ECON 311
MICROECONOMIC THEORY I

THE THEORY OF THE FIRM

Cost Theory

What choices should a firm make?

Cost Minimization

- Now that we know ***what choices a firm can make***, lets consider ***what choices a firm should make*** based on the behavioral assumption that firms will want to minimize cost (or maximize output) … ***maximize profit***
  - Coming from the mathematical principle that any constrained ***minimization problem*** has associated with it a dual problem in constrained ***maximization*** that focuses attention on the constraints in the original problem

- To solve the cost-minimization problem, it is necessary to define the various combination of inputs that the firm can purchase with a fixed sum of money (expenditure)
  - Analogous to the budget line of a consumer

Explicit and Implicit Cost

- However, it is important to differentiate between Explicit and Implicit cost.

- Explicit cost are outlays made by a firm for purchased or hired factors of production
  - payments for raw materials, utilities, wages and rent

- Implicit cost are the imputed cost of self own, self employed factors of production
  - salary of an entrepreneur, return on capital owned by owner

- In evaluating a firms implicit cost, economists are guided by the term: opportunity cost – the cost of an input to a firm is that value forgone by not employing it in its best alternative use
  - Farmers wage if he had worked in another job

Economic Cost

- Limiting ourselves to only explicit cost and only two inputs that are homogenous, we define total cost as

$$C = w_1 x_1 + w_2 x_2$$

  where w_1 and w_2 are the prices of input x_1 and x_2 respectively

  - The above is an isocost equation

- We can generate a line from this equation to show all combinations of land and labour that the firm can purchase with the fixed amount of money

Primal Problem of a Firm

- The primal (fundamental) problem of a firm is to minimize the total cost of inputs used to produce a given level of output.

- However, the *mathematical principle of duality* - a constrained minimization problem has an associated dual problem in constrained maximization that focuses attention on the constraints in the original (primal) problem

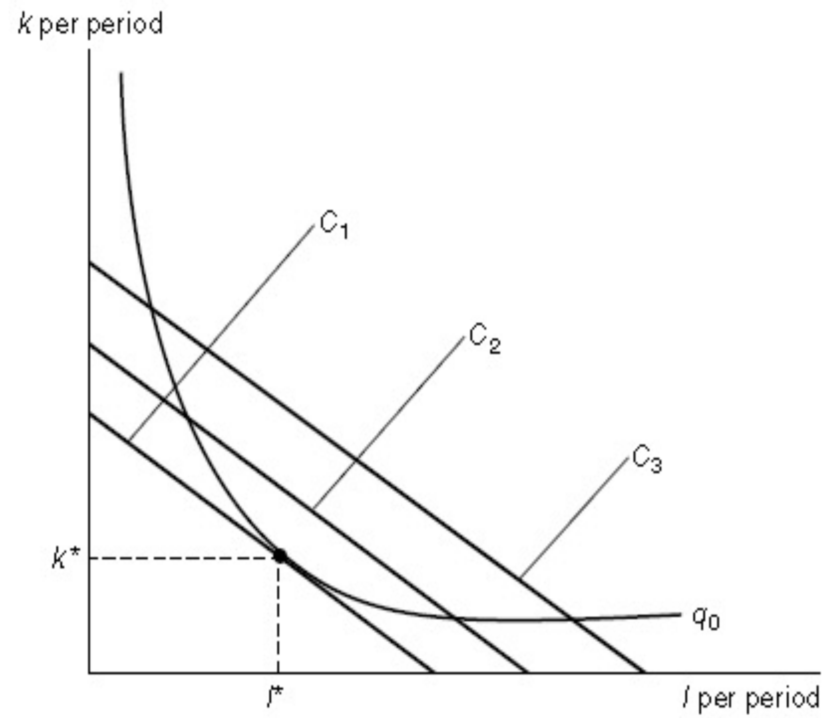
- Therefore, the dual problem of the minimization problem is to maximize output for a given cost of inputs purchased

Cost Minimization

- Intuitively, Equate the rate at which *k* can be traded for *l* in production to the rate at which they can be traded in the marketplace

Slope of isocost is equal to slope of isoquant: $MRTS_{k,l} = \frac{w}{r}$

This is, therefore, a constrained optimization problem



Cost Minimization

- The Isocost equation can be expressed as an Isocost line.

If the total cost, $C = w_1 x_1 + w_2 x_2$

where w_1 and w_2 are the prices of input x_1 and x_2

Making x_2 the subject of the equation leads to:

$$x_2 = \frac{c}{w_2} - \frac{w_1}{w_2} x_1$$

where w_1/w_2 is the slope and the constant c/w_2 is the vertical intercept

Cost Minimization

- Assuming a production function of the form $\boldsymbol{q = f(k, l)}$ and the isocost equation is expressed as $\boldsymbol{C = wl + vk}$, the cost minimizing input choice can mathematically be derived by setting up a Lagrangian equation of the form:

$$\boldsymbol{L = wl + vk + \lambda[q_0 - f(k, l)]}$$

- Taking the First Order Condition (FOC)

$$\frac{\partial L}{\partial l} = w - \lambda \frac{\partial f}{\partial l} = 0 \ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial L}{\partial k} = v - \lambda \frac{\partial f}{\partial k} = 0 \ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial L}{\partial \lambda} = q_0 - f(k, l) = 0 \ldots\ldots\ldots\ldots (3)$$

Cost Minimization

- Dividing the first two equations, i.e. (1)÷ (2), yields

$$\frac{w}{v} = \frac{df/dl}{df/dk} = RTS\ (l\ for\ K)$$

- This expression simply says that the cost-minimizing firm should equate the RTS for the two inputs to the ratio of their prices

- The same equation can be re-written as:

$$\frac{f_k}{v} = \frac{f_l}{w}$$

Cost Minimization

- For costs to be minimized, the marginal productivity per dollar spent should be the same for all inputs.

- This cost minimizing problem is similar to the individual's expenditure-minimization problem.

Dual Problem: Output Maximization

- Mathematically, we set up a lagrangian expression for output maximization, given $\boldsymbol{TC = wl + vk}$, of the form

$$L = f(k, l) + \lambda(TC - wl - vk)$$

- Taking the First Order Condition (FOC)

$$\frac{\partial L}{\partial l} = \frac{\partial f}{\partial l} - \lambda w = 0 \ldots\ldots\ldots\ldots(1)$$

$$\frac{\partial L}{\partial k} = \frac{\partial f}{\partial k} - \lambda v = 0 \ldots\ldots\ldots\ldots..(2)$$

$$\frac{\partial L}{\partial \lambda} = TC - wl - vk = 0 \ldots\ldots\ldots\ldots(3)$$

Dual Problem: Output Maximization

- Dividing the first two equations, i.e. (1)÷(2) yields

$$\frac{w}{v} = \frac{df/dl}{df/dk} = RTS\ (l\ for\ K)$$

- Therefore, maximum output is attainable by equating the RTS for the two inputs to the ratio of their prices

- Note the similarity between the cost minimization approach and the output maximization approach;
  - the only difference is whether we hold production constant or cost constant

Input Demand Function

- Solving the FOC for each input generates the respective input demand function

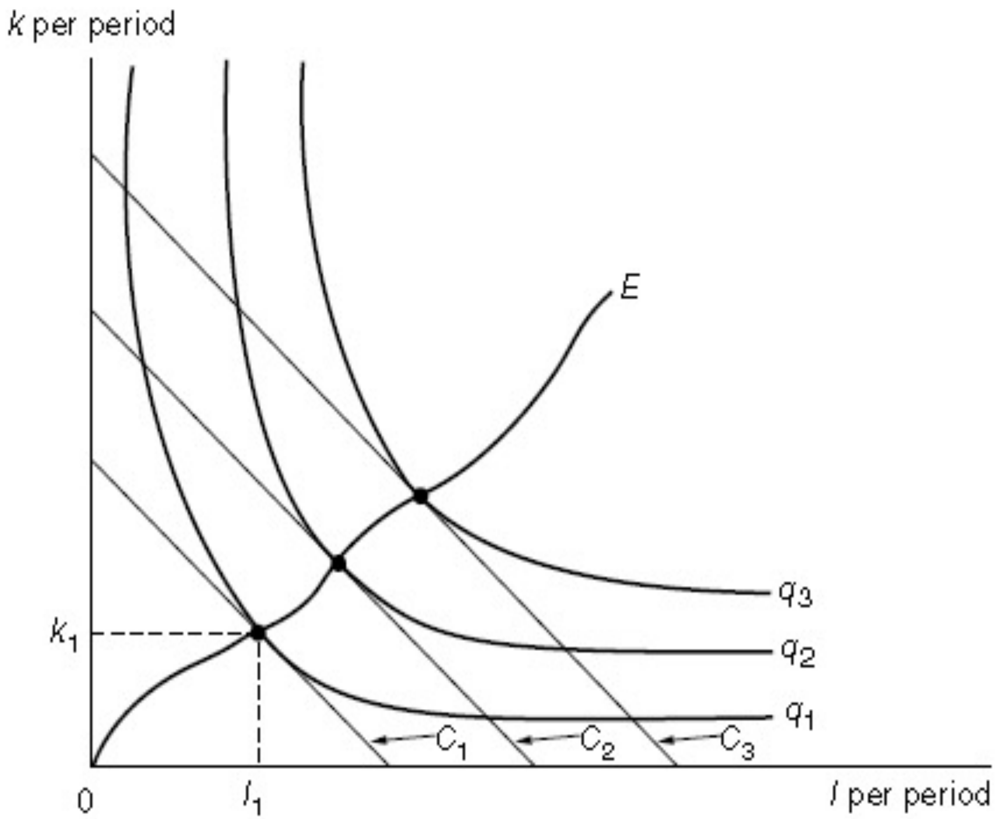
- These input demand functions will depend on the quantity of output that the firm chooses to produce and are therefore the demand for input is a derived demand

The Expansion Path

- As a first step towards accomplishing the ultimate objective of profit maximization, firms necessarily repeat the analysis of cost minimization for each level of production or each output level
- In other words, for each level of output, the firm must determine the optimal input combination
- Once the firm solves the cost minimization problem for each level of output, it can proceed with the determination of the optimal output

Expansion Path

- The locus of cost-minimizing tangencies gives the firms expansion path.



Expansion Path

- This expansion path is analogous to the Income Consumption Curve

- Thus, in the same way we can use the ICC to distinguish between normal and inferior goods … we can also use the expansion path to distinguish between *normal and inferior inputs*
  - By definition, a normal input is one whose quantity increases with output while an inferior input is one whose quantity falls as output expands

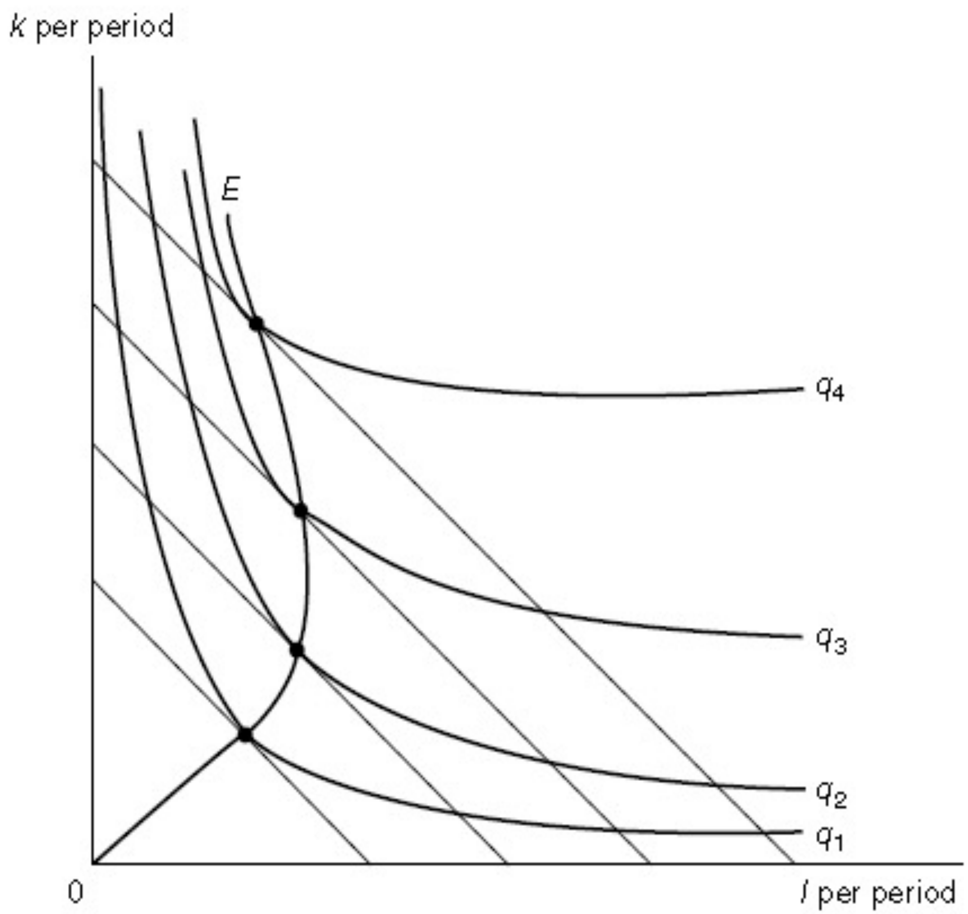
Expansion Path

- Because cost minimization requires that the RTS always be set equal to input price ratio, and because this ratio is assumed constant, the shape of the expansion path will be determined by where a particular RTS occurs on successively higher isoquants.

- If the production function exhibits constant returns to scale, then the expansion path will be a straight line because in that case the RTS depends only on the ratio of k to l.
  - That ratio would be constant along such a linear expansion path.

Expansion Path

- The expansion path need not be positive.



Increases of output beyond q_2 causes the quantity of labor used to decrease.

In this range, labor would be said to be an inferior input.

The occurrence of inferior inputs is a theoretical possibility that may happen, even when isoquants have their usual convex shape.

Expansion Path

- Mathematically, the output expansion path is obtainable by the equation below:

$$\frac{w}{v} = \frac{df/dl}{df/dk} = RTS\ (l\ for\ K)$$

Some Notes on Cost

- Private Cost
  - These are costs paid/incurred by individual firms for the purchase of goods and services (inputs) from the market

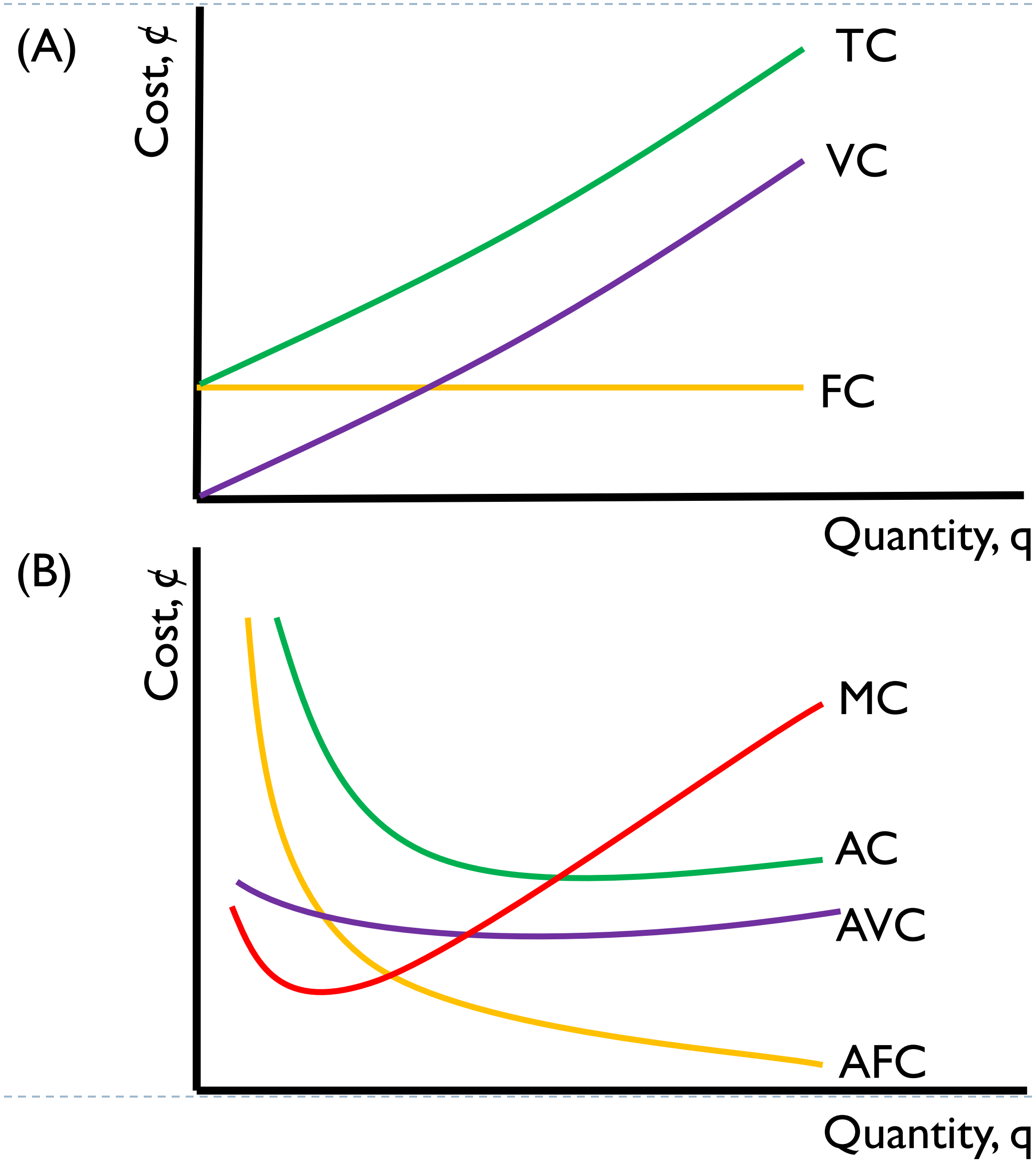
- Social Cost
  - These are costs which the society bears

Short Run Cost Theory

- Short run refers to a period of time during which some inputs are fixed and unadjustable
  - This lack of adjustability of factor inputs is the distinguishing feature of the short run from the long run
  - In the short run, inputs are classified into fixed and variable

- Short run total cost therefore consists of two components
  - Total Fixed Cost - part of total cost that remains constant at all levels of output. It is unavoidable even at zero output.
  - Total Variable Cost – part of total cost that the firm can avoid completely by closing down. Only variable cost is relevant for decision making

Short Run Cost Theory



What determines the shape of the marginal, average variable and average cost curves?

Long Run Cost Theory

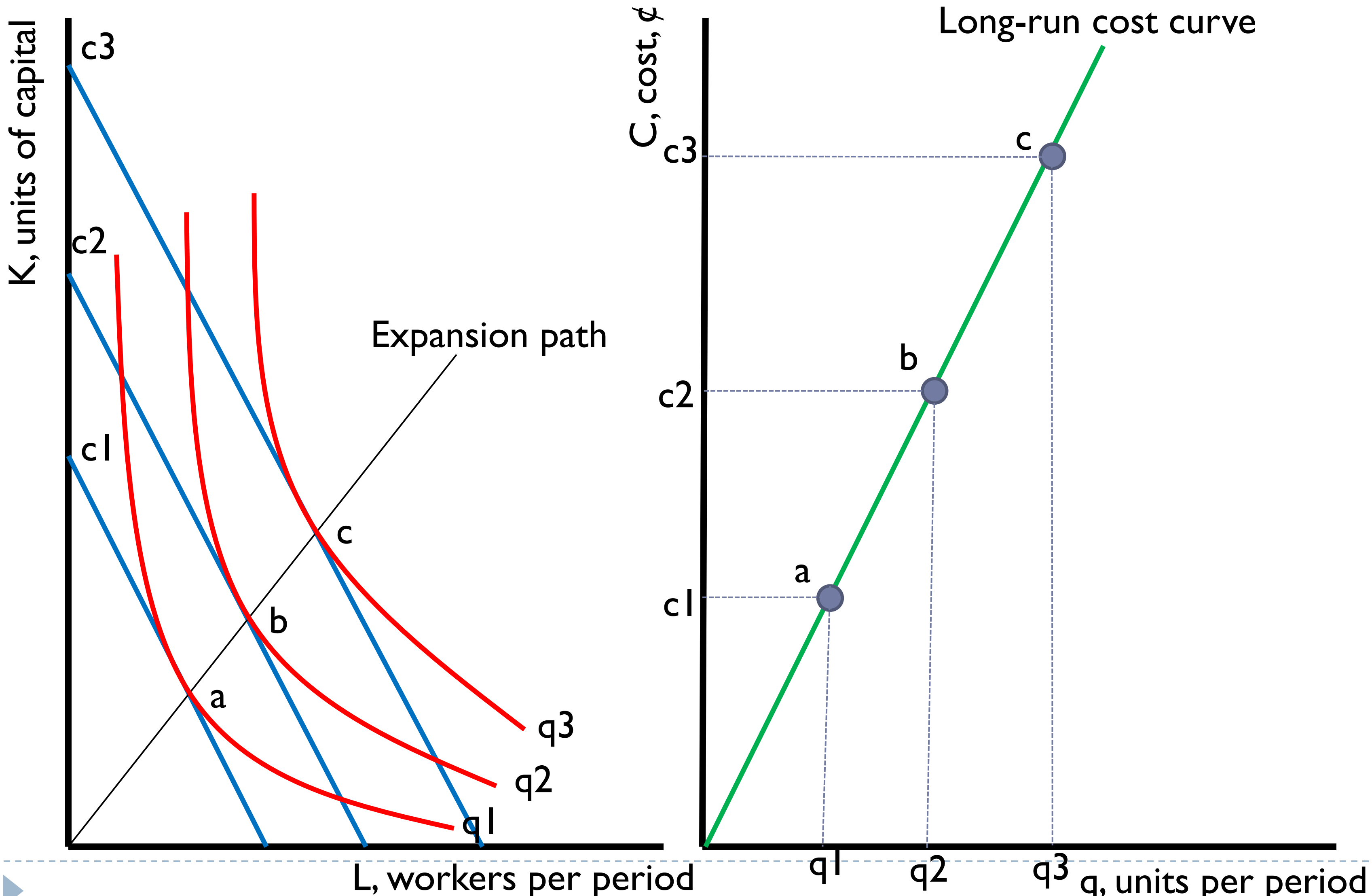
- Long run refers to the time period when firms enjoy complete adjustability of all inputs

- The long run cost curves to be dealt with are:
  - Long-run total cost curve (LTC)
  - Long-run average cost curve (LAC)
  - Long-run marginal cost curve (LMC)

Long Run Cost Theory

- Long Run Total Cost (LTC) Curve
  - Basic long-run cost concept – the least cost of producing a certain quantity of total output
  - It shows relationship between long-run total cost and alternative quantities of output
  - The expansion path gives all the information necessary for the derivation of the LTC curve
    - Since an isoquant gives the amount of output produced and an isocost line gives the least cost way of producing that output, the intersection of the two is required for the least cost way of producing an output.

Long Run Cost Theory



Long Run Cost Theory

- Long Run Total Cost (LTC) Curve
  - Has 2 important properties
    - Starts at the origin: when output is zero, long run total cost is also zero. Firm is at planning stage
    - Has a positive slope: least cost way of producing a larger output is higher than the least cost way of producing a lower output
  - The typical LTC curve is concave at lower levels of output and eventually becomes convex

Long Run Cost Theory

- **Long Run Average Cost (LAC) Curve**
  - The long run cost per unit of output
  - Mathematically, $LAC = \frac{Total\ Cost}{Total\ Output}$
  - LAC has a positive intercept
  - The minimum of the LAC occurs to the right of the inflection point on the LTC
  - For any output level, we can derive the LTC from the LAC

$$LTC = output \times LAC$$

Economies and Diseconomies of Scale

- In the LR, the shape of the LAC cannot be explained by DMR
- The precise shape of the LAC curve reflects the characteristics of the underlying technology or simply returns to scale
- A cost function exhibits economomies of scale when there is increasing returns to scale – an increase in input casues more than a proportionate change in output.
- A firm experiences diseconomies of scale if average costs rise with increasing output
- What happens if an increase in output has no effect on average cost?

Long Run Cost Theory

- Long Run Marginal Cost (LMC) Curve
  - The addition to LTC that results from a unit increase in total output
  - Mathematically, LMC $= \frac{\Delta LTC}{\Delta Q}$
  - Graphically, the LMC is given by the slope of the LTC curve at the current total output
  - As we travel on the concave portion of the LTC, LMC declines
  - As we travel on the convex portion of the LTC, LMC rises

Long Run Cost Theory

- Relationship between LMC and LAC
  - Both have the same positive intercept
  - When the LMC is lower than the average, LAC tends to fall as output increases … when the LMC is higher than the average, LAC tends to increase as output increases
  - LAC reaches its minimum at a point where it is intersected by the LMC
  - LMC reaches its minimum at a lower output than the LAC

Economic Cost and Economic Profit

- Thus Economic Profit is the difference between a firms total revenue and total cost

$$\pi = Total\ Revenue\ - Total\ Cost$$
$$= pq - w_1 x_1 - w_2 x_2$$
$$pf(k, l) - w_1 x_1 + w_2 x_2$$

- Economic profit is therefore a function of the amount of capital and labour employed

- If we assume firms seek maximum profit, then we can study the behaviour of *K* and *L* to achieve that