CSE416A hw3

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October 26, 2018

1 Question 1

(a)

Code for generating Random Graph:

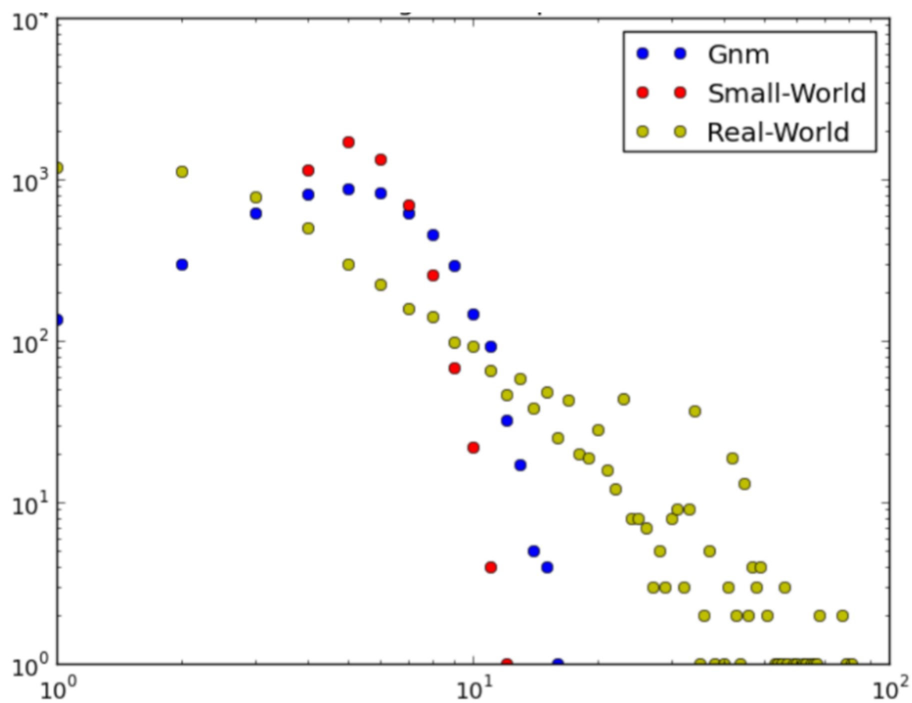
```
import networkx as nx

#Generate gnm graph
gnm = nx.gnm_random_graph(5242, 14496)

#define a function Generate small world random network:
def small_world(n, e):
    graph = nx.Graph()
    #Generate ring
    for i in range(n):
        if (i == n-1):
            sw.add_edge(i,0)
            sw.add_edge(i,1)
        else if (i == n-2):
            graph.add_edge(i, i+1)
            graph.add_edge(i,0)
        else:
            graph.add_edge(i, i+2)
            graph.add_edge(i, i+1)
    #Add Random Edges
    j = 0
    while (j < e):
        j_1, j_2 = randint(0, n -1), randint(0, n-1)
        if(j_1 != j_2 and (j_1,j_2) not in sw.edges() and (j_2,j_1) not in graph.edges()):
          sw.add_edge(j_1,j_2)
          j = j + 1
    return graph

# Generate with desired parameter:
sw = small_world(5242, 4000)
```

(b)

(i)

The following plot gives the degree distribution for all three graph.

Degree Distribution



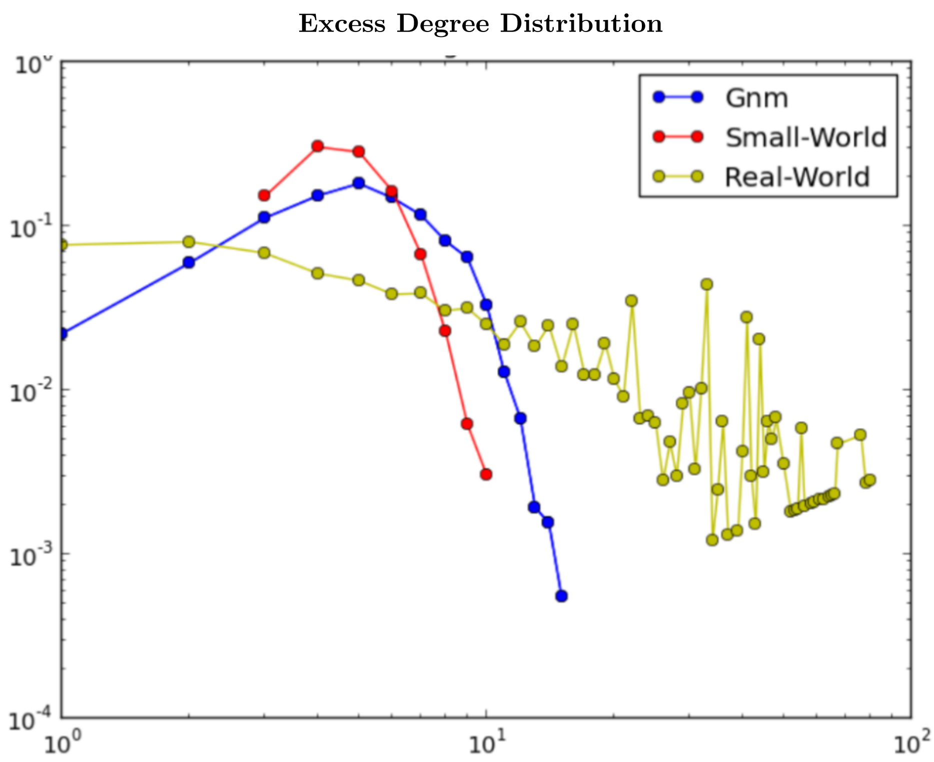
(ii)

It looks like compared to the plot we simulated, the real world graph has more vertices with higher degree counts, and also greater max degree counts. Which might suggest the real world graph is more "connected" compared to the graph we randomly generated.

(c)

(i)

The following plot gives the degree distribution for all three graph.



(ii)

Similar to that of degree distribution it looks like compared to the plot we simulated, the real world graph has more vertices with higher excess degree counts, and also greater max excess degree counts. Also, there are more variations at the "tail" of excess degree counts in real world graph compared to that of simulated graph.

(iii)

The following table summarize the expected degree of all three networks: All three graph has same expected degree but the

	Random Graph	Small-World	Real-World
Expected Degree	5.5	5.5	5.5
Expected Excess Degree	5.6	4.8	15.9

real world graph seems to have significantly higher expected excess degree (which is the "average number of collaborators of collaborators"). Since if we start from an author to one of his collaborator, we will reach those "hub" (who collaborate with others a lot) many times and they have high degree counts, so it is reasonable that in the real-world graph, the expected excess degree is higher than randomly generated graph and also higher than expected degree.

1.4: Clustering Coefficient

The following table summarize the expected degree of all three networks:

	Random Graph	Small-World	Real-World
Average Clustering Coefficient	0.0006	0.28	0.53

The real world graph has the highest clustering coefficient, which is as expected since in real world there will be some "central" collaborator who will collaborate with lots of researcher and those "hubs" make the component of the graph more "close". The small world graph has similar clustering coefficient since node is connected in a ring and there are edges between nodes which make average path between nodes shorter.

1.5

Compared to the complex structures of the real world model, we know very clearly how the null models are generated and what there structures are. Therefore, by comparing the coefficient of the null model we generated and the real world network, we can get a clearer sense about the actual structure behind the real world network.

Question 2

2.1

$$\int_{k_{min}}^{\infty} Zk^{-\alpha} dk = 1$$

$$\frac{Zk_{min}^{1-\alpha}}{1-\alpha} = 1$$

$$Z = (\alpha - 1)k_{min}^{\alpha-1}$$

$$P(k) = (\alpha - 1)k_{min}^{\alpha-1}k^{-\alpha}$$

2.2

$$< k >= \int_{k_{min}}^{\infty} kZk^{-\alpha} dk$$

$$< k >= \int_{k_{min}}^{\infty} Zk^{1-\alpha} dk$$

$$< k >= \frac{\alpha - 1}{\alpha - 2} k_{min}$$

For $\alpha < 2$ The integral diverges.

$$< k >= \infty$$

2.3

The varience is infinity. This means we can not estimate the expected value from mean value of samples.

2.4

$$P(X \geq k) = \int_{k}^{\infty} ZX^{-\alpha} dX$$

$$P(X \geq k) = (\frac{k}{k_{min}})^{(-\alpha+1)}$$

Question 3

See Code Submission