Assignment 6+7

For this assignment we have decided to celebrate the recent first ever direct detection of Gravitational Waves (GW) by having you mimic one of the methods used by the Laser Interferometer Gravitational-Wave Observatory (LIGO) for detecting GWs, thus experiencing the excitement of analyzing GW signals.

We will be using data generated by real numerical relativity simulations kindly made publicly available by one of the worldwide leading groups in simulating Binary Black Hole mergers: the Center for Computational Relativity and Gravitation (see http://ccrg.rit.edu/downloads/waveforms – Campanelli, Lousto, Nakano, and Zlochower, Phys.Rev. D79:084010, 2009)

An oversimplified representation of LIGO's "matched filtered algorithm" could be described as follow:

1. Consider a bank of waveforms generated by numerical relativity simulations. Each wave form is given as a complex-valued time series.
2. Collect a set of GW signals from the detector (also complex-valued time series);
3. Perform a power spectrum analysis, correlating the GWs signals with the signals in the simulated waveform bank. This correlation analysis is performed in the Fourier domain to make the results more robust, as the signals are usually embedded in noise;
4. Select candidate events with high correlation coefficient.

Assignment

In this assignment, we will be implementing something similar. We will considering just one predicted GW from simulations of the merger of two black holes, in the file `GWprediction.rat`, and 32 mock detector signals, in the files `detection01.rat` ... `detection32.rat`. The rat format is a text format that rarray uses for input and output, which will be further described below.

You will have to compute the correlation of the power spectra of the prediction with each mock detection. The power spectrum F of a signal f is related to the fourier tranform of that signal: for each wave number k, the power spectrum is the square norm of the fourier component with wave vector k, i.e.

$$F_k = |\hat{f}_k|^2 = \hat{f}_k \hat{f}_k^* \tag{1}$$

Note that the power spectrum is real.

The correlation of two power spectra F and G is given by

$$CC_{F,G} = \frac{< F, G >}{\sqrt{< F, F >< G, G >}} \tag{2}$$

where $<,>$ denotes the inner product in fourier space, which is simply

$$< F, G >= \sum_k F_k G_k^* \tag{3}$$

(where * denotes complex conjugation, which has no effect for the real power spectra).

The denominator in Eq. (2) is a normalization factor such that when the functions are highly correlated the correlation coefficient $CC_{F,G}$ is close to 1.

The waveform files are just ascii text files, here in rat format (for 'rarray text'). Each contains the time series of the measurements of a complex quantity representing the GW signal. The rat format first lists all the times at which samples are taken as a one-dimensional set, separated by commas and enclosed in curly braces, and then lists the complex values of the signal at each of these points, as pairs "(r,i)", also separated by commas and enclosed by curly braces. This format is supported by the rarrayio header in the latest version of rarray (please update your version), and can simply be read in as shown in the following example:

```
#include <ifstream>
#include <rarray>
#include <rarrayio>
int main() {
    // open the file
    std::ifstream f("GWprediction.rat");
    // create empty arrays
    rarray<double,1> times;
    rarray<complex<double>,1> signal;
    // read in the signal
    f >> times;
    f >> signal;
}
```

The structure of the algorithm you have to implement is the following:

1. Read the predicted GW signal from `GWprediction.rat`.
2. Read one of the GW signal from observations `detection01.rat` ... `detection32.rat`.
3. Compute the FFT of the two complex quantities, using FFTW,
4. Compute the power spectrum of both signals,
5. Compute the correlation coefficient between the power spectra as defined in Eq. (1), using a `?dot` BLAS call for the inner product from Eq. (3).
6. Output the correlation coefficient
7. Repeat steps 2-to-6 for each of the signals in the observation set.
8. Finally, determine the 5 most significant candidates (those with the 5 largest values of the correlation coefficient) from the observations set.

Using rarrays with cblas calls

The lectures on linear algebra and pdes contained examples of calling blas routines, through the cblas interface, using plain C-style arrays. It is, perhaps not surprisingly, also possible to use rarrays in calling the cblas routines, if you use the following member functions. For a given rarray `A`:

`A.extent(0)` gives the first dimension of the rarray. For 1d rarrays, i.e., vectors, this is the size, for 2d rarrays, this is the number of rows.

`A.extent(1)` gives the second dimension of the rarray. For 1d rarrays, this has no meaning, while for 2d rarrays, this is the number of colums.
Note: this is also the value to use for the LDA parameters.

`A.data()` gives the pointer to the first element of the rarray; this is the pointer that the cblas routines expect.

If a cblas routine requires an "INC" parameter, it is usually 1.

Background

The following information is not necessary for solving the assignment, but just in case you are interested, notice that the GW signal is a complex quantity, named ψ_4, composed of $h_+ + ih_x$, where h_+ and h_x are quadrupole modes of the gravitational wave, i.e., of the perturbation of the metric describing the space-time far away from the source of the event. This event could, for instance, be the merger of two black holes.