THE CHINESE UNIVERSITY OF HONG KONG

Department of Mathematics

2024-25 Term 2 MATH2221A Mathematics Laboratory II

Lab Assignment 2 Suggested Solutions

- Full Mark: 40
- You are required to hand in both of the following:
  - For the written part, write your answers in the space provided. Hand in the lab assignment worksheet by the end of the lab session.
  - For the coding part, submit a zip file containing all the required code files to Blackboard (under "Lab Assignment 2") by the end of the lab session.

1. (a) (3 marks) Let

$$A = \begin{bmatrix} 5 & 1 & 2 \\ 1 & 8 & 2 \\ 2 & 2 & 9 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}.$$

Solve the equation $Ax = b$ using the backslash operator \ in MATLAB. Write down both your commands and the answer obtained.

Solution:

```
>> A = [5, 1, 2; 1, 8, 2; 2, 2, 9];
>> b = [3; 5; 6];
>> x = A\b
x =
    0.3094
    0.4625
    0.4951
```

(b) (3 marks) Note that the least-squares solution to an overdetermined system $My = c$ can be obtained by solving the normal equation $(M^T M)y = M^T c$,

which is a square linear system. Consider the overdetermined system:

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

By solving the normal equation $(M^T M)y = M^T c$ in MATLAB, find the least-squares solution `y` to the above overdetermined system. Write down both your commands and the answer `y` obtained.

Solution:

```
>> M = [1, 2; 1, 4; 1, 8];
>> c = [1; 2; 3];
>> y = (M'*M)\(M'*c)
y =
    0.5000
    0.3214
```

(c) (3 marks) In fact, we can use the backslash operator \ in MATLAB to solve overdetermined systems directly. Run the command `z = M\c` and write down your answer. Also, run the command `norm(y-z)` to check the Euclidean norm of the difference between `y` and `z`. Write down the value obtained.

Solution:

```
>> z = M\c
z =
    0.5000
    0.3214
>> norm(y-z)
ans =
    1.2816e-15
```

Remark: We can see that the 2-norm difference is very small ($\sim 10^{-15}$). Note that this number may vary slightly on different computers with different MATLAB versions.

2. It is well known that for any triangle with side length a, b, c, its area A can be calculated using the Heron's formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = \frac{1}{2}(a+b+c)$.

(a) (3 marks) Write a MATLAB function `A = heron(a,b,c)` that takes any numbers a, b, c as input and returns the area A using the above formula. Include the code file `heron.m` in your submission.

Solution:

```
function A = heron(a,b,c)
s = (a+b+c)/2;
A = sqrt(s*(s-a)*(s-b)*(s-c));
end
```

(b) (2 marks) Run the following commands and write down your answers:

(i) `A1 = heron(3,4,5)`

(ii) `A2 = heron(1,1,1)`

Solution:

```
>> A1 = heron(3,4,5)
A1 =
    6
>> A2 = heron(1,1,1)
A2 =
    0.4330
```

(c) (4 marks) Write a MATLAB function `[a,b,c] = calculate_length(P,Q,R)` that takes three vertices $P, Q, R \in \mathbb{R}^3$ as input and output the three edge lengths a, b, c of the triangle ΔPQR. Include the code file `calculate_length.m` in your submission.

Solution:

```
function [a,b,c] = calculate_length(P,Q,R)
a = sqrt(sum((Q-R).^2));
b = sqrt(sum((P-R).^2));
c = sqrt(sum((P-Q).^2));
end
```

(d) (3 marks) Based on the above functions, write a MATLAB script `q2.m` to compute the area of the triangle formed by three vertices $(1, 2, 3), (1, 1, 2), (3, 5, 8)$ in $\mathbb{R}^3$. Write down the triangle area obtained in the space provided. Include the code file `q2.m` in your submission.

Solution:

The content of `q2.m`:

```
P = [1,2,3];
Q = [1,1,2];
R = [3,5,8];
[a,b,c] = calculate_length(P,Q,R);
A = heron(a,b,c)
```

Running `q2.m`:

```
>> q2
A =
    1.7321
```

3. As discussed in the lecture, another way of writing MATLAB functions is by creating function handles using `@`.

(a) (3 marks) Create a function handle `f` with

$$f(x) = \frac{1}{1+x}.$$

Write down the values of $f(1)$, $f(f(1))$, $f(f(f(1)))$, $f(f(f(f(1))))$, and $f(f(f(f(f(1)))))$ in the space provided.

Solution:

```
>> f = @(x) 1/(1+(x));
>> f(1)
ans =
    0.5000
>> f(f(1))
ans =
    0.6667
>> f(f(f(1)))
ans =
    0.6000
>> f(f(f(f(1))))
ans =
    0.6250
>> f(f(f(f(f(1)))))
ans =
    0.6154
```

Remark: If we further repeat the process, the answer will converge to $\frac{-1+\sqrt{5}}{2} \approx 0.6180$.

(b) (4 marks) Write a MATLAB script `q3b.m` to do the following:

- Create two function handles `g` and `h` with

$$g(x) = x^2 - x + 1, \qquad h(x) = \sin(x) - \exp(x^2).$$

  Use entrywise operators so that both g and h can take a vector as the input x.

- Generate a vector `v` containing 7 equally spaced points between 0 and 1 (including the endpoints).

- Verify the inequality $g^2 + h^2 \geq 2gh$ using the vector `v` and the relational operator `>=`. Display the resulting logical array.

Write down the resulting logical array in the space provided. Include the code file `q3b.m` in your submission.

Solution:

The content of `q3b.m`:

```
g = @(x) x.^2 - x + 1;
h = @(x) sin(x) - exp(x.^2);
v = linspace(0,10,7);
g(v).^2+h(v).^2 >= 2*g(v).*h(v)
```

Running `q3b.m`:

```
>> q3b
ans =
  1×7 logical array
  1 1 1 1 1 1 1
```

4. Let $a = 1$, $b = 2$, $c = 3$. For each of the following MATLAB commands, write down the answer and explain why the command returns that answer.

   (a) (2 marks) `a == abs(b-c)`

Solution:

```
>> a == abs(b-c)
ans =
  logical
    1
```

The command checks whether a is equal to the absolute value of $b - c$. Since $a = 1$ and $|b - c| = |-1| = 1$, it returns the logical value 1.

   (b) (2 marks) `(b > a) & (b >= c)`

Solution:

```
>>(b > a) & (b >= c)
ans =
  logical
```

```
    0
```

The command checks whether both $b > a$ and $b \geq c$ are true. Since $b = 2 > 1 = a$ and $b = 2 < 3 = c$, it returns the logical value 0.

(c) (2 marks) `(a ~= b) | (b == c)`

Solution:

```
>> (a ~= b) | (b == c)
ans =
  logical
    1
```

The command checks whether a is not equal to b or b is equal to c. Since $a = 1 \neq 2 = b$, it returns the logical value 1.

5. (6 marks) Write a MATLAB script `q5.m` that does the following:

   - It first asks the user to input a number N in the command window. You may use the `input` function as follows (the user will then input the number in the command window and press the Enter key to continue):
     `N = input('Please input a number N: ');`
   - It then checks whether both N and $N + 2$ are prime numbers (you may use the `isprime` function). If both of them are prime numbers, display "N and N+2 are twin primes.".
   - If not both of N and $N + 2$ are prime numbers, check whether N is a prime number. If yes, display "N is a prime but N+2 is not.".
   - Otherwise, display "N is not a prime.".

   Include the code file `q5.m` in your submission. Also, run your script with each of the following inputs and write down the outputs:

   (i) 13

   (ii) 57

   (iii) 9857

Solution:

The content of q5.m:

```
N = input('Please input a number N: ');
if isprime(N) && isprime(N+2)
    disp('N and N+2 are twin primes.');
elseif isprime(N)
    disp('N is a prime but N+2 is not.');
else
    disp('N is not a prime.');
end
```

Running q5.m:

```
>> q5
Please input a number N: 13
N is a prime but N+2 is not.
>> q5
Please input a number N: 57
N is not a prime.
>> q5
Please input a number N: 9857
N and N+2 are twin primes.
```

End of Lab Assignment