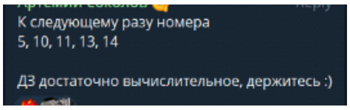


Т5. $p_\xi(t) = \begin{cases} \lambda e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$ $\quad \varphi(\xi) = \xi^k$ – возр

$$p_{\xi^k}(t) = p_\xi(t^{\frac{1}{k}}) \cdot \frac{1}{|k t^{\frac{k-1}{k}}|} = \frac{\lambda e^{-\lambda t^{1/k}}}{k t^{\frac{k-1}{k}}}$$

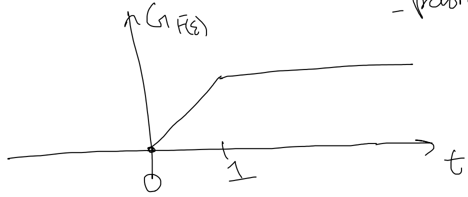
Т11. $F_{\xi^2}(t) = P(\xi^2 \leq t) = P(-\sqrt{t} \leq \xi \leq \sqrt{t}) = F_\xi(\sqrt{t}) - F_\xi(-\sqrt{t})$

$$p_{\xi^2}(t) = F'_{\xi^2}(t) = p_\xi(\sqrt{t}) \cdot \frac{1}{2\sqrt{t}} + p_\xi(\sqrt{t}) \cdot \frac{1}{2\sqrt{t}} = \frac{1}{\sqrt{t}} I(b \leq \sqrt{t} \leq a)$$

Т.14. F – непр. Тогда $G_{F(\xi)}(t) = P(F(\xi) \leq t) = \begin{cases} \text{если } t < 0, \text{ то} \\ P(F(\xi) \leq t) = P(\varnothing) = 0 \\ \text{если } t > 1, \text{ то} \\ P(F(\xi) \leq t) = P(\Omega) = 1 \end{cases} \overset{t \in (0,1)}{=} P(\xi \leq F^{-1}(t)) \ominus$

$$\ominus F(F^{-1}(t)) = t \Rightarrow$$

– равн. распр на $[0,1]$.



Т.10. 1) $p(x) = \lambda e^{-\lambda x} I(x > 0)$

$\varphi(x) = \frac{1}{\lambda} \ln x$

$\varphi^{-1}(x) = e^{\lambda x}$

$$p_\varphi(x) = \frac{p(e^{\lambda x})}{\frac{1}{\lambda} e^{-\lambda x}} = \lambda e^{-\lambda(e^{\lambda x} - x)}$$

2) $F_\eta(x) = P(\{\xi\} \leq x)$ $\quad \begin{array}{|l} x \geq 1 \;\; F_\eta(x) = 1 \\ x \leq 0 \;\; F_\eta(x) = 0 \end{array}$

$\{\xi\} \leq x \Rightarrow \xi \in [k, k+x]$

$$F_\eta(x) = \sum_{k=1}^{\infty} P(\xi \in [k, k+x]) = \sum_{n=1}^{\infty} (F_\xi(k+x) - F_\xi(k)) \ominus$$

$$\ominus \sum_{n=1}^{\infty} e^{-\lambda k}(1 - e^{-\lambda x}) = (1 - e^{-\lambda x}) \frac{1}{1 - e^{-\lambda}} = \frac{F_\xi(x)}{1 - e^{-\lambda}}$$

$$p_\eta(x) = \frac{p_\xi(x)}{1 - e^{-\lambda}} I_{[0,1)}(x)$$

3) $\varphi(\xi) = 1 - e^{-\alpha \xi}$ – возрастает

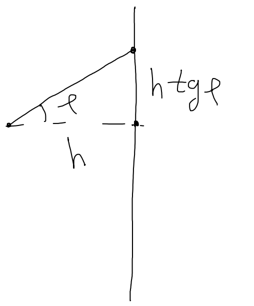
$\varphi^{-1}(t) = -\frac{1}{\alpha} \ln(t-1)$

$$p_\varphi(t) = \lambda e^{+\lambda\left(\frac{1}{\alpha} \ln(t-1)\right)} \cdot \frac{1}{\alpha} \cdot \frac{-1}{t-1} = -\frac{\lambda}{\alpha} e^{-\frac{\lambda}{\alpha} \ln(t-1)} \cdot \frac{1}{t-1} \ominus$$

$$\ominus -\frac{\lambda}{\alpha}\left(\frac{1}{t-1}\right)(t-1)^{-\frac{\lambda}{\alpha}} = -\frac{\lambda}{\alpha}(t-1)^{-\frac{\lambda}{\alpha} - 1}$$

$\Leftrightarrow -\frac{\lambda}{2}\left(\frac{1}{t-1}\right)(t-1)^{-\frac{1}{2}} = -\frac{\lambda}{2}(t-1)^{\alpha-1}$

T. 13. 1) $p_\varphi = \frac{I(-\frac{\pi}{2}, \frac{\pi}{2})}{\pi}$ $\quad l = h\,tg\,\varphi \to l^{-1}(t) = arctg\left(\frac{t}{h}\right)$



$p_l = \frac{I(tg(-\frac{\pi}{2}), tg(\frac{\pi}{2}))}{\pi} \cdot \frac{h}{t^2+h^2} = \frac{1}{\pi}\frac{h}{t^2+h^2}$ // − распр Коши.

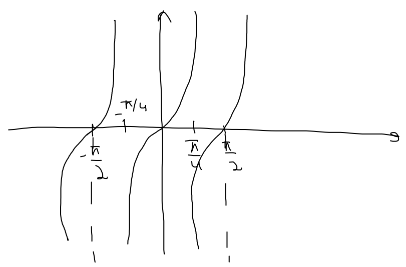
2) Так как p_ξ − распр Коши явл композицией равн распр $\varphi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ и $\psi = tg\,\varphi$

то 1. $q = \frac{\xi^2}{1+\xi^2} = \frac{tg^2\varphi}{1+tg^2\varphi} = tg^2\varphi \cdot \cos^2\varphi = \sin^2\varphi \Rightarrow$

$\Rightarrow p_q = \frac{I([0,1])}{\pi} \cdot 2 \cdot \frac{1}{2\sqrt{t-t^2}} = \frac{I([0,1])}{\pi\sqrt{t-t^2}}$?

2. $q = \frac{1}{1+\xi^2} = \cos^2\varphi \Rightarrow p_q = -\frac{I([0,1])}{\pi} \cdot 2\frac{-1}{2\sqrt{t-t^2}} = \frac{I([0,1])}{\pi\sqrt{t-t^2}}$

3. $q = \frac{2\xi}{1-\xi^2} = tg\,2\varphi \Rightarrow p_q = \frac{I([0,+\infty))}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} + \frac{I((-\infty,0))}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} \oplus$



$\oplus \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} \ominus$

$\ominus \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{1+t^2} = \frac{1}{\pi(1+t^2)}$

4. $q = \frac{1}{\xi} = ctg\,\varphi \Rightarrow p_q = \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{|-1+t^2|} = \frac{1}{\pi(1+t^2)}$