Minimax, Alpha-Beta Pruning, and Expectimax Algorithms

Algorithm 1 Minimax Algorithm

```
1: function MINIMAX(node, depth, maximizingPlayer)
        if depth == 0 or node is a terminal node then
3:
            return Evaluate(node)
        end if
4:
       \mathbf{if} \ \mathrm{maximizingPlayer} \ \mathbf{then}
5:
           bestValue \leftarrow -\infty
6:
            for each child in node's children do
7:
                value \leftarrow MINIMAX(child, depth - 1, false)
8:
               bestValue \leftarrow max(bestValue, value)
9:
10:
            return bestValue
11:
        else
12:
            bestValue \leftarrow \infty
13:
            for each child in node's children do
14:
15:
                value \leftarrow MINIMAX(child, depth - 1, true)
               bestValue \leftarrow min(bestValue, value)
16:
            end for
17:
18:
           return bestValue
       end if
19:
20: end function
```

Algorithm 2 Alpha-Beta Pruning Algorithm

```
1: function AlphaBeta(node, depth, \alpha, \beta, maximizingPlayer)
        if depth == 0 or node is a terminal node then
             return Evaluate(node)
 3:
        end if
 4:
        if maximizingPlayer then
 5:
 6:
             bestValue \leftarrow -\infty
             {f for} each child in node's children {f do}
 7:
                 value \leftarrow AlphaBeta(child, depth - 1, \alpha, \beta, false)
 8:
 9:
                 bestValue \leftarrow max(bestValue, value)
                 \alpha \leftarrow \max(\alpha, \text{bestValue})
10:
                 if \beta \leq \alpha then
11:
12:
                     break
                                                                                 ▶ Beta cutoff
13:
                 end if
             end for
14:
             return bestValue
15:
16:
        else
            bestValue \leftarrow \infty
17:
             {f for} each child in node's children {f do}
18:
                 value \leftarrow AlphaBeta(child, depth - 1, \alpha, \beta, true)
19:
                 bestValue \leftarrow min(bestValue, value)
20:
21:
                 \beta \leftarrow \min(\beta, \text{ bestValue})
                 if \beta \leq \alpha then
22:
                     break
                                                                               \triangleright Alpha cutoff
23:
                 end if
24:
             end for
25:
            return bestValue
26:
27:
        end if
28: end function
```

Algorithm 3 Expectimax Algorithm

```
1: function Expectimax(node, depth, maximizingPlayer)
 2:
        if depth == 0 or node is a terminal node then
            return Evaluate(node)
 3:
        end if
 4:
 5:
        if \ {\rm maximizingPlayer} \ {\bf then}
 6:
            bestValue \leftarrow -\infty
 7:
            {f for} each child in node's children {f do}
                value \leftarrow Expectimax(child, depth - 1, false)
 8:
 9:
               bestValue \leftarrow max(bestValue, value)
10:
            end for
            return bestValue
11:
        else
12:
            sumValues \leftarrow 0
13:
            numChildren \leftarrow number \ of \ children
14:
            {f for} each child in node's children {f do}
15:
               value \leftarrow Expectimax(child, depth - 1, true)
16:
17:
               sumValues \leftarrow sumValues + value
            end for
18:
            return sumValues / numChildren
                                                                      \triangleright Expected value
19:
        end if
20:
21: end function
```