

$$3)b) \frac{d^2 y}{dt^2} = -g + \frac{k}{m}(l-y) - \frac{k}{m} \frac{l_0(l-y)}{\sqrt{x^2 + (l-y)^2}}$$

$$\tau = \sqrt{\frac{l}{g}}, L=l$$

Substitute

$$t = \tau \bar{t}, y = L \bar{y}, x = L \bar{x}$$

$$\frac{L d^2 \bar{y}}{\tau^2 d\bar{t}^2} = -g + \frac{k}{m}(l - L \bar{y}) - \frac{k}{m} \frac{l_0(l - L \bar{y})}{\sqrt{L^2 \bar{x}^2 + (l - L \bar{y})^2}}$$

$$g \frac{d^2 \bar{y}}{d\bar{t}^2} = -g + \frac{k}{m}(l - l \bar{y}) - \frac{k}{m} \frac{l_0(l - l \bar{y})}{\sqrt{l^2 \bar{x}^2 + (l - l \bar{y})^2}}$$

divide both sides by g and factor l out:

$$\frac{d^2 \bar{y}}{d\bar{t}^2} = -1 + \frac{k l (1 - \bar{y})}{g m} - \frac{k l l_0 (1 - \bar{y})}{m g \sqrt{l^2 \bar{x}^2 + l^2 (1 - \bar{y})^2}}$$

$$\frac{d^2 \bar{y}}{d\bar{t}^2} = -1 + \frac{1}{\sigma} (1 - \bar{y}) - \frac{1}{\sigma} \frac{l_0 (1 - \bar{y})}{l \sqrt{\bar{x}^2 + (1 - \bar{y})^2}}$$

$$l_0 = l(1 - \sigma) \rightarrow \frac{l_0}{l} = (1 - \sigma), \text{ substitute for } \frac{l_0}{l} \text{ to get:}$$

$$\frac{d^2 \bar{y}}{d\bar{t}^2} = -1 + \frac{1}{\sigma} (1 - \bar{y}) - \frac{1}{\sigma} (1 - \sigma) (1 - \bar{y}) / \sqrt{\bar{x}^2 + (1 - \bar{y})^2}$$

non-dimensional equation