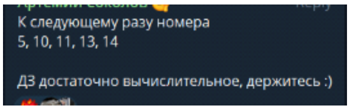

К следующему разу номера
5, 10, 11, 13, 14

ДЗ достаточно вычислительное, держитесь :)

T5. $p_\xi(t)=\begin{cases}\lambda e^{-\lambda t}, & t\ge 0\\ 0, & t<0\end{cases}$ $\qquad \varphi(\xi)=\xi^k$ – возр

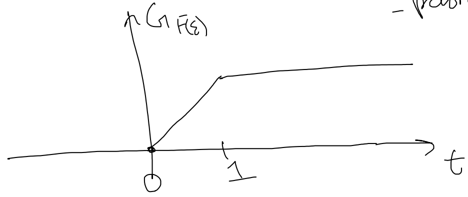
$$p_{\xi^k}(t) = p_\xi(t^{\frac{1}{k}})\cdot \frac{1}{|k t^{\frac{k-1}{k}}|} = \frac{\lambda e^{-\lambda t^{1/k}}}{k t^{\frac{k-1}{k}}}$$

T11. $F_{\xi^2}(t)=P(\xi^2\le t)=P(-\sqrt t\le\xi\le\sqrt t)=F_\xi(\sqrt t)-F_\xi(-\sqrt t)$

$$p_{\xi^2}(t)=F'_{\xi^2}(t)=p_\xi(\sqrt t)\cdot\frac{1}{2\sqrt t}+p_\xi(\sqrt t)\cdot\frac{1}{2\sqrt t}=\frac{1}{\sqrt t}I(b\le\sqrt t\le a)$$

T.14. F-непр. Тогда $G_{F(\xi)}(t)=P(F(\xi)\le t)=\begin{cases}\text{если } t<0, \text{ то}\\ P(F(\xi)\le t)=P(\varnothing)=0\\ \text{если } t>1, \text{ то}\\ P(F(\xi)\le t)=P(\Omega)=1\end{cases} \overset{t\in(0,1)}{=} P(\xi\le F^{-1}(t)) =$

$= F(F^{-1}(t))=t \Rightarrow$ – равн. распр на [0, 1].



T.10. 1) $p(x)=\lambda e^{-\lambda x}I(x>0)$

$\varphi(x)=\frac{1}{\lambda}\ln x$

$\varphi^{-1}(x)=e^{\lambda x}$

$$p_\varphi(x)=\frac{p(e^{\lambda x})}{\frac{1}{\lambda}e^{-\lambda x}}=\lambda e^{-\lambda(e^{\lambda x}-x)}$$

2) $F_\eta(x)=P(\{\xi\}\le x)$ $\quad\Big|\;$ $x\ge1$ $F_\eta(x)=1$; $x\le 0$ $F_\eta(x)=0$

$\{\xi\}\le x \Rightarrow \xi\in[k,k+x]$

$$F_\eta(x)=\sum_{k=1}^{\infty}P(\xi\in[k,k+x])=\sum_{n=1}^{\infty}(F_\xi(k+x)-F_\xi(k))=$$

$$=\sum_{n=1}^{\infty}e^{-\lambda k}(1-e^{-\lambda x})=(1-e^{-\lambda x})\frac{1}{1-e^{-\lambda}}=\frac{F_\xi(x)}{1-e^{-\lambda}}$$

$$p_\eta(x)=\frac{p_\xi(x)}{1-e^{-\lambda}}I_{[0,1]}(x)$$

3) $\varphi(\xi)=1-e^{-\alpha\xi}$ – возрастает

$\varphi^{-1}(t)=-\frac{1}{\alpha}\ln(t-1)$

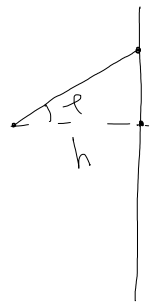
$$p_\varphi(\cancel{t})=\lambda e^{+\lambda(\frac{1}{\alpha}\ln(t-1))}\cdot\frac{1}{\alpha}\cdot\frac{-1}{t-1}=-\frac{\lambda}{\alpha}e^{-\frac{\lambda}{\alpha}\ln(t-1)}\cdot\frac{1}{t-1}=$$

$$=-\frac{\lambda}{\alpha}\left(\frac{1}{t-1}\right)(t-1)^{-\frac{\lambda}{\alpha}}=-\frac{\lambda}{\alpha}(t-1)^{-\frac{\lambda}{\alpha}-1}$$

$\Leftrightarrow -\frac{\lambda}{\alpha}\left(\frac{1}{t-1}\right)(t-1)^{-\frac{1}{\alpha}} = -\frac{\lambda}{\alpha}(t-1)^{\alpha-1}$

T. 13. 1)

$p_\varphi = \frac{I\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)}{\pi}$ $\quad l = h\,tg\,\varphi \to l^{-1}(t) = arctg\left(\frac{l}{h}\right)$



$p_l = \frac{I\left(tg\left(-\frac{\pi}{2}\right), tg\left(\frac{\pi}{2}\right)\right)}{\pi} \cdot \frac{h}{l^2+h^2} = \frac{1}{\pi}\frac{h}{l^2+h^2}$ // – распр Коши.

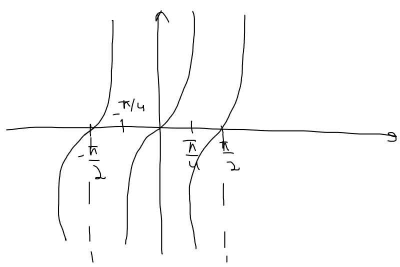
2) Так как p_ξ – распр Коши явл композицией равн распр $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ и $\psi = tg\,\varphi$

то 1. $q = \frac{\xi^2}{1+\xi^2} = \frac{tg^2\varphi}{1+tg^2\varphi} = tg^2\varphi \cdot \cos^2\varphi = \sin^2\varphi \Rightarrow$

$\Rightarrow p_q = \frac{I([0,1])}{\pi} \cdot 2 \cdot \frac{1}{2\sqrt{t-t^2}} = \frac{I([0,1])}{\pi\sqrt{t-t^2}}$?

2. $q = \frac{1}{1+\xi^2} = \cos^2\varphi \Rightarrow p_q = -\frac{I([0,1])}{\pi} \cdot 2\frac{-1}{2\sqrt{t-t^2}} = \frac{I([0,1])}{\pi\sqrt{t-t^2}}$

3. $q = \frac{2\xi}{1-\xi^2} = tg\,2\varphi \Rightarrow p_q = \frac{I([0,+\infty))}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} + \frac{I((-\infty,0))}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} \oplus$



$\oplus \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{1+t^2} \cdot \frac{1}{2} \ominus$

$\ominus \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{1+t^2} = \frac{1}{\pi(1+t^2)}$

4. $q = \frac{1}{\xi} = ctg\,\varphi \Rightarrow p_q = \frac{I([-\infty,+\infty])}{\pi} \cdot \frac{1}{|-1+t^2|} = \frac{1}{\pi(1+t^2)}$