Physical constants

Quantity	**Symbol**	**Value**	**Unit**	**Relative Standard Uncertainty**
Speed of light in vacuum	c	2.99792458×10^{8}	m/s	exact
Elementary charge	e	$1.602176634 \times 10^{-19}$	C	exact
Planck constant	h	$6.62607015 \times 10^{-34}$	J s	exact
Reduced Planck constant	$\hbar$	$1.0545718 \times 10^{-34}$	J s	1.5×10^{-8}
Electron mass	m_e	$9.10938356 \times 10^{-31}$	kg	5.0×10^{-8}
Proton mass	m_p	$1.672621898 \times 10^{-27}$	kg	2.0×10^{-8}
Fine-structure constant	α	$7.2973525693 \times 10^{-3}$	1	1.5×10^{-10}
Bohr radius	a_0	$5.2917721067 \times 10^{-11}$	m	1.9×10^{-10}
Rydberg constant	R_∞	$1.0973731568508 \times 10^{7}$	m^{-1}	6.6×10^{-12}

Table 1: Physical Constants

Table of Integrals

Integral	Result
$\int e^x \, dx$	$e^x + C$

Table of Polynomial Families

Polynomial Family	Definition
Hermite polynomials	$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$
Laguerre polynomials	$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}(e^{-x} x^n)$
Generalised Laguerre polynomials	$L_n^{(\alpha)}(x) = \frac{d^\alpha}{dx^\alpha} L_n(x) = \sum_{m=0}^{n} (-1)^m \frac{n+\alpha)!}{(\alpha+m)!(n-m)!m!} x^m$
Legendre polynomials	$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell}[(x^2-1)^\ell]$
Associated Legendre polynomials	$P_\ell^m(x) = \frac{1}{2^\ell \ell!} \left(1-x^2\right)^{m/2} \frac{d^{m+\ell}}{dx^{m+\ell}} \left(x^2-1\right)^\ell$

Table 2: Common Polynomial Families

Hydrogenic wavefunctions

$$|n\ell m\rangle = \psi_{nlm}(r,\theta,\phi) = R_{n\ell}(r)Y_\ell^m(\theta,\phi)$$

$$R_{n\ell}(r) = -\left\{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}\right\}^{1/2} e^{-Zr/na_0}\left(\frac{2Zr}{na_0}\right)^\ell L_{n+\ell}^{2\ell+1}(2Zr/na_0)$$

$$Y_\ell^m(\theta,\phi) = (-1)^{(m+|m|)/2}\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-|m|)!}{(\ell+|m|)!}}P_\ell^m(\cos\theta)e^{im\phi}$$

ℓ	m	$Y_\ell^m(\theta,\phi)$
0	0	$\frac{1}{\sqrt{4\pi}}$
1	0	$\sqrt{\frac{3}{4\pi}}\cos\theta$
1	±1	$\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$
2	0	$\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$
2	±1	$\mp\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta e^{\pm i\phi}$
2	±2	$\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$
3	0	$\sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta-3\cos\theta\right)$
3	±1	$\mp\sqrt{\frac{21}{64\pi}}\sin\theta(5\cos^2\theta-1)e^{\pm i\phi}$
3	±2	$\sqrt{\frac{105}{32\pi}}\sin^2\theta\cos\theta e^{\pm 2i\phi}$
3	±3	$\mp\sqrt{\frac{35}{64\pi}}\sin^3\theta e^{\pm 3i\phi}$

Table 3: Angular wavefunctions

n	ℓ	$R_{n,l}(r)$
1	0	$2\left(\frac{Z}{a_0}\right)^{3/2}e^{-Zr/a_0}$
2	0	$2\left(\frac{Z}{2a_0}\right)^{3/2}\left(1-\frac{Zr}{2a_0}\right)e^{-Zr/2a_0}$
2	1	$\frac{1}{\sqrt{3}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)e^{-Zr/2a_0}\cos\theta$
3	0	$2\left(\frac{Z}{3a_0}\right)^{3/2}\left(1-\frac{2Zr}{3a_0}+\frac{2}{27}\left(\frac{Zr}{a_0}\right)^2\right)e^{-Zr/3a_0}$
3	1	$\frac{4\sqrt{2}}{9}\left(\frac{Z}{3a_0}\right)^{3/2}\frac{Zr}{a_0}\left(1-\left(\frac{Zr}{6a_0}\right)\right)e^{-Zr/3a_0}$
3	2	$\frac{2\sqrt{2}}{27\sqrt{5}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}$

Table 4: Radial wavefunctions

Quantum mechanical formulae

$$E_n^{(1)} = \left\langle n\left|H'\right|n\right\rangle$$

$$E_n^{(2)} = \sum_{n\neq m}\frac{\left|\left\langle n^{(0)}\left|H'\right|m^{(0)}\right\rangle\right|^2}{\left(E_n^{(0)}-E_m^{(0)}\right)}$$

$$E_0 \leq \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}$$

$$J_\pm = J_x \pm iJ_y$$

$$J_\pm|j\,m_j\rangle = \hbar\left[j(j+1)-m_j(m_j\pm1)\right]^{1/2}|j\,m_j\pm1\rangle$$

n	ℓ	m	$\psi_{n,\ell,m}(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}\left(\frac{Z}{2a_0}\right)^{3/2}\left(1-\frac{Zr}{2a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\frac{1}{2\sqrt{\pi}}\left(\frac{Z}{2a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right) e^{-Zr/2a_0}\cos\theta$
2	1	±1	$\mp\frac{1}{2\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right) e^{-Zr/2a_0}\sin\theta e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(1-\frac{2Zr}{3a_0}+\frac{2}{27}\left(\frac{Zr}{a_0}\right)^2\right) e^{-Zr/3a_0}$
3	1	0	$\frac{2\sqrt{2}}{3\sqrt{3\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\frac{Zr}{a_0}\left(1-\left(\frac{Zr}{6a_0}\right)\right) e^{-Zr/3a_0}\cos\theta$
3	1	±1	$\mp\frac{2}{3\sqrt{3\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\frac{Zr}{a_0}\left(1-\left(\frac{Zr}{6a_0}\right)\right) e^{-Zr/3a_0}\sin\theta e^{\pm i\phi}$
3	2	0	$\frac{1}{27\sqrt{2\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}(3\cos^2\theta-1)$
3	2	±1	$\mp\frac{\sqrt{3}}{27\sqrt{\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}\sin\theta\cos\theta e^{\pm i\phi}$
3	2	±2	$\frac{\sqrt{3}}{54\sqrt{\pi}}\left(\frac{Z}{3a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}\sin^2\theta e^{\pm 2i\phi}$

Table 5: Complete wavefunctions of hydrogenic atoms