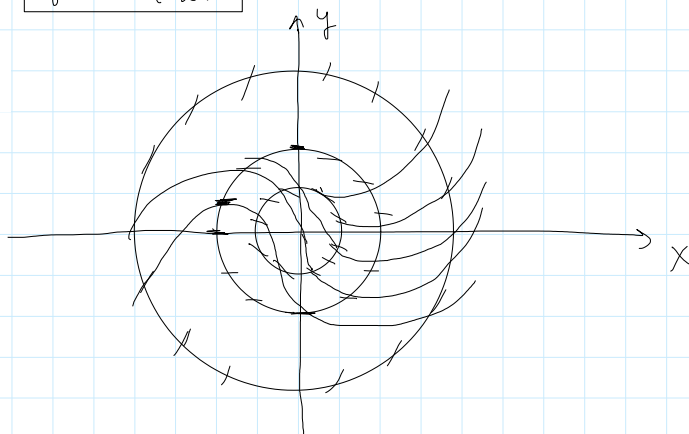


7. $2x^2 + Cy^2 = 1$. $4x dx + 2Cy dy = 0$

$$4xy dx + 2Cy^2 dy = 0$$

$$4xy dx + (1 - 2x^2) dy = 0$$

$$y' = -\frac{4xy}{1 - 2x^2}$$



37. $y' = x^2 + y^2 - 1$.

$$y' = -1 \Rightarrow x^2 + y^2 = 0$$

$$y' = -0.75 \Rightarrow x^2 + y^2 = 0.25$$

$$y' = 0 \Rightarrow x^2 + y^2 = 1$$

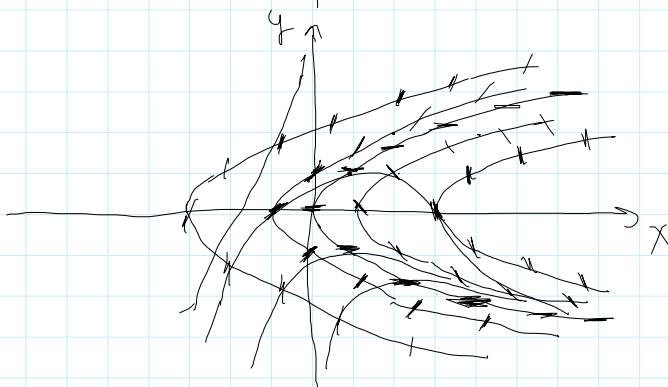
$$y' = 3 \Rightarrow x^2 + y^2 = 4$$

$$y' = 8 \Rightarrow x^2 + y^2 = 9$$

$$y' = y^2 - x$$

$$y'' = 2yy' - 1$$

$$67. \text{ at } y' = 0 \Rightarrow y'' = -1$$



4. $y' \cos x + y(1 + y) \sin x = 0$.

$$\frac{dy}{y(1+y)} = -\tan x dx$$

$$\int \frac{dy}{y(1+y)} = \int dy \left(\frac{1}{y} - \frac{1}{1+y} \right) = \ln y - \ln(1+y) = -\int \tan x dx = +\ln \cos x$$

$$C + \ln \frac{y}{1+y} = +\ln \cos x \Rightarrow \boxed{\frac{y}{1+y} = \cos x \cdot C}$$

17. $(1 + \cos x)yy' = (1 + y^2) \sin x$.

$$\frac{y dy}{1+y^2} = \frac{\sin x dx}{1+\cos x} \Rightarrow \frac{1}{2} \int \frac{dy^2}{1+y^2} = \frac{1}{2} \ln(1+y^2) = -\int \frac{d\cos x}{1+\cos x} = -\ln(1+\cos x) + C$$

$$\boxed{\frac{1}{\sqrt{1+y^2}} = (1+\cos x) \cdot C}$$

31. $3x(x+1)y' = (x+2)y$, $y(1) = -1$.

$$3 \frac{dy}{y} = \frac{x+2}{x(x+1)} dx = 2 \cdot \frac{2(x+1) - x}{2(x+1)x} dx = \left(\frac{2}{x} - \frac{1}{x+1} \right) dx$$

$$3 \ln y = 2 \ln x - \ln(x+1) + C \Rightarrow y^3 = C \cdot \frac{x^2}{x+1} \Rightarrow C = -2 \Rightarrow \boxed{y^3 = -\frac{2x^2}{x+1}}$$

43. $y = C \sin x - 2$.

$$4+2.$$

$$1 - 1, \dots, 1, \dots$$

43. $y = C \sin x - 2$.

$$y' = C \cos x = \frac{y+2}{\sin x} \cos x = (y+2) \operatorname{ctg} x \Leftrightarrow -\frac{1}{y+2} = (y+2) \operatorname{ctg} x$$

$$- \operatorname{ctg} x dx = (y+2) dy$$

$$\boxed{+ \ln \cos x = \frac{y^2}{2} + 2y + C}$$

59. $x dy = (y + \sqrt{x^2 + y^2}) dx, x \geq 0$.

$$\left\{ \begin{aligned} y = xz &\Rightarrow dy = z dx + x dz \\ xz dx + x^2 dz &= (xz + \sqrt{x^2 + x^2 z^2}) dx \\ xz dx + x^2 dz &= (z + \sqrt{1+z^2}) dx \\ x dz &= \sqrt{1+z^2} dx \\ \frac{dz}{\sqrt{1+z^2}} &= \frac{dx}{x} \Rightarrow \ln \left(\frac{z + \sqrt{1+z^2}}{x} \right) = \ln x + C \Rightarrow z + \sqrt{1+z^2} = Cx \end{aligned} \right.$$

$$\boxed{\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = Cx}$$

72. $(x+y+1)dx + (x-y+3)dy = 0$.

$$\left. \begin{aligned} x+y+1=0 \\ x-y+3=0 \end{aligned} \right\} \begin{aligned} x &= -2 \\ y &= 1 \end{aligned} \Rightarrow \left. \begin{aligned} t &= x+2 \\ p &= y-1 \end{aligned} \right\} \begin{aligned} (t-2+p+1+1)dt + (t-2-p-1+3)dp &= 0 \\ (t+p)dt + (t-p)dp &= 0 \end{aligned}$$

$$t dt + p dt + t dp - p dp = 0$$

$$d\frac{t^2}{2} + d(pt) - d\frac{p^2}{2} = 0$$

$$\frac{t^2}{2} + pt - \frac{p^2}{2} = C \Rightarrow \boxed{\frac{(x+2)^2}{2} + (y-1)(x+2) - \frac{(y-1)^2}{2} = C}$$

27. $4y' + 12x^2y = 3x^2$.

$$4y' = 3x^2(1-4y)$$

$$\int \frac{4dy}{1-4y} = \int 3x^2 dx \Rightarrow -\ln(1-4y) = x^3 + C \Rightarrow \boxed{1-4y = C e^{-x^3}}$$

35. $(1+y^2)dx + (2xy-1)dy = 0$.

$$\frac{\partial(1+y^2)}{\partial y} = 2y = \frac{\partial(2xy-1)}{\partial x} \Rightarrow \exists F(x,y) \text{ т.ч. } dF = (1+y^2)dx + (2xy-1)dy$$

$$\begin{cases} \frac{\partial F}{\partial x} = 1+y^2 \\ \frac{\partial F}{\partial y} = 2xy-1 \end{cases} \Rightarrow F(x,y) = (1+y^2)x + f(y) \Rightarrow \frac{\partial F}{\partial y} = f'(y) + 2xy = 2xy-1 \Rightarrow f'(y) = -1 \Rightarrow f(y) = -y + C$$

$$\boxed{F(x,y) = (1+y^2)x - y = \text{const}}$$

45. $x^2 y' + y = 4, y(-1) = 5$.

$$z = y - 4 \Rightarrow x^2 z' + z = 0 \Rightarrow \frac{x^2 dz}{dx} = -z \Rightarrow -\frac{dz}{z} = \frac{dx}{x^2} \Rightarrow -\ln z = -\frac{1}{x} + C \Rightarrow$$

$$\Rightarrow z = C e^{\frac{1}{x}} \Rightarrow y = 4 + C e^{\frac{1}{x}} \Rightarrow \begin{cases} 5 = 4 + C/e \\ C = e \end{cases} \Rightarrow \boxed{y = 4 + e^{\frac{1+x}{x}}}$$

64. $y' \cos x + y \sin x + 3y^2 \cos x = 0$.

1) $y=0$ - решение //

2) $y \neq 0$

$$\left. \begin{aligned} y' + y \operatorname{tg} x + 3y^2 &= 0 \\ z = 1/y \\ y' = -\frac{1}{z^2} z' \end{aligned} \right\} \begin{aligned} -\frac{1}{z^2} z' + \frac{1}{z} \operatorname{tg} x + 3 \frac{1}{z^2} &= 0 \\ -z' + z \operatorname{tg} x + 3 &= 0 \end{aligned}$$

 Перем $z' = z \operatorname{tg} x$

$$\frac{dz}{z} = \operatorname{tg} x dx \Rightarrow \ln z = -\ln \cos x + C$$

$$z = C(x)/\cos x$$

Тогда: $z' = \frac{C'(x)}{\cos x} + \frac{C(x) \sin x}{\cos^2 x} \Rightarrow \frac{C'}{\cos x} + \frac{C \sin x}{\cos^2 x} - \frac{C \sin x}{\cos^2 x} - 3 = 0$

$$C' = 3 \cos x \Rightarrow C(x) = 3 \sin x + B \Rightarrow \boxed{\frac{1}{y} = \frac{3 \sin x + B}{\cos x} = 3 \operatorname{tg} x + \frac{B}{\cos x}}$$

84. $4y' = y^2 + \frac{4}{x^2}$.

Пусть $y = \frac{A}{x} \Rightarrow -\frac{4A}{x^2} = \frac{A^2}{x^2} + \frac{4}{x^2} \Rightarrow A^2 + 4A + 4 = 0 \Rightarrow A = -2 \Rightarrow y = -\frac{2}{x}$ - частное реш.

Тогда $y = z - \frac{2}{x}$
 $y' = z' + \frac{2}{x^2}$

$$\left. \begin{aligned} 4z' + \frac{8}{x^2} &= z^2 - \frac{4z}{x} + \frac{4}{x^2} + \frac{4}{x^2} \\ \Rightarrow 4z' &= z^2 - \frac{4z}{x} \end{aligned} \right\} \boxed{z=0 \text{ - решение}}$$

Заведя $t = \frac{1}{z}, z \neq 0: -\frac{4}{t^2} t' = \frac{1}{t^2} - \frac{4}{tx} \Rightarrow 4t' = \frac{4t}{x} - 1$

Перем $4t' = \frac{4t}{x} \Rightarrow \frac{dt}{t} = \frac{dx}{x} \Rightarrow t = Cx \mid t = F(x)x \sim t' = F'(x)x + F(x)$

Тогда $4F'(x)x + 4F(x) = 4F(x) - 1 \Rightarrow F'(x) = -\frac{1}{4x} \Rightarrow F(x) = -\frac{1}{4} \ln x + C$

$$\frac{1}{z} = \left(-\frac{1}{4} \ln x + C\right)x \Rightarrow \boxed{y = \frac{1}{x(-\frac{1}{4} \ln x + C)} - \frac{2}{x} \mid y = -\frac{2}{x}}$$

94. $x^2 y' - 5xy + x^2 y^2 + 8 = 0$. (???)

$y = \frac{A}{x} \Rightarrow x^2 \left(-\frac{A}{x^2}\right) - \frac{5Ax}{x} + x^2 \frac{A^2}{x^2} + 8 = 0$

$$-A - 5A + A^2 + 8 = 0$$

$$A^2 - 6A + 8 = 0$$

$$\begin{cases} A=4 \\ A=2 \end{cases}$$

$y = z + \frac{2}{x}$
 $y' = z' - \frac{2}{x^2}$

$$\left. \begin{aligned} x^2(z' - \frac{2}{x^2}) - 5x(z + \frac{2}{x}) + z^2 + \frac{4}{x^2} + \frac{2z}{x} + 8 &= 0 \\ z'x^2 - 5xz + z^2 + \frac{2z}{x} &= 0 \end{aligned} \right\} \boxed{z=0 \text{ - решение}}$$

$t = \frac{1}{z} \Rightarrow -\frac{1}{t^2} t' x^2 - \frac{5x}{t} + \frac{1}{t^2} + \frac{2}{xt} = 0$

$$x^2 t' + 5xt - 1 - \frac{2t}{x} = 0$$

Перем $x^2 t' = \left(\frac{2}{x} - 5x\right)t$

$$\frac{dt}{t} = \frac{2-5x^2}{x^3} dx$$

$$\ln t = -\frac{1}{x^2} - 5 \ln x + C$$

$$z = C e^{\frac{1}{x^2} + 5 \ln x} \Rightarrow \begin{cases} y = \frac{2}{x} \\ y = C e^{\frac{1}{x^2} + 5 \ln x} + \frac{2}{x} \end{cases}$$

$$18. \left(1 + \frac{2x}{y^3}\right) dx + \left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right) dy = 0.$$

$$\frac{\partial \left(1 + \frac{2x}{y^3}\right)}{\partial y} = -\frac{6x}{y^4} = \frac{\partial \left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right)}{\partial x}$$

$$\text{Тогда } \begin{cases} \frac{\partial F}{\partial x} = 1 + \frac{2x}{y^3} \Rightarrow F(x, y) = x + \frac{x^2}{y^3} + f(y) \Rightarrow \\ \frac{\partial F}{\partial y} = \frac{1}{y^2} - \frac{3x^2}{y^4} \Rightarrow \frac{\partial F(x, y)}{\partial y} = -\frac{3x^2}{y^4} + f'(y) = \frac{1}{y^2} - \frac{3x^2}{y^4} \Rightarrow \end{cases}$$

$$\Rightarrow f(y) = -\frac{1}{y} + C \Rightarrow F(x, y) = x + \frac{x^2}{y^3} - \frac{1}{y} + C \Rightarrow \boxed{x + \frac{x^2}{y^3} - \frac{1}{y} = \text{const}}$$

$$33. \left(\frac{1}{x} - y\right) dx = \frac{1}{y} dy.$$

$$\left\{ d\left(\frac{x}{y}\right) = \frac{dx}{y} - \frac{x dy}{y^2} \right\}$$

$$\frac{dx}{x} - y dx = \frac{dy}{y}$$

$$\frac{dx}{y} - x dx = \frac{x dy}{y^2}$$

$$d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right) \Rightarrow \boxed{\frac{x}{y} = \frac{x^2}{2} + C}$$

$$59. x^2 y y' + x^3 = (x^2 + y^2)^2.$$

$$t = x^2 + y^2 \Rightarrow t' = 2x + 2y y'$$

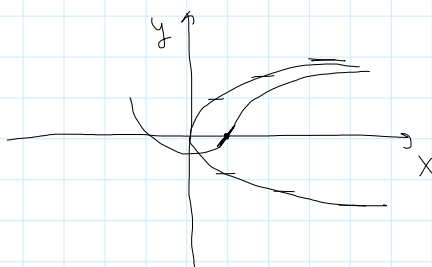
$$x^2 \left(\frac{t' - 2x}{2}\right) + x^3 = t^2$$

$$\frac{x t'}{2} - x^3 + x^3 = t^2 \Rightarrow \frac{dt}{2t^2} = \frac{dx}{x^2} \Rightarrow -\frac{1}{2t} = -\frac{1}{x} + C \Rightarrow \frac{1}{t} = \frac{2}{x} + C$$

$$\boxed{\frac{1}{x^2 + y^2} = \frac{2}{x} + C}$$

$$\text{II. 1. } y' = x - y^2, y(1) = 0$$

1) при каких $y' \geq 0 \Leftrightarrow x - y^2 \geq 0 \Leftrightarrow y^2 \leq x$, т.е. при $x \geq y^2$ $y(x)$ увеличивается.



2) $y'' = 1 - 2y y' = \begin{cases} y' = 0 \end{cases} = 1$ - на графике $y^2 = x$ $y(x)$ имеет максимум.

3) Так как $y(1) = 0$, то точка $(1, 0)$ лежит выше $y^2 = x \Rightarrow y(x) -$ убывает, тогда $y(x)$ не может пересечь $y^2 = x$ в точке при $x > 1$. А противно! Если пересечется в (x_0, y_0) $y_0^2 = x_0$, то в точке (x_0, y_0) есть макс, т.е. $y'(x_0) = 0$ при $x = x_0$ $y(x) \uparrow$, а при $x > x_0$ $y(x) \downarrow \Rightarrow y'(x > x_0) \leq 0 \Rightarrow y^2(x > x_0) \geq x > x_0 = y_0^2 \Rightarrow y > y_0$ - в точке макс. максимум - противор.

тогда $y(x)^2 \leq x$ - неравенство непрерывно выполн.

$$2. \quad y = \psi(x) \quad f(x, y) = \frac{\psi'(x)(y - \varphi(x)) - \varphi'(x)(y - \psi(x))}{\psi(x) - \varphi(x)}$$

$$\text{при } y = \psi(x): y' = \psi'(x) = \frac{\psi'(x)(\psi(x) - \varphi(x)) - \varphi'(x)(\psi(x) - \psi(x))}{\psi(x) - \varphi(x)} = \psi'(x) //$$

аналог. с $y = f(t)/f$

4. 1)

	x	sin x
0	0	0
1	1	1
2	0	0
3	0	-1

$n \geq 4$

2) $\cos x$ и $\sin x$
 $\cos x = \sin x \rightarrow x = \frac{\pi}{4} + \pi k$
 случаи 0 не рассматриваем,
 \exists y $n \geq 1$

3) $\cos x^2 = 1 - \frac{x^4}{2} + \frac{x^8}{24}$

	$\cos x^2$
1	1
0	0
0	0
0	-1

$n \geq 4$

4)

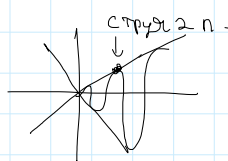
	sin x	1
0	1	1
1	0	0
2	-1	0

$n \geq 3$

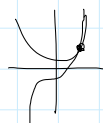
5)

	x sin x	x
0	+	+
1	+	+
1	~	~

$n \geq 3$



6) 1 n-го сл. $n \geq 2$



7)

	$x e^x$	x
0	0	0
1	1	1
2	0	0
3	1	0

$n \geq 3$

$$x e^x = x \left(1 + \frac{x^2}{2} + \frac{x^3}{6} \right) = x + \frac{x^3}{2} + \frac{x^4}{6}$$

III. а) $\begin{cases} \dot{x} = x \sin y \\ \dot{y} = 0 \end{cases} \Rightarrow y = y_0$
 $\dot{x} = x \sin y_0 \Rightarrow \ln x = \sin y_0 t + \ln x_0 \Rightarrow x = x_0 e^{t \sin y_0}$
 $g^t \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 e^{t \sin y_0} \\ y_0 \end{pmatrix}$

б) $\begin{cases} \dot{x} = x \cos y \\ \dot{y} = 1 \end{cases} \Rightarrow y = t + y_0$
 $\dot{x} = x \cos(t + y_0) \Rightarrow \ln x = \sin(t + y_0) + C \Rightarrow x = C e^{\sin(t + y_0)}$
 $\Rightarrow x = x_0 e^{\frac{\sin(t + y_0) - \sin y_0}{\sin y_0}}$
 $g^t \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 e^{\frac{\sin(t + y_0) - \sin y_0}{\sin y_0}} \\ t + y_0 \end{pmatrix}$

в) $\begin{cases} \dot{y} = y - x + 1 \\ \dot{x} = 1 \end{cases} \Rightarrow x = t + x_0$
 $\dot{y} = y - t - x_0 + 1 \Rightarrow y = C e^t + t + x_0, C = (y_0 - x_0)$
 $g^t \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} t + x_0 \\ (y_0 - x_0) e^t + t + x_0 \end{pmatrix}$

IV. 1. $xy'' + xy'^2 + y' = 0, x \neq 0$.

$y' = z: x z' + x z^2 + z = 0$

$t = \frac{1}{z} \Rightarrow z' = -\frac{1}{t^2} t' \Rightarrow x \left(-\frac{1}{t^2} \right) t' + x \frac{1}{t^2} + \frac{1}{t} = 0$
 $-x t' + x + t = 0$

$t' = 1 + \frac{t}{x}$

$$\left. \begin{aligned} t' &= 1 + \frac{t}{x} \\ t' &= \frac{t}{x} \Rightarrow t = Cx \end{aligned} \right\} \begin{aligned} t' &= C'x + C \Rightarrow -x(C'x + C) + x + Cx = 0 \\ &-C'x^2 + x = 0 \\ C' &= \frac{1}{x} \Rightarrow C = \ln x + A \end{aligned}$$

$$\frac{1}{z} = (\ln x + A)x \Rightarrow z = \frac{1}{x(\ln x + A)}$$

$$y = \int x(A + \ln x) dx = \frac{Ax^2 + x^2 \ln x}{2} - \frac{x^2}{4} + C$$

10. $(y^3 + y)y'' - (3y^2 - 1)y' = 0$.

$$y' = t \Rightarrow y'' = t' t$$

$$(y^3 + y)tt' - (3y^2 - 1)t^2 = 0$$

$$\frac{t'}{t} = \frac{3y^2 - 1}{y^3 + y} \Rightarrow -\ln y + 2 \ln(y^2 + 1) + C = \ln t \Rightarrow y' = C \left(\frac{y^2 + 1}{y} \right) \Rightarrow$$

$$\Rightarrow \frac{yy'}{(y^2 + 1)^2} = C \Rightarrow y = -\frac{C}{2x^2 + 2} + A$$

24. $y'' + y'^2 = y'e^y, y(0) = 0, y'(0) = \frac{1}{2}$.

$$z = y' \Rightarrow y'' = zz'$$

$$zz' + z^2 = ze^y$$

$$z' + z = e^y$$

$$y' = z = Ce^{-y} + \frac{e^y}{2} \Rightarrow \frac{1}{2} = \frac{1}{2} + C \Rightarrow C = 0 \Rightarrow y' = \frac{e^y}{2} \Rightarrow e^{-y} dy = \frac{dx}{2} \Rightarrow$$

$$\Rightarrow -e^{-y} = \frac{x}{2} + C \Rightarrow C = -1 \Rightarrow e^{-y} = \frac{x}{2} - 1$$

40. $yy'' - y'^2 + y^2 \sin x = 0$.

$$z = \frac{y'}{y} \Rightarrow y' = zy \Rightarrow y'' = z'y + zy' = z'y + z^2y = (z' + z^2)y$$

$$y^2(z' + z^2) - z^2y^2 + y^2 \sin x = 0$$

$$z' + z^2 - z^2 + \sin x = 0$$

$$\frac{y'}{y} = z = \cos x + C \Rightarrow \ln y = \sin x + C_1 x + C_2$$

56. $xyy'' - xy'^2 + y'(y' + y) \sin x = 0, y(1) = 1, y'(1) = -1$.

$$z = \frac{y'}{y} : (z' + z^2)y^2x - xz^2y^2 + zy(z'y + y) \sin x = 0 \mid z(1) = \frac{-1}{1} = -1$$

$$x(z' + z^2) - xz^2 + z(z+1) \sin x = 0$$

$$z' + \frac{\sin x}{x} z(z+1) = 0 \quad z = -1 - \text{переносим}$$

$$x(z+z') - xz' + z(z+1)\sin x = 0$$

$$z' + \frac{\sin x}{x} z(z+1) = 0 \quad z = -1 - \text{переносим}$$

$$z = -1 = \frac{y'}{y} \rightarrow \ln y = -x \rightarrow y = e^{-x} + C = e^{-x} - e^{-1}$$

65. $x^2 y'' + 2x^2 y y' + 2xy^2 - 2y = 0$,

a) $y(1) = 2, y'(1) = 0$,

$$\begin{cases} x = e^t & y' = (z' - z) e^{-2t} & xy = z \rightarrow x=1 \rightarrow t=0 \\ y = (e^t)^{-1} z(t) & y'' = (z'' - 3z' + 2z) e^{-3t} & 2 = y(1) = z(0) \end{cases}$$

$$\downarrow$$

$$z'' - 3z' + 2z = 0$$

$$\begin{cases} p = z' \\ z'' = p p' \end{cases} \rightarrow \begin{cases} p p' - 3p + 2z p = 0 \\ p' - 3 + 2z = 0 \end{cases} \rightarrow dp = (-2z + 3) dz \rightarrow p = -z^2 + 3z + C = z'$$

$$y'(1) = 0 \Rightarrow (z'(0) - z(0)) e^{-0} = 0 \Rightarrow z'(0) = z(0) \Rightarrow z_0 = -z_0^2 + 3z_0 + C \Rightarrow C = z_0^2 - 2z_0 = 0$$

$$z' = -z^2 + 3z$$

$$\frac{dz}{3z - z^2} = dt \rightarrow \frac{1}{3} \ln \left(\frac{z}{3-z} \right) = t + C \rightarrow \frac{z}{3-z} = C e^{3t}, C = 2$$

$$\frac{z}{3-z} = 2 e^{3t} \rightarrow \frac{z/e^t}{3/e^t - z/e^t} = 2 e^{3t} \rightarrow \frac{y}{\frac{3}{x} - y} = 2x^3$$

$$\boxed{xy = 2x^3(3 - xy)}$$

229. Могут ли графики двух решений данного уравнения на плоскости x, y пересекаться в некоторой точке (x_0, y_0)

а) для уравнения $y' = x + y^2$? Нет

б) для уравнения $y'' = x + y^2$? Да

230. Могут ли графики двух решений данного уравнения на плоскости x, y касаться друг друга в некоторой точке (x_0, y_0)

а) для уравнения $y' = x + y^2$? Нет

б) для уравнения $y'' = x + y^2$? Нет

в) для уравнения $y''' = x + y^2$? Да

231. Сколько существует решений уравнения $y^{(n)} = x + y^2$, удовлетворяющих одновременно двум условиям: $y(0) = 1, y'(0) = 2$? Рассмотреть отдельно случаи $n = 1, 2, 3$.

$n=1$: 1 решение

$n=2$: $y''(0) = 0 + 1 = 1 \rightarrow 1$ решение

$n=3$: y''' — любое позитивное бесконечно малое реш.

234. При каких n уравнение $y^{(n)} = f(x, y, y', \dots, y^{(n-1)})$ с непрерывно дифференцируемой функцией f может иметь среди своих решений две функции: $y_1 = x, y_2 = \sin x$?

при $n \geq 4$, давал раньше.