II. Матрицы

15.2 1) $3\begin{pmatrix}1&2\\1&2\end{pmatrix}-\begin{pmatrix}3&2\\3&2\end{pmatrix}-4\begin{pmatrix}0&1\\0&1\end{pmatrix}=\begin{pmatrix}3&6\\3&6\end{pmatrix}-\begin{pmatrix}3&2\\3&2\end{pmatrix}-\begin{pmatrix}0&4\\0&4\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

15.5 1) $(2\ -3\ 0)\begin{pmatrix}4\\3\\1\end{pmatrix}=(-1)$

2) $\begin{pmatrix}4\\3\\1\end{pmatrix}(2\ -3\ 0)=\begin{pmatrix}8&-12&0\\6&-9&0\\2&-3&0\end{pmatrix}$

15.10. 1) $\begin{pmatrix}1&2\\3&4\end{pmatrix}(1\ 2)\begin{pmatrix}2\\1\end{pmatrix}\nexists$

2) $\begin{pmatrix}2\\4\end{pmatrix}(1\ 2)\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}2&4\\4&8\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}8\\16\end{pmatrix}$

15.11. 1) $I=\begin{pmatrix}1&1\\1&1\end{pmatrix}$; $I^2=\begin{pmatrix}1&1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\1&1\end{pmatrix}=\begin{pmatrix}2&2\\2&2\end{pmatrix}=2I$

Пусть $I^n=2^nI$, тогда $I^{n+1}=2^nI^2=2^{n+1}I$ – по инд.

Ответ: $I^n=2^nI$

III. Системы ур.

17.1 4) $\begin{cases}y+3z=-1\\2x+3y+5z=3\\3x+5y+7z=6\end{cases}\to\left(\begin{array}{ccc|c}0&1&3&-1\\2&3&5&3\\3&5&7&6\end{array}\right)\sim\left(\begin{array}{ccc|c}2&3&5&3\\3&5&7&6\\0&1&3&-1\end{array}\right)\begin{matrix}-3\\2\end{matrix}\sim\left(\begin{array}{ccc|c}2&3&5&3\\0&1&-1&3\\0&1&3&-1\end{array}\right)(-1)\sim$

$\sim\left(\begin{array}{ccc|c}2&3&5&3\\0&1&-1&3\\0&0&4&-4\end{array}\right)\Rightarrow\begin{matrix}z=-1\\y-z=3\Rightarrow y=2\\2x+3y+5z=3\Rightarrow x=1\end{matrix}$

19.1. 5) $\begin{matrix}x+2y+3z=-4\\2x+3y+4z=1\\3x+4y+5z=6\end{matrix}\to\left(\begin{array}{ccc|c}1&2&3&-4\\2&3&4&1\\3&4&5&6\end{array}\right)\begin{matrix}-2&-3\end{matrix}\sim\left(\begin{array}{ccc|c}1&2&3&-4\\0&-1&-2&9\\0&-2&-4&18\end{array}\right)(-2)\sim\left(\begin{array}{ccc|c}1&2&3&-4\\0&1&2&-9\end{array}\right)\to$

$\to\begin{matrix}x+2y+3z=-4\\y+2z=-9\end{matrix}\Rightarrow\begin{matrix}y=-9-2z\\x=-4-3z+18+4z=z+14\end{matrix}\Rightarrow$ Отв: $\begin{pmatrix}z+14\\-9-2z\\z\end{pmatrix}$

I. Определители.

14.4 2) $\begin{vmatrix}3&4\\0&7\end{vmatrix}=21-20=1$

5) $\begin{vmatrix}\cos\alpha&-\sin\alpha\\\sin\alpha&\cos\alpha\end{vmatrix}=\cos^2\alpha+\sin^2\alpha=1$

14.7. 1) $\begin{vmatrix}0&0&1\\0&1&0\\1&0&0\end{vmatrix}=1\begin{vmatrix}0&1\\1&0\end{vmatrix}=-1$

3) $\begin{vmatrix}1&2&2\\2&1&-2\\2&-2&1\end{vmatrix}=(1-8-8)-(4+4+4)=-27$

IV. Линейные соотношения.

№1.6

$a(-5,-1)$ $\frac{-5}{1}\neq-\frac{1}{3}$, то a лин. незав. к b.

№1.6

$a(-5,-1)$, $b(-1,3)$, т.к. $\frac{-5}{-1} \neq -\frac{1}{3}$, то a лин. незав. к b.

1) $c(-1,2) = A\cdot a + Bb$ $\begin{cases} -5A - B = -1 \\ -A + 3B = 2 \end{cases}$ $A = \frac{1}{16}$ $B = \frac{11}{16}$

$c = \frac{a}{16} + \frac{11b}{16}$

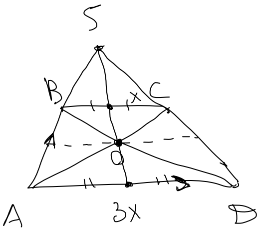
2) $d(2,-6) = Aa + Bb$ $\begin{cases} -5A - B = 2 \\ -A + 3B = -6 \end{cases}$ $A = 0$, $B = -2$

$d = -2b$

№1.11 (2,3) 2) $l = a+b+c$, $m = b+c$, $n = -a+c$ $\to \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 1 - 1 - (-1) = 1$ – ЛН

3) $l = c$, $m = a-b-c$, $n = n-b+c$ $\to n - m - 2l = 0$ – ЛЗ.

$n-b+c-a+b+c-2c = 0$

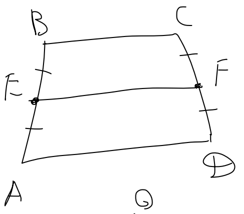


№1.16

$\vec{AC} = \vec{AB} + \vec{AD}/3$

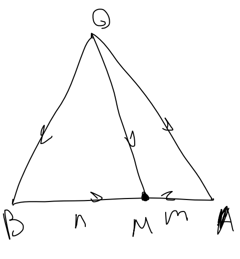
$\vec{AO} = \vec{AC} \cdot \frac{2x \cdot 3x}{x + 3x} = \frac{3}{2}\vec{AB} + \frac{1}{2}\vec{AD}$

$\vec{AS} = 3\vec{AB}$



№1.17.

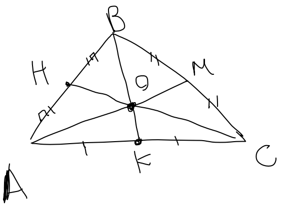
$\begin{cases} \vec{EF} = \vec{EB} + \vec{BC} + \vec{CF} \\ \vec{EF} = -\vec{EB} + \vec{AD} - \vec{FD} \end{cases}$ $\vec{EF} = \frac{\vec{AD} + \vec{BC}}{2}$



№1.24(1)

$\begin{cases} \vec{OM} = \vec{OA} + \vec{AB} \cdot \frac{m}{n+m} \\ \vec{OM} = \vec{OB} - \vec{AB} \cdot \frac{n}{m+n} \end{cases}$ $\frac{\vec{OM}}{m} + \frac{\vec{OM}}{n} = \frac{\vec{OA}}{m} + \frac{\vec{OB}}{n}$

$\boxed{\vec{OM} = \frac{n}{m+n}\vec{OA} + \frac{m}{m+n}\vec{OB}}$



№1.37.

1) Пусть O – т. пер. медиан.

$\vec{KB} = \vec{OB} \cdot \frac{3}{2} = \vec{KA} + \vec{AB} = \frac{\vec{AC}}{2} + \vec{CB}$

$\vec{MA} = \vec{OA} \cdot \frac{3}{2} = \frac{\vec{BC}}{2} + \vec{CA}$

$\vec{HC} = \vec{OC} \cdot \frac{3}{2} = \frac{\vec{AB}}{2} + \vec{BC}$

$(\vec{OA} + \vec{OB} + \vec{OC})\frac{3}{2} = \frac{\vec{AB}}{2} + \frac{\vec{BC}}{2} - \frac{\vec{AC}}{2} = 0$

$\vec{OA} + \vec{OB} + \vec{OC} = 0$

2) т.к. при $O \notin (ABC)$ нев. тетраэдр $OABC$, то $(\vec{OA}, \vec{OB}, \vec{OC})$ – не компланарны $\Rightarrow \nexists\ \lambda_1, \lambda_2, \lambda_3$ т.ч. $\lambda_1\vec{OA} + \lambda_2\vec{OB} + \lambda_3\vec{OC} = 0$ $\Rightarrow$ нет таких т.ч. $\vec{OA} + \vec{OB} + \vec{OC} = 0$ ($\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 1$)

V. Замена базиса и системы координат

4.3. $O, e_1, e_2 \mid O', e_1', e_2'$ $OO' = (-1,3)$ $T = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$

1) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

2) $\begin{cases} e_1' = 2e_1 + 3e_2 \\ e_2' = e_1 + e_2 \end{cases}$ $e_1 = -e_1' + 3e_2'$, $e_2 = e_1' - 2e_2'$ $T^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$ | $O'O$ во 2 ДСК: $OO' = C^{-1}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \end{pmatrix}\right] =$

$= \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \end{pmatrix}$

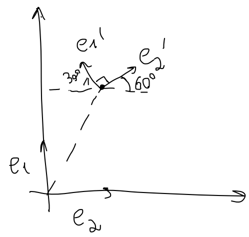
2) $e_1' = 2e_1 + 3e_2$, $e_2' = e_1 + e_2$ $\Big\}$ $e_1 = -e_1' + 3e_2'$, $e_2 = e_1' - 2e_2'$ $\quad T' = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ 9 \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -4 \\ 9 \end{pmatrix}$

3) $O = (-4; 9)$ $e_1 = (-1; 3)$ $e_2 = (1; -2)$

4.26(1)

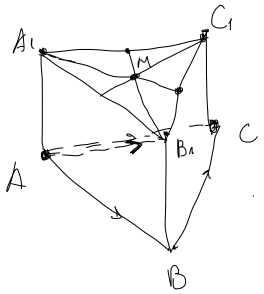


$e_2' = e_1 \sin 60° + e_2 \cos 60° = \frac{e_1\sqrt{3}}{2} + \frac{e_2}{2}$

$e_1' = e_1 \sin 30° - e_2 \cos 30° = \frac{e_1}{2} - \frac{e_2\sqrt{3}}{2}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

№4.19.



$\overrightarrow{AA_1} = \overrightarrow{AB_1} - \overrightarrow{AB}$

$\overrightarrow{A_1B} = \overrightarrow{AB} - \overrightarrow{AA_1} = \overrightarrow{AB} - \overrightarrow{AB_1} + \overrightarrow{AB} = 2\overrightarrow{AB} - \overrightarrow{AB_1}$

$\overrightarrow{A_1C} = \overrightarrow{A_1A} + \overrightarrow{AC} = \overrightarrow{AB} - \overrightarrow{AB_1} + \overrightarrow{AC}$

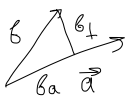
$\overrightarrow{A_1M} = \frac{2}{3}\left(\overrightarrow{AB} + \frac{\overrightarrow{BC}}{2}\right) = \frac{2\overrightarrow{AB}}{3} + \frac{\overrightarrow{AC}}{3} - \frac{\overrightarrow{AB}}{3} = \frac{\overrightarrow{AB}}{3} + \frac{\overrightarrow{AC}}{3}$

$T = \begin{pmatrix} 2 & 1 & 1/3 \\ 0 & 1 & +1/3 \\ -1 & -1 & 0 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1/3 \\ 0 & 1 & 1/3 \\ -1 & -1 & 0 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

Скалярное произведение.

№2.27(2) $\vec{a}(1,-1,2)$ $\vec{b}(1,1,2)$



$\vec{b_a} = \frac{(\vec{a}\vec{b})}{|\vec{a}|^2}\vec{a} = (1,-1,2)\cdot\frac{4}{6} = \frac{2}{3}(1,-1,2)$

$\vec{b_\perp} = \vec{b} - \vec{b_a} = (1,1,2) - \left(\frac{2}{3}, -\frac{2}{3}, \frac{4}{3}\right) = \left(\frac{1}{3}, \frac{5}{3}, \frac{2}{3}\right)$

№2.35. $a(1,-1,1)$ $b(5,1,1)$ $c(x,y,z)$

$x^2+y^2+z^2=1$

$(ac)=0$: $x-y+z=0$

$\sqrt{\frac{2}{27}} = \frac{bc}{|b|}$: $\sqrt{\frac{2}{27}} = \frac{5x+y+z}{\sqrt{27}} \Rightarrow 5x+y+z=\sqrt{2}$ $\Big\}$ $z = \frac{\sqrt{2}}{2} - 3x$, $y = \frac{\sqrt{2}}{2} - 2x$ $\Big\}$ $\left(\frac{\sqrt{2}}{2} - 3x\right)^2 + \left(\frac{\sqrt{2}}{2} - 2x\right)^2 + x^2 = 1$

$\left[\begin{array}{l} x = 0 \\ x = \frac{5\sqrt{2}}{14} \end{array}\right.$

$\boxed{\begin{array}{l} c\left(0; \frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2}\right) \\ c\left(\frac{5\sqrt{2}}{14}; -\frac{3\sqrt{2}}{14}; -\frac{4\sqrt{2}}{7}\right) \end{array}}$

№2.21 $|e_1| = 3$, $|e_2| = \sqrt{2}$, $|e_3| = 4$ $\quad \angle(e_1,e_2) = \angle(e_2,e_3) = 45°$, $\angle(e_1,e_3) = 60°$

$G = \begin{pmatrix} e_1e_1 & e_1e_2 & e_1e_3 \\ e_2e_1 & e_2e_2 & e_2e_3 \\ e_3e_1 & e_3e_2 & e_3e_3 \end{pmatrix} = \begin{pmatrix} 9 & 3 & 6 \\ 3 & 2 & 4 \\ 6 & 4 & 16 \end{pmatrix}$

$\left(\begin{smallmatrix} 9 & 3 & 6 \end{smallmatrix}\right)\left(\begin{smallmatrix} 1 \\ \cdot \end{smallmatrix}\right) = (0\ {-3}\ {-6})\begin{pmatrix} 1 \\ -3 \end{pmatrix} = 9 \Rightarrow |a| = 3$

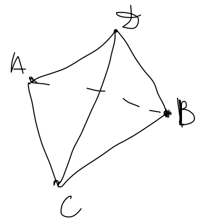
$$e_3e_1 \quad e_3e_2 \quad e_3e_3 \qquad 6 \quad 4 \quad 16$$

$a(1,-3,0)$ $\quad |a|^2 = (1,-3,0)\begin{pmatrix} 9 & 3 & 6 \\ 3 & 2 & 4 \\ 6 & 4 & 16 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = (0,-3,-6)\begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = 9 \Rightarrow |a| = 3$

$b(-1,2,1)$ $\quad |b|^2 = (-1,2,1)\begin{pmatrix} 9 & 3 & 6 \\ 3 & 2 & 4 \\ 6 & 4 & 16 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = (3,5,18)\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 25 \Rightarrow |b| = 5$

$$(ab) = (1\,-3\,0)\begin{pmatrix} 9 & 3 & 6 \\ 3 & 2 & 4 \\ 6 & 4 & 16 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = (0\,-3\,-6)\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = -12 \Rightarrow \cos\varphi = -\frac{12}{15} = -\frac{4}{5}$$

№2.45.



$AC \cdot BD = 0 \quad AB = AC + CB$

$BC \cdot AD = 0 \quad DC = DA + AC$

$AB \cdot DC \overset{?}{=} 0$

$\rightarrow AB \cdot DC = AC \cdot DA + AC^2 + \underbrace{CB \cdot DA}_{0} + CB \cdot AC =$

$= AC(DA + CB) + AC^2 =$

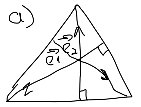
$= AC(DA + CB + AC) = AC(DC + CB) =$

$= AC \cdot DB = 0$

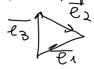
T.1. Найдите сумму ортогональных проекций вектора $\vec{a}$ на прямые, перпендикулярные

а) сторонам некоторого правильного треугольника (если $\vec{a}$ лежит в плоскости этого треугольника);

б)* граням некоторого правильного тетраэдра.

а) 

e_1, e_2, e_3 обр. равн.-тр.



$\Gamma = \begin{pmatrix} e_1e_1 & e_1e_2 \\ e_2e_1 & e_2e_2 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}$

$a = (\lambda_1, \lambda_2)$, тогда $\text{пр}_{e_1} a = (\lambda_1 \lambda_2)\begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = (\lambda_1 - \frac{\lambda_2}{2}; \lambda_2 - \frac{\lambda_1}{2})\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda_1 - \frac{\lambda_2}{2}$

$\text{пр}_{e_2} a = (\lambda_1 \lambda_2)\Gamma\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (\lambda_1 - \frac{\lambda_2}{2}; \lambda_2 - \frac{\lambda_1}{2})\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \lambda_2 - \frac{\lambda_1}{2}$

$\text{пр}_{e_3} a = (\lambda_1 \lambda_2)\Gamma\begin{pmatrix} -1 \\ -1 \end{pmatrix} = (\lambda_1 - \frac{\lambda_2}{2}; \lambda_2 - \frac{\lambda_1}{2})\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \frac{\lambda_2}{2} - \lambda_1 + \frac{\lambda_1}{2} - \lambda_2 = -\frac{\lambda_2}{2} - \frac{\lambda_1}{2}$

$\sum \text{Пр} = (\lambda_1 - \frac{\lambda_2}{2}, 0) + (0, \lambda_2 - \frac{\lambda_1}{2}) + (\frac{\lambda_2}{2} + \frac{\lambda_1}{2}; \frac{\lambda_2}{2} + \frac{\lambda_1}{2}) = (\frac{3\lambda_1}{2}; \frac{3\lambda_2}{2}) = \frac{3}{2}(\lambda_1; \lambda_2) = \frac{3}{2}\vec{a}$

VII. Векторное и смешанное произведение.

№3.1(1) $\begin{pmatrix} i & j & k \\ 3 & -1 & 2 \\ 2 & -3 & -5 \end{pmatrix} = \begin{pmatrix} 11 \\ -19 \\ -7 \end{pmatrix}$

№3.7(2) Решаем аналог. 2.35 $\quad c = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}; \frac{1}{7\sqrt{2}}\begin{pmatrix} 5 \\ -3 \\ -8 \end{pmatrix}$

1) $\begin{vmatrix} 1 & -1 & 1 \\ 5 & 1 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{vmatrix} = 5\sqrt{2} > 0$ – тройки сонапр. – ед. решение.

2) $\begin{vmatrix} 1 & -1 & 1 \\ 5 & 1 & 1 \\ \frac{5}{7\sqrt{2}} & -\frac{3}{7\sqrt{2}} & -\frac{8}{7\sqrt{2}} \end{vmatrix} = -5\sqrt{2} < 0$ – тройки не совп

№3.12. Д-ть: $[a \times b] = [b \times c] = [c \times a] \Leftrightarrow a + b + c = 0$

1) $a + b + c = 0 \Rightarrow [a + b + c \times a] = 0 \Rightarrow \underbrace{[a \times a]}_{0} + [b + c \times a] = 0 \Rightarrow$

$\Rightarrow [b \times a] = -[c \times a] \Rightarrow [a \times b] = [c \times a]$

Аналогично для умн на b и c. ЧТД

2) $[a \times b] = [b \times c]$

$[a + c \times b] = 0$

$[a + b + c \times b] = 0 \Rightarrow a + b + c \parallel b$ $\Big\}$ $a + b + c = 0$ т.к. (a, b, c) – неколлинеарны

$[a+c \times b] = 0$

$[a+b+c \times b] = 0 \Rightarrow a+b+c \parallel b$

аналог. получим, что $a+b+c \parallel a$, $a+b+c \parallel c$ $\Big\}$ $a+b+c=0$ т.к. (a,b,c) - неколлинеарны

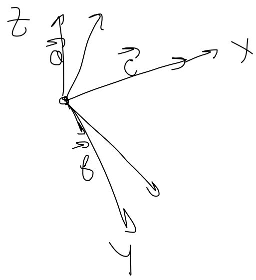
№3.13 (1,2)

1) $|[a \times b]|^2 = \begin{vmatrix} (a,a) & (a,b) \\ (a,b) & (b,b) \end{vmatrix} = \underline{|a|^2|b|^2 - (a,b)^2}$

$\parallel$

$|[a\times b]|^2 = |a|^2|b|^2 \sin^2\alpha = |a|^2|b|^2 - |a|^2|b|^2\cos^2\alpha = \underline{|a|^2|b|^2 - (a,b)^2}$

2) $[a \times [b \times c]] = b(ac) - c(ab)$



$c(x_3, 0, 0)$

$b(x_2, y_2, 0)$

$a(x_1, y_1, z_1)$

1) $(ac) = x_1 x_3$, $(ab) = x_1x_2 + y_1y_2$ $\Big\}$ $b(ac) - c(ab) = (x_1x_2x_3, x_1y_2x_3, 0) - (x_1x_2x_3 + x_3y_1y_2, 0, 0) =$

$= (-y_1y_2x_3, x_1y_2x_3, 0)$

$\parallel$

2) $\begin{vmatrix} i & j & k \\ x_2 & y_2 & 0 \\ x_3 & 0 & 0 \end{vmatrix} = x_3 \begin{vmatrix} j & k \\ y_2 & 0 \end{vmatrix} = -x_3y_2\vec{k} = \begin{pmatrix} 0 \\ 0 \\ -x_3y_2 \end{pmatrix}$

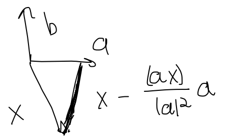
$\begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ 0 & 0 & -x_3y_2 \end{vmatrix} = -x_3y_2 \begin{vmatrix} i & j \\ x_1 & y_1 \end{vmatrix} = -x_3y_2y_1\vec{i} + x_3y_2x_1\vec{j} = (-y_1y_2x_3, x_1y_2x_3, 0)$

№3.16. $[x \times a] = b$

$[a \times [x \times a]] = [a \times b]$

$x|a|^2 - a(ax) = [a \times b]$

$\vec{n}(x) = x - \frac{(ax)}{|a|^2}a = \frac{[a\times b]}{a^2}$



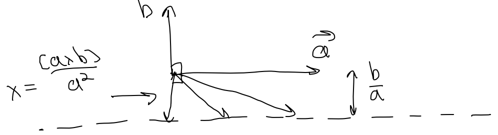
$\frac{(ax)}{|a|^2}a$ — проекция x на a

нормаль

$n(x) = \frac{[a\times b]}{a^2} = const$, т.е годограф всех решений - прямая.

$|n(x)| = \frac{b}{a}$.

при $x \parallel [a \times b]$, $x \perp a$, $x \perp b$. $|x| = \frac{b}{a}$



№3.20(1) $\begin{vmatrix} 2 & 3 & 5 \\ 7 & 1 & -1 \\ 3 & -5 & -11 \end{vmatrix} = (-22 - 175 - 9) - (15 + 10 - 231) = 0$ - в-ра компланарны

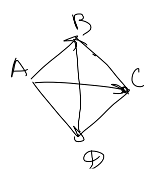
№3.23. $A(2,1,-1)$ $B(3,0,2)$ $C(5,1,1)$ $D(0,-1,3)$

$\vec{AB} = (1,-1,3)$ $6V_{ABCD} = \begin{vmatrix} 1 & -1 & 3 \\ 3 & 0 & 2 \\ -2 & -2 & 4 \end{vmatrix} = 2 \Rightarrow \underline{V_{ABCD} = \frac{2}{6} = \frac{1}{3}}$

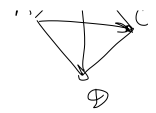
$\vec{AC}(3,0,2)$

$\vec{AD}(-2,-2,4)$ $2S_{ABC} = |\vec{AB} \times \vec{AD}| = \sqrt{AB^2AD^2 - (AB\cdot AD)^2} =$

$= \sqrt{11 \cdot 24 - 12^2} = 2\sqrt{30}$

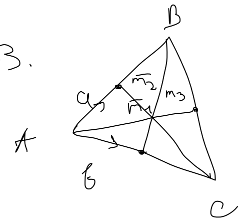


$h = \frac{6V_{ABD}}{2S_{ABC}} = \frac{2}{2\sqrt{30}} = \underline{\frac{1}{\sqrt{30}}}$



$h = \frac{6V_{ABCD}}{2S_{ABC}} = \frac{2}{2\sqrt{30}} = \frac{1}{\sqrt{30}}$

№3.33.



$\overline{m_1} = \vec{a} + \vec{b}$

$\overline{m_2} = -2\vec{b} + \vec{a}$

$\overline{m_3} = -2\vec{a} + \vec{b}$

1) т.к. $\overline{m_1} + \overline{m_2} + \overline{m_3} = 0$ то из мед. можно сост. треуг, не меняя вектора. $\overline{m_1}, \overline{m_2}, \overline{m_3}$

2) $S_{ABC} = \frac{1}{2}|[2a \times 2b]| = 2|[a \times b]|$

3) $S_{m_1m_2m_3} = \frac{1}{2}|[\overline{m_2} \times \overline{m_3}]| = \frac{1}{2}|4[\bar{a} \times \bar{b}] - [\bar{a} \times \bar{b}]| = \frac{3}{2}|[a \times b]|$

$$\frac{S_{m_1m_2m_3}}{S_{ABC}} = \frac{3}{4}$$

№3.28. 1) $[a \times b], [b \times c], [c \times a]$ - компл, то a, b, c - компл.

$([a \times b], [[b \times c] \times [c \times a]]) = 0$

$([a \times b], c(a, [b \times c]) - a\underbrace{(c, [b \times c])}_{0}) = 0$

$([a \times b], c)(a, [b \times c]) = 0$

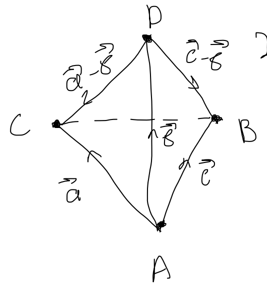
$(a, b, c) = 0$

2) $q_1 = [a, b]$ пусть α - пл. обр (a, b, c) тогда т.к. $q_1 \perp a, b$, то $q_1 \perp \alpha$

$q_2 = [b, c]$ анал. $q_2 \perp \alpha$

$q_3 = [c, a]$ $q_3 \perp \alpha$ $\Big\}$ $q_1 \parallel q_2 \parallel q_3$

Т.2. Для каждой грани тетраэдра построен вектор, направленный перпендикулярно грани вне тетраэдра и равный по длине площади грани. Докажите, что сумма четырех построенных векторов равна $\vec{0}$.



$2\Sigma\vec{d} = [\vec{c} \times \vec{b}] + [\vec{b} \times \vec{a}] + [\vec{a} \times \vec{c}] + [\vec{c} - \vec{b} \times \vec{a} - \vec{b}] =$

$= \cancel{[\vec{c} \times \vec{b}]} + \cancel{[\vec{b} \times \vec{a}]} + \cancel{[\vec{a} \times \vec{c}]} + \cancel{[\vec{c} \times \vec{a}]} - \cancel{[\vec{c} \times \vec{b}]} - \cancel{[\vec{b} \times \vec{a}]} + \underbrace{(\vec{b} \times \vec{b}]}_{0} = 0$

№3.29. 1) $b_1 = \alpha_1[a_2, a_3]$ $(b_1 a_1) = 1$; $(a_1, a_2, a_3) = \frac{1}{\alpha_1} \neq 0$

$b_2 = \alpha_2[a_1, a_3]$ $(b_2 a_2) = 1$: $(a_1, a_3, a_2) = \frac{1}{\alpha_2}$ $\Rightarrow$ $\alpha_1 = \alpha_3$, $\alpha_2 = -\alpha_1$

$b_3 = \alpha_3[a_1, a_2]$ $(b_3 a_3) = 1$: $(a_1, a_2, a_3) = \frac{1}{\alpha_3}$

2) $b_1 = \frac{[a_2, a_3]}{(a_1, a_2, a_3)}$ $b_2 = \frac{[a_1, a_3]}{(a_1, a_3, a_2)}$ $b_3 = \frac{[a_1, a_2]}{(a_1, a_2, a_3)}$

3) $(b_1, b_2, b_3) = \frac{1}{\underbrace{(a_1, a_2, a_3)^3}_{>0}}([a_2, a_3], [[a_1, a_3], [a_1, a_2]]) =$

$= \frac{1}{\alpha_1^3}([a_2, a_3], a_1([a_1, a_3], a_2)) = \frac{1}{\alpha_1^3}(a_1, a_2, a_3)^2 = \frac{1}{(a_1, a_2, a_3)}$

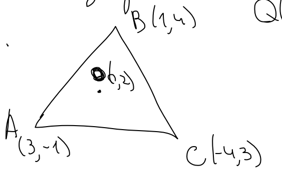
Только один ориентирован.

Аффинные задачи IX:

$B(1,4)$ $Q(0,0)$: $QO = \frac{QA + QB + QC}{3}$ $\frac{}{(3,-1) + (1,4) + OC}$

Афинные задачи IX!

№5.15. $Q(0,0)$: $QO=\frac{QA+QB+QC}{3}$



$(0,2)=\frac{(3,-1)+(1,4)+OC}{3}$

$(0,6)=(4,3)+OC$

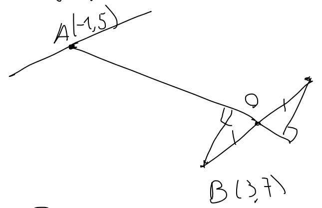
$(-4,3)=OC \Rightarrow C=(-4,3)$

$\vec{AC}(-7,2) \Rightarrow AC: \frac{x-3}{-7}=\frac{y+1}{2}$

$\vec{AB}(-2,5) \Rightarrow AB: \frac{x-3}{-2}=\frac{y+1}{5}$

$\vec{BC}(-5,-1) \Rightarrow BC: \frac{x-1}{-5}=\frac{y-4}{-1}$

№5.19.



$C(1,-1)$

$l_1: l_1 \parallel BC: \frac{x+1}{2}=\frac{y+8}{5}$

$l_2: O(2,3) \Rightarrow \vec{AO}(3,2) \Rightarrow l: \frac{x-2}{3}=\frac{y-3}{2}$

$\vec{BC}=(-2,-8)$

№6.16(2) $\begin{cases} x-y+2z+4=0 \\ -2x+y+z+3=0 \end{cases} \Rightarrow \begin{matrix} -x+3z+\cancel{9}7=0 \\ -y+5z+11=0 \end{matrix} \Rightarrow \begin{cases} x=3z+7 \\ y=5z+11 \\ z=z \end{cases}$ $\quad \frac{x-7}{3}=\frac{y-11}{5}=\frac{z}{1}$

№6.29(2) $M(3,2,-1)$, $\vec{a}(1,5,3)$, $\vec{b}(-2,1,2)$

$\alpha: \begin{vmatrix} x-3 & y-2 & z+1 \\ 1 & 5 & 3 \\ -2 & 1 & 2 \end{vmatrix}=0$

$(x-3)\cdot 7-(y-2)\cdot 8+(z+1)\cdot 11=0$

$7x-8y+11z+6=0$

№6.38(2) $A(-1,1,-1)$

$l_1: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{-1}$

$l_2: \frac{x}{4}=\frac{y+5}{-5}=\frac{z-3}{2}$

$\alpha_2 \supset l_2$, $\ni A$: $\begin{vmatrix} x & y+5 & z-3 \\ 4 & -5 & 2 \\ -1 & 6 & -4 \end{vmatrix}=0$ $\quad \alpha_2: 8x+14y+19z-44=0$

– пересеч. этих пл.-ся прямая

$\alpha_1 \supset l_1$, $\ni A$: $\begin{vmatrix} x-1 & y-2 & z \\ 2 & 3 & -1 \\ -2 & -1 & -1 \end{vmatrix}=0$ $\quad \alpha_1: x-y-z+4=0$

X. Метрические задачи

№5.30. $2=\frac{|4x-3y|}{\sqrt{16+9}} \Rightarrow |4x-3y|=10$

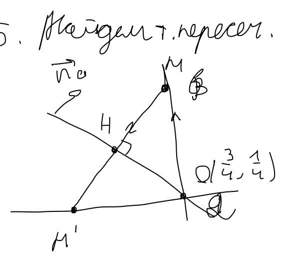
$2x-3y+4=0 \Rightarrow 4x=6y-8$

$\begin{cases} |6y-8-3y|=10 \\ |3y-8|=10 \end{cases}$

$y=6 \quad y=-\frac{2}{3}$

$x=7 \quad x=-3$

№5.35. Найдем т. пересеч. $\begin{cases} x+y=1 \\ 3x-y-2=0 \end{cases} \Rightarrow \begin{matrix} x=\frac{3}{4} \\ y=\frac{1}{4} \end{matrix}$ $\quad O(\frac{3}{4},\frac{1}{4})$



$M \in B$: $M(1,1) \Rightarrow \vec{OM}(\frac{1}{4},\frac{3}{4})$

$\vec{MH}=-\vec{n_a}\frac{(\vec{OM}\,\vec{n_a})}{n_a^2}=-(1,1)\cdot\frac{1}{2}=(-\frac{1}{2},-\frac{1}{2})$

$\vec{MM'}=-(1,1) \Rightarrow \vec{OM'}=(-\frac{3}{4},-\frac{1}{4})$

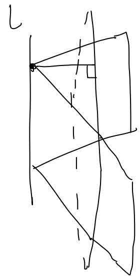
Тогда прямая: OM': $\frac{x-\frac{3}{4}}{-\frac{3}{4}}=\frac{y-\frac{1}{4}}{-\frac{1}{4}}$

№6.49(2)

1) $x+3y-z+2=0$, $\vec{n}(1,3,-1)$ $\quad x-3 \quad y \quad \frac{z-6}{}$

когда прямая ... $-\frac{3}{4}$ $-\frac{1}{4}$

№6.49(2)



1) $x+3y-z+2=0$, $\vec{n}(1,3,-1)$

2) $\begin{cases}2x-y+z=0\\ x+2y+z-3=0\end{cases} \Rightarrow \begin{matrix}x=3y-3\\ y=y\\ z=-5y+6\end{matrix} \Rightarrow \frac{x+3}{3}=\frac{y}{1}=\frac{z-6}{-5}$

т. $M(-3,0,6)\in l$

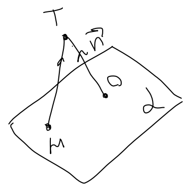
$\vec{a}(3,1,-5)$ – н.в.

$\alpha: \begin{vmatrix}x+3 & y & z-6\\ 3 & 1 & -5\\ 1 & 3 & -1\end{vmatrix}=0 \Leftrightarrow \underline{7x-y+4z-3=0}$

№6.60 $l: \frac{x-2}{3}=\frac{y+1}{1}=\frac{z-2}{4}$; $5x-y+z-4=0$; $\Rightarrow l: \begin{cases}x=3t+2\\ y=t-1\\ z=4t+2\end{cases}$

1) M – т. пересеч: $5(3t+2)-(t-1)+(4t+2)-4=0 \Rightarrow t=-\frac{1}{2} \Rightarrow M\left(\frac{1}{2},-\frac{3}{2},0\right)$

2) $T\in l$: $T(2,-1,2)$, опред. с какой стороны T от пл-ти: $\overrightarrow{MT}\left(\frac{3}{2},\frac{1}{2},2\right)$



$5\cdot 2+1+2-4=9>0$

$\vec{n}(5,-1,1)\perp\alpha$, $\overrightarrow{OT}=\vec{n}\frac{(\vec{n},\overrightarrow{MT})}{n^2}=(5,-1,1)\cdot\frac{9}{27}=\left(\frac{5}{3},-\frac{1}{3},\frac{1}{3}\right)$

$\overrightarrow{MT'}=\overrightarrow{MT}-\overrightarrow{OT}=\left(\frac{3}{2},\frac{1}{2},2\right)-\left(\frac{5}{3},-\frac{1}{3},\frac{1}{3}\right)=\left(-\frac{1}{6},\frac{5}{6},\frac{10}{6}\right)$

$MT': \frac{x-\frac{1}{2}}{-1}=\frac{y+\frac{3}{2}}{5}=\frac{z}{10}$

№6.61(2) $\begin{matrix}x-y+2z-1=0\\ 3x-y+2z+2=0\end{matrix}$ $\begin{cases}x=-\frac{3}{2}\\ y=2z-\frac{5}{2}\\ z=z\end{cases} \Rightarrow -\frac{3}{2}+10z-\frac{25}{2}-z-25=0 \Rightarrow z=\frac{13}{3}$, $x=-\frac{3}{2}$, $y=\frac{37}{6}$ – т. пересеч. $O\left(-\frac{3}{2},\frac{37}{6},\frac{13}{3}\right)$

$\alpha: x+5y-z-25=0$

$M\in l: M\left(-\frac{3}{2},-\frac{5}{2},0\right)$

при этом $\alpha(M)<0$

$\overrightarrow{OM}\left(0,-\frac{26}{3},-\frac{13}{3}\right)$

?

$\vec{n}_\alpha(1,5,-1)$: $пр_{\vec{n}_\alpha}=\frac{(n_\alpha\cdot\overrightarrow{OM})}{n_\alpha^2}\vec{n}_\alpha=\frac{-39}{27}(1,5,-1)=-\frac{13}{9}(1,5,-1)$

$пр_\alpha l = \overrightarrow{OM}+пр_{\vec{n}_\alpha}\overrightarrow{OM}=\left(0,-\frac{26}{3},-\frac{13}{3}\right)+\left(\frac{13}{9},\frac{65}{9},-\frac{13}{9}\right)=\left(\frac{13}{9},\frac{91}{45},-\frac{52}{9}\right)\parallel(65,91,-260)$

ур. пр: $\frac{x+\frac{3}{2}}{65}=\frac{x-\frac{37}{6}}{91}=\frac{x-\frac{13}{3}}{-260}$

№6.64(1) $4x+4y-7x+1=0 \Rightarrow \vec{n}_\beta=(4,4,-7)$

$\begin{matrix}x+y+z+1=0\\ 2x+y+3z+2=0\end{matrix} \Leftrightarrow \begin{matrix}x=-2z-1\\ y=z\\ z=z\end{matrix} \Rightarrow \vec{a}=(-2,1,1)$

$\sin\alpha=\frac{\vec{n}_\beta\cdot\vec{a}}{|\vec{n}_\beta||\vec{a}|}=\frac{-11}{\sqrt{6}\cdot\sqrt{81}}=\frac{-11}{9\sqrt{6}}$

№6.72(1) $\frac{x-4}{3}=\frac{y+1}{6}=\frac{z-1}{-2}$ и $\frac{z-5}{-6}=\frac{y}{-12}=\frac{z}{4}$

$-\frac{1}{2}=\frac{3}{-6}=\frac{6}{-12}=\frac{-2}{4} \Rightarrow l_1\parallel l_2$, пусть $M\in l_1: M(4,-1,1)$; $M'\in l_2: M'(5,0,0)$; $\overrightarrow{MM'}(1,1,-1)$

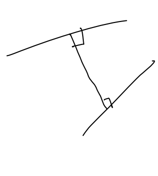
$\vec{a}(-6,-12,4)\parallel(3,6,-2)$

$\rho(l_1,l_2)=\rho(M,l_2)=\frac{|[\overrightarrow{MM'}\times\vec{a}]|}{|a|}=\frac{|(4,-1,3)|}{\sqrt{9+36+4}}=\frac{\sqrt{26}}{\sqrt{49}}=\frac{\sqrt{26}}{7}$

№6.73(3) $l_1: \frac{x-6}{1}=\frac{y-1}{2}=\frac{z-10}{-1}$ $\vec{a}=(1,2,-1)$; $M_1(6,1,10)$

$l_2: \frac{x+4}{-7}=\frac{y-3}{2}=\frac{z-4}{3}$ $\vec{b}=(-7,2,3)$; $M_2(-4,3,4)$

$\overrightarrow{M_2M_1}(10,-2,6)$



$\rho(l_1,l_2)=\frac{(\overrightarrow{M_2M_1},\vec{a},\vec{b})}{|[\vec{a},\vec{b}]|}=\frac{168}{\sqrt{336}}=2\sqrt{21}$

$\vec{d}=2\sqrt{21}\,\overline{(8,4,16)}$

$\vec{n}(l_1,l_2)=[\vec{a},\vec{b}]=\overline{(8,4,16)}$

$l_1\parallel\vec{a}$, $l_1\parallel\vec{n}$; $l_1\subset\alpha_1$: $\begin{vmatrix}x-6 & y-1 & z-10\\ 8 & 4 & 16\end{vmatrix}=0 \Rightarrow -3x+2y+z+6=0$

$\vec{n}(l_1,l_2)=[\vec{a},\vec{b}]=(8,4,16)$

$\alpha_1 \parallel \vec{a},\ \alpha_1 \parallel \vec{n},\ l_1 \subseteq \alpha_1:\ \begin{vmatrix} x-6 & y-1 & z-10 \\ 8 & 4 & 16 \\ 1 & 2 & -1 \end{vmatrix} = 0 \Rightarrow -3x+2y+z+6=0$

$\alpha_2 \parallel \vec{b},\ \alpha_2 \parallel \vec{n},\ l_2 \subseteq \alpha_2:\ \begin{vmatrix} x+4 & x-3 & x-4 \\ -7 & 2 & 3 \\ 8 & 4 & 16 \end{vmatrix} = 0 \Rightarrow 20x+136y-44z-152=0 \Rightarrow$

$\Rightarrow 5x+34y-11z-38=0$

другая п-тели
– задачи
перпенд.

$l_1 \cap \alpha_2:\ -3(-7t-4)+2(2t+3)+(3t+4)+6=0$

$t=-1;\ x=3,\ y=1,\ z=1$

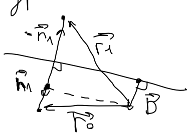
$l_2 \cap \alpha_1:\ 5(t+6)+34(2t+1)-11(-t+10)-38=0$

$t=1;\ x=7,\ y=3,\ z=9$

№6.30. ~~что такое биссекторная плоск?~~ в конце сделал ↓

Векторные ур нения

№5.4(2)



$\vec{n}=\vec{b}+\frac{(\vec{r}_0,\vec{b})}{b^2}\vec{b}$

$\vec{r}_1=\vec{r}_0-2\vec{n}_1=\vec{r}_0-2\vec{b}\left(1+\frac{(\vec{r}_0\vec{b})}{b^2}\right),\ \vec{b}=\frac{\vec{n}D}{n}$

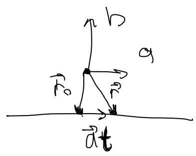
$\vec{r}_1=\vec{r}_0-2\frac{\vec{n}}{n}D\left(1+\frac{(\vec{r}_0\vec{n})\frac{D}{n}}{b^2}\right)=\vec{r}_0-2\frac{\vec{n}D}{n}\left(1+\frac{(\vec{r}_0\vec{n})}{nD}\right)$

№6.1(1, 3, 4) 1) $r=r_0+au+bv$

$(r,n)=(r_0,n)+\underbrace{(a,n)}_{0}u+\underbrace{(b,n)}_{0}v$

$(r,n)=\underbrace{(r_0,n)}_{D}$

3) $[r,a]=b,\ a\perp b,\ \vec{r}\perp b$



$\vec{r}_0=\frac{[a,b]}{a^2} \Rightarrow r=\frac{[a,b]}{a^2}+at$

4) $\left\{\begin{array}{l}(r,n_1)=D_1\\ (r,n_2)=D_2\end{array}\right.$

$\vec{a}=[\vec{n}_1,\vec{n}_2];\ [r\times[n_1\times n_2]]=n_1(rn_2)-n_2(rn_1)=n_1D_2-n_2D_1=b$

$b=n_1D_2-n_2D_1$

№6.3. $\left.\begin{array}{l}r=r_1+a_1t\\ r=r_2+a_2t\end{array}\right\}\ r_1-r_2=(a_2-a_1)t$

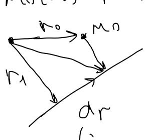
1) $(r_1-r_2)\parallel(a_2-a_1)\ \{a_2-a_1\neq 0\}$

2) $(r_1-r_2)\nparallel(a_2-a_1)$

3) $r_1\neq r_2,\ a_1=a_2$

4) $r_1=r_2,\ a_1=a_2$

№6.9(1) 1) $M_0(\vec{r}_0)\ \ r=\vec{r}_1+\vec{a}t\quad \vec{a}_r=\frac{(\vec{r}_0,\vec{a})}{a^2}\vec{a}$



$r_{M_0}=\vec{r}_1+\vec{a}_r=\vec{r}_1+\frac{(\vec{r}_0,\vec{a})}{a^2}\vec{a}$

№6.10(1, 3, 4) 1) $\vec{r}=\vec{r}_0+\vec{a}t,\ B:(\vec{r},\vec{n})=D$

$(r-r_0,a,n)=0$ – пл. α: $\alpha\perp D,\ l\subseteq\alpha$.

тогда пр: $\left\{\begin{array}{l}(\vec{r}-\vec{r}_0,\vec{a},\vec{n})=0\\ (\vec{r},\vec{n})=D\end{array}\right.$

3) $l_1: r = r_1 + a_1 t \Leftrightarrow (r - r_1, a_1) = 0$ $\quad \alpha_1: (r - r_0, a_1, r_1 - r_0) = 0$ — скрещ. пересек-
$l_2: r = r_2 + a_2 t \Leftrightarrow (r - r_2, a_2) = 0$ $\quad \alpha_2: (r - r_0, a_2, r_2 - r_0) = 0$ прямая.
$M(r_0)$

4) $\bar r = \bar r_1 + \bar a_1 t$ $\quad \vec a = [\bar a_1, \bar a_2] \Rightarrow \alpha_1: (\bar r - \bar r_1, [\bar a_1, \bar a_2], \bar a_1) = 0$ — пересеч.
$\bar r = \bar r_2 + \bar a_2 t$ $\quad \alpha_2: (\bar r - \bar r_2, [\bar a_1, \bar a_2], \bar a_2) = 0$

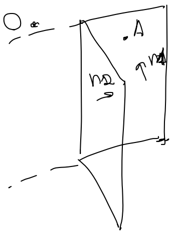
№6.11. (3, 4, 8) 3) $(r_0, n) = D_1$, $(r_0', n) = D_2$

$$p = \frac{(r_0' - r_0, n)}{|n|} = \frac{|D_2 - D_1|}{|n|}$$

4) $$p = \frac{|[r_0 - r_1, a]|}{|a|}$$

8) $\left.\begin{matrix} r = r_1 + a_1 t \\ r = r_2 + a_2 t \end{matrix}\right\}$ $$p = \frac{|(\bar r_1 - \bar r_2, \bar a_1, \bar a_2)|}{|[a_1, a_2]|}$$

№6.80.



$\alpha_1: x - z - 5 = 0$
$\alpha_2: 3x + 5y + 4z = 0$
$A: (1, 1, 1)$

$\alpha_1(A)$ $1 - 1 - 5 = -5 < 0$
$\alpha_2(A) = 3 + 5 + 4 = 12 > 0$

значит A лежит как показано на рис.

Тогда $\vec n = \frac{\vec n_1 + \vec n_2}{2}$, $\vec n_1 = \frac{(1, 0, -1)}{\sqrt 2}$

$\vec n_2 = \frac{(3, 5, 4)}{5\sqrt 2}$

$$\vec n = \frac{(4, \frac{5}{2}, -\frac{1}{2})}{5\sqrt 2} = \frac{1}{10\sqrt 2}(8, 5, -1) \Rightarrow \vec n'(8, 5, -1)$$

Тогда $M \in \alpha_1$, $M \in \alpha_2$: $M(5, -3, 0)$: Биссе: $8x + 5y - z + D = 0$
$8 \cdot 5 + (-5) \cdot 3 \cdot 0 + D = 0$
$40 - 15 + D = 0$
$D = -25$

Биссе: $8x + 5y - z - 25 = 0$