grounds remains one of the outstanding problems of fish behaviour, to which further studies on schooling could make a significant contribution.

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Amplitude Variations in Pulsed Radio Sources

by

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Refraction by irregularities in the interstellar medium may cause the long period fluctuations in the amplitudes of radio signals from the pulsed sources.

THE radio pulses from the sources discovered by Hewish *et al.*[1,2] show extremely regular repetition rates but quite erratic amplitude fluctuations, over time scales ranging from less than a second[1] to hours[3] or even days[1]. The rapid pulse-to-pulse variations appear to be well correlated on different frequencies[3], and, as will be shown, they are most probably an intrinsic property of the source. The long-period variations show little, if any, correlation between different frequencies[3], and it seems difficult to imagine a source of wide-band pulses which have a very complex spectrum that is preserved over hundreds or thousands of pulses. Such complex frequency structure rather suggests that the emission process is modulated by interference phenomena, either between a number of coherent pulsed sources on the surface of a star, or else between a number of images of the source caused by irregular refraction in an interstellar medium. The latter possibility is considered in this article.

For detailed discussion of scattering in irregular media the reader is referred to Uscinski[4] and for the correlation of scintillations at different wavelengths to Little[5], and to references quoted in these. Only rough calculations will be attempted here.

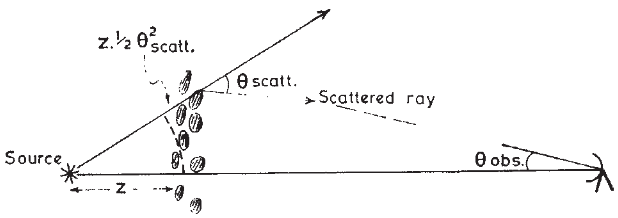

Fig. 1. Geometry of source, screen and observer.

The interstellar medium contains large regions of ionized gas with $10^{-1\pm1}$ electrons cm^{-3}; the clearest evidence for the existence of such a general distribution of free electrons comes from the rotation measures of polarized radio sources and from the dispersion of the pulses from the pulsed radio sources themselves[1,3]. Suppose the medium between us and the source is not perfectly regular, but has irregularities characterized by an electron density perturbation ΔN and a linear scale a; the phase perturbation on passing through one such irregularity is

$$\delta\varphi = \Delta N r_0 \lambda a \tag{1}$$

at wavelength λ, where r_0 is the classical electron radius ($2{\cdot}8\times10^{-13}$ cm). On passing through a depth L of such irregularities the root mean square phase deviation becomes

$$\Delta\varphi \simeq (L/a)^{\frac{1}{2}}\delta\varphi \simeq (La)^{\frac{1}{2}}\Delta N r_0 \lambda \tag{2}$$

Because the observed amplitude variations are large, any scattering screen which plays an important part must have $\Delta\varphi > \pi$ radians, so that nearly all the incident radiation is scattered at least once.

For most purposes the scattering is equivalent to that of a physically thin screen, causing phase deviations $\Delta\varphi$ given by equation (2), still on a linear scale a, and placed somewhere near the middle of the depth L of the actual screen[6]. Because $a \gg \lambda$ and we require $\Delta\varphi > \pi$, geometrical optics may be used to estimate the effect of the screen; incident rays are refracted through angles of the order of

$$\theta_{scat} \simeq (\lambda/2\pi)\Delta\varphi/a = (1/2\pi)(L/a)^{\frac{1}{2}}\Delta N r_0 \lambda^2 \tag{3}$$

(If the density perturbation in each irregularity has a Gaussian spatial variation $\exp(-r^2/a^2)$, and θ_{scat} and ΔN are interpreted as root mean square values, the correct result is

$$\theta_{scat} = 2^{-1/4}\pi^{-3/4}(L/a)^{\frac{1}{2}}\Delta N r_0 \lambda^2 \tag{3a}$$

Note that if only a fraction f of space in the scattering layer is occupied by irregularities, the only effect is to replace ΔN by $\Delta N\sqrt{f}$ throughout our formulae.)

If the screen is placed at a distance z from the source, where $z \ll$ (distance of source from Earth), scintillations will be observed subject to the following conditions:

(i) $\Delta\varphi > \pi$, as noted earlier.

(ii) Amplitude variations (as opposed to mere refraction) will be observed only if several interfering beams reach the observer, that is, only if the region of the screen contribution to the observed radiation is larger than the size of one irregularity

$$z\theta_{scat} > a$$

(iii) The scintillations will be similar at wavelengths λ and $\lambda + \Delta\lambda$ if the phase differences between the various interfering beams are the same at $\lambda + \Delta\lambda$ as at λ, to within π radians, say. The phase differences are made up of two parts; the first, caused by beams passing through different parts of the screen, is about $\Delta\varphi$; the second, caused by different physical path lengths, is of the order of $(2\pi/\lambda)(\frac{1}{2}\theta^2_{scat}z)$. From equation (3) and condition (ii) it is easily seen that the latter contribution is greater. Thus the scintillations are correlated over a bandwidth

$\Delta\lambda$ given by

$$\Delta\lambda/\lambda = \lambda/(z\theta^2_{scat}) \tag{4}$$

(The estimate (4) corresponds to making Little's[5] parameter $K = \pi$, so that his "bandwidth visibility" is about 0·5.) This condition is much stronger than a similar one, that the rise time of the pulses must not be drawn out more than a few ms by arriving by different paths.

Formally identical restrictions may be derived by similar arguments when the screen is at a distance z from the observer and the source is much further away.

The conditions (i) to (iii) show what ranges of ΔN and a could produce large amplitude variations coherent over a bandwidth $\Delta\lambda$. They may be represented uniquely in terms of the dimensionless scaled quantities

$$\Delta N^* = \Delta N L^{\frac{1}{2}} r_0 \lambda^{5/4} z^{1/4} \tag{5}$$
$$a^* = a z^{-\frac{1}{2}} \lambda^{-\frac{1}{2}}$$

when they become

$$\text{(i)}\ \Delta N^* a^{*\frac{1}{2}} > \pi; \quad \text{(ii)}\ \Delta N^*/a^{*3/2} > (2\pi^3)^{1/4};$$
$$\text{(iii)}\ \Delta\lambda/\lambda = (2\pi^3)^{\frac{1}{2}} a^*/(\Delta N^*)^2 \tag{6}$$

as shown in Fig. 2.

Scintillation by the Interstellar Medium

In the case of the general interstellar medium $z \simeq L \simeq$ (distance of source), and Fig. 3 shows the conditions (i), (ii) and (iii) for this case, taking the distance to be $1{\cdot}5 \times 10^{20}$ cm and $\lambda = 370$ cm. It seems that even very small fluctuations in an assumed mean density of 0·1 electrons cm^{-3} can cause significant scintillation.

Observations of other radio sources already place severe limits on the irregularities in the interstellar gas; the strongest of these is the observation that the small diameter source in the Crab nebula has an apparent diameter less than 0·4″ arc at 38 MHz, so that $\theta_{scat} < 2 \times 10^{-6}$ radians for $L = 3 \times 10^{21}$ cm, $\lambda = 790$ cm, and therefore $\theta_{scat} < 10^{-7}$ radians for $L = z = 1{\cdot}5 \times 10^{20}$ cm, $\lambda = 370$ cm. This restriction is also shown in Fig. 3; it adds nothing to the condition that scintillations correlate over at least 0·5 MHz near 81·5 MHz.

Time Scale of the Scintillations

Two further restrictions must now be imposed: the scintillations must not be smoothed out by the finite diameter of the source, and the time scale τ of the scintillations must correspond with the time scale of some observed amplitude fluctuations.

The irregular screen produces a diffraction pattern in space; fluctuations are observed only because either the Earth moves through the diffraction pattern, or because relative motion of the source and the screen sweeps the pattern past us. It will be assumed that the relevant

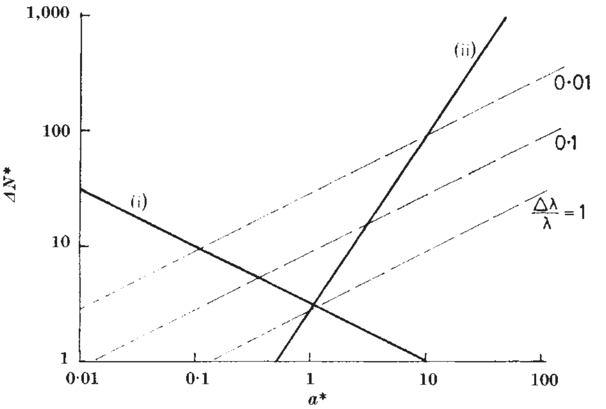

Fig. 2. The conditions required for large amplitude fluctuations, in terms of the scaled parameters ΔN^* and a^*. The point representing the screen must lie above lines (i) and (ii); the bandwidth over which scintillations are similar is shown by the dashed lines. Note that deep scintillation necessarily implies $(\Delta\lambda/\lambda) < 1$.

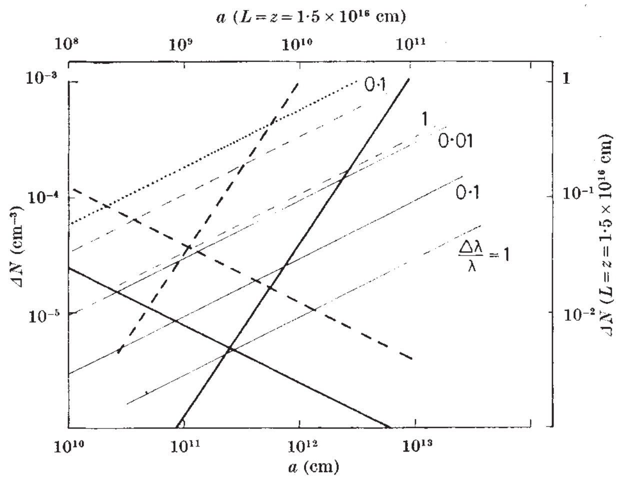

Fig. 3. Conditions required for large amplitude fluctuations in a source observed through $1{\cdot}5 \times 10^{20}$ cm of interstellar medium, shown as in Fig. 2. Full lines: 81·5 MHz; dashed lines: 408 MHz. The dotted line shows the upper limit on the strength of interstellar irregularities obtained from observations of the Crab Nebula. If only a region of radius $1{\cdot}5 \times 10^{16}$ cm around the source causes the scintillations, the conditions for the medium are given by the same diagram when used with the top and right-hand scales for ΔN and a.

velocities v do not greatly exceed 30 km s^{-1} in either case.

First consider the Earth's motion.

$$v\tau = \text{scale of diffraction pattern} = \lambda/\theta_{obs}$$

where θ_{obs} is the angular spectrum observed at the Earth; it is clear from Fig. 1 that θ_{obs} may be less than θ_{scat} but it cannot be greater. By the obvious analogue of equation (4)

$$\Delta\lambda/\lambda \leqslant \lambda/(z\theta^2_{obs})$$

where z is the distance of the screen from the Earth. Hence

$$v\tau = \lambda/\theta_{obs} \geqslant (z\Delta\lambda)^{\frac{1}{2}} \tag{7}$$

Thus, with $\Delta\lambda = 5$ cm, $z = 1{\cdot}5 \times 10^{20}$ cm, and $v < 30$ km s^{-1} we should have $v\tau \geqslant 3 \times 10^5$ km, or $\tau > 10^4$ s.

A similar argument leads to the same result when motion of the source is considered. z then represents the distance from the screen to the source in relation (7). With a screen close enough to the source, and perhaps associated with it, one might then hope to account for short-period variations. As shown by the alternative scales on Fig. 3, scintillations can still be achieved with tolerable values of ΔN and a for $L = z = 1{\cdot}5 \times 10^{16}$ cm, giving $\tau \simeq 100$ s if other parameters in relation (7) are left unchanged. One cannot account for still shorter variations (such as pulse-to-pulse variations), however, for the finite size of the source now becomes important. The argument is simple and general: suppose that a point source must move a distance x to shift the observed diffraction pattern by its own correlation length; then the time scale of scintillations is given by $v\tau = x$. But if the source consists of incoherent elements spread out over a region of diameter $d > x$, the simultaneous presence of their displaced diffraction patterns smoothes out any possible scintillations. The time scale of scintillation must thus exceed the time it takes the source to move by its own diameter. For a source of 6,000 km diameter (white dwarf ?) and $v = 30$ km s^{-1}, $\tau > 200$ s. To obtain pulse-to-pulse variations we should require a source on the scale of 10 km with a screen at a distance not much greater than 10^{11} cm containing (compare with Fig. 3 and relations (5)) 10–100 km irregularities with electron density fluctuations of 10–100 cm^{-3}; or else a very large relative velocity between source and screen. (Note that most kinds of orbital motion are already excluded, for the corresponding displacements in pulse times have not been observed[1].)

Amplitude Distribution of Pulses

When a plane wave is scattered by a weakly scattering screen ($\Delta\varphi \ll 1$) the diffracted field amplitude has a Rice distribution. For thicker screens ($\Delta\varphi > 1$) the amplitude distribution approximates to a Rayleigh distribution, and consequently the distribution of intensity I becomes exponential.

$$\mathrm{d(probability)} = \exp(-I/I_0)\mathrm{d}I/I_0 \qquad (8)$$

The distribution of individual pulse heights from *CP* 1919 at 81·5 MHz has been plotted by Collins (private communication) and is not exponential, having a much longer tail of very large pulses. The same appears to be true of *CP* 0950. So far as longer-term variations are concerned, an adequate statistical sample is not yet available to me, but the observations to date suggest that here, too, the high intensity tail of the distribution is considerably larger than in an exponential distribution. Over intermediate periods, in which there is no correlation between successive intervals, the distribution of the running means necessarily approaches a Gaussian distribution about a mean.

Though the amplitude distributions are inconsistent with scattering by a single screen, it should be noted that long-tailed amplitude distributions could arise from more complex scintillation processes. As a very simplified example, consider a screen (with $\Delta\varphi > 1$) close to the source, which projects a diffraction pattern on to a second screen (also with $\Delta\varphi > 1$) further away from the source, and suppose also that all that part of the second screen which contributes to the radiation observed by us lies within one correlation length of the diffraction pattern from the first screen. The modulation of the observed intensity is then the product of two exponentially distributed random variables. The result is easily shown to be a distribution of the form

$$\mathrm{d(probability)} = K_0(\sqrt{2I/I_0})\mathrm{d}I/I_0 \qquad (9)$$

where K_0 is a modified Bessel function. The function (9) is not (and is not expected to be) a quantitative fit to the data, but it shows the required qualitative features, that is, much larger proportions of very large and very small intensities than an exponential distribution.

Discussion

Pulse-to-pulse variations are probably not caused by a scintillation mechanism, because:

(*a*) There are severe difficulties with the time scale. The screen would have to be fairly dense and close to the source, and the source would have to be either a neutron star or a source of highly directional radiation to have a sufficiently small effective size.

(*b*) It has been reported that pulse-to-pulse variations are closely correlated over a wide range of frequencies[3]. Fig. 2 shows that this is incompatible with deep scintillations.

(*c*) The distribution of pulse amplitudes has a much longer tail than an exponential distribution.

Scintillation is a much more attractive explanation for the long-period variations, but there are severe difficulties in this case too:

(*a*) Time scale. The 1 min variations in *CP* 1919 at 81·5 MHz, and the variations observed in three sources at 151 MHz with time scales of a few minutes[3], are too fast to be explained in terms of the general interstellar medium, unless the source moves with a speed of several thousand km s^{-1}.

(*b*) A screen which produces strong scintillation ($\Delta\varphi > \pi$) at 922 MHz[3] has $\lambda/(z\theta^2_{\mathrm{scat}}) < 1$ at this frequency, and therefore has $\Delta\lambda/\lambda = \lambda/(z\theta^2_{\mathrm{scat}}) < (81{\cdot}5/922)^3$ at 81·5 MHz; that is, scintillations near 81·5 MHz become uncorrelated within 80 kHz. Large amplitude variations are observed with a receiver bandwidth of 1 MHz, however, and indeed in the case of *CP* 0834 the amplitude is coherent over a range of 4 MHz near 80 MHz[7]. Thus no diffracting screen can account for both the high frequency and the low frequency variations.

The question still arises whether $\Delta\lambda/\lambda$ is necessarily so small at low frequencies if high and low frequency scintillations are produced by different regions of the interstellar medium. It turns out that a consistent model can be devised, but only if one screen is very much closer to the source than the other. Suppose screen 1, producing deep scintillation at a short wavelength λ^+, is at distance z_1 from the source, and screen 2, at distance $z_2 > z_1$ from the source and z from the observer, is capable of producing deep scintillations at a long wavelength λ. The radiation falling on screen 2 has an angular spectrum of width $(z_1/z_2)\theta_{\mathrm{scat}1}$, and will behave just like an incoherent source with this angular diameter if we observe with a receiver the bandwidth of which exceeds $\Delta\lambda/\lambda$ at the long wavelength. This angular diameter will blur out the scintillation caused by screen 2 unless

$$(z_1/z_2)\theta_{\mathrm{scat}1}z < \text{(scale of diffraction pattern at observer)}$$

$$\simeq \lambda/\left\{\left(\frac{z_1}{z_1+z}\right)\theta_{\mathrm{scat}\ 2}\right\}$$

Combining this with the conditions that screens 1 and 2 can produce deep scintillations at wavelengths λ^+ and λ, respectively, we find that

$$\frac{z_1}{z+z_2} < \frac{z_2}{z}\left(\frac{\lambda^+}{\lambda}\right)^3 \qquad (10)$$

Another type of model is also possible. Suppose screen 1 produces the long wave scintillations (and has little effect at the short wavelength λ^+), while screen 2 produces the scintillation at wavelength λ^+. The long wave scintillations caused by screen 2 have $(\Delta\lambda/\lambda) < (\lambda^+/\lambda)^3$ and will be smoothed out by any normal receiver bandwidth, but the amplitude variations caused by screen 1 will be observed provided the scale of the diffraction pattern falling on screen 2 is larger than that part of screen 2 contributing to the radiation observed at any one time; that is, if

$$\frac{\lambda}{(z_1/z_2)\theta_{\mathrm{scat}1}} > z\frac{z_2}{z+z_2}\theta_{\mathrm{scat}2}$$

Taken together with the conditions for deep scintillation at each screen, this leads again to the condition (10) obtained for the first model.

The second of these models seems the more plausible as an approximation to a real situation. For screen 2 we might take the general interstellar medium, which could easily produce scintillations at frequencies of the order of 1,000 MHz with time scales of some hours, as observed[3]. Screen 2 would then have to be the medium within 0·01 to 0·1 pc of the source to account for scintillations at frequencies around 100 MHz. The smaller distance of the screen from the source would permit the shorter time scale of these variations, and the strength of the irregularities required (see Fig. 3), though a good deal greater than that of the general interstellar medium, would not be implausibly great. If this model is in any sense correct, then additional amplitude variations should be found at low frequencies when the sources are observed with very narrow-band receivers.

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