7. $2x^2 + Cy^2 = 1.$

$$4x\,dx + 2Cy\,dy = 0$$

$$4xy\,dx + 2Cy^2\,dy = 0$$

$$4xy\,dx + (1-2x^2)\,dy = 0$$

$$\boxed{y' = -\frac{4xy}{1-2x^2}}$$

37. $y' = x^2 + y^2 - 1.$

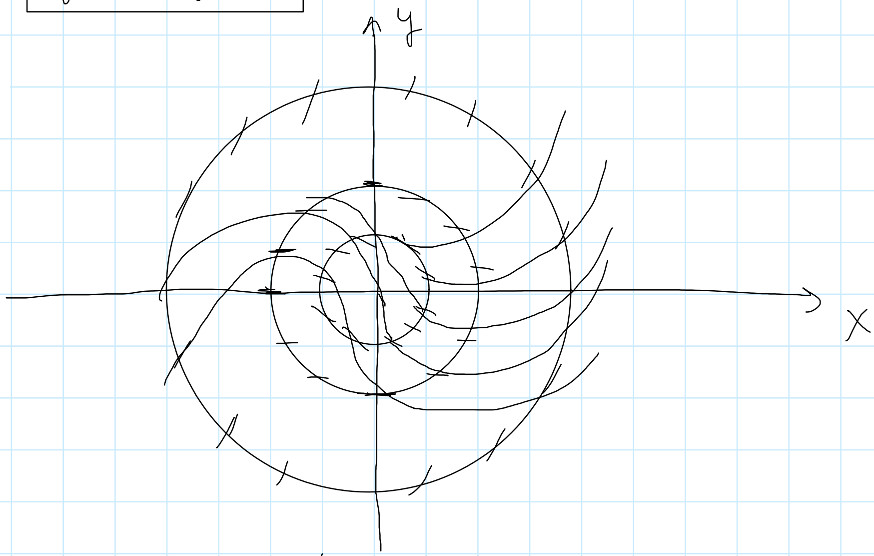
$y' = -1 \Rightarrow x^2+y^2=0$

$y' = -0{,}75 \Rightarrow x^2+y^2=0{,}25$

$y' = 0 \Rightarrow x^2+y^2=1$

$y' = 3 \Rightarrow x^2+y^2=4$

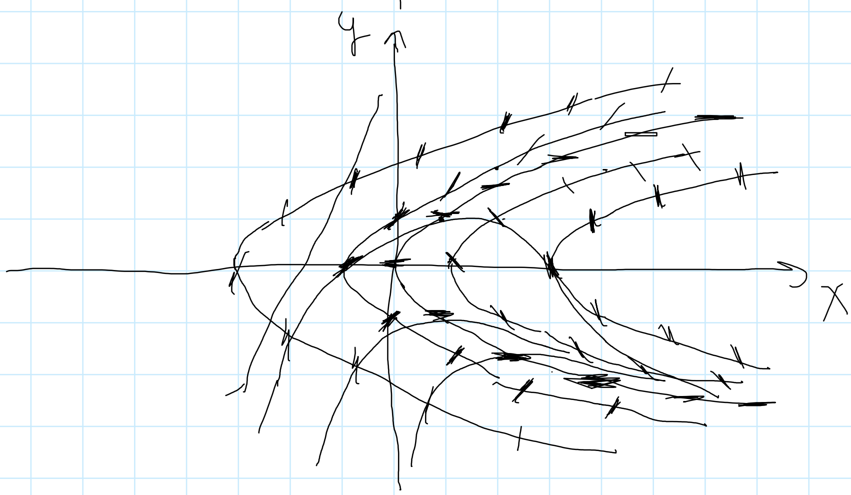
$y' = 8 \Rightarrow x+y^2=9$



$y' = y^2 - x$

$y'' = 2yy' - \underline{1}$

в т. ге $y' = 0 \Rightarrow y'' = -1$ ∩



4. $y'\cos x + y(1+y)\sin x = 0.$

$$\frac{dy}{y(1+y)} = -\operatorname{tg}x\,dx$$

$$\int\frac{dy}{y(1+y)} = \int dy\left(\frac{1}{y} - \frac{1}{1+y}\right) = \ln y - \ln(1+y) = -\int \operatorname{tg}x\,dx = +\ln\cos x$$

$$C + \ln\frac{y}{1+y} = +\ln\cos x \Rightarrow \boxed{\frac{y}{1+y} = \cos x \cdot C}$$

17. $(1+\cos x)yy' = (1+y^2)\sin x.$

$$\frac{y\,dy}{\ldots} = \frac{\sin x\,dx}{\ldots} \to \frac{1}{2}\int\frac{dy^2}{\ldots} = \frac{1}{2}\ln(1+y^2) = -\int\frac{d\cos x}{\ldots} = -\ln(1+\cos x) + C$$

17. $(1+\cos x)yy' = (1+y^2)\sin x.$

$$\frac{y\,dy}{1+y^2} = \frac{\sin x\,dx}{1+\cos x} \Rightarrow \frac{1}{2}\int\frac{dy^2}{1+y^2} = \frac{1}{2}\ln(1+y^2) = -\int\frac{d\cos x}{1+\cos x} = -\ln(1+\cos x) + C$$

$$\boxed{\frac{1}{\sqrt{1+y^2}} = (1+\cos x)\cdot C}$$

31. $3x(x+1)y' = (x+2)y,\ y(1) = -1.$

$$3\frac{dy}{y} = \frac{x+2}{x(x+1)}dx = 2\cdot\frac{2(x+1)-x}{2(x+1)x}dx = \left(\frac{2}{x} - \frac{1}{x+1}\right)dx$$

$$3\ln y = 2\ln x - \ln(x+1) + C \Rightarrow y^3 = C\cdot\frac{x^2}{x+1} \Rightarrow C = -2 \Rightarrow \boxed{y^3 = -\frac{2x^2}{x+1}}$$

43. $y = C\sin x - 2.$

$$y' = C\cos x = \frac{y+2}{\sin x}\cos x = (y+2)\,\mathrm{ctg}\,x \Leftrightarrow -\frac{1}{y'} = (y+2)\,\mathrm{ctg}\,x$$

$$-\mathrm{tg}\,x\,dx = (y+2)\,dy$$

$$\boxed{+\ln\cos x = \frac{y^2}{2} + 2y + C}$$

59. $x\,dy = \left(y + \sqrt{x^2+y^2}\right)dx,\ x \geqslant 0.$

$$\{y = xz \Rightarrow dy = z\,dx + x\,dz\}$$

$$xz\,dx + x^2dz = \left(xz + \sqrt{x^2+x^2z^2}\right)dx$$

$$z\,dx + x\,dz = \left(z + \sqrt{1+z^2}\right)dx$$

$$x\,dz = \sqrt{1+z^2}\,dx$$

$$\frac{dz}{\sqrt{1+z^2}} = \frac{dx}{x} \Rightarrow \ln\left(z + \sqrt{1+z^2}\right) = \ln x + C \Rightarrow z + \sqrt{1+z^2} = Cx$$

$$\boxed{\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = Cx}$$

72. $(x+y+1)dx + (x-y+3)dy = 0.$

$$\left.\begin{matrix} x+y+1=0 \\ x-y+3=0\end{matrix}\right\} \begin{matrix} x=-2 \\ y=1\end{matrix} \Rightarrow \left.\begin{matrix} t = x+2 \\ p = y-1\end{matrix}\right\} \begin{matrix}(t-2+p+1+1)dt + (t-2-p-1+3)dp = 0 \\ (t+p)dt + (t-p)dp = 0\end{matrix}$$

$$t\,dt + p\,dt + t\,dp - p\,dp = 0$$

$$d\frac{t^2}{2} + d(pt) - d\frac{p^2}{2} = 0$$

$$\frac{t^2}{2} + pt - \frac{p^2}{2} = C \Rightarrow \boxed{\frac{(x+2)^2}{2} + (y-1)(x+2) - \frac{(y-1)^2}{2} = C}$$

27. $4y' + 12x^2y = 3x^2$.

$$4y' = 3x^2(1-4y)$$

$$\int \frac{4dy}{1-4y} = \int 3x^2dx \Rightarrow -\ln(1-4y) = x^3 + C \Rightarrow \boxed{1-4y = Ce^{-x^3}}$$

35. $(1 + y^2)dx + (2xy - 1)dy = 0$.

$$\frac{\partial(1+y^2)}{\partial y} = 2y = \frac{\partial(2xy-1)}{\partial x} \Rightarrow \exists F(x,y) \text{ т.ч. } dF = (1+y^2)dx + (2xy-1)dy$$

$$\begin{cases} \dfrac{\partial F}{\partial x} = 1+y^2 \\ \dfrac{\partial F}{\partial y} = 2xy-1 \end{cases} \Rightarrow F(x,y) = (1+y^2)x + f(y) \Rightarrow \frac{\partial F}{\partial y} = f'(y) + 2xy = 2xy - 1 \Rightarrow$$

$$\Rightarrow f(y) = -y + C$$

$$\boxed{F(x,y) = (1+y^2)x - y = const}$$

45. $x^2y' + y = 4$, $y(-1) = 5$.

$$z = y - 4 \Rightarrow x^2z' + z = 0 \Rightarrow \frac{x^2dz}{dx} = -z \Rightarrow -\frac{dz}{z} = \frac{dx}{x^2} \Rightarrow -\ln z = -\frac{1}{x} + C \Rightarrow$$

$$\Rightarrow z = Ce^{\frac{1}{x}} \Rightarrow y = 4 + Ce^{1/x} \Rightarrow \begin{Bmatrix} 5 = 4 + C/e \\ C = e \end{Bmatrix} \Rightarrow \boxed{y = 4 + e^{1+\frac{1}{x}}}$$

64. $y' \cos x + y \sin x + 3y^2 \cos x = 0$.

1) $y = 0$ – решение //

2) $y \neq 0$

$$\left.\begin{matrix} y' + y\,tg x + 3y^2 = 0 \\ z = 1/y \\ y' = -\frac{1}{z^2}z' \end{matrix}\right\} \quad \begin{matrix} -\frac{1}{z^2}z' + \frac{1}{z}tg x + 3\frac{1}{z^2} = 0 \\ -z' + z\,tg x + 3 = 0 \end{matrix}$$

Решим $z' = z\,tg x$

$$\frac{dz}{z} = tg x\,dx \Rightarrow \ln z = -\ln\cos x + C$$

$$z = C(x)/\cos x$$

Тогда: $z' = \dfrac{C'(x)}{\cos x} + \dfrac{C(x)\sin x}{\cos^2 x} \Rightarrow \dfrac{c'}{\cos x} + \dfrac{C\sin x}{\cos^2 x} - \dfrac{c\sin x}{\cos^2 x} - 3 = 0$

$$C' = 3\cos x \Rightarrow C(x) = 3\sin x + B \Rightarrow \boxed{\frac{1}{y} = \frac{3\sin x + B}{\cos x} = 3tg x + \frac{B}{\cos x}}$$

84. $4y' = y^2 + \dfrac{4}{x^2}$.

Пусть $y = \dfrac{A}{x} \Rightarrow -\dfrac{4A}{x^2} = \dfrac{A^2}{x^2} + \dfrac{4}{x^2} \Rightarrow A^2 + 4A + 4 = 0 \Rightarrow A = -2 \Rightarrow y = -\dfrac{2}{x}$ – частное реш.

Тогда $y = z - \dfrac{2}{x}$ $\Big\}$ $4z' + \dfrac{8}{x^2} = z^2 - \dfrac{4z}{x} + \dfrac{4}{x^2} + \dfrac{4}{x^2} \Rightarrow$

Тогда $\left.\begin{aligned} y &= z - \frac{2}{x} \\ y' &= z' + \frac{2}{x^2} \end{aligned}\right\}$ $4z' + \frac{8}{x^2} = z^2 - \frac{4z}{x} + \frac{4}{x^2} + \frac{4}{x}2 \Rightarrow$

$\Rightarrow 4z' = z^2 - \frac{4z}{x}$, $\boxed{z=0 - \text{решение}}$

Замена $t = \frac{1}{z}$, $z \neq 0$! $-\frac{4}{t^2}t' = \frac{1}{t^2} - \frac{4}{tx} \Rightarrow +4t' = \frac{4t}{x} - 1$

Решим $4t' = \frac{4t}{x} \Rightarrow \frac{dt}{t} = \frac{dx}{x} \Rightarrow t = Cx$ | $t = F(x)x \rightsquigarrow t' = F'(x)x + F(x)$

Тогда $4F'(x)x + 4F(x) = 4F(x) - 1 \Rightarrow F'(x) = -\frac{1}{4x} \Rightarrow F(x) = -\frac{1}{4}\ln x + C$

$\frac{1}{z} = \left(-\frac{1}{4}\ln x + C\right)x \Rightarrow \boxed{y = \frac{1}{x\left(-\frac{1}{4}\ln x + C\right)} - \frac{2}{x} \;\Big|\; y = -\frac{2}{x}}$

94. $x^2y' - 5xy + x^2y^2 + 8 = 0.$???

$y = \frac{A}{x} \Rightarrow x^2\left(\frac{-A}{x^2}\right) - \frac{5Ax}{x} + x^2\frac{A^2}{x^2} + 8 = 0$

$-A - 5A + A^2 + 8 = 0$

$A^2 - 6A + 8 = 0$

$\left[\begin{aligned} A &= 4 \\ A &= 2 \end{aligned}\right.$

$\left.\begin{aligned} y &= z + \frac{2}{x} \\ y' &= z' - \frac{2}{x^2} \end{aligned}\right\}$ $x^2\left(z' - \frac{2}{x^2}\right) - 5x\left(z + \frac{2}{x}\right) + z^2 + \frac{4}{x^2} + \frac{2z}{x} + 8 = 0$

$z'x^2 - 5xz + z^2 + \frac{2z}{x} = 0$ | $z=0$ - решение

$t = \frac{1}{z} \Rightarrow -\frac{1}{t^2}t'x^2 - \frac{5x}{t} + \frac{1}{t^2} + \frac{2}{xt} = 0$

$x^2t' + 5xt - 1 - \frac{2t}{x} = 0$

Решим $x^2t' = \left(\frac{2}{x} - 5x\right)t$

$\frac{dt}{t} = \frac{2 - 5x^2}{x^3}dx$

$\ln t = -\frac{1}{x^2} - 5\ln x + C$

$z = Ce^{\frac{1}{x^2} + 5\ln x} \Rightarrow \left[\begin{aligned} y &= \frac{2}{x} \\ y &= Ce^{\frac{1}{x^2} + 5\ln x} + \frac{2}{x} \end{aligned}\right.$

18. $\left(1 + \frac{2x}{y^3}\right)dx + \left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right)dy = 0.$

$\frac{\partial\left(1 + \frac{2x}{y^3}\right)}{\partial y} = -\frac{6x}{y^4} = \frac{\partial\left(\frac{1}{y^2} - \frac{3x^2}{y^4}\right)}{\partial x}$

Тогда $\begin{cases} \frac{\partial F}{\partial x} = 1 + \frac{2x}{y^3} \Rightarrow F(x,y) = x + \frac{x^2}{y^3} + f(y) \Rightarrow \\ \frac{\partial F}{\partial y} = \frac{1}{y^2} - \frac{3x^2}{y^4} \quad \Rightarrow \frac{\partial F(x,y)}{\partial y} = -\cancel{\frac{3x^2}{y^4}} + f'(y) = \frac{1}{y^2} - \cancel{\frac{3x^2}{y^4}} \Rightarrow \end{cases}$

$$\begin{cases} \dfrac{\partial F}{\partial y} = \dfrac{1}{y^2} - \dfrac{3x^2}{y^4} \end{cases} \Rightarrow \frac{\partial F(x,y)}{\partial y} = -\cancel{\frac{3x^2}{y^4}} + f'(y) = \frac{1}{y^2} - \cancel{\frac{3x^2}{y^4}} \Rightarrow$$

$$\Rightarrow f(y) = -\frac{1}{y} + C \Rightarrow F(x,y) = x + \frac{x^2}{y^3} - \frac{1}{y} + C \Rightarrow \boxed{x + \frac{x^2}{y^3} - \frac{1}{y} = \text{const}}$$

33. $\left(\frac{1}{x} - y\right) dx = \frac{1}{y}\, dy.$

$$\left\{ d\left(\frac{x}{y}\right) = \frac{dx}{y} - \frac{x\,dy}{y^2} \right\}$$

$$\frac{dx}{x} - y\,dx = \frac{dy}{y}$$

$$\frac{dx}{y} - x\,dx = \frac{x\,dy}{y^2}$$

$$d\left(\frac{x}{y}\right) = d\left(\frac{x^2}{2}\right) \Rightarrow \boxed{\frac{x}{y} = \frac{x^2}{2} + C}$$

59. $x^2yy' + x^3 = (x^2 + y^2)^2.$

$$t = x^2 + y^2 \Rightarrow t' = 2x + 2yy'$$

$$x^2\left(\frac{t' - 2x}{2}\right) + x^3 = t^2$$

$$\frac{x^2t'}{2} - \cancel{x^3} + \cancel{x^3} = t^2 \Rightarrow \frac{dt}{2t^2} = \frac{dx}{x^2} \Rightarrow -\frac{1}{2t} = -\frac{1}{x} + C \Rightarrow \frac{1}{t} = \frac{2}{x} + C$$

$$\boxed{\frac{1}{x^2+y^2} = \frac{2}{x} + C}$$