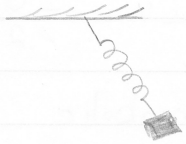
non-dimensionalization

3) b)

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x + \frac{k}{m}\frac{l_0 x}{\sqrt{x^2+(l-y)^2}}$$



$\tau = \sqrt{\frac{l}{g}}$, $L = l$ replace all ts

$t = \bar{t}\tau$, $x = L\bar{x}$ (l) replace all xs , $\sigma = \frac{mg}{kl}$, $l_0 = l(1-\sigma)$

$$\frac{L}{\tau^2}\frac{d^2\bar{x}}{d\bar{t}^2} = \frac{-k}{m}L\bar{x} + \frac{k}{m}\frac{l_0 L\bar{x}}{\sqrt{L^2\bar{x}^2+(l-L\bar{y})^2}}$$

$$\frac{lg}{l}\frac{d^2\bar{x}}{d\bar{t}^2} = \frac{-k}{m}l\bar{x} + \frac{k}{m}\frac{l_0 l\bar{x}}{l\sqrt{\bar{x}^2+(1-\bar{y})^2}}$$ use

$$\frac{d^2\bar{x}}{d\bar{t}^2} = \frac{-kl}{mg}\bar{x} + \frac{k}{mg}\frac{l_0\bar{x}}{\sqrt{\bar{x}^2-(1-\bar{y})^2}}$$ use the hints

$$\frac{d^2\bar{x}}{d\bar{t}^2} = \frac{-1}{\sigma}\bar{x} + \frac{kl}{mg}\frac{(1-\sigma)\bar{x}}{\sqrt{\bar{x}^2-(1-\bar{y})^2}}$$

$$\boxed{\frac{d^2\bar{x}}{d\bar{t}^2} = \frac{-1}{\sigma}\bar{x} + \frac{1}{\sigma}\frac{(1-\sigma)\bar{x}}{\sqrt{\bar{x}^2-(1-\bar{y})^2}}}$$ non-dimensional equation