

Assignment 1: Debye, Drude, Sommerfeld, and chemistry

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Due: 1700, September 13, 2021

Exercise 1 *Debye* (6 points)

- (i) What are the assumptions of the Debye model?
- (ii) Write an expression for the number of modes in a two-dimensional system, and thus determine the *Debye wavenumber* (the wavenumber which corresponds to the Debye frequency).
- (iii) Provide a brief discussion of which elements you would expect to have a high Debye temperature, and which elements you would expect to have a low Debye temperature.

Note: your reasoning is important, not the actual elements!

Exercise 2 *Drude: mixed ion-electron conductors (MIECs)* (8 points)

MIECs are a fascinating brand of material with the property that both electrons and ions can be charge carriers. You can probably imagine that this would be a desirable property under some situations, for example, in a fuel cell. An example MIEC is SrTiO_3 , the material of which I showed an atomic resolution micrograph in the introductory video.

Within a MIEC, assume that free electrons and ions have a densities $n_{e,i}$, scattering times $\tau_{e,i}$, masses $m_{e,i}$ and charges $-e, +e$ respectively.

- (i) Calculate the electrical resistivity σ
- (ii) Calculate the thermal conductivity κ
- (iii) What is the Weidemann-Franz law? Does it hold in this situation?
- (iv) If we consider a magnetic field applied in the $+z$ direction, we need only consider the conductivity in x and y and can write

$$\rho = \begin{pmatrix} \frac{m}{nq^2\tau} & \frac{Bq}{n} \\ -\frac{Bq}{n} & \frac{m}{nq^2\tau} \end{pmatrix}$$

which is true for both charge carrier species. Use this result to calculate the resistivity matrix in the case of a MIEC.

Exercise 3 *Sommerfeld* (10 points)

- (i) In your own words, explain what is the Fermi energy, Fermi temperature and the Fermi surface
- (ii) Write an expression for the number of states for a gas of free electrons in three dimension and use this to calculate the Fermi wavenumber and Fermi Energy
- (iii) Using the previous result, estimate the value of the Fermi energy for Caesium
- (iv) Obtain an expression for the density of states at the Fermi surface of a **two-dimensional** free-electron gas.

- (v) Using the above result, show that for a two-dimensional free-electron gas that the chemical potential μ is independent of temperature when $T \ll \mu$

Exercise 4 *Chemistry* (6 points)

- (i) Explain using the simplest language you can muster why is there periodic table important?
- (ii) Choose a naturally occurring element with a high atomic number and use Madelung's Rule to deduce the shell filling configuration.

For lols: the highest unique atomic number will be awarded an additional prize

- (iii) Using any tools at your disposal (i.e. use a computer) produce a plot of the energy eigenstate described by $|5, 2, 0\rangle$. You must include your code and you will be partially assessed on presentation: producing content that is digestible and visually pleasing is an important part of modern science!