begin{cases} \alpha_e e^{-(m_e^2 + l^2)^{1/2} x}, & x > 0, \\ \alpha_0 e^{(m_0^2 + l^2)^{1/2} x}, & x < 0, \end{cases} \quad (20)$$

where we require that $(m_0^2 + l^2)^{1/2} > 0$ and $(m_e^2 + l^2)^{1/2} > 0$. The restrictions on $(m_0^2 + l^2)^{1/2}$ and $(m_e^2 + l^2)^{1/2}$ imply that we are considering surface waves.

Across the interface $x = 0$ it is necessary that both $\hat{v}_x(x)$ and $\hat{p}_T(x)$ be continuous (see Roberts, 1981). Thus $\alpha_e = \alpha_0$, and continuity of \hat{p}_T at $x = 0$ gives

$$\rho_0(k^2 v_A^2 - \omega^2)(m_e^2 + l^2)^{1/2} + \rho_e(k^2 v_{Ae}^2 - \omega^2)(m_0^2 + l^2)^{1/2} = 0, \quad (21)$$

valid for $(m_e^2 + l^2)^{1/2}$ and $(m_0^2 + l^2)^{1/2}$ both positive. Equation (21) is the dispersion relation for surface waves at a single magnetic interface; it has been obtained by Wentzel (1979) and Roberts (1980, 1981). Equation (21) can be rewritten in terms of the propagation angle θ as follows:

$$\rho_0(K^2 v_A^2 \cos^2 \theta - \omega^2)(m_e^2 + l^2)^{1/2} + \rho_e(K^2 v_{Ae}^2 \cos^2 \theta - \omega^2)(m_0^2 + l^2)^{1/2} = 0, \quad (22)$$

where

$$m_e^2 + l^2 = \frac{(K^2 c_e^2 \cos^2 \theta - \omega^2)(K^2 v_{Ae}^2 \cos^2 \theta - \omega^2)}{(c_e^2 + v_{Ae}^2)(K^2 c_T^2 \cos^2 \theta - \omega^2)} + K^2 \sin^2 \theta, \quad (23)$$

$$m_0^2 + l^2 = \frac{(K^2 c_0^2 \cos^2 \theta - \omega^2)(K^2 v_A^2 \cos^2 \theta - \omega^2)}{(c_0^2 + v_A^2)(K^2 c_T^2 \cos^2 \theta - \omega^2)} + K^2 \sin^2 \theta. \quad (24)$$

Equation (22) can be re-arranged in the form (Roberts, 1981)

$$\frac{\omega^2}{K^2} = \cos^2 \theta \left[v_A^2 - \frac{R}{R+1} (v_A^2 - v_{Ae}^2) \right] = \cos^2 \theta \left[v_{Ae}^2 + \frac{1}{R+1} (v_A^2 - v_{Ae}^2) \right], \quad (25)$$

where

$$R = \left(\frac{\rho_e}{\rho_0} \right) \left(\frac{m_0^2 + l^2}{m_e^2 + l^2} \right)^{1/2} > 0; \quad (26)$$

note that R is a function of ω^2/K^2 and θ .

3.2. PROPERTIES OF THE MODES

It is clear from the form (25) of the dispersion relation that the phase-speed, $c_{ph} \equiv \omega/K$, of a magnetoacoustic surface wave lies between $v_A \cos \theta$ and $v_{Ae} \cos \theta$, and its longitudinal phase-speed, ω/k , lies between v_A and v_{Ae} . (Thus, in particular, if one side of the interface is field-free, say $v_{Ae} = 0$, then $\omega/K \leq v_A \cos \theta$.) Hence, it follows immediately that *magnetoacoustic surface waves are unable to propagate perpendicular to the applied magnetic field* (i.e., magnetoacoustic surface waves are unable to propagate when $\theta = \pi/2$). The dispersion relation (22), subject to the constraint $(m_0^2 + l^2)^{1/2} > 0$ and $(m_e^2 + l^2)^{1/2} > 0$, may possess either no solution, one solution or two solutions, depending upon the ordering of the sound speeds and the Alfvén speeds in the two media (Roberts, 1981). Where two waves arise, these are referred to as *slow* and *fast* magnetoacoustic surface waves. Their properties are explored in Section 3.2.2 and thereafter.

Also, we may note from Equation (25) that surface waves at an interface are *non-dispersive* (i.e., ω/K is independent of K). This is to be expected since in such a medium no natural length-scale exists, and so the propagation speeds of all waves depend only on θ .

3.2.1. Incompressible Limit

There are a number of interesting features of the general dispersion relation (21). To begin with, note the reduction of the dispersion relation for the case of an incompressible fluid ($c_s^2 \rightarrow \infty$). The incompressible form of Equation (21) is obtained by considering the limiting forms of m_e^2 and m_0^2 as $c_0, c_e \rightarrow \infty$: m_0^2 and $m_e^2 \rightarrow K^2 \cos^2 \theta = k^2$. Then Equation (21) reduces to

$$\frac{\omega^2}{K^2} = c_k^2 \cos^2 \theta; \quad \text{i.e.,} \quad \omega^2 = k^2 c_k^2. \quad (27)$$

Equation (27) is the dispersion relation for a hydromagnetic surface wave in an incompressible medium.

3.2.2. Low- β Plasma

A possibly more interesting case arises when we consider the case of a low- β plasma, corresponding to both Alfvén speeds greatly exceeding the sound speeds. The pressure balance condition for a low- β plasma implies $\rho_0 v_A^2 \approx \rho_e v_{Ae}^2$, and from Equations (23) and (24) we have $m_0^2 + l^2 \approx K^2 - \omega^2/v_A^2$ and $m_e^2 + l^2 \approx K^2 - \omega^2/v_{Ae}^2$. Thus Equation (22) reduces to (Roberts, 1981; Uberoi, 1982)

$$v_{Ae}(v_A^2 \cos^2 \theta - c_{ph}^2)(v_{Ae}^2 - c_{ph}^2)^{1/2} + v_A(v_{Ae}^2 \cos^2 \theta - c_{ph}^2)(v_A^2 - c_{ph}^2)^{1/2} = 0. \quad (28)$$

Now the phase-speed c_{ph} must lie between $v_A \cos \theta$ and $v_{Ae} \cos \theta$. Additionally, both the Alfvén speeds should be greater than the phase-speed c_{ph} for the surface wave solutions to be evanescent in both media ($(v_{Ae}^2 - c_{ph}^2)^{1/2} > 0$, $(v_A^2 - c_{ph}^2)^{1/2} > 0$). Thus, if v_A is taken to be larger than v_{Ae} , then for a given θ , $v_A \cos \theta > c_{ph}$ and $v_{Ae} \cos \theta < c_{ph} < v_{Ae}$. If $l \gg k$, then Equation (28) reduces to

$$\frac{\omega^2}{K^2} = c_k^2 \cos^2 \theta; \quad \text{i.e.,} \quad \omega^2 = k^2 c_k^2. \quad (29)$$

A more quantitative description of Equation (28) may be given by squaring it, which leads to a quadratic equation in c_{ph}^2 :

$$c_{ph}^4 - (v_A^2 + v_{Ae}^2)c_{ph}^2 + v_A^2 v_{Ae}^2 \cos^2 \theta (1 + \sin^2 \theta) = 0. \quad (30)$$

Solving Equation (30) gives the roots

$$2c_{ph}^2 = (v_A^2 + v_{Ae}^2) \pm [(v_A^2 + v_{Ae}^2)^2 - 4v_A^2 v_{Ae}^2 \cos^2 \theta (1 + \sin^2 \theta)]^{1/2}. \quad (31)$$

The positive root is unacceptable since $c_{ph}^2 < v_A^2, v_{Ae}^2$. Thus,

$$2c_{ph}^2 = (v_A^2 + v_{Ae}^2) - [(v_A^2 - v_{Ae}^2)^2 + 4v_A^2 v_{Ae}^2 \sin^4 \theta]^{1/2}. \quad (32)$$

Now, for small θ (strictly, for $\sin^4 \theta \ll 1$) Equation (32) gives

$$2c_{ph}^2 = (v_A^2 + v_{Ae}^2) - |v_A^2 - v_{Ae}^2| \left[1 + \frac{2v_A^2 v_{Ae}^2 \sin^4 \theta}{(v_A^2 - v_{Ae}^2)^2} \right], \quad (33)$$

and so

$$c_{ph}^2 = \min(v_A^2, v_{Ae}^2) - \frac{v_A^2 v_{Ae}^2}{|v_A^2 - v_{Ae}^2|} \sin^4 \theta, \quad (34)$$

provided v_A is not too close to v_{Ae} .

At the other extreme, of propagation close to perpendicular to the magnetic field, $\sin \theta \approx 1$, and Equation (32) yields

$$c_{ph}^2 \approx \left(\frac{v_A^2 v_{Ae}^2}{v_A^2 + v_{Ae}^2} \right) \cos^2 \theta (1 + \sin^2 \theta); \quad (35)$$

that is,

$$c_{ph}^2 \approx c_k^2 \cos^2 \theta [1 - \frac{1}{2} \cos^2 \theta]. \quad (35)'$$

Thus, as noted earlier, there are no surface modes able to propagate across the magnetic field (i.e., when $\theta = \pi/2$). Additionally, we can see from Equation (34) that no surface wave is able to propagate at angle $\theta = 0^\circ$, for c_{ph}^2 must lie between v_A^2 and v_{Ae}^2 (see also Roberts, 1981). Altogether, if both sides of the magnetic interface are low- β plasmas then there are no surface waves for either $\theta = 0^\circ$ or $\theta = \pi/2$. However, a (fast) surface mode exists for $0 < \theta < \pi/2$, with phase-speed given by Equation (32). Also, if only one side of the magnetic interface has a low beta then a surface mode may arise even for $\theta = 0^\circ$; see Section 4 below.

3.2.3. Parallel Propagation ($\theta = 0^\circ$)

Consider the reduction of Equation (22) for $l = 0$, corresponding to parallel propagation:

$$\rho_0(k^2 v_A^2 - \omega^2)m_e + \rho_e(k^2 v_{Ae}^2 - \omega^2)m_0 = 0. \quad (36)$$

Case (a): $v_A > v_{Ae}$

The condition $m_e > 0$ implies that $\omega/k < c_{Te}$ or $\min(c_e, v_{Ae}) < \omega/k < \max(c_e, v_{Ae})$, while the condition $m_0 > 0$ implies $\omega/k < c_T$ or $\min(c_0, v_A) < \omega/k < \max(c_0, v_A)$. Additionally, the longitudinal phase-speed ω/k is such that $v_{Ae} < \omega/k < v_A$. Thus there arises the possibility of $\omega/k < \min(c_e, c_T)$; this is the *slow surface wave*. For $c_0 \rightarrow \infty$ and $c_e \rightarrow \infty$, it can easily be seen from Equation (29) that the slow surface wave tends to c_k , the phase-speed of a surface wave in an incompressible medium.

In Figure 2(a) we exhibit the dependence on c_0/c_e of the phase-speed ω/k for $v_A/c_e = 1.8$ and $v_{Ae}/c_e = 0.4$. We may note that there exists a critical value of c_0/c_e below

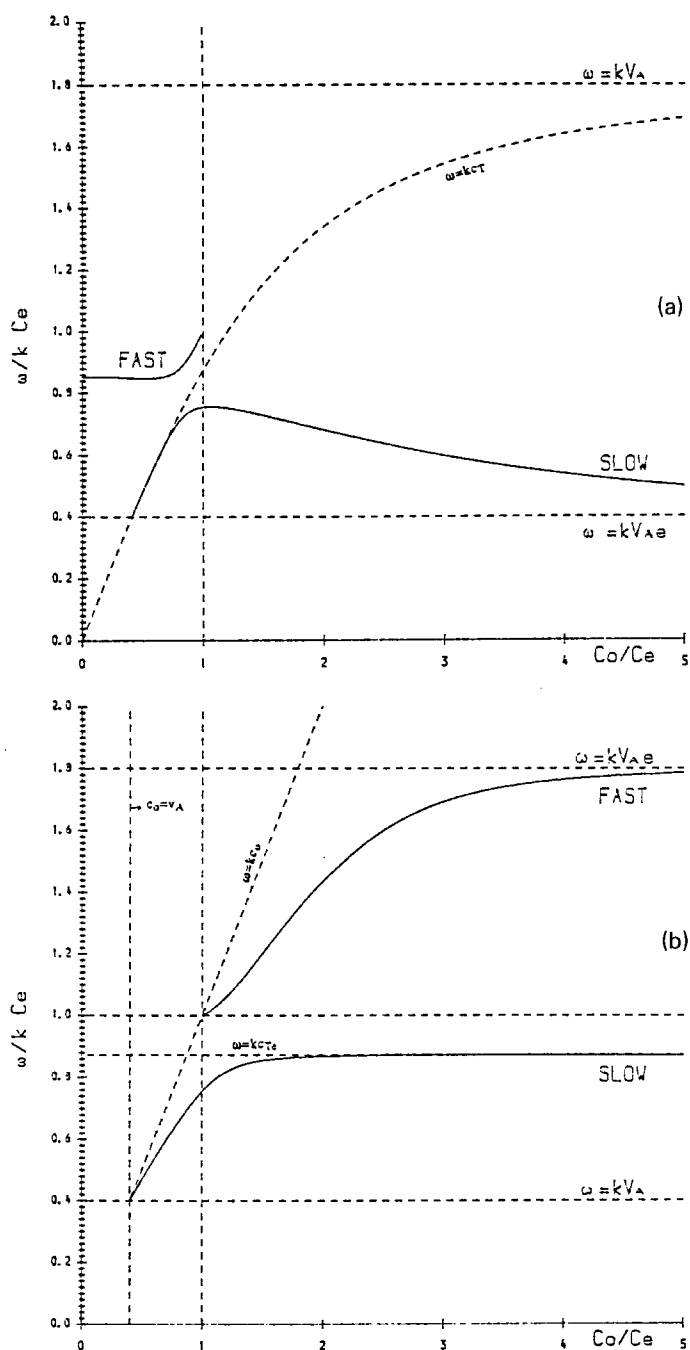


Fig. 2. The variation of the phase-speed (in units of sound speed, c_e) of the slow and fast magnetoacoustic surface waves, propagating parallel to the magnetic field, with the parameter c_0/c_e for the cases (a) $v_A > v_{Ae}$, and (b) $v_A < v_{Ae}$. In case (a) we have chosen $v_A/c_e = 1.8$ and $v_{Ae}/c_e = 0.4$; in case (b) we have taken $v_A/c_e = 0.4$ and $v_{Ae}/c_e = 1.8$. Observe that both modes have phase-speeds between v_{Ae} and v_A . Here (and elsewhere in the figures) we have set $\gamma = \frac{5}{3}$.

which the slow surface wave is unable to propagate. The critical value of c_0/c_e is given by the intersection of the curves $\omega/k = c_T$ and $\omega/k = v_{Ae}$. For the free-field case (i.e., when $v_{Ae} = 0$) the critical value is zero, and the slow surface mode exists for all c_0/c_e . As illustrated in Figure 2(a), if the β of the plasma is small on one side of the interface but large on the other side, then we may have either a *fast* mode or a *slow* mode or both *slow* and *fast* surface modes co-existing, depending upon the ratio c_0/c_e . When the sound speed (c_0) of the low- β plasma exceeds the sound speed (c_e) of the high- β plasma, then only the slow surface mode exists. For a low- β situation where $v_A > c_0$ and $v_{Ae} > c_0$, only a fast mode exists.

Case (b): $v_{Ae} > v_A$

This case is considered to trace the evolution from two surface modes to no mode. The longitudinal phase-speed satisfies the condition $v_A < \omega/k < v_{Ae}$. The slow surface wave has phase-speed $v_A < \omega/k < c_{Te}$. Figure 2(b) shows the variation of ω/k as a function of c_0/c_e . In this case, the critical value of c_0/c_e below which the slow surface wave does not exist is given by the intersection of the curves $\omega/k = v_A$ and $c_0 = v_A$. Figure 2(b) illustrates our earlier conclusion that there are no surface waves when both sides of the magnetic interface are low- β plasmas ($v_{Ae} > c_e$ and $v_A > c_0$). The slow surface wave exists for all values of $c_0 > v_A$ and the fast surface wave starts appearing as soon as c_0 exceeds c_e . This is to be expected because when $v_{Ae} > v_A$, the condition for the fast mode to exist is $v_{Ae} > c_e$ and $c_e < c_0$.

It is interesting to consider the penetration depths, the distances from the interface to which surface waves are able to penetrate (see also Miles and Roberts, 1989). Fast and slow magnetoacoustic waves penetrate a distance of the order of m_0^{-1} into the region $x < 0$ and a distance m_e^{-1} into the region $x > 0$. Figure 3 displays the dependence of penetration depths ($(\lambda m_e)^{-1}$ into the region $x < 0$ and $(\lambda m_0)^{-1}$ into the region $x > 0$ for a wavelength $\lambda \equiv 2\pi/k$) of the slow and fast magnetoacoustic surface waves on c_0/c_e .

Note from Figure 3(a–b) that as c_0 exceeds a critical value (corresponding to $c_0 = v_{Ae}$ in Figure 3(a) and $c_0 = v_A$ in Figure 3(b)), the slow surface modes penetrate more into the weak magnetic field region than into the strong field region, whereas the fast surface modes penetrate more into the strong magnetic field region than into the weak field region.

4. Surface Waves on a Magnetic–Nonmagnetic Interface

Consider the case of non-parallel propagation ($\theta \neq 0$) on an interface one side of which is field-free. With $v_{Ae} = 0$, Equation (22) reduces to

$$\rho_0(K^2 v_A^2 \cos^2 \theta - \omega^2)(m_e^2 + l^2)^{1/2} - \rho_e \omega^2(m_0^2 + l^2)^{1/2} = 0, \quad (37)$$

where now

$$m_e^2 + l^2 = K^2 - \frac{\omega^2}{c_e^2}. \quad (38)$$

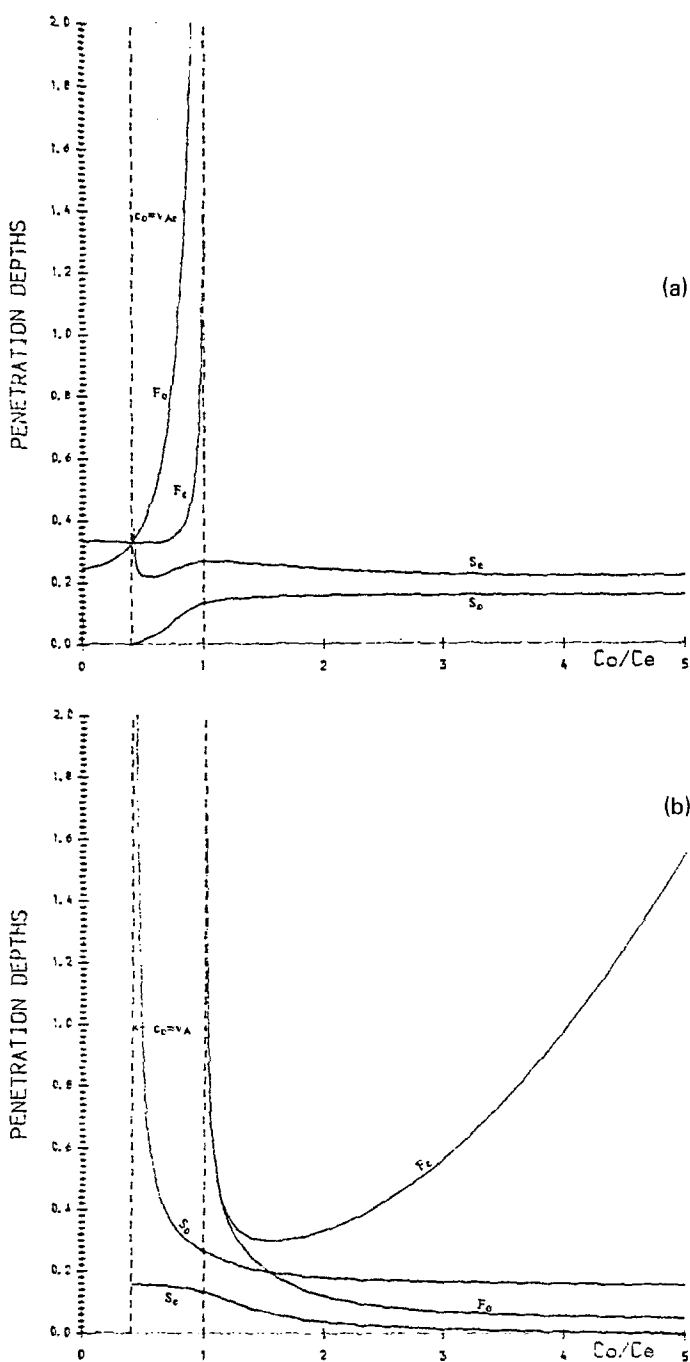


Fig. 3. The variation with c_0/c_e of the penetration depths of the fast and slow surface waves (propagating parallel to the field) for the cases (a) $v_A/c_e = 1.8$, $v_{Ae}/c_e = 0.4$ and (b) $v_{Ae}/c_e = 0.4$, $v_A/c_e = 1.8$. Depths are measured in units of wavelength $2\pi/k$. ' S_0 ' - slow surface wave into the region (v_A, c_0) ; ' S_e ' - slow surface wave into the region (v_{Ae}, c_e) ; ' F_0 ' - slow surface wave into the region (v_A, c_0) ; ' F_e ' - slow surface wave into the region (v_{Ae}, c_e) .

Equation (37) may be cleared of radicals by squaring, the result being a quartic in $c_{ph}^2 \equiv \omega^2/K^2$ (Roberts, 1981):

$$\begin{aligned} & (c_0^2 + v_A^2)(c_T^2 \cos^2 \theta - c_{ph}^2)(v_A^2 \cos^2 \theta - c_{ph}^2)(c_e^2 - c_{ph}^2) = \\ & = \left(\frac{\rho_e}{\rho_0}\right)^2 c_{ph}^4 c_e^2 [(c_0^2 \cos^2 \theta - c_{ph}^2)(v_A^2 \cos^2 \theta - c_{ph}^2) + \\ & + \sin^2 \theta (c_0^2 + v_A^2)(c_T^2 \cos^2 \theta - c_{ph}^2)], \end{aligned} \quad (39)$$

with $m_e^2 + l^2 > 0$ and $m_0^2 + l^2 > 0$. It should be noted that spurious roots may be introduced by squaring; these satisfy Equation (39) but not the original dispersion relation (37) and its conditions for an exponentially decaying solutions.

The condition $m_e^2 + l^2 > 0$ implies that $c_{ph} < c_e$, and the condition $m_0^2 + l^2 > 0$ implies $c_{ph} < c_T \cos \theta$ or $\min(c_0 \cos \theta, v_A \cos \theta) < c_{ph} < \max(c_0 \cos \theta, v_A \cos \theta)$. Also, $c_{ph} \leq v_A \cos \theta$. Thus, there exists the possibility of a slow surface wave with phase-speed $c_{ph} < \min(c_T \cos \theta, c_e)$. In fact, the slow surface wave is present irrespective of the magnitudes of c_e , c_0 , and v_A (Roberts, 1981; Miles and Roberts, 1989). But if $v_A > c_0$ and the field-free medium is warmer than the magnetic medium, so that $c_e > c_0$, then a fast surface wave may propagate with phase-speed satisfying $c_0 \cos \theta < c_{ph} < \min(c_e, v_A \cos \theta)$.

The variation with v_A/c_e of the phase-speed ω/K of surface waves propagating at various angles θ to the applied field is shown in Figure 4 for two values of c_0/c_e . It can easily be seen from this figure that fast and slow modes are both present only for $v_A > c_0$. When the gas inside the magnetised region is warmer than the field-free medium (i.e., when $c_0 > c_e$), the fast wave is absent (Figure 4(b)). Also apparent in Figure 4 is that both slow and fast surface waves propagate with phase-speeds which *decrease* with increasing angle θ , for a given c_0/c_e . In each case we note that $\omega/Kc_e \rightarrow 0$ as $v_A/c_e \rightarrow 0$ for the slow surface wave. Also, $\omega/Kc_e \rightarrow 0$ as $\theta \rightarrow \pi/2$ for both slow and fast surface waves; this is to be expected since the phase-speed for a surface wave must lie in the range $0 < \omega/K < v_A \cos \theta$ when there is a field-free region on one side of the interface.

Figure 5 illustrates the dependence on c_0/c_e for two different values of v_A/c_e . Once again, the fast wave propagates only when $c_0 < c_e$. Here also the phase-speeds of slow and fast modes *decrease* with increasing θ as a function of c_0/c_e for a given v_A/c_e .

In Figure 6 we exhibit the dependence on v_A/c_e of the penetration depth for angles $\theta = 0^\circ$ and 60° , with c_0/c_e taken to be 0.75 (Figure 6(a-b)) or 1.4 (Figure 6(c-d)). For large v_A/c_e , we see that the fast mode penetrates more than the slow mode into either region and both the modes penetrate deeper into the field-free region than into the field. It is interesting to note the decrease in the penetration depths with the increase in angle θ for large v_A/c_e . Also, the penetration depths of slow and fast surface waves become equal as $\theta \rightarrow \pi/2$; this is due to the fact that both $(m_0^2 + l^2)^{-1/2}$ and $(m_e^2 + l^2)^{-1/2}$ tend to K when θ approaches $\pi/2$.

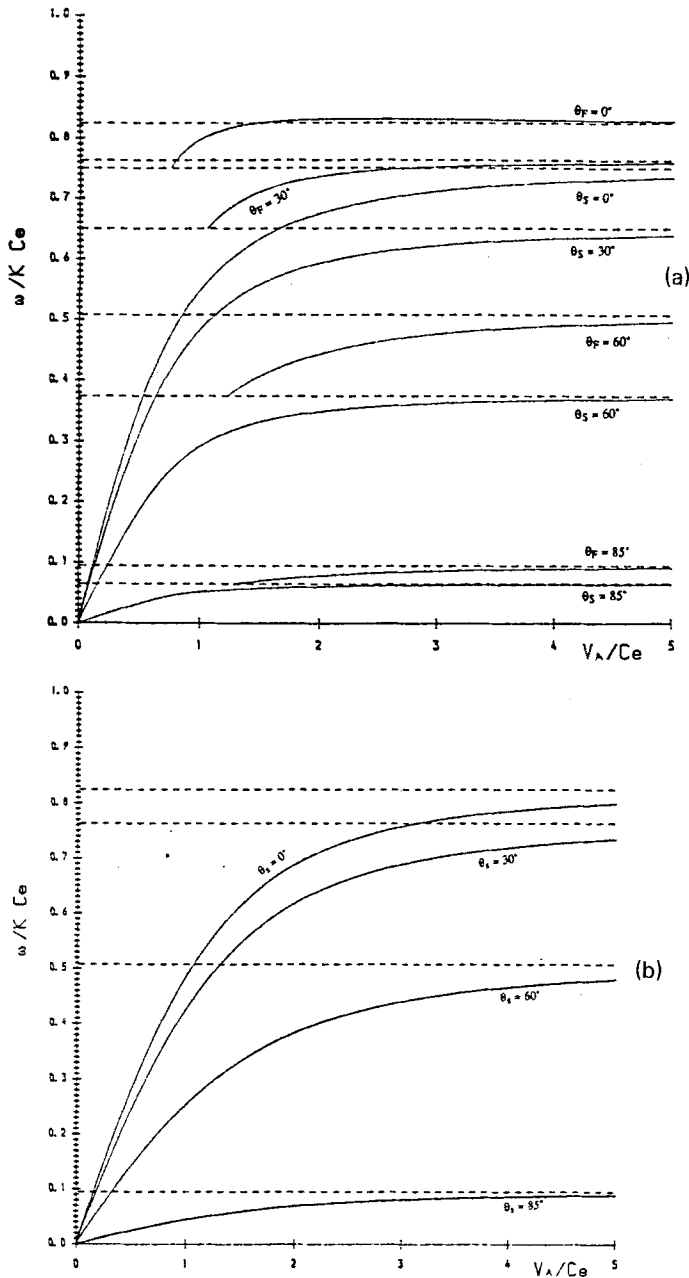


Fig. 4. The variation of the dimensionless phase-speed ω/Kc_e of the fast and slow surface waves with the parameter v_A/c_e , for angles of propagation $\theta = 0^\circ, 30^\circ, 60^\circ$, and 85° . Cases (a) $c_0/c_e = 0.75$; (b) $c_0/c_e = 1.4$. The subscript 'S' denotes 'slow' surface wave and 'F' denotes 'fast' surface wave. The fast wave exists only when $v_A > c_0$ and $c_0 < c_e$. For large v_A/c_e the slow wave asymptotes to the smaller of the values $(c_0/c_e) \cos \theta$ and

$$\left\{ \frac{2 \cos^4 \theta}{\gamma^2} \left[\left(1 + \frac{\gamma^2}{\cos^4 \theta} \right)^{1/2} - 1 \right] \right\}^{1/2}.$$

The fast surface wave (when permitted to propagate) asymptotes to the larger of these two values.

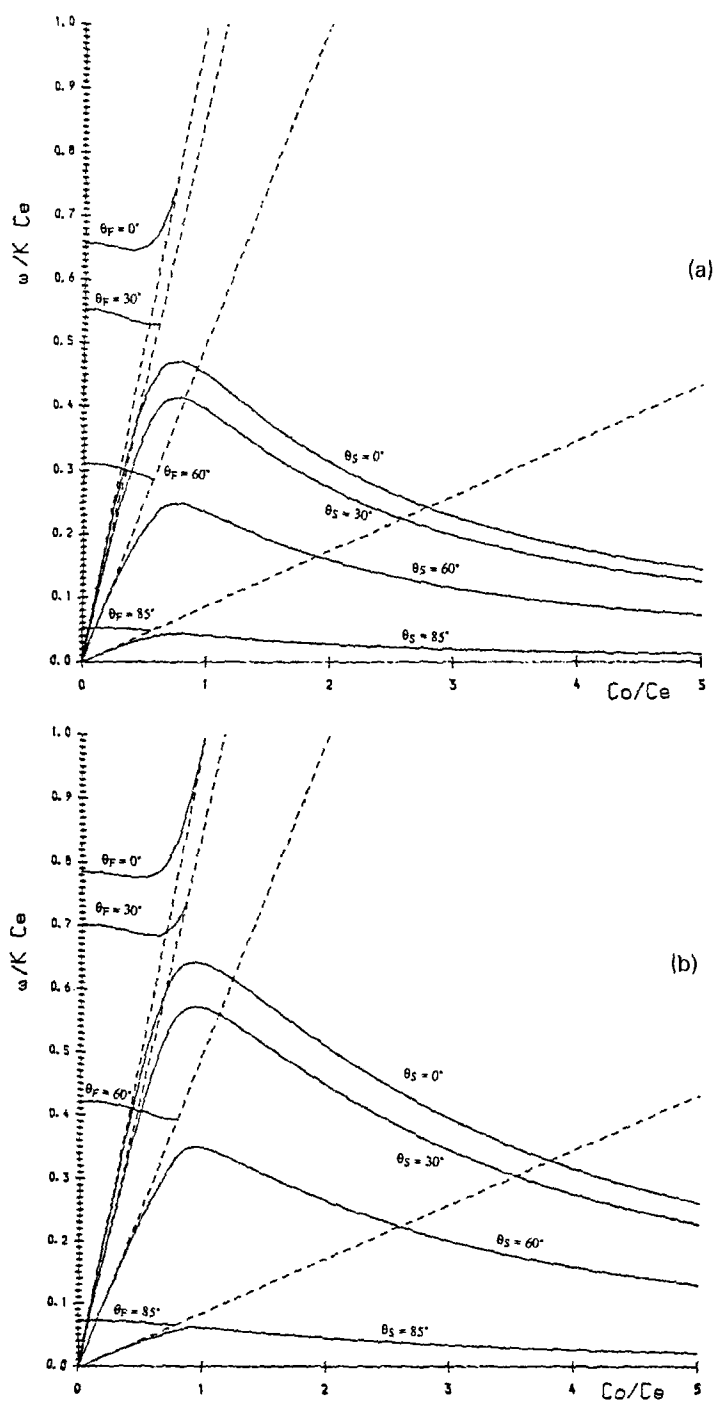


Fig. 5. The variation of the phase-speed of the fast and slow surface waves with the parameter c_0/c_e , for $\theta = 0^\circ, 30^\circ, 60^\circ,$ and 85° . Cases (a) $v_A/c_e = 0.75$; and (b) $v_A/c_e = 1.4$. The sloping asymptotes are the lines $\omega = K c_0 \cos \theta$ and various θ .

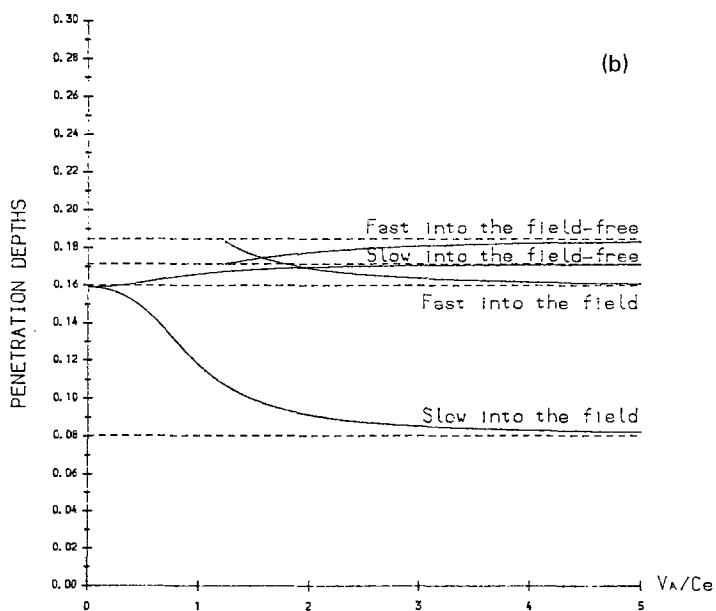
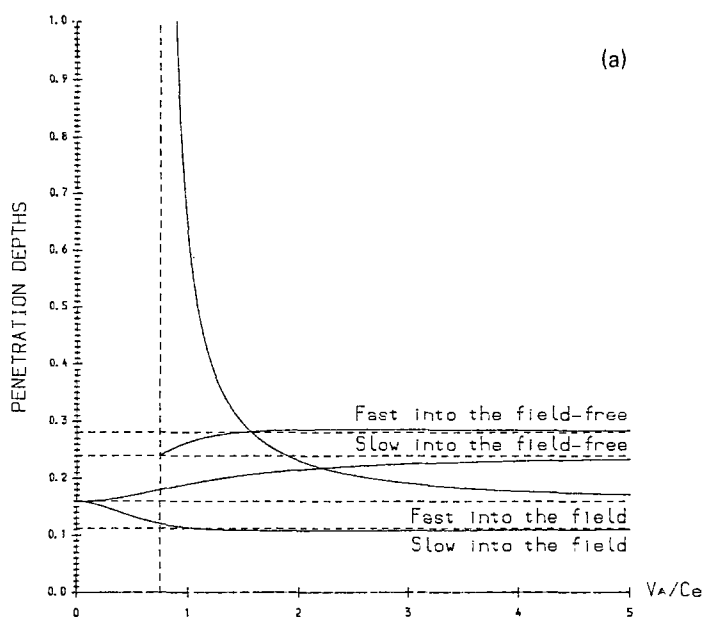


Fig. 6a, b.

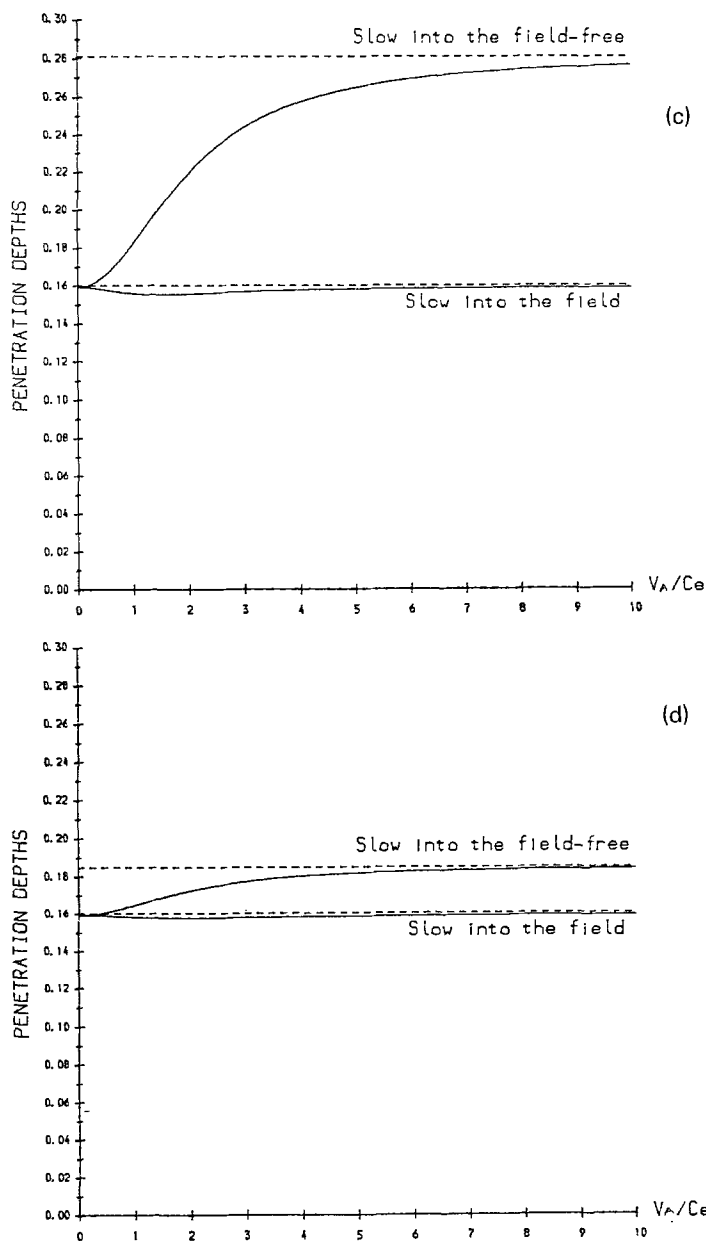


Fig. 6c, d.

Fig. 6. The variation with v_A/c_e of the penetration depths (in units of $2\pi/K$) of the fast and slow surface waves into the field-free and magnetic regions for $\theta = 0^\circ$ and 60° . (a) $\theta = 0^\circ$, $c_0/c_e = 0.75$, (b) $\theta = 0^\circ$, $c_0/c_e = 1.4$; (c) $\theta = 60^\circ$, $c_0/c_e = 0.75$; (d) $\theta = 60^\circ$, $c_0/c_e = 1.4$. For large values of v_A/c_e , the slow surface wave penetration depth asymptotes to $1/2\pi(1 - (c_0^2/c_e^2)\cos^2\theta)^{1/2}$ in the field-free medium, and to

$$\frac{(\gamma/2)(c_0^2/c_e^2)}{2\pi(1 - (c_0^2/c_e^2)\cos^2\theta)^{1/2}}$$

in the field medium. The penetration depth of fast surface wave asymptotes (for large v_A/c_e) to

$$\frac{1}{2\pi \left\{ 1 - \frac{2\cos^4\theta}{\gamma^2} \left[\left(1 + \frac{\gamma^2}{\cos^4\theta} \right)^{1/2} - 1 \right] \right\}^{1/2}}$$

in the field-free medium, and to $(2\pi)^{-1}$ in the field-medium.

5. Discussion

Fast and slow magnetoacoustic surface waves may arise whenever there is a discontinuity in the temperature or in the magnetic field. In the case of an interface one side of which is field-free, a slow surface wave can always propagate but a fast surface wave will propagate only if the sound speed in the field is less than the Alfvén speed and the field-free medium is warmer than the field medium (Roberts, 1981). At a magnetic–magnetic interface, however, both slow and fast magnetoacoustic surface waves can arise but there exists a critical value of c_0/c_e below which the slow surface wave does not propagate.

At the magnetic, nonmagnetic interface the fast surface wave penetrates more than the slow surface wave into either media and both penetrate more into the field-free medium than into the field for large v_A/c_e .

Acknowledgements

One of us (RJ) wishes to thank Alan J. Miles and Andrew de Ville for helpful discussions.

References

- Cadez, V. M. and Okretic, V. K.: 1989, *J. Plasma Phys.* **41**, 23.
 Cally, P. S. and Adam, J. A.: 1983, *Solar Phys.* **85**, 97.
 Chandrasekhar, S.: 1961, *Hydrodynamic and Hydromagnetic Stability*, Clarendon Press, Oxford, Section 97.
 Davila, J. M.: 1990, in E. R. Priest and V. Krishan (eds.), 'Basic Plasma Processes on the Sun', *IAU Symp.* **142**, 149.
 Gerwin, R.: 1967, *Phys. Fluids* **10**, 2164.
 Giovanelli, R. G.: 1972, *Solar Phys.* **27**, 71.
 Goedbloed, J. P.: 1971, *Physica* **53**, 412.
 Hollweg, J. V.: 1986, in B. Buti (ed.), *Advances in Space Plasma Physics*, World Scientific, Singapore, p. 77.
 Hollweg, J. V.: 1987a, *Astrophys. J.* **312**, 880.
 Hollweg, J. V.: 1987b, *Astrophys. J.* **320**, 875.
 Hollweg, J. V.: 1990, *Comp. Phys. Reports* **12**, 205.
 Ionson, J. A.: 1978, *Astrophys. J.* **226**, 650.
 Ionson, J. A.: 1985, *Solar Phys.* **100**, 289.
 Kruskal, M. and Schwarzschild, M.: 1954, *Proc. Roy. Soc.* **A223**, 348.
 Lee, M. A. and Roberts, B.: 1986, *Astrophys. J.* **301**, 430.
 Miles, A. J. and Roberts, B.: 1989, *Solar Phys.* **119**, 257.
 Nye, A. H. and Thomas, J. H.: 1974, *Solar Phys.* **38**, 399.
 Nye, A. H. and Thomas, J. H.: 1976, *Astrophys. J.* **204**, 582.
 Rae, I. C. and Roberts, B.: 1981, *Geophys. Astrophys. Fluid Dyn.* **18**, 197.
 Rae, I. C. and Roberts, B.: 1983, *Solar Phys.* **84**, 99.
 Roberts, B.: 1980, *Ann. Phys. Fr.* **5**, 453.
 Roberts, B.: 1981, *Solar Phys.* **69**, 27.
 Roberts, B.: 1986, in W. Deinzer, M. Knölker, and H. H. Voigt (eds.), *Small Scale Magnetic Flux Concentrations in the Solar Photosphere*, Vandenhoeck and Ruprecht, Göttingen, p. 169.
 Rytova, M. P.: 1990, in J. O. Stenflo (ed.), 'Solar Photosphere: Structure, Convection and Magnetic Fields', *IAU Symp.* **138**, 229.
 Small, L. M. and Roberts, B.: 1984, *The Hydromagnetics of the Sun*, ESA SP-200, p. 257.
 Somasundaram, K. and Uberoi, C.: 1982, *Solar Phys.* **81**, 19.

- Spruit, H. C.: 1983, in J. O. Stenflo (ed.), *Solar and Stellar Magnetic Fields: Origins and Coronal Effects*, D. Reidel Publ. Co., Dordrecht, Holland, p. 41.
- Spruit, H. C. and Roberts, B.: 1983, *Nature* **304**, 401.
- Thomas, J. H.: 1985, in H. U. Schmidt (ed.), *Theoretical Problems in High Resolution Solar Physics*, Max-Planck-Institut für Astrophysik, Garching, p. 126.
- Uberoi, C.: 1982, *Solar Phys.* **78**, 351.
- Wentzel, D. G.: 1979, *Astrophys. J.* **227**, 319.
- Zhugzhda, Y. D. and Dzhaliilov, N. S.: 1984, *Astron. Astrophys.* **133**, 333.
- Zirin, H. and Stein, A.: 1972, *Astrophys. J.* **178**, L85.