Pulsar scintillations from corrugated reconnection sheets in the ISM

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ABSTRACT

We show that surface waves along interstellar current sheets closely aligned with the line of sight lead to pulsar scintillation properties consistent with those observed. By contrast with previously considered scintillation drivers, our mechanism naturally produces the length and density scales of the ISM scattering lenses that are required to explain the magnitude and dynamical spectrum of the scintillations. In our picture, the parts of warm ionized interstellar medium that are responsible for the scintillations are relatively quiescent, with scintillation and scattering resulting from weak waves propagating along magnetic domain boundary current sheets, which are both expected from helicity conservation and have been observed in numerical simulations. The model quantitatively predicts the spacing and amplitudes of inverted parabolic arcs seen in Fourier-transformed dynamical spectra of strongly scintillating pulsars. Multi-frequency, multi-epoch low frequency VLBI observations can quantitatively test this picture. If successful, in addition to mapping the ISM, this opens the door to precise nanoarcsecond pulsar astrometry, distance measurements, and emission studies using these 100AU interferometers in the sky.

Key words: Interstellar Medium, reconnection

1 INTRODUCTION: OBSERVATIONS AND THEORETICAL CHALLENGES

Compact radio sources provide a precision probe of the ionized interstellar medium (IISM). The propagation speed of radio waves depends on the density of free electrons, and therefore the spatial inhomogeneity of the IISM may result in refractive and diffractive abberation and scattering. This causes scintillation (time-variability) of the compact radio-sources (Scheuer 1968; Blandford et al. 1986; Rickett et al. 2006) . Observations of the pulsar scintillations are particularly interesting for infering the IISM properties, due to the brightness of many pulsars that have been observed for other purposes over long time intervals.

However, despite of considerable effort, the small-scale structure of the IISM has remained enigmatic. At a first glance, one expects the scattering to be caused by density inhomogeneities produced from turbulent motions of the IISM. However, this picture is in contradiction with the last decade of the pulsar scintillation data. There, a major observational progress of the ISM scintillations has been achieved through the Stinebring et al. (2001) detection of parabolic structures in the Fourier-transformed dynamical spectra of

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strongly scintillating pulsars. These parabolic structures imply that for these pulsars the radio-wave scattering occurs mostly within one or several thin screens (Walker et al. (2004); Cordes et al. (2006); Walker et al. (2008)). Moreover, the multiple "inverted parabolae" (Hill et al. (2005)) show that the scattering inside the screen is strongly inhomogeneous and occurs in localized clumps. The latter was recently confirmed by Brisken et al. (2010) who have obtained the VLBI scattering image PSR B0834+06. They have found that not only the scattering image was clumpy but that the clumps lined up along a thin line. All of these observational facts present a major challenge for theoretical interpretation where the scattering is caused by density inhomogeneities produced in a turbulent cascade. In particular, (1) the origin of the screens is unexplained, and (2) the strongly non-gaussian scattering requires AU-size regions which are over-pressurized by factors of $\sim 10^3$ relative to the ambient warm ISM(Romani et al. 1987); no conventional physical mechanism has been proposed for how such regions may be formed.

In this paper, we advocate a scenario in which the scattering is produced by a refractive structure, with turbulence playing a secondary role. Radio-wave scattering by non-turbulent large-scale refractive structures has been previously considered by Romani, Blandford, & Cordes (1987), mostly in the context of the so-called extreme-

scattering events observed by ?. Recently, Goldreich & Sridhar (2006) proposed that the image of the SgrA* radiosource is strongly scattered by several reconnection sheets that are closely aligned with the line of sight to SgrA*. In this paper, we develop further the Goldreich & Sridhar's (2006) idea [see also Pen & King (2012)] and apply it to construct a quantitative picture of pulsar scintillations. Namely, we consider a scenario where the pulsar radio-wave scattering occurs due to several weakly corrugated reconnection sheets that are closely aligned with the line of sight to the pulsar; see figure 1. We show that this scenario provides compelling explanations for previously unexplained features of the scintillations: (1) the "scattering screens" are simply effective descriptions of such sheets; their location is marked approximately by the sheets' intersections with the line of sight, (2) "the scattering clumps" correspond to those parts of sheet folds where the sheet is parallel to the line of sight; the strength of the scattering follows a strongly non-Gaussian distribution, even though the corrugation itself is assumed to be a realization of a Gaussian distribution, and (3) the strong non-isotropy of the Brisken et al. 2010 image is a consequence of the sheet's inclination, with the clump locations aligned along the sheet's line of nodes. The plan of the paper is as follows: in the next section we briefly describe the origin of the reconnection sheets, in section 3 we derive the model for the fold statistics, in section 4 we derive the lensing by the corrugated sheet, and in section 5 we compute the Fourier-transformed dynamical spectrum and demonstrate the parabolic structures. In section 6 we conclude.

2 ASTROPHYSICAL PICTURE

2.1 Two Regimes of Lensing: Diffractive vs Refractive

Two regimes to generate pulsar scintillation have been considered. In the diffractive regime, the scattering/lensing angle is determined by the ratio of the wavelength λ of the radio-waves, and the characteristic spatial scale L of the scattering structure: $\theta \sim \lambda/L$. The brightness of the scattered image is determined by the amplitude of the wavefront modulation caused by the scattering structure. To explain the observed angles in the range 1-100 mas at wavelength \sim 1m, requires structures in the ISM on scales of order 10⁶⁻⁸m. This imposes unexpected properties on the IISM, since this scale much smaller than the coloumb mean free path of protons and thus compressive perturbations of this scale are overdamped and decay esponentially on the soundcrossing time-scale. If one still assumes that somehow these perturbations are created and maintained, then the angular image of a pulsar, and therefore its dynamic spectrum, are superpositions of thousands of AU/10⁸m weak structures (for a 1-d scattering image), and are expected to be roughly Gaussian. The length scale is given by the path length difference, which in turn is inferred from the inverse scintillation correlation frequency. The parabolically structured 2-D power spectrum of the dynamic spectrum, and the VLBI image of the scattering disk consisting of several prominent clumps, are inconsistent with such a picture. The number of 10⁶⁻⁸m eddies along the line of sight can be large, possibly

 10^{13} for a pulsar at a distance of \sim kpc. The total scattered power comes from the cumulative projected variation in refractive index, which grows as the square root of the number of eddies. Each eddie only needs to change the refractive index by a part in a million of the cumulative change, so a refractive index variation of a part in $\sim 10^{12}$ could account for the strong scintillation. But the superposition of such a large number of eddies would surely look very Gaussian from the central limit theorem. An intermittency of 10^{13} to appear intermittent in projection would lead to highly overpressurized eddies.

A second mechanism is due to refractive lensing. In this scenario, the bending angle is determined by Snell's law, i.e. the change in refractive index and the angles of incidence. The refractive index of a plasma is $n = 1/\sqrt{1 - \omega_p^2/\omega^2}$, with plasma frequency $\omega_{\rm p}=\sqrt{n_e e^2/\epsilon_0 m_e}$. Pulsar observations are done at frequencies much higher than the plasma frequency, for which we expand $n-1 \sim \frac{\omega_p^2}{2\omega^2} = 1.8 \times 10^{-8} n_e$ at wavelengths of a meter. The observed scattered images at 20 mas require deflection angles of at least 40 mas, corresponding to $n_e \sim 10/\text{cm}^3$ neglecting geometric alignment factors. However, the mean density of the IISM is determined to be of the order 10^{-2} , as measured from the pulsars' dispersion measure. Therefore, the refractive picture is also challenging to reconcile with the data, since the observed scattering angles would naïvely require changes in free electron density of $\sim 10^3$ which are difficult to understand or confine. This contraint, however, is alleviated when one considers refractive sheets that are closely aligned with the line of sight (Goldreich & Sridhar (2006)).

Historically, refractive lensing was used to interpret long time variability, and diffraction for the minute time scale effects. Recently, it was understood that the refractive images result in an interference fringe pattern (Walker et al. 2004) which have similar time and frequency scales for flux modulation as diffractive effects. The frequency and time scaling was historically interpreted as related to an underlying stochastic diffractive process driven by turbulence. Goldreich & Sridhar (2006) showed that refractive lensing by aligned sheets results in scintillation similar to the diffractive picture.

2.2 Physical origin of the reconnection sheets

The interstellar medium is stirred on scales of parsecs by various energetic processes, including supernovae, ionization fronts, spiral density waves, and other phenomena. These processes are generally short lived, and we conjecture after the stirring, the warm medium relaxes to an nearequilibrium configuration on small (several AU) scales. Current numerical simulations of the supernova-driven turbulence in the warm ISM of the Galaxy (e.g., Hill et al. 2011) do not have the resolution to tell how realistic this assumption is. In the presence of helicity, the equilibrium magnetic fields are configured as interlaced twisted tori, which are long lived (Braithwaite & Spruit 2004). It has been shown by (Gruzinov 2009) that a "generic magnetic equilibrium of an ideally conducting fluid contains a volume-filling set of singular current layers." In this picture, the magnetic fields are locally almost parallel, with discontinuous interface regions, a bit like magnetic domains in a ferromagnet. Singular current sheets have also been seen in the simulations of Schekochihin et al. (2004).

At the boundary between between magnetic field configurations, current sheets maintain the discontinuities. Depending on the nature of reconnection, these current sheet may be self-enforcing due to inflow of fresh fields, maintaining a thickness potentially as thin as an electron gyromagnetic radius. At constant temperature, Pressure equilibrium in the direction perpendicular to the sheet implies that the electron density inside the sheet is enhanced by a factor R given by

$$R - 1 \sim \frac{B_{\text{out}}^2 - B_{\text{in}}^2}{4\pi P_{\text{out}}},$$
 (1)

where $B_{\rm in}$ and B_{rmout} is the magnetic field in and outside the sheet, and $P_{\rm out}$ is the non-magnetic pressure outside the sheet. The right-hand side of the above equation is thought to be of order 1 for the IISM, but may be significantly larger if the IISM is magnetic-pressure dominated.

Depending on the ratio of heating time due to reconnection to cooling time, the thin sheet may have an enhanced temperature, in which case it can be underdense, with $R \to 0$ in the limit of a strong entropy injection.

The lensing geometry is shown in Figure 1. A crucial ingredient in our model is that the reconnection sheet is assumed to be weakly corragated. Let is assume that the corrugation pattern is fixed, and vary the inclination angle. In this case, as can be seen from figure 1, there is characteristic inclination angle α at which the perturbations in the sheet can generate caustics in the projected surface density. This angle α is the ratio of the wave peak displacement to its wavelength. We envision ranges of $\alpha \sim 10^{-3}-10^{-2}$. When the caustics are present, they become effective refractive scattering centers for the radio waves. The resulting lense features multiple images that closely line up along the direction perpendicular to the line of nodes of the scattering sheet.

2.3 Surface Dynamics

We assume that the current sheet is physically thin, \leq AU. Theoretically, the thickness of current sheets is not understood, so we choose this scale to be thin enough to explain the smallest scale observed structures. On each side the magnetic field points in a different direction. The change in alvenic properties gives rise to surface wave (Jain & Roberts 1991; Joarder et al. 2009), whose amplitude decays exponentially with the distance to the current sheet and which are mathematically analogous to deep water ocean waves. The restoring force is due to the difference in magnetic field component projected along the wave vector. Like ocean waves, these waves penetrate about a wavelength into each side. We will be considering wavelengths of hundreds to thousands of AU whose projected wavelength is related to the observed lensing structures ~several AU, so the thickness of the current sheet itself is neglible as far as the dynamics of the waves are concerned. Seen in projection along the aligned sheet, the projected wavelengths will be \sim AU, reduced by the alignment angle α .

The displacements are transverse to the wave vector, and perpendicular to the sheet. While alvenic in nature, the surface modes possess only one polarization, unlike bulk

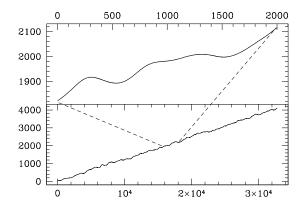


Figure 2. Sheet with transverse perturbations. The upper panel shows an zoomed version of a short section.

waves. These waves resemble a flag blowing in the wind. Disturbances travelling along the sheet are decoupled from bulk waves. Being confined to a sheet, the amplitude away from a source drops as $\propto 1/r$ instead of the normal inverse square law. A amplitude of order $\alpha\lambda_{\rm wave}$, or about 10^{-3} – 10^{-2} of the wavelength of the surface wave, is sufficient to cause the sheet to appear folded in projection; here $\lambda_{\rm wave}$ is the wavelength of the surface wave.

3 FOLD STATISTICS

We expect a minimum wavelength of these surface waves, which is substantially larger than the thickness of the sheet. Short wavelength perturbations are not bound to the surface, and can dissipate into the bulk. The exact cutoff depends on unknown factors, including the distance to the source, and vertical structure of the current sheet. Nonlinear effects also cause short wavelength waves to dissipate, just like sound waves in the air. Primarily waves with amplitude larger than α contribute to scattering. The strength of damping depends on the distance to the source in units of wavelength, with shorter wavelengths being more damped.

We model the waves as a displacement function $\zeta(x)$ which is a Gaussian random field with a correlation function that is a Gaussian, $\xi(r) = \langle \zeta(x)\zeta(x+r)\rangle = A^2 \exp(-r^2/2/\sigma^2)$, where A is the mean amplitude of the displacement and σ is the surface-wave cogerence scale. Figure 2 shows a realization of a sheet with a random fluctuations.

The current sheet has a reduced magnetic pressure and thus has a refractive index different from the ambient ISM. For simplicity, we assume here that the sheet has constant thickness, and consider its optical depth as is relevant for refractive lensing. In projection along the line of sight, the column density of the sheet results in a highly non-Gaussian distribution. Figure 3 shows the column density distribution in a simulation. Folds occur when the gradient of the displacement is equal to $-\alpha$. Let θ be the angle between the screen and the line of sight Then the optical depth is $\rho \propto 1/\theta$ for $\theta \ll 1$, and $P_{\text{screen}}(\rho) \propto 1/\rho^2$, where $P_{\text{screen}}(\rho)$ is the probability of a piece of screen to have the optical depth of ρ . However, we are interested in the probability density with respect to the impact parameter relative to the line of

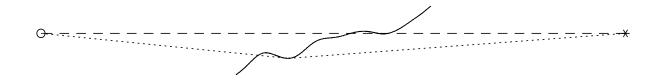


Figure 1. lensing geometry. The earth is at left, pulsar at right. A section of the scattering sheet is in the middle. The dashed line shows the unperturbed light path. The dotted line shows the path of an image lensed by a fold caustic of the projected sheet.

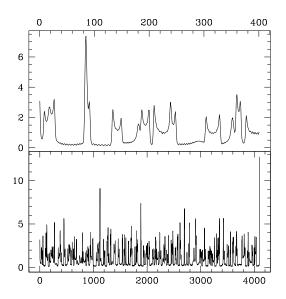


Figure 3. projected density. The upper panel is a zoom of the central portion of the lower panel. Each time the sheet folds in projection, we see a caustic structure in density.

sight, and not with respect to the location on the screen. For nearly aligned screens, this is not the same thing. In particular, part of the screen with low θ occupies less of the impact-parameter space than the part of the screen with the same area but high θ . Thus,

$$P_{\rm impactparameter}(\rho) \propto 1/\rho^3$$
 (2)

which is a strongly non-Gaussian distribution.

Equation (2) describes the 1 point PDF in figure 3. The deflection angle is ρ'

The convergence $\kappa \propto \rho''$ is given by the second derivative of the density. The magnification $\mu = 1/(1-2\kappa)$.

!The regions near the locations where the screen is parallel to the line-of-sight, which the call the caustics, give rise to the localized clumps in the pulsar's scattering image, which produce the inverted parabolic arcs in pulsar secondary spectra.

4 LENSING

The lensing of this density sheet can be computed in analogy to Pen & King (2012).

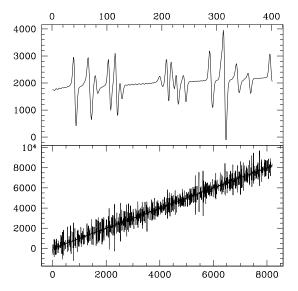


Figure 4. deflection angle mapping. The horizontal axis is angle on the sky, and the vertical axis is the intersection of this light ray on the source plane. Whenever multiple different directions on the sky intersect on the same position in the source plane, multiple images are formed, which form a coherent interference pattern.

Given the projected column density distribution in Figure 3, and assuming that the sheet has a fixed optical depth along its normal, we can compute the mapping of apparent angle on the sky to position in the source (pulsar) plane. The caustics in the projected density distribution lead to large angle deflections, and multiple imagine, whenever the sheets are aligned closer than the amplitude of the perturbations. This explains why only a small fraction of (edge-on) current sheets contribute to scintillation.

This model predicts the number of images and their fluxes as a function of separation (time lag). It has a single free dimensionless parameter, which is the maximum projected magnification. There is a dimensional scaling of time units, which is a function of pulsar transverse velocity, projected screen size, and observing frequency. We show a histogram of image magnifications in Figure 5, which can be compared to holographic flux measurements Walker et al. (2008).

Other predictable observables include the density of arclets as a function of separation, shown in figure 6

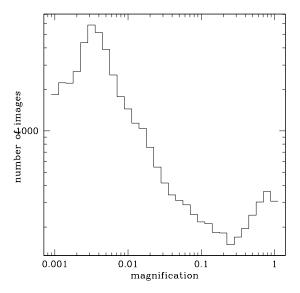


Figure 5. PDF of image magnifications. The peak near 1 are images at the unscattered positions, while the population on the left are lensed images.

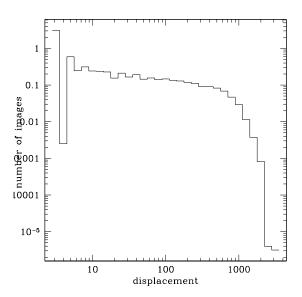


Figure 6. PDF of image positions per logarithmic distance interval. The images are scattered out to a characteristic scattering scale.

5 SIMULATED DYNAMIC SPECTRA

With the density field, we can solve the lens equations to simulate dynamic spectra. By adding the voltages on each image with their appropriate amplitude and phases, we simulate the dynamic spectrum, shown in figure 8.

A 2-D fourier transform maps this dynamic spectrum into a secondary spectrum, shown in figure 9.

We find that the interference of these discrete, co-linear images forms the inverted parabolic arcs, qualitatively similar to those that are observed in Stinebring et al. (2001) and Hill et al. (2005).

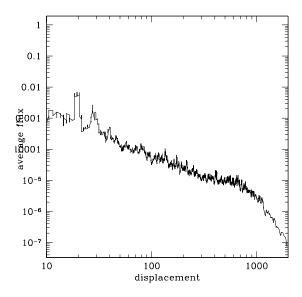


Figure 7. Average Spatial Distribution of flux per logarithmic distance interval.

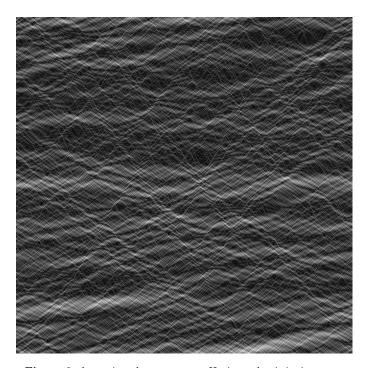


Figure 8. dynamic pulsar spectrum. Horizontal axis is time, vertical axis is frequency. We reproduce the characteristic criss-cross pattern observed in real scintillation spectra.

6 DISCUSSION

We can estimate the length scales involved in making the current sheet that would produce the observed scintillation pattern. This theory requires as input a current sheet thickness, inclination angle, curvature, amplitude of waves, and dissipation scale.

The thickness of the sheets is determined by the magnification of images. As shown in Pen & King (2012), the flux is roughly the thickness divided by the impact parameter.



Figure 9. secondary pulsar spectrum. The inverted parabolic arcs arise naturally in this model.

This follows from flux conservation of lensing: the net flux is conserved, and flux changes by order unity at impact parameters of order the physical size of the lens, so the typical flux off-axis is roughly the ratio of the furthest distance at which at image forms, to the size the lens. The caustic itself only contains a small fraction of the integrated flux.

It suggests a typical thickness of $h \lesssim 0.1$ to explain the scattering amplitudes observed in Brisken et al. (2010). A wave of projected wavelength $\alpha \lambda_{\rm wave}$ enhances projected densities by a factor $w = \sqrt{\alpha \lambda/h}$ The projected wavelengths are known to be of order ~ 10 AU, so $w \sim 10$. The largest deflection angles are $\gamma \sim 0.1$ ", which requires a projected electron density change $\delta n_e/(w\alpha) \sim 100$.

As discussed in Pen & King (2012), the ESE phenomenology prefers underdense lenses. In an underdense sheet, the maximal change of density is the density itself. For the mean interstellar medium densities determined from pulsar dispersion, we obtain $\alpha \sim 10^{-2}$. The probability of seeing a sheet at such an angle is $\sim \alpha^2$, requiring the existence of $\sim 1/\alpha^2$ sheets along the line of sight: if we imagine each "sheet" to be a thin disk, the probability of seeing a disk at angle α gets one contribution from the intrinsic alignment distribution, and one more from the reduced geometric cross section of an aligned sheet.

For B0834+06, the distance is ~ 1 kpc, giving a typical sheet separation of $s \sim 0.1$ pc. If we assume the curvature to be separation (it could be smaller), we find a limiting projection angle $\alpha_{\rm min} = \sqrt{h/s} \sim 0.002$. The wave amplitudes must be $> \alpha \lambda$ in order to form projected caustics. This is a small amplitude, which rules out large amplitude universal turbulence. The latter would deform the sheet too much. In a Kolmogorov spectrum, the energy power spectrum scales as $k^{-5/3}$. Scaling from the alven damping scale to the fresnel scale, this results in density fluctuations $< 10^{-4}$. This is insufficient to create scintillation in pulsar fluxes.

These estimates are qualitative. One expects current sheets to come in a range of sizes, curvature and perturbation amplitude. The thickness might also vary.

One of the attractive features of our mechanism is that it explains very naturally the 1-d image of Brisken et al. (2010). The reduced dimensionality of the scattering image comes from the fact that the deflection created by the screen is mostly in the direction perpendicular to the screen's line of nodes. Therefore, the scattering clumps will also form a line that is perpendicular to the line of nodes. This is demonstrated in Fig. 9, where we show a simulated

Figure 10. Simulated scattering image of the pulsar. This compares favourably with the VLBI images of Brisken et al. (2010)

2-d scattering image of a pulsar. The alignment of the scattering clumps is apparent.

7 FUTURE POTENTIAL

In our picture, pulsar scintillation is dominated by a small number of magnetic discontinuities highly aligned to the line of sight. Surface waves will move very slowly in this projected geometry, allowing for a precise determination of the geometric properties. This allows the use of these sheets as lenses to study both pulsars and the ISM.

7.1 ISM dynamics

Free parameters in our model include the thickness of the sheet and the inclination angle. These can be inferred by broad band measurements of pulsar scintillation, as follows. Firstly, the apparent position of images is expected to shift by a distance of order of the sheet width, as one decreases the observing frequency from the highest critical frequency at which the image forms, to a factor of two below. Secondly, the co-linearity of the images is related to the inclination angle of the sheet: the more aligned is the sheet with the line of sight, the greater is the Aspect ratio of the scattering image.

7.2 Pulsar Emission imaging

A straightforward application is the study of the reflex motion of the emission region of the pulsar across the pulse phase. One expects the apparent emission region to move by distance of order of the effective emission height multiplied by the ratio of the pulse width to the rotation period.

7.3 Distance Measurement

It is tempting to use these lenses to obtain precision parallax distances to pulsars. The challenges in obtaining the parallax without the lense is that over the period of a year, which is needed to obtain earth-sun parallax, typical pulsars move by much more than an astronomical unit, making the differential measurements challenging. In the sheet lensing scenario, one can imagine obtaining widely separated scattering images: the lensing angle scales as $\propto \lambda^2$, so at low frequencies, for example with LOFAR LBA, hundreds of AU are probed. Over the course of a year, persistent images should remain visible, and the interference pattern could be studied as a

function of annual modulation. This would result in direct parallax distances with nano arc second precision, enough to determine pulsar distances for coherent gravity wave detection.

8 CONCLUSIONS

We have presented a quantitative theory of pulsar scintillation inverse parabolic arcs. These extend recent ideas of Goldreich & Sridhar (2006) and Pen & King (2012) about thin current sheets as the scattering objects in the ISM, which naturally explain the large angle scattering observed in pulsars and some extragalactic sources.

This picture could explain all scintillation phenomena from refractive lensing, all with structures greater than 0.1 AU. The apparent diffractive structure results from the interference between refractive images, and no diffractive scattering is needed.

In this scenario, VLBI monitoring of pulsars on time scales of weeks, at multiple low frequencies, can allow forecasts of the scattering behaviour. This in turn could improve gravity wave timing residuals. The same scenario also enables the coherent use of the scattered images as a gigantic interstellar interferometer to map the motions of the pulsar emission regions.

9 ACKNOWLEDGEMENTS

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