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KYA322

Atomic physics

Andy McCulloch

NMR apparatus play a crucial role in structure determination

Transient effects

Foot Ch. 7 // Steck Ch. 4, 5

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Learning outcomes

Week 5, lecture 1

Foot §7.3 – 7.5 // Steck §5.2, 5.4, 5.5:

Time evolution of the two-level atom

- Time dependent solutions of the optical Bloch equations
  - Rabi oscillations
- Bloch sphere
  - Geometric interpretation of states

The optical Bloch equations (slide 1 of 1)

The optical Bloch equations in the rotating wave approximation are

$$\dot{\rho}_{11} = \frac{i\Omega}{2}\,(\sigma_{21} - \sigma_{12}) + \Gamma\rho_{22}$$
$$\dot{\rho}_{22} = -\dot{\rho}_{11} = -\frac{i\Omega}{2}\,(\sigma_{21} - \sigma_{12}) - \Gamma\rho_{22}$$
$$\dot{\sigma}_{21} = i(\omega - \omega_0)\sigma_{21} - \frac{i\Omega}{2}\,(\rho_{22} - \rho_{11}) - \gamma_\perp\sigma_{21}$$
$$\dot{\sigma}_{12} = \dot{\sigma}_{21}^{*} = -i(\omega - \omega_0)\sigma_{12} + \frac{i\Omega}{2}\,(\rho_{22} - \rho_{11}) - \gamma_\perp\sigma_{12}$$

where the density matrix elements are related to the slowly-rotating components

$$\rho_{21}(t) = \sigma_{21}(t)e^{-i\omega t}$$
$$\rho_{12}(t) = \rho_{21}^{*}(t) = \sigma_{12}(t)e^{i\omega t}, \qquad \text{with} \quad \sigma_{21} = \sigma_{12}^{*}$$

and the Rabi frequency is

$$\boldsymbol{d}\cdot\boldsymbol{E}_0 = \hbar\Omega$$

Transient transience

All effects have thus far have been *steady-state effects*, that is, with no time dependence. We now turn to *transient* effects.

From the OBEs, we have:

$$\dot{\rho}_{22} = -\dot{\rho}_{11} = -\frac{i\Omega}{2}(\sigma_{21} - \sigma_{12})$$

$$\dot{\sigma}_{12} = \dot{\sigma}_{21}^{*} = -i\Delta\sigma_{21} + \frac{i\Omega}{2}(\rho_{22} + \rho_{11})$$

How to solve these equations?

Ultimately looks a bit like the differential equation for an exponential ⇒ look for a solution of the form

$$\sigma_{ij}(t) = \sigma_{ij}(0)e^{\lambda t}$$

We are using a notational convenience: here we are simply defining

$$\sigma_{11} \equiv \rho_{11}$$
$$\sigma_{22} \equiv \rho_{22}$$

Crank the handle

Putting the trial solution into the OBEs yields

$$\begin{pmatrix} -\lambda & 0 & -\frac{i\Omega}{2} & \frac{i\Omega}{2} \\ 0 & -\lambda & \frac{i\Omega}{2} & -\frac{i\Omega}{2} \\ -\frac{i\Omega}{2} & \frac{i\Omega}{2} & -\lambda & -\Delta \\ \frac{i\Omega}{2} & -\frac{i\Omega}{2} & \Delta & -\lambda \end{pmatrix} \begin{pmatrix} \rho_{11}(0) \\ \rho_{22}(0) \\ \sigma_{12}(0) \\ \sigma_{21}(0) \end{pmatrix} = 0$$

which in turn yields

$$\lambda^2(\lambda^2 + \Delta^2 + \Omega^2) = 0$$

$$\Rightarrow \lambda = 0, \pm i\Omega' \text{ where } \Omega'^2 \equiv \Omega^2 + \Delta^2$$

The solution to the density matrix elements will then be a sum of these solutions

$$\sigma_{ij}(t) = \sigma_{ij}^{(0)} + \sigma_{ij}^{(i\Omega')} e^{i\Omega' t} + \sigma_{ij}^{(-i\Omega')} e^{-i\Omega' t}$$

Time to get cranking

Ω' is the *generalised Rabi frequency* and is used as the dynamics of a system with detuning Δ are the same as a system with no detuning and a Rabi frequency $\Omega = \Omega'$

Rabi oscillations

Assuming that the excited state population is initially zero, the transient excited state population is given by

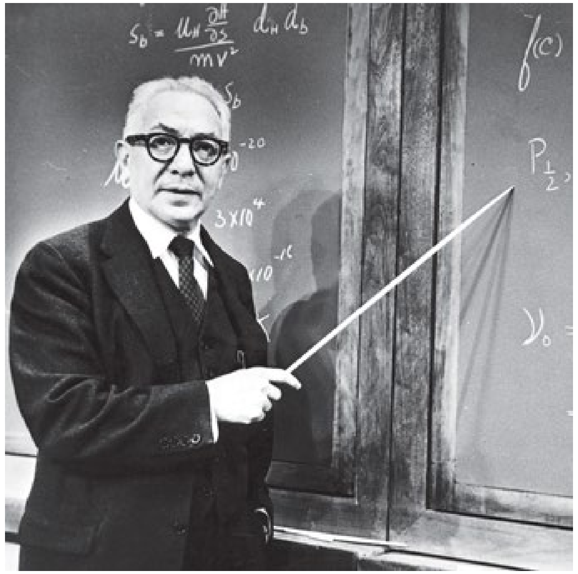
$$\boxed{\rho_{22}(t) = \left(\frac{\Omega}{\Omega'}\right)^2 \sin^2\left(\frac{\Omega' t}{2}\right)}$$

Looking at case of resonance ($\Delta = 0$):

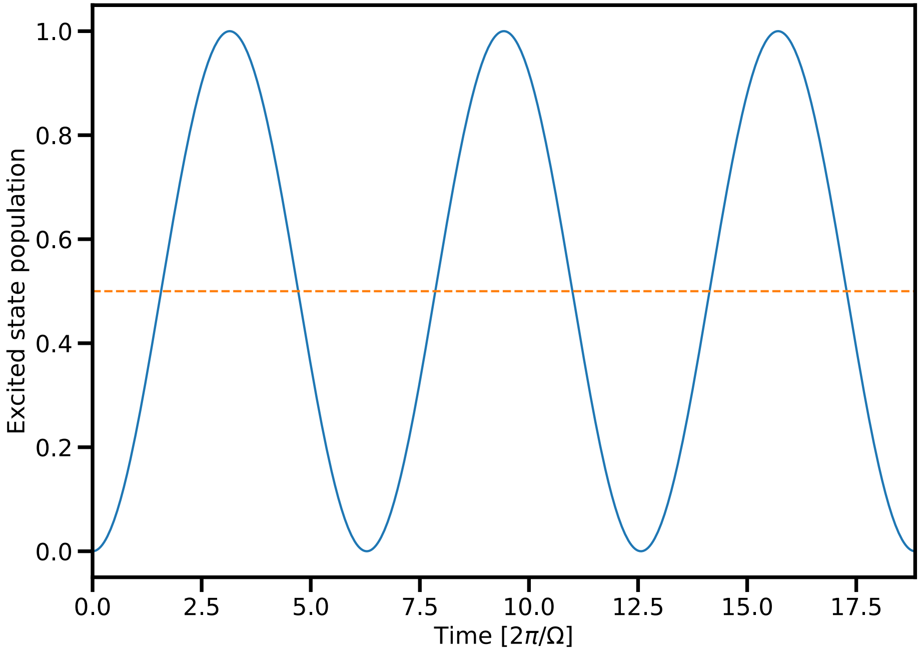
$$\rho_{22}(t) = \sin^2\left(\frac{\Omega t}{2}\right)$$

This should strike you as odd for a number of reasons:

- A coupling should drive atomic excitation, shouldn't it?
- This suggests that I can put a system into a state whereby I can guarantee that excited-state population is 100%
- I don't recall seeing atomic systems undergoing oscillations indefinitely



Isidor Isaac Rabi ...



... and his oscillations: Rabi oscillations

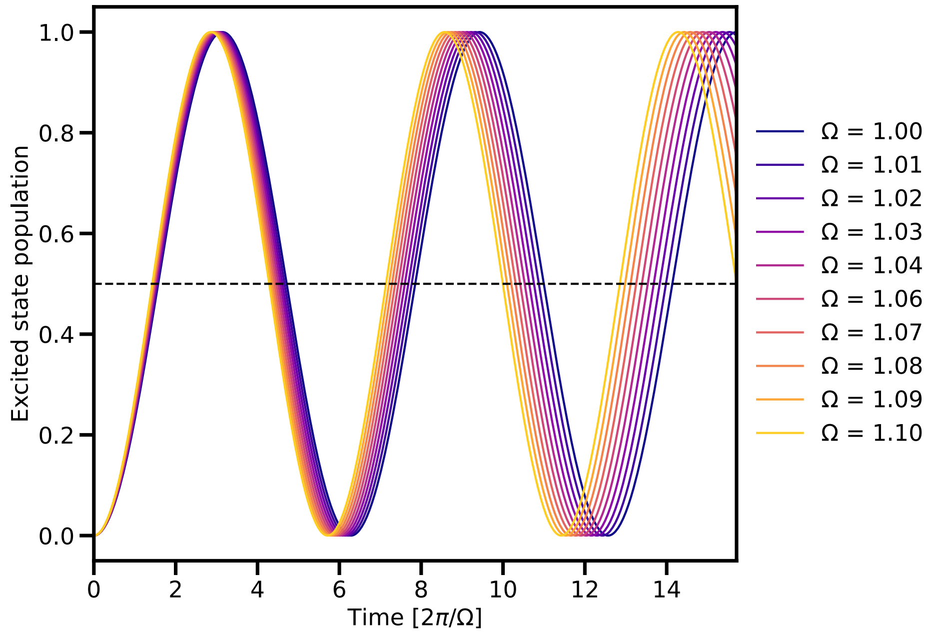
Tuning the oscillations

Implications of the relation

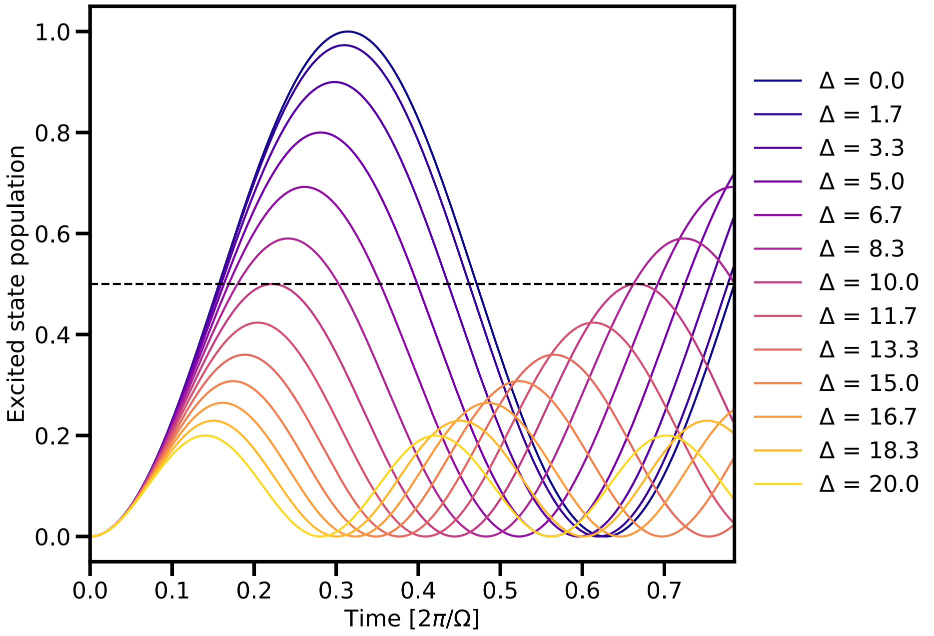
$$\rho_{22}(t) = \left(\frac{\Omega}{\Omega'}\right)^2 \sin^2\left(\frac{\Omega' t}{2}\right)$$

We can control the excited state population, and its evolution!

- Changing the coupling between the atoms and field (Ω) changes the rate at which oscillations occur
- Introducing a detuning (Δ) reduces the efficiency of excitation and induces faster oscillations

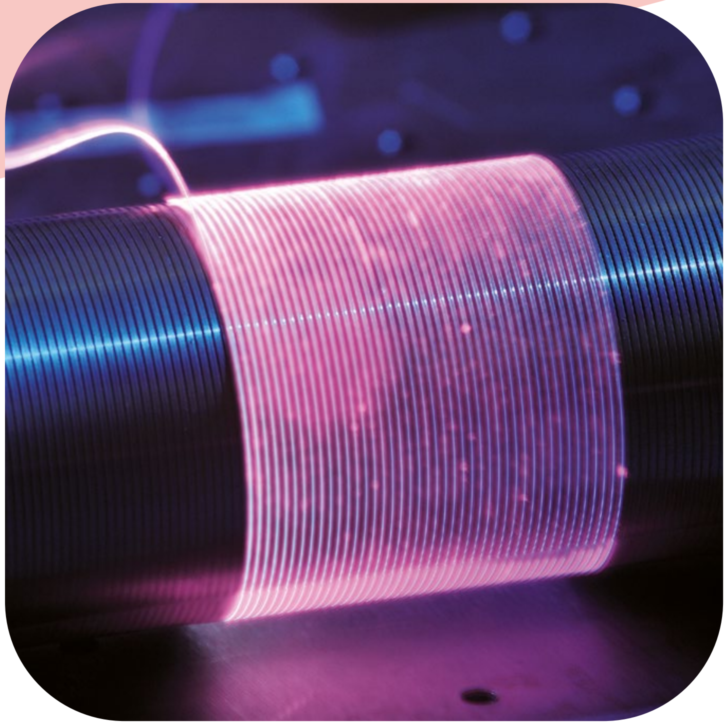


Rabi oscillations for different coupling strengths

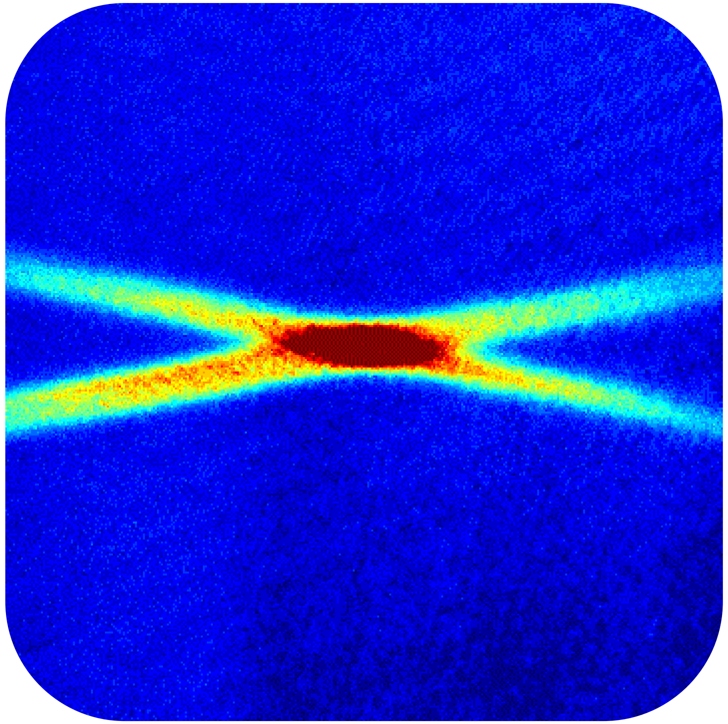


Rabi oscillations for different detunings

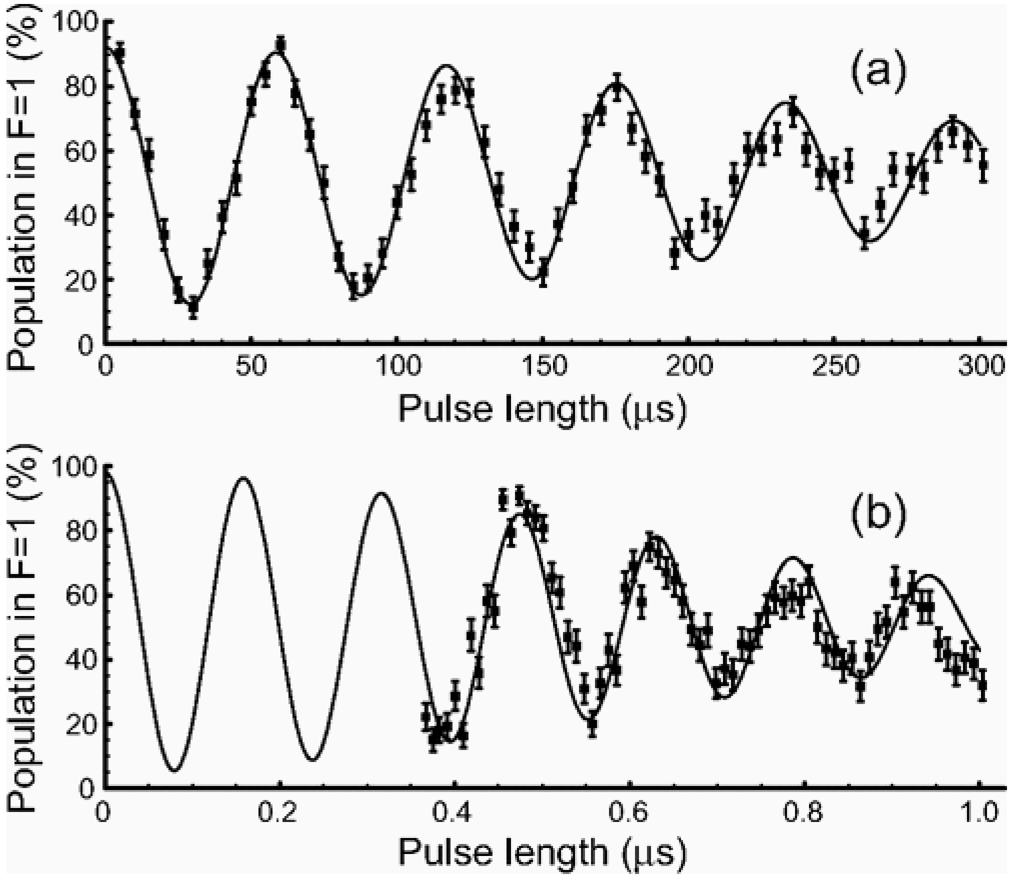
Oscillations of the population don't actually occur, right?



Fibre lasers can be extremely powerful



One can trap (single) atoms in an optical trap



Rabi oscillations of a single atom

Fast quantum state control of a single trapped neutral atom

The cycle continues, well, at least for some time

Rabi oscillations are an inherently quantum phenomenon, and it stands to reason they would not continue indefinitely.

In constructing the optical Bloch equations, we phenomenologically baked in relaxation via

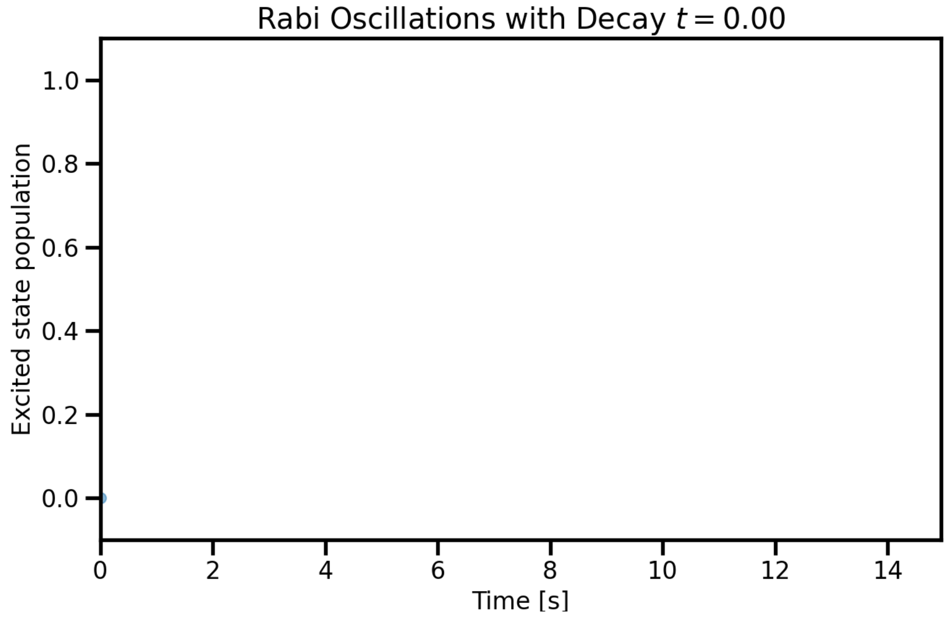
$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}(t), \hat{\rho}\right] + \hat{\mathcal{L}}_{\mathrm{relax}}$$

Explicitly for a two-level atom, we have

$$\dot{\rho}_{22} = -\Gamma \rho_{22}$$
$$\dot{\sigma}_{12} = -\gamma_{\perp} \sigma_{12}$$

The question is how does this work for a single atom?

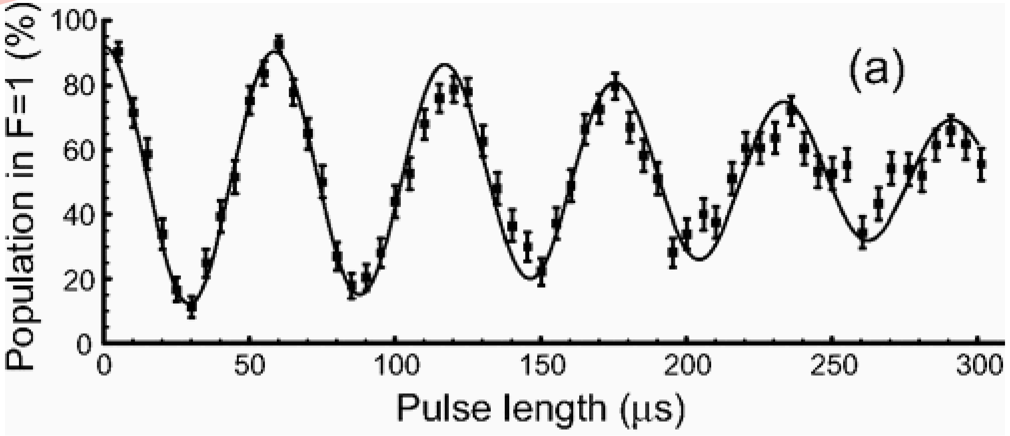
- Introduce Poisson decay process
- Exponential decay with rate parameter Γ



The evolution of a single atom with decay

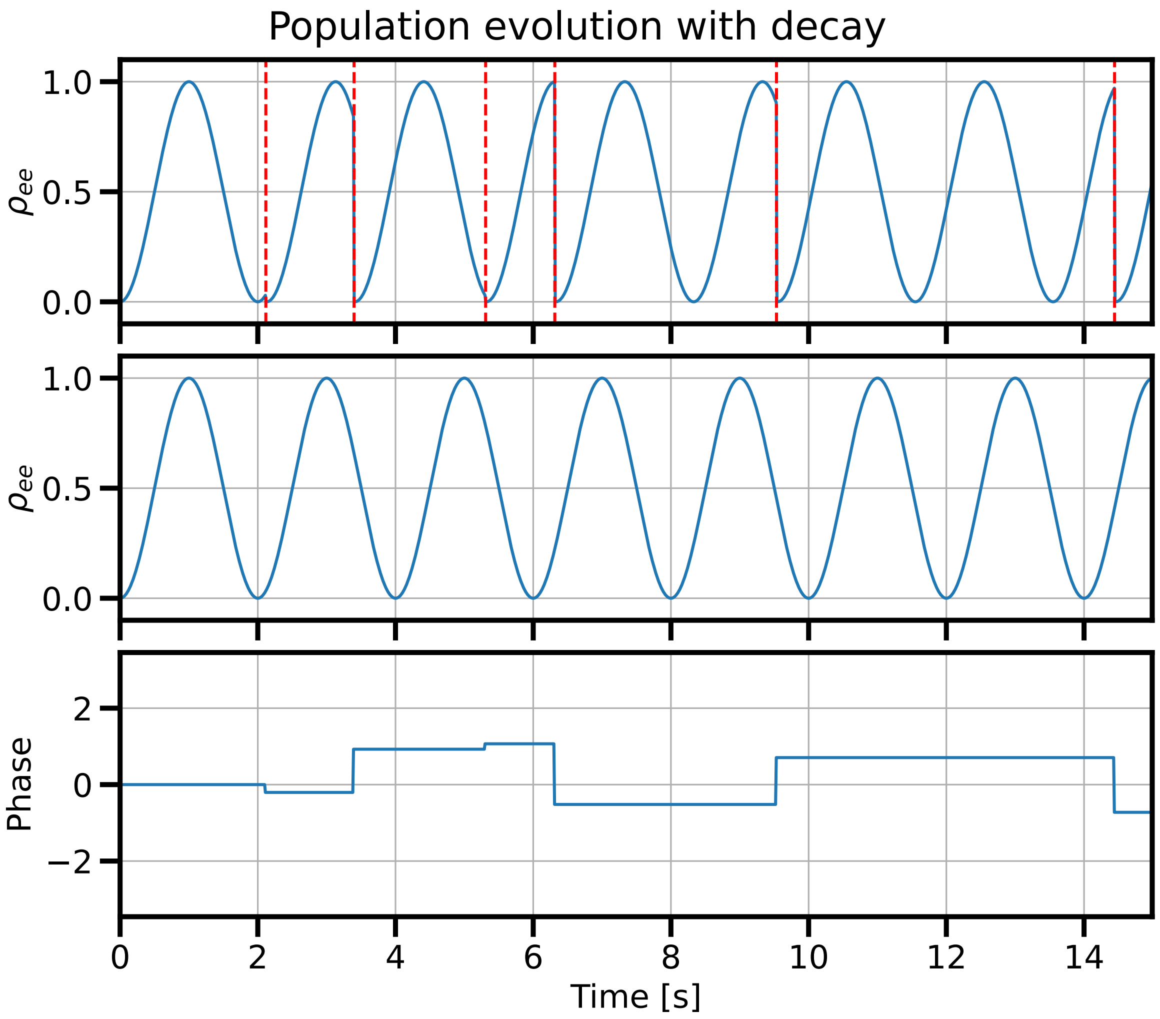
The ensemble evolves

But why do the oscillations look smooth?



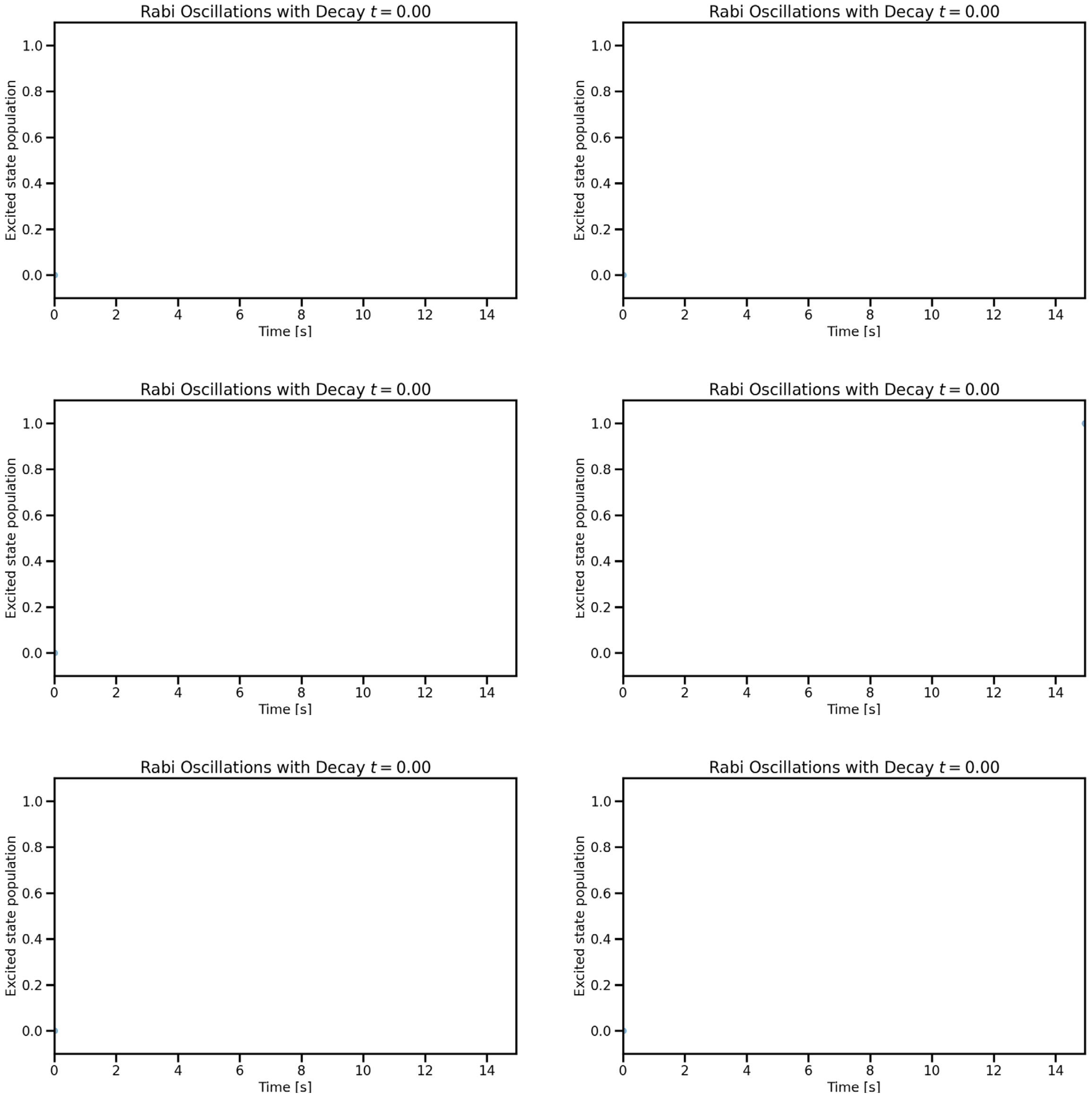
Rabi oscillations of Rubidium

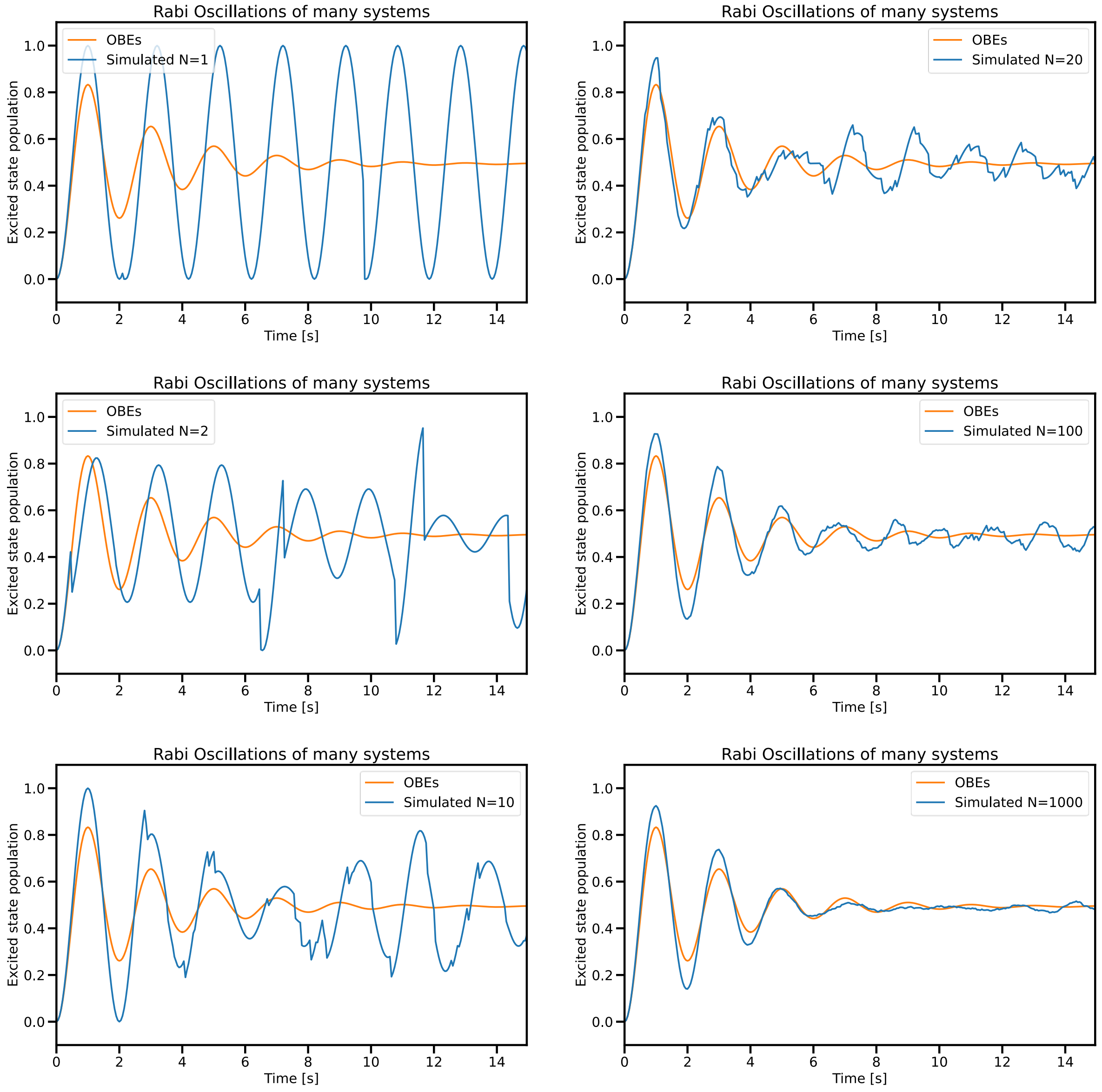
The evolution of a single atom may have discontinuities, but in an ensemble, a decay is equivalent to dephasing relative to the initial Rabi oscillation

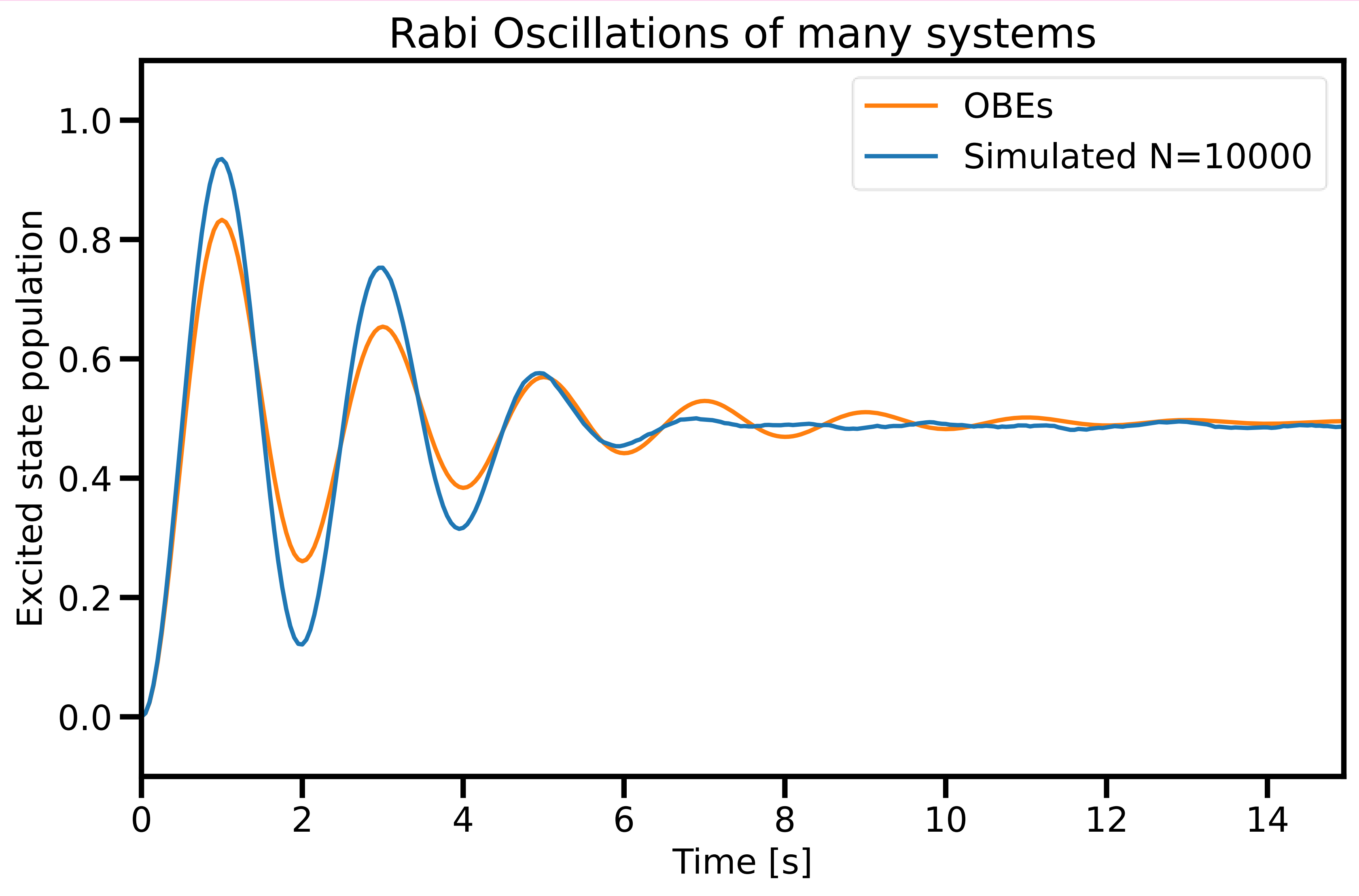


Atoms with and without decay, and associated phase shift

Compare the pair



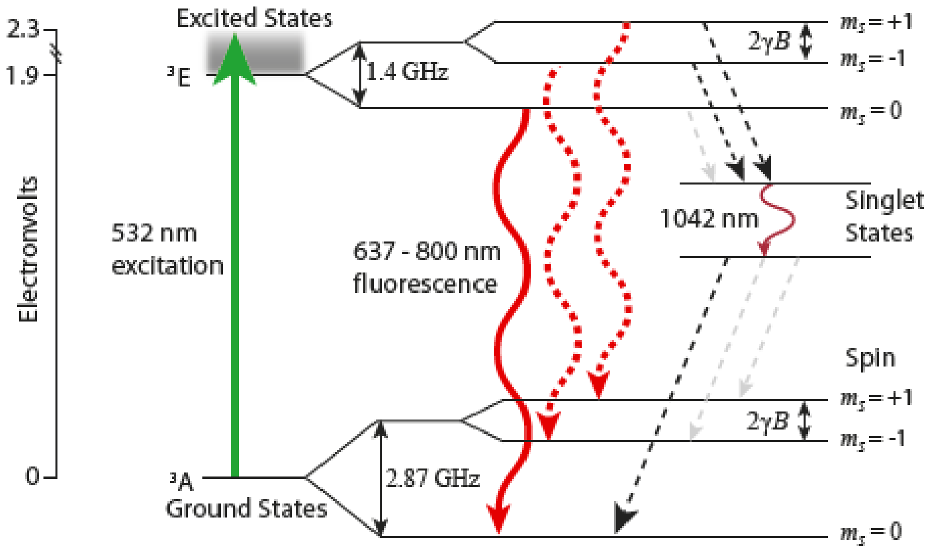



Rabi Oscillations of many systems
OBEs
Simulated N=10000
Excited state population
Time [s]
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10
12
14

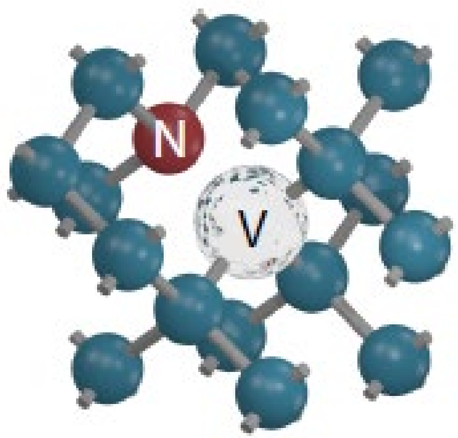
Does this mean my Rabi cycles are doomed?

Decay makes the quantum go away: what can we do?

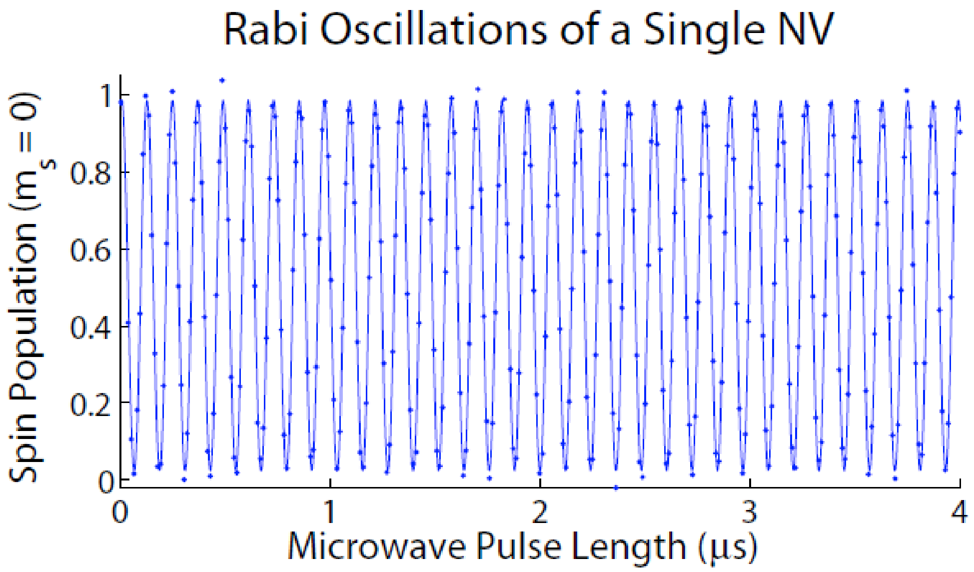
- Small linewidth/long lifetime
- Good environment



Energy-level diagram of the nitrogen vacancy centre in diamond



The structure of nitrogen vacancy centres



Rabi oscillations of a nitrogen vacancy centre in diamond: very little decay

Dephasing

The time to *dephase* – lose coherence between states – is characterised by $T_2 = 1/\gamma_\perp$.

Whereas $T_1 = 1/\Gamma$ is a property of the atom, T_2 is greatly influenced by the environment

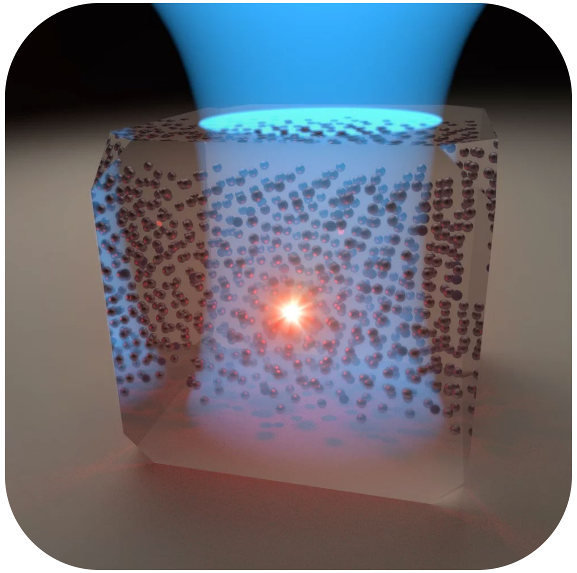
- Magnetic field fluctuations and inhomogeneities
- Temperature changes
- Interactions with neighbouring spins

⇒ Make stuff small, use big fields, cool stuff down, find "magic" materials

E.g. Praseodymium-doped Yttrium Orthosilicate (Pr^{3+}:Y_2SiO_5) has coherence times on the order of many hours!

Efficient quantum memory for light

Yttrium orthosilicate



Single praseodymium ions in a yttrium orthosilicate have extremely long T_2 times

The problem

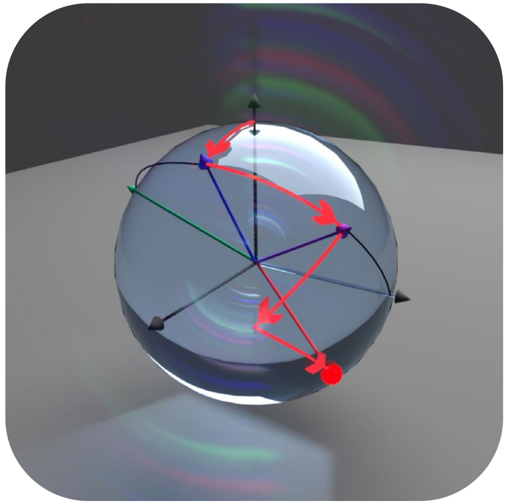
To this point, we have really only looked at following the excited- (and ground-) state population.

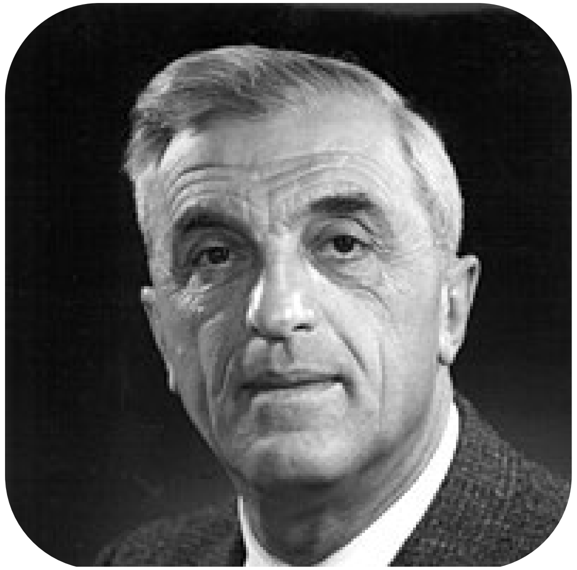
The symbol to summon Bloch

We have (complex-valued) coherences which determine properties of the system, and we want to visualise how the system evolves. How can we possibly do this?

The answer:

The Bloch sphere





Felix Bloch

To be expected

$$\sigma = |2\rangle\langle 1|, \qquad \rho = \begin{pmatrix} \rho_{11} & \rho_{21} \\ \rho_{12} & \rho_{22} \end{pmatrix}$$

If we compute expectation values:

$$\langle \sigma \rangle = \mathrm{tr}(\sigma\rho) = \sigma_{21}$$
$$\langle \sigma^\dagger \rangle = \sigma_{12}$$
$$\langle \sigma^\dagger \sigma \rangle = \rho_{22}$$
$$\langle \sigma \sigma^\dagger \rangle = \rho_{11}$$

Then

$$\langle \sigma_x \rangle = \langle \sigma \rangle + \langle \sigma^\dagger \rangle = \sigma_{21} + \sigma_{12}$$
$$\langle \sigma_y \rangle = i(\sigma_{21} - \sigma_{12})$$
$$\langle \sigma_z \rangle = (\rho_{22} - \rho_{11})$$

Optical Bloch equations:

$$\dot{\rho}_{22} = -\dot{\rho}_{11} = -\frac{i\Omega}{2}(\sigma_{21} - \sigma_{12})$$
$$\dot{\sigma}_{12} = \dot{\sigma}_{21}^* = -i\Delta\sigma_{21} + \frac{i\Omega}{2}(\rho_{22} + \rho_{11})$$

$$\frac{d\langle \sigma_x \rangle}{dt} = \dot{\sigma}_{21} + \dot{\sigma}_{12}$$
$$= i\Delta(\sigma_{21} - \sigma_{12}) - i\Omega(\rho_{22} - \rho_{11})$$
$$= \Delta\langle \sigma_y \rangle - i\Omega\langle \sigma_z \rangle$$

$$\frac{d\langle \sigma_y \rangle}{dt} = -\Delta\langle \sigma_x \rangle$$
$$\frac{d\langle \sigma_z \rangle}{dt} = -\Omega\langle \sigma_y \rangle$$

Measuring up

We are close... All that remains is to look at the length of the expectation value of σ:

$$|\langle\sigma\rangle^2| = \langle\sigma_x\rangle^2 + \langle\sigma_y\rangle^2 + \langle\sigma_x\rangle^2$$

$$= (\sigma_{21} + \sigma_{12})^2 - (\sigma_{21} - \sigma_{12})^2 + (\rho_{22} - \rho_{11})^2$$

$$= 4\sigma_{21}\sigma_{12} + \rho_{22}^2 + \rho_{11}^2 - 2\rho_{22}\rho_{11}$$

$$= (\rho_{22} + \rho_{11})^2 = 1$$

We assume here that we have a pure state; in this case:

$$\boxed{\rho_{11}\rho_{22} = \sigma_{21}\sigma_{12}}$$

In the case of a mixed state, this value will be less than one, and for a completely mixed state, it will be zero.

Z is for…

So, we have defined an operator, with associated vector

$$(\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_Z\rangle)$$

Why!?

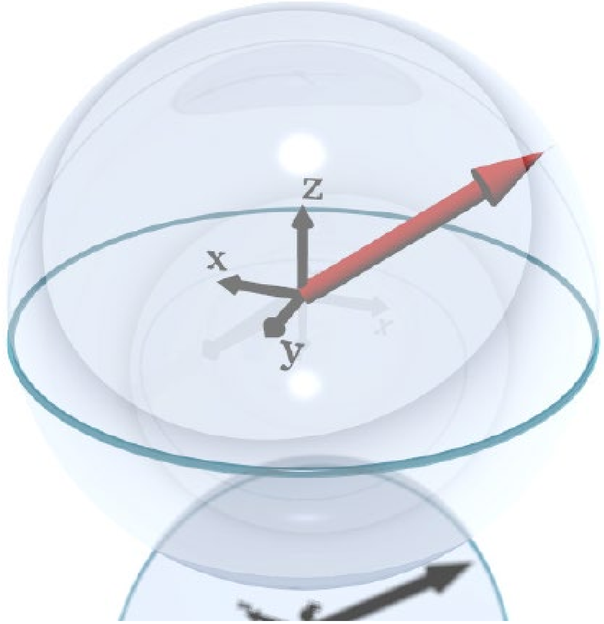
The component $\langle\sigma_z\rangle = (\rho_{22} - \rho_{11})$ quantifies the excited state population

Poles indicate system is either in state $|1\rangle$ (south) or $|2\rangle$ (north)

Note: in other conventions, the poles are reversed

$$\sigma = |2\rangle\langle 1|$$

$$\langle\sigma_x\rangle = \sigma_{21} + \sigma_{12}$$
$$\langle\sigma_y\rangle = i(\sigma_{21} - \sigma_{12})$$
$$\langle\sigma_z\rangle = (\rho_{22} - \rho_{11})$$



Enter the Bloch sphere: visualising states

X and Y are for…

Recall the dipole operator, which couples states $|1\rangle$ and $|2\rangle$, is

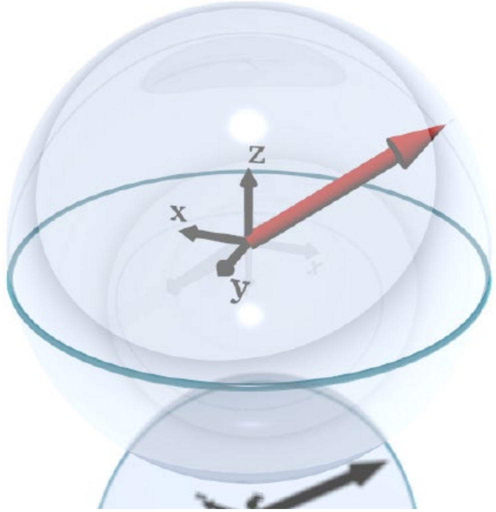
$$d = \begin{pmatrix} 0 & d_{12} \\ d_{21} & 0 \end{pmatrix}$$

The expectation value $\langle d \rangle$ is then

$$\begin{aligned} \mathrm{tr}(d\rho) &= d_{12}(\rho_{12} + \rho_{21}) \\ &= d_{12}\left(\sigma_{12}e^{i\omega t} + \sigma_{21}e^{-i\omega t}\right) \\ &= d_{12}[(\sigma_{12} + \sigma_{21})\cos(\omega t) + i(\sigma_{12} - \sigma_{21})\sin(\omega t)] \\ &= d_{12}\left(\underbrace{\langle\sigma_x\rangle\cos(\omega t)}_{\text{In phase}} - \underbrace{\langle\sigma_y\rangle\sin(\omega t)}_{\text{In quadrature}}\right) \end{aligned}$$

We can decompose a sinusoid into two components: one *in phase* with $\sin(x)$, and one *in quadrature*, that is, exactly out of phase with $\sin(x)$, or in phase with $\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$

$$\sin(x + \phi) = \sin(x)\cos(\phi) + \sin\left(x + \frac{\pi}{2}\right)\sin(\phi)$$



The Bloch sphere contains much information

Meaning of the Bloch vector

The Bloch vector is defined as $\mathbf{r} = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$, where the components have the physical interpretations:

$\langle\sigma_z\rangle \equiv w$ is the degree of atomic excitation

$\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\end{pmatrix} \equiv \begin{pmatrix}u\\ v\end{pmatrix}$ are the components of $\langle\hat{d}\rangle$ which oscillate in phase and quadrature with the coupling field.

What does that mean?
That there will only be a dipole moment when the atom is a superposition of $|1\rangle$ and $|2\rangle$

$$\chi = \frac{2nd_{12}^2}{\varepsilon_0\hbar\Omega}\sigma_{12}$$

We have seen this before in the case of the susceptibility

The density matrix can be constructed from the Bloch vector via

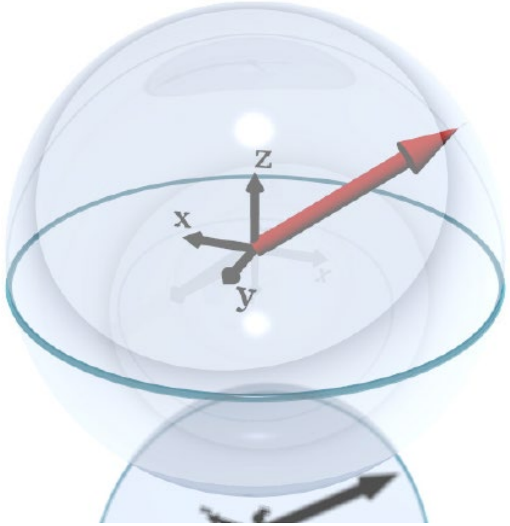
$$\rho = \frac{1}{2}(\mathbf{1} + \mathbf{r}\cdot\boldsymbol{\sigma})$$

u component: oscillates in phase with the coupling field, represents one part of the coherence between states $|1\rangle$ and $|2\rangle$

v component: oscillates in quadrature with the coupling field, represents the other part of the coherence.

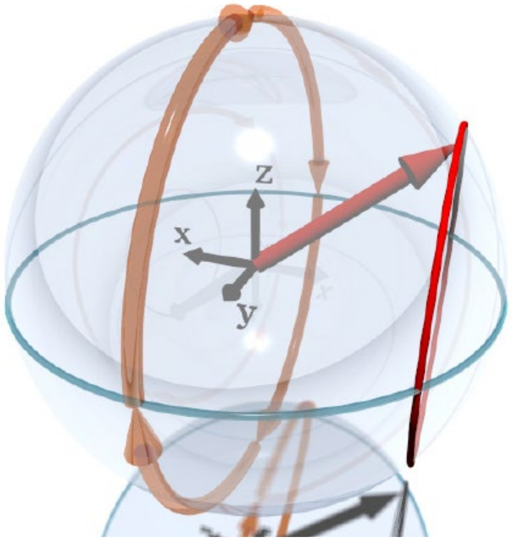
Implication: there is a dipole moment when the atom is in a superposition of states $|1\rangle$ and $|2\rangle$ because the coherence (u and v) allows the atom to interact with the coupling field and exhibit a dipole moment.

States and operations on the Bloch sphere

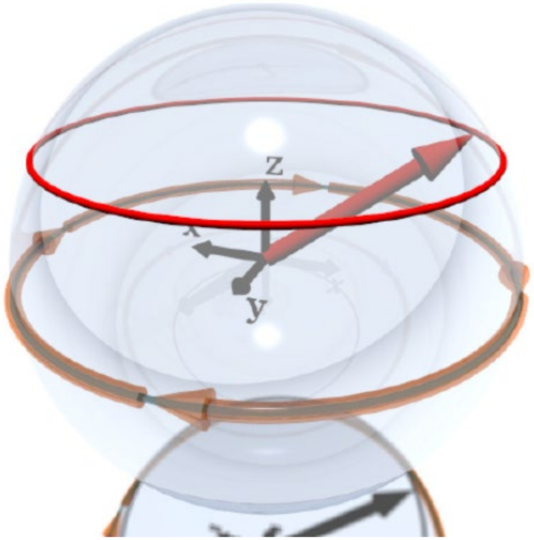


General state

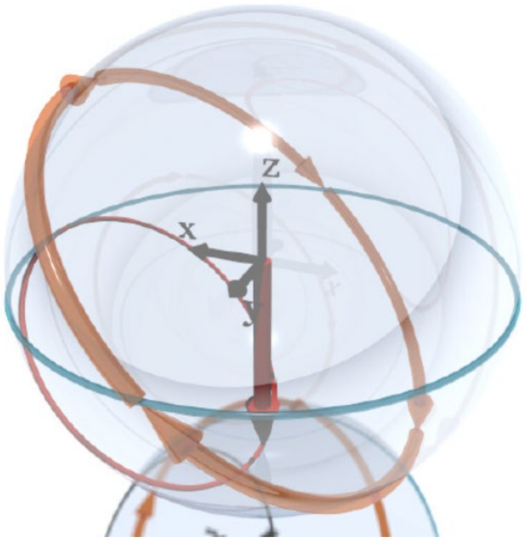
Point not on the surface of the sphere? Mixed



On-resonant excitation



Free evolution



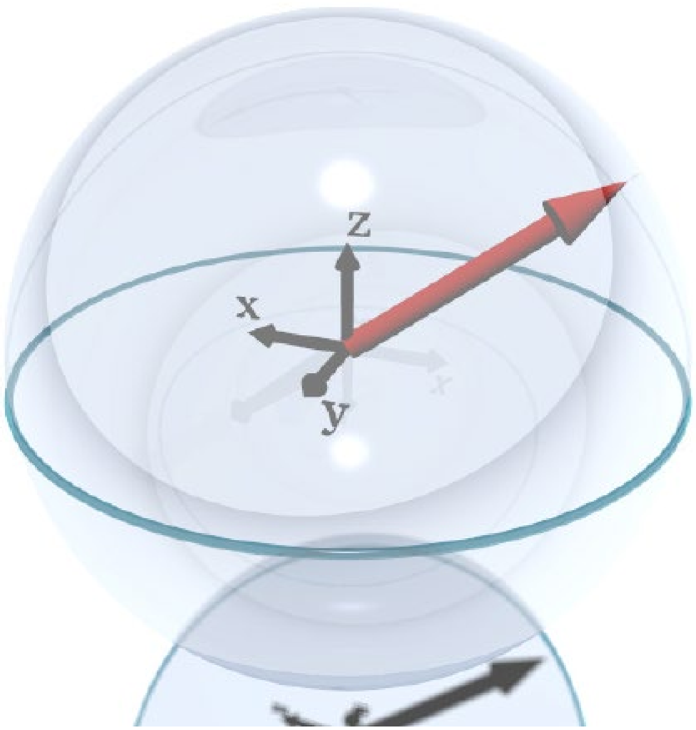
Off-resonant excitation

Bloch says, relax

Recall that in the absence of a coupling field:

$$\dot{\rho}_{22} = -\Gamma\rho_{22}$$
$$\dot{\sigma}_{12} = -\gamma_{\perp}\sigma_{12}$$
$$\Gamma = \frac{1}{T_1}, \gamma_{\perp} = \frac{1}{T_2}$$

What does this look like on the Bloch sphere?



Cheat sheet

Fundamental quantities

Name	Symbol	Unit
Linewidth	Γ	Hertz
Transverse relaxation rate	$\gamma_\perp$	Hertz
Resonant frequency	ω_0	rad/s

Derived quantities

Name	Symbol	Definition
Detuning	Δ	$\omega - \omega_0$
Rabi frequency	Ω	$\hbar\Omega = \boldsymbol{d} \cdot \boldsymbol{E}_0$
Generalised Rabi frequency	Ω'	${\Omega'}^2 = \Omega^2 + \Delta^2$
Saturation parameter	s	$\dfrac{\Omega^2/\gamma_\perp\Gamma}{1 + (\Delta/\gamma_\perp)^2}$
T_1 time	T_1	$1/\Gamma$
T_2 time	T_2	$1/\gamma_\perp$

The Greek alphabet attacks!

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Summary

Problems

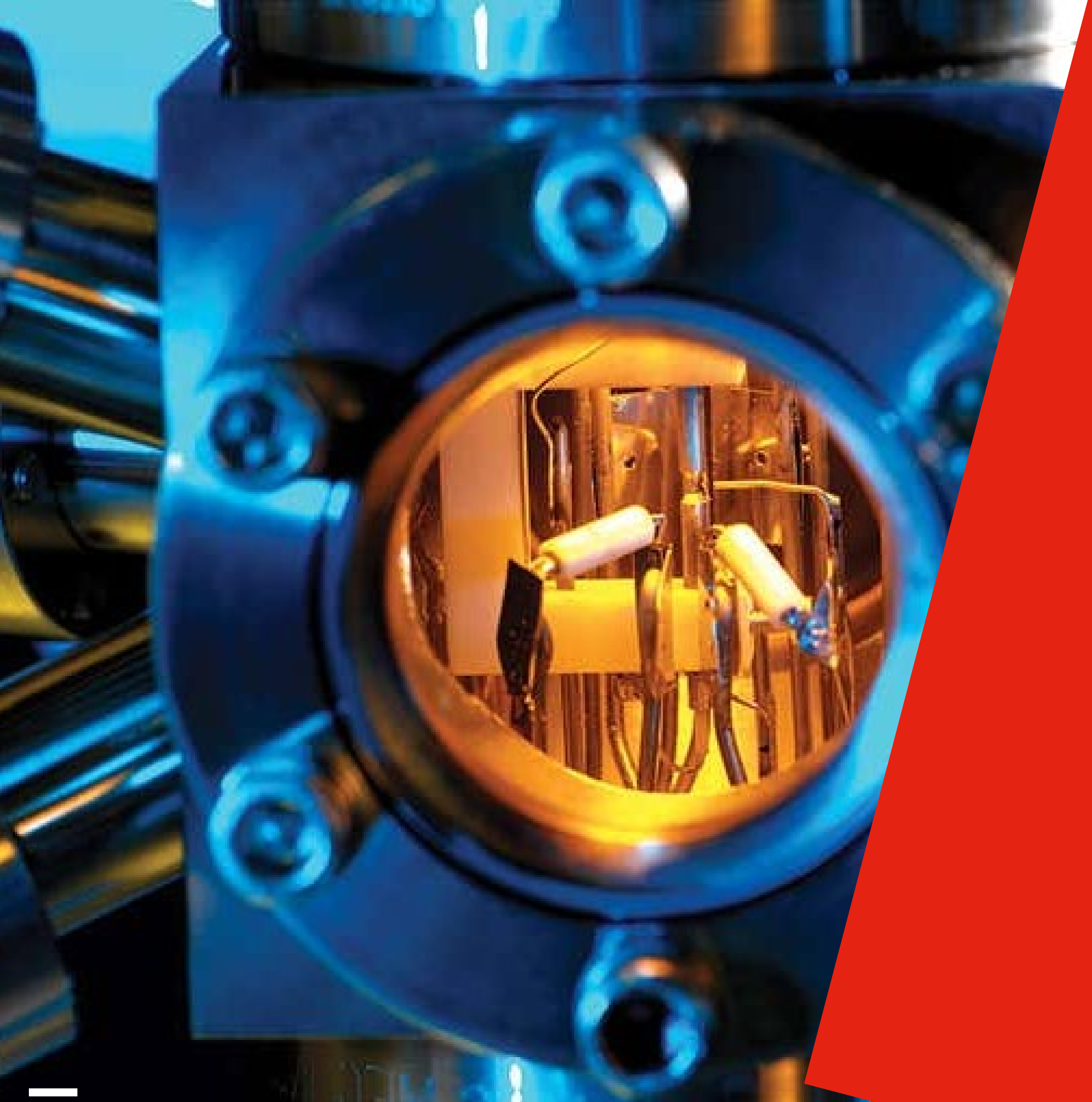
- Time-dependent solutions to the optical Bloch equations
  - Probabilities evolve in time sinusoidally in the absence of decay, e.g.

$$\rho_{22}(t) = \left(\frac{\Omega}{\Omega'}\right)^2 \sin^2\left(\frac{\Omega' t}{2}\right)$$

  - Rate and excitation probability altered by coupling strength and detuning from resonance
  - Pulsing the coupling can deterministically shift atomic populations

- Decay and dephasing
  - Need to understand the link between individual atomic behaviour and ensemble behaviour
  - Decay and interaction leads to a loss of coherence, quantum behaviour dissipates

- The Bloch sphere
  - Provides a way to visualise states and their evolution



A strontium lattice clock is the most accurate timepiece

Atomic timekeeping

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Learning outcomes

Week 5, lecture 1

Foot §7.4 // Steck §5.4 :

Atomic timekeeping

- Frequency measurement
  - How do I accurately measure frequency?
  - Understanding atomic evolution on the Bloch sphere
  - Experimental considerations for measuring frequency
  - Why do we care about frequency standards?

Frequency is king

The fundamental superpower of atomic physics is to understand and predict the structure of atoms

The most fundamental thing we care about is energy:

- Energy splitting
- Emitted photons
- Absorbed photons

Whilst we can always talk in energy, it is natural to work in units of frequency: we measure the frequency of light which interacts with atoms

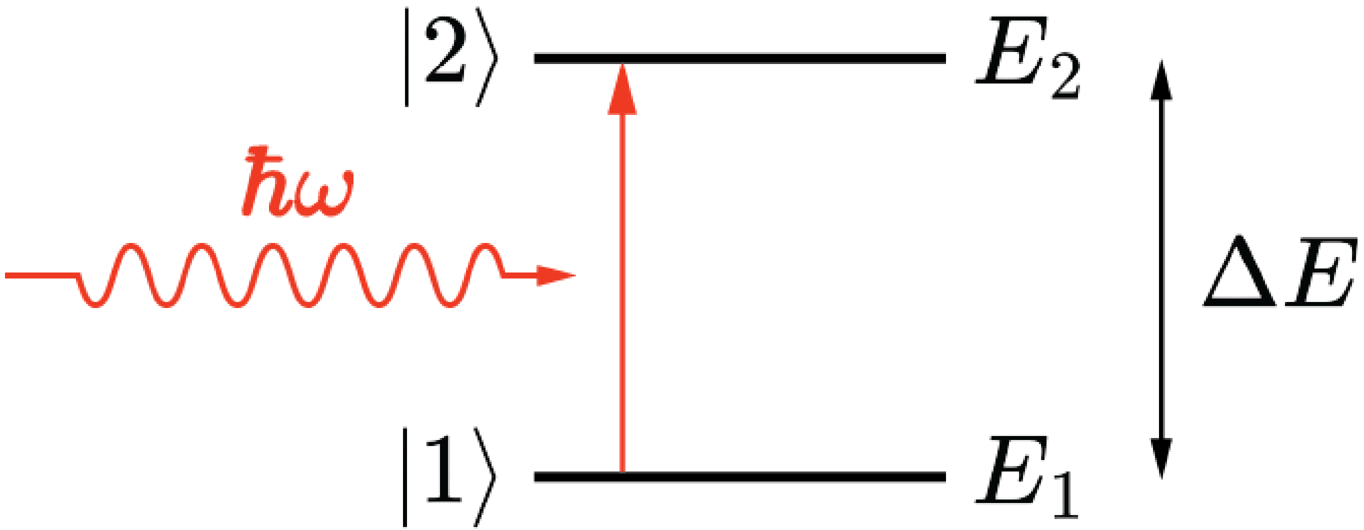
Our goal: to understand how we can both accurately and precisely measure frequency, and then what we can do with such measurements

Just measure it, right?

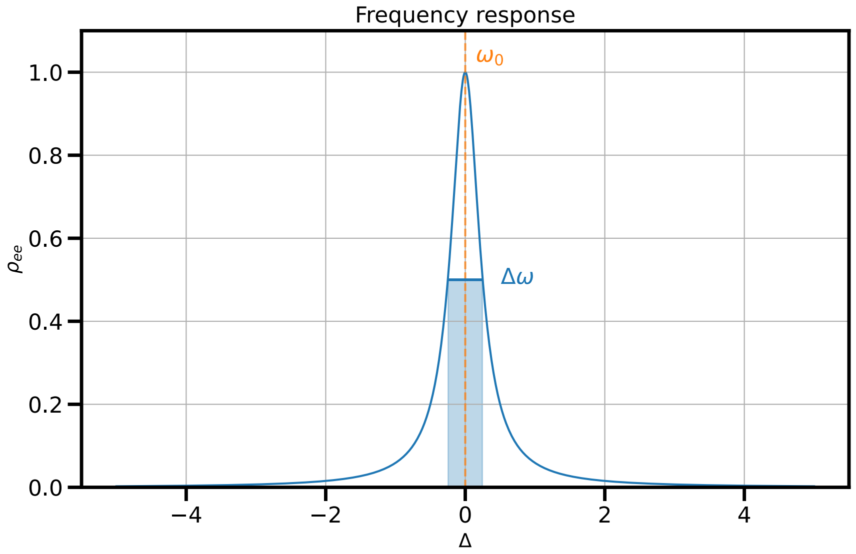
What is it that we want to measure?

Fundamentally, we want to measure the energy splitting between 2 states

In order to get a precise measurement, we require a sharp resonance



A two-level atom with monochromatic radiation



The frequency response of an atomic resonance

But how do we actually make measurements?

Recall that

$$\Delta E = \hbar A_{21} = \frac{\hbar}{\tau} = \hbar\Gamma = \hbar\Delta\omega$$

Caesium 101

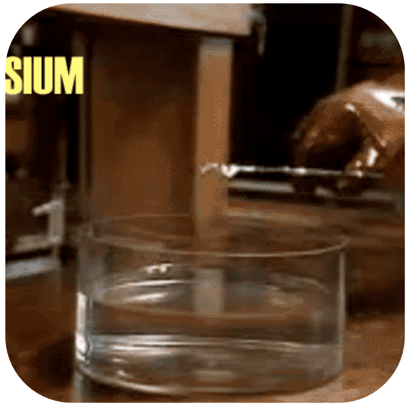
Group I element ($Z = 55$) $\Rightarrow$ single valence shell electron: $6s$

Discovered by Kirchhoff and Bunsen in 1860, named from the Latin *caesius* (blueish grey)

Spectroscopy power couple Gustav Kirchhoff and Robert Bunsen

Caesium flame

Also, goes (more) boom:



Caesium in water

Caesium and Rubidium

The Rabi method

To measure an energy splitting $\Delta E = \hbar\omega_0$, we need a source of atoms

High temperatures lead to Doppler broadening, making the resonance peak broader $\Rightarrow$ worse measurement of ω_0

How to get "cold" atoms?

- Many methods to cool beams atoms, but common to use an atomic beam.

Given an atomic beam with velocity v and interaction region of length L with our coupling field (i.e. laser), the optical Bloch equations give

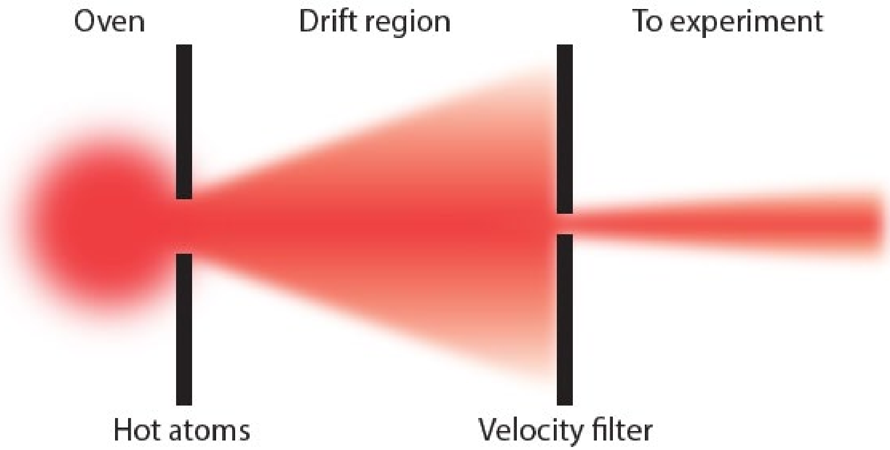
$$\rho_{22} = \frac{1}{1+\left(\frac{\Delta}{\Omega}\right)^2}\sin^2\left(\frac{L}{v}\frac{\sqrt{\Omega^2+\Delta^2}}{2}\right)$$

For maximum excitation ($\rho_{22} = 1$) $\Rightarrow \Omega'\left(\frac{L}{v}\right) = \pi$

Due to saturation, width of the transition goes as Ω

$\Rightarrow$ want low v, large L, and correspondingly small Ω

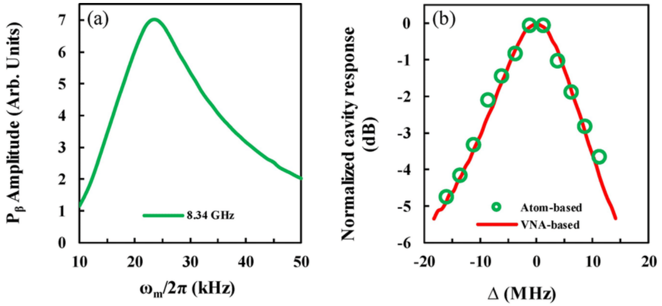
At atomic beam: cook and go



Velocity filter cooling, aka atoms in the bin

The Rabi apparatus

1. The beam is spin separated
2. Passed through a highly-uniform static field with a weak coupling field
3. The beam is spin separated

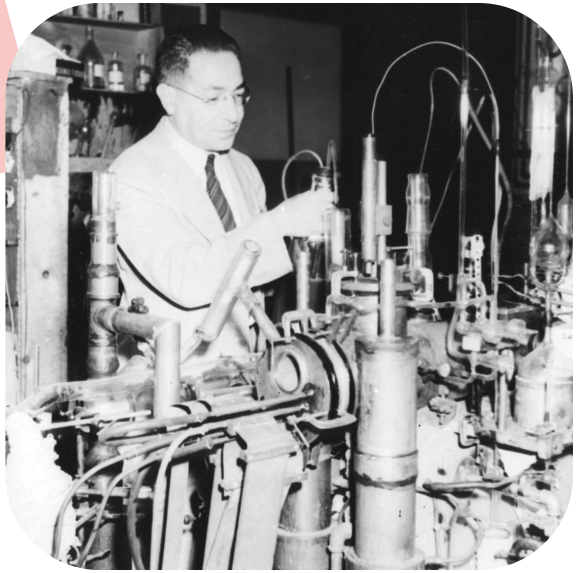


A Rabi resonance measurement

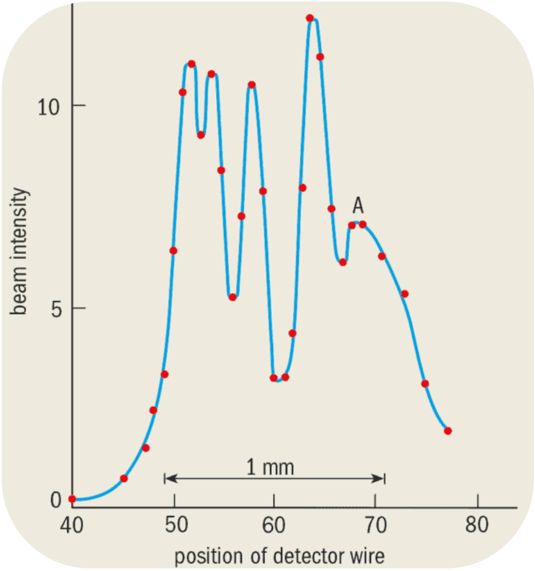
Simple method to directly measure the transition frequency

Allows for the characterisation of the strength of interactions between the atom and external field

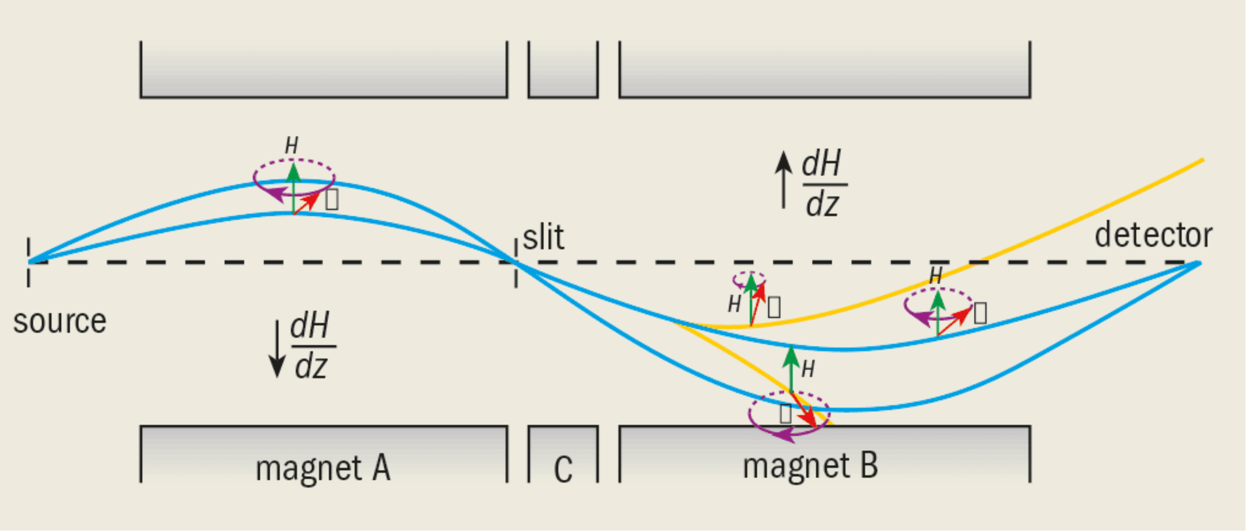
The method is limited by the linewidth of the resonance



Rabi and his apparatus



Measurement of Na, showing $I = 3/2$



A schematic of the apparatus

The Ramsey method

Ramsey's method is an expansion on Rabi's method

1. Prepare atoms in a defined state (e.g. $|0\rangle$)
2. Apply a $\pi/2$-pulse
3. Wait some time T
4. Apply another $\pi/2$-pulse
5. Measure the atomic state

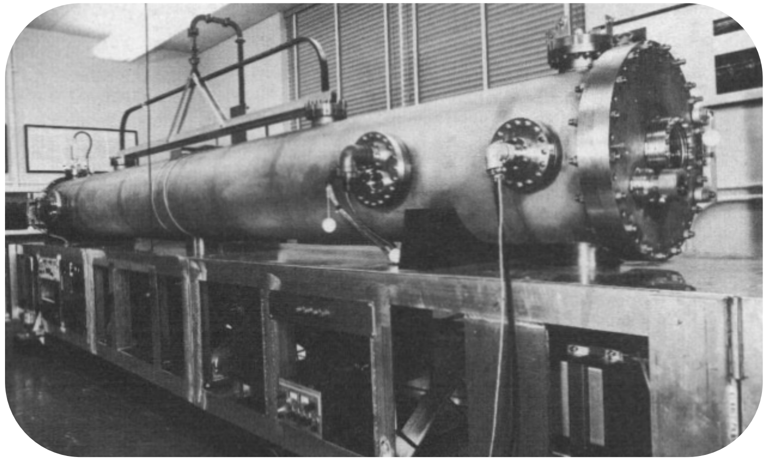
By varying T and measuring ρ_{22}, we *drastically* improve the accuracy of the measurement of ω_0.

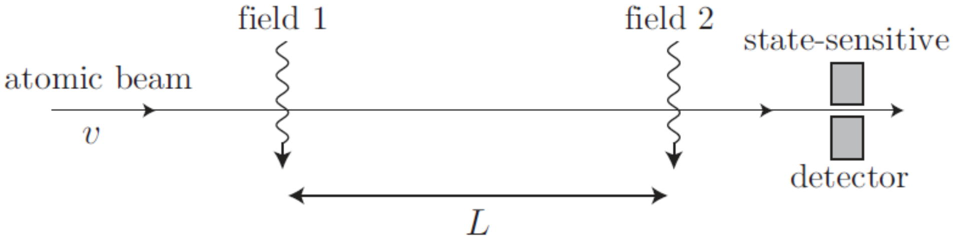
Discussion

Why might this be?

Primer: what does the $\pi/2$ pulse do? What will happen during the free evolution? What will the next $\pi/2$ pulse do?

Norman Ramsey

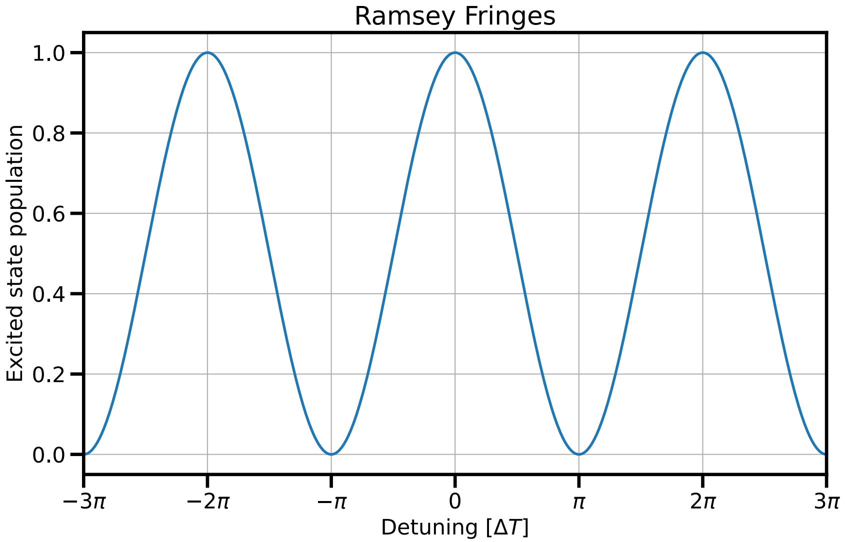




Ramsey apparatus and schematic

Ramsey fringes

The precession (phase accumulation) leads to a variation in output signal (excited state population) which is sinusoidal in T $(= L/v)$, with period $2\pi/\Delta$

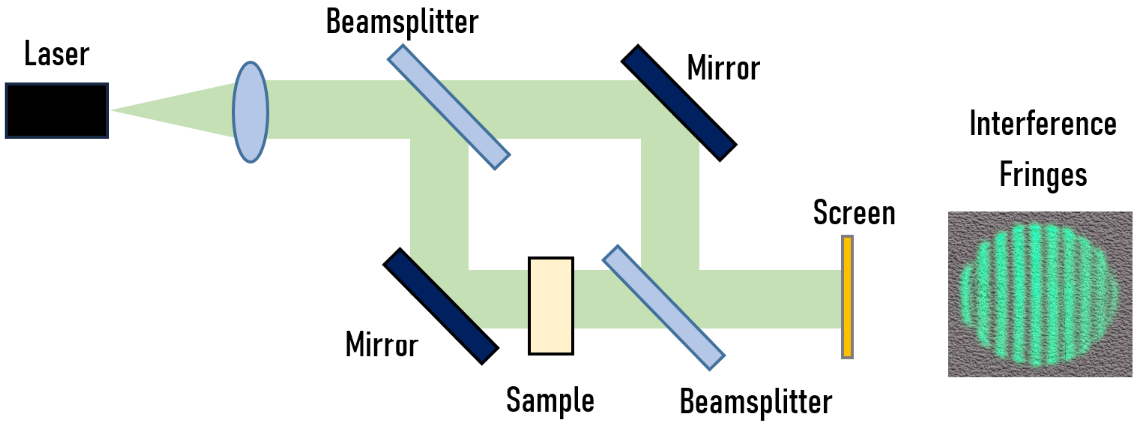


A Ramsey interferometer signal when $\Delta \ll \Omega$

To extract ω_0, vary Δ and fit to ρ_{22}

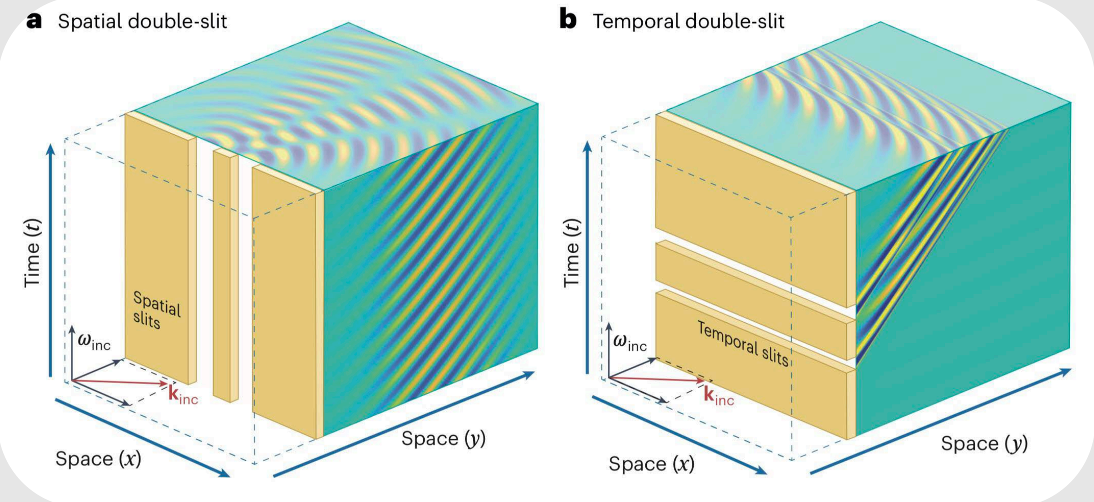
$$\rho_{22} = \cos^2\left(\frac{\Delta T}{2}\right)$$

Effectively, we have built an (Mach–Zehnder) interferometer, but the arms of the interferometer are the internal states of the atom, and the beamsplitters correspond to the $\pi/2$-pulses.



A (Mach-Zehnder) interferometer

Equivalently, we can think Young's double slit experiment, but with slits are separated in time, hence we observe fringes in frequency



The OG tick tockers

If we are fitting a sinusoid, surely, we want lots of fringes?

$\Rightarrow \Delta$ will get big, and the approximation $\Delta \ll \Omega$

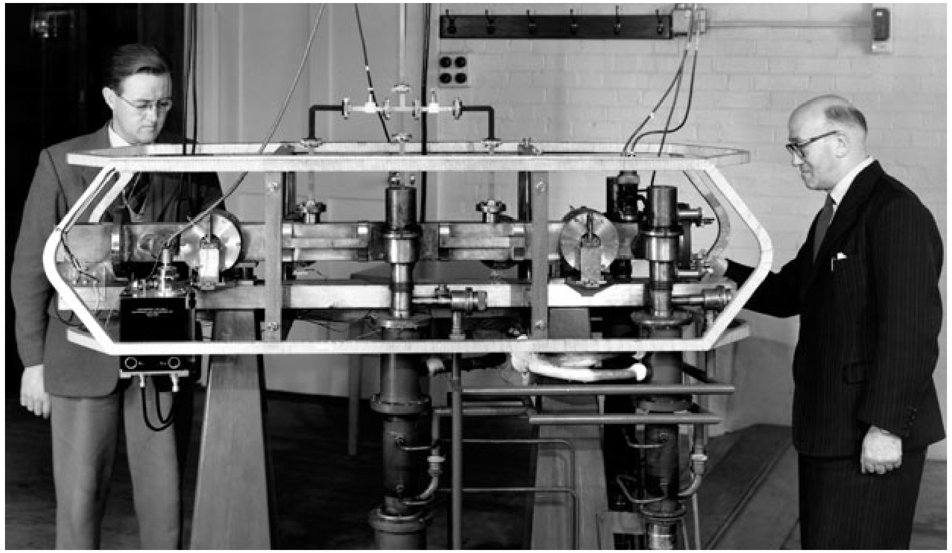
The full solution for the excited state population for a general Δ is

$$\rho_{22} = 4\left(\frac{\Omega}{\Omega'}\right)\sin^2\left(\frac{\Omega'\tau}{2}\right)\left[\cos\left(\frac{\Delta T}{2}\right)\cos\left(\frac{\Omega'\tau}{2}\right) - \frac{\Delta}{\Omega'}\sin\left(\frac{\Delta T}{2}\right)\sin\left(\frac{\Omega'\tau}{2}\right)\right]^2$$

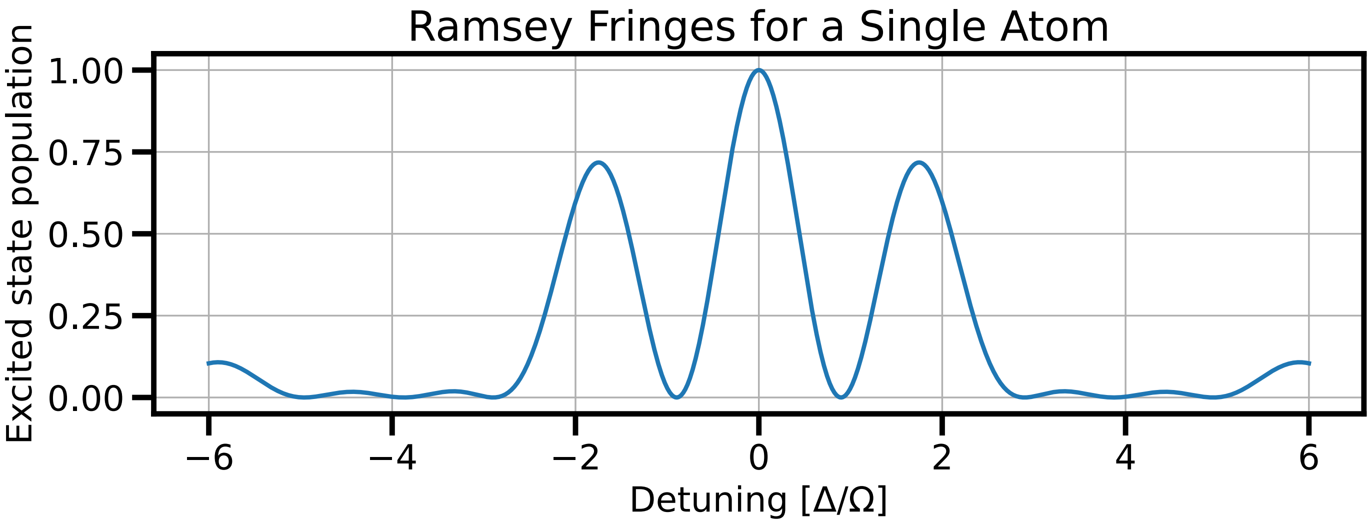
where

- τ is the interaction time of the $\pi/2$ pulses of Rabi frequency Ω ($\tau = \ell/v$)
- T is the free evolution time ($T = L/v$)

Since 1967, "*The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.*"



The first caesium atomic clock (National Physical Laboratory, London)



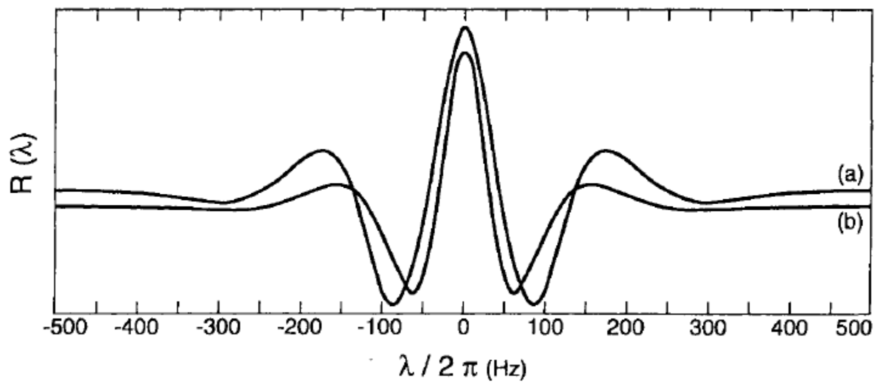
Theoretical Ramsey fringes in all their glory

Well measured

To obtain the best measurement possible, we want

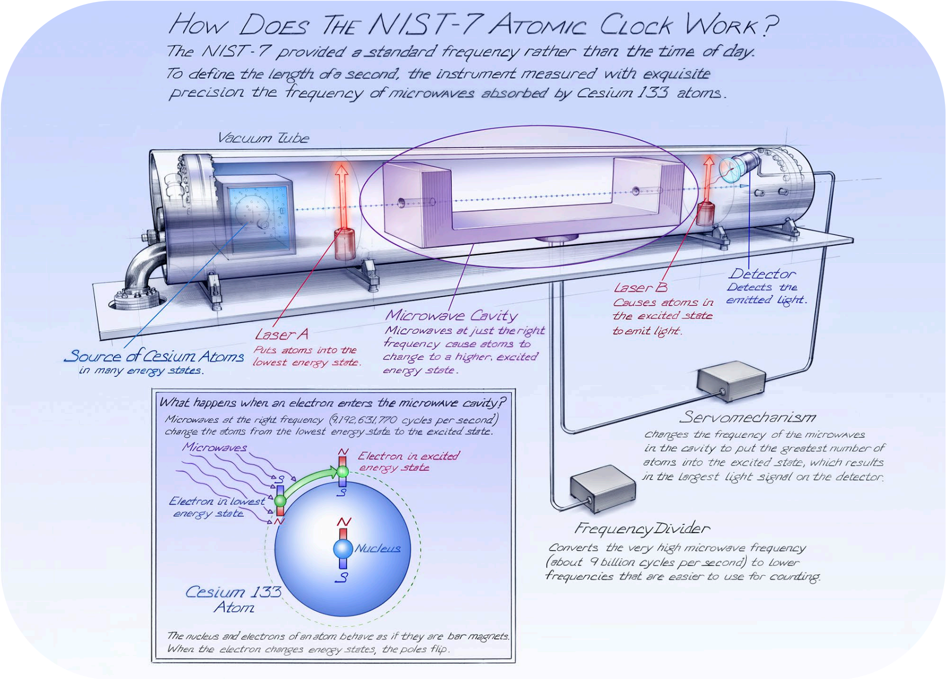
- Small ℓ (reduce inhomogeneities, minimise velocity effects, reduced systematics, a well-defined $T \Rightarrow$ better fringe contrast)
- Large L (more phase accumulated $\Rightarrow$ narrower fringes)
- Small Ω (AC stark shift)

The NIST-7 clock has $L = 1.53\ \mathrm{m}$, $\ell = 2.3\ \mathrm{cm}$ and $\langle v \rangle = 230\ \mathrm{m/s}$, and had an uncertainty of 5×10^{-15}



Ramsey fringes from NIST-7

The NIST-7, the "best" clock from 1993-1999



A schematic of the NIST-7

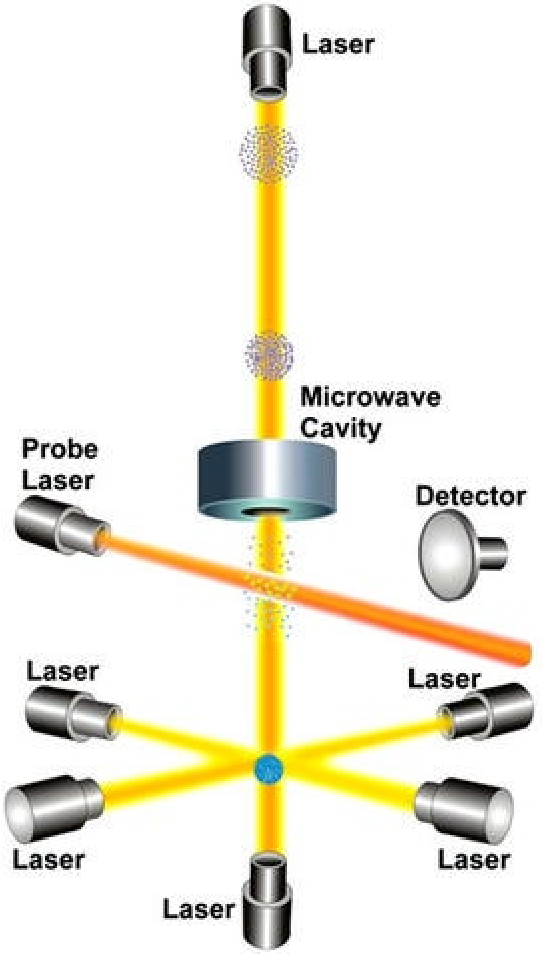
Can we do better?

- Small ℓ: hard to optimise
- Large L: why not $2L$?
- Small Ω: Hard to optimise

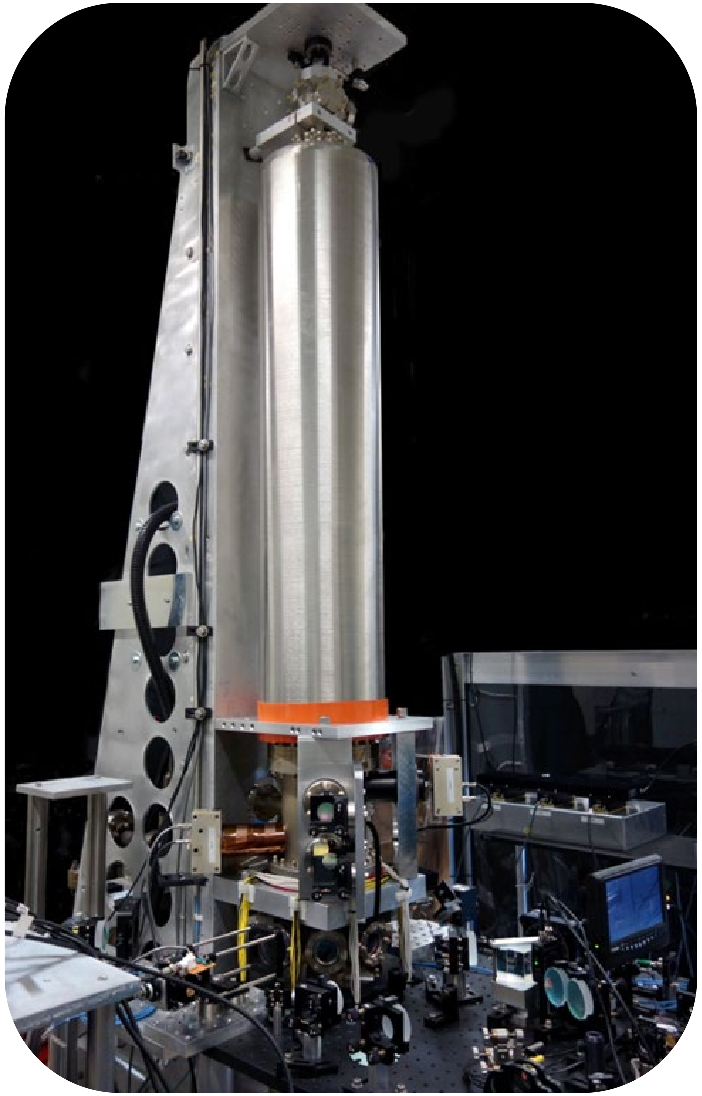
Anything else?

- Reduce v

Atomic-fountain clocks use laser cooling to make cold atoms which are then launched vertically, with the $\pi/2$-pulse performed by the *same* cavity

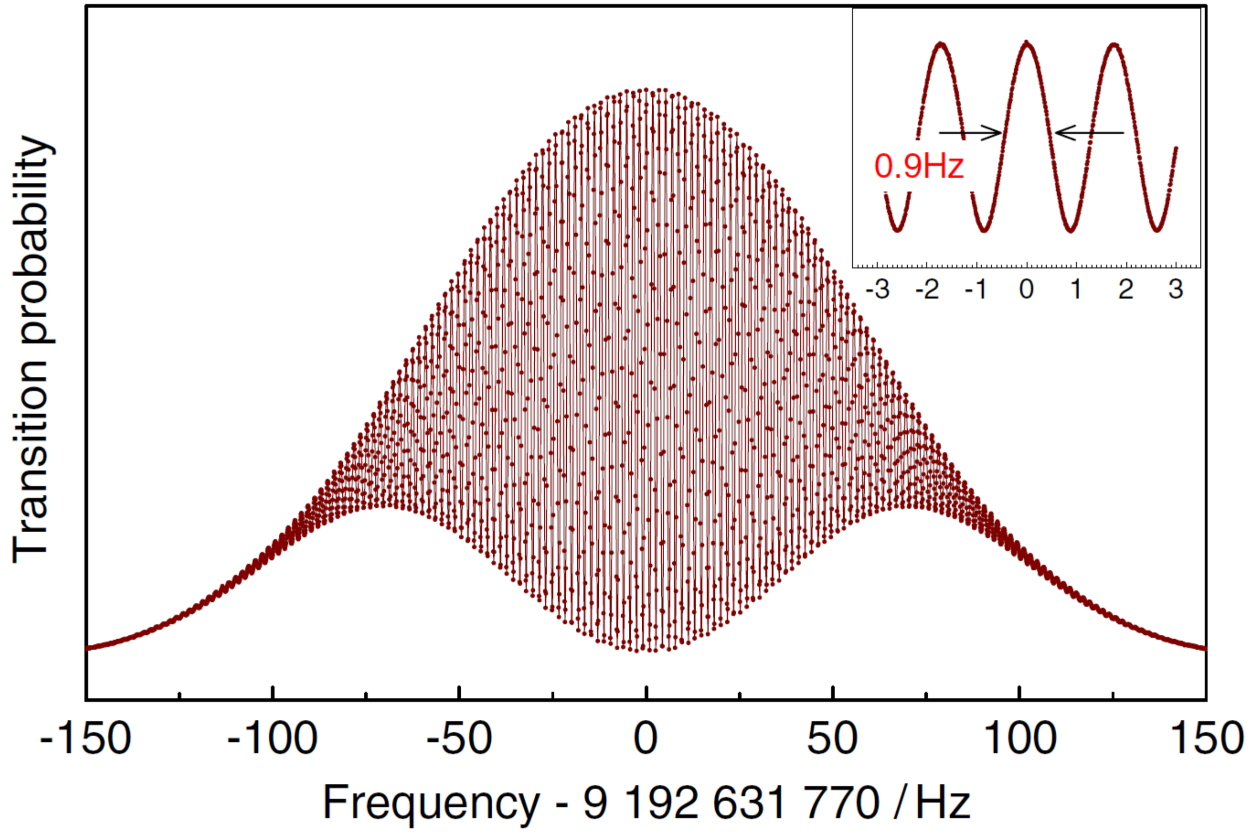


A schematic of a fountain clock



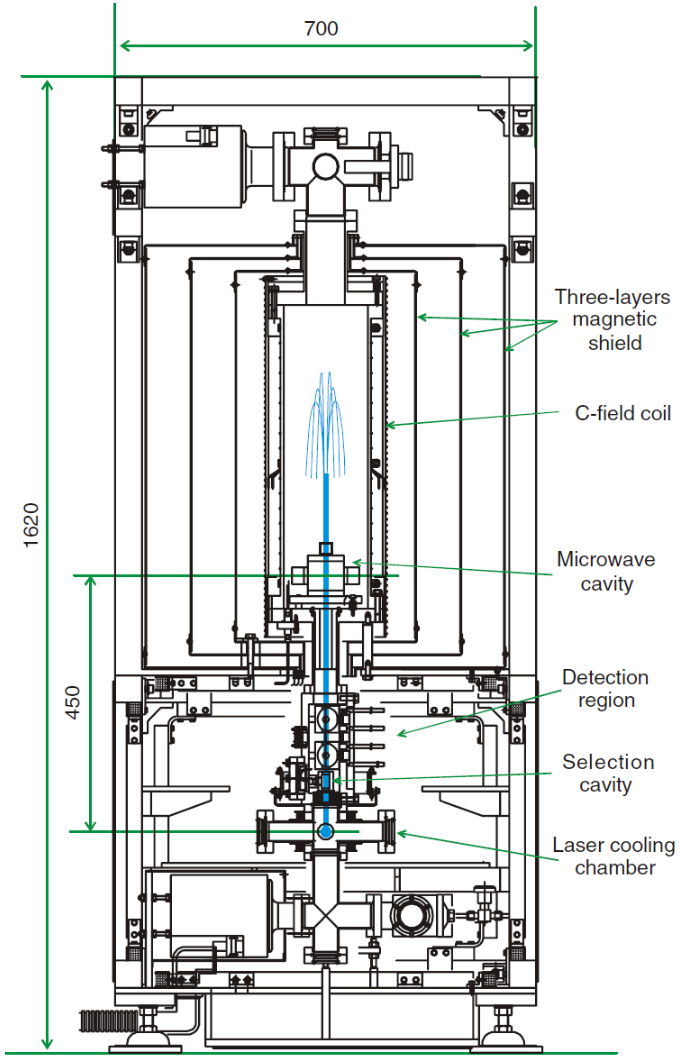
NIST-F4

Fringe benefits



Ramsey fringes from the NICT-CsF1 clock

Uncertainties approaching 1×10^{-16}



A schematic, and maybe a photo, of the NICT-CsF1 clock

¯_(ツ)_/¯

Optical clocks

The measure of clock stability is $\omega_0/\delta\omega$, where $\delta\omega$ is the frequency uncertainty

We know how to make $\delta\omega$ small, but what about making ω_0 big?

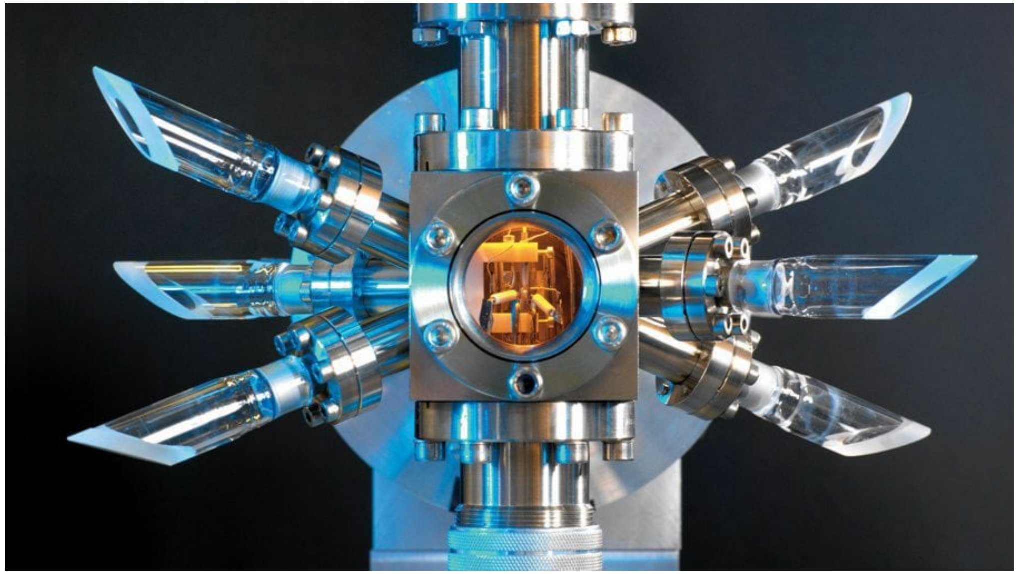
Microwave $\Rightarrow$ optical transitions

- Option 1: same instrument (beam + Ramsey) but optical frequency
- Option 2: use atomic physics magic

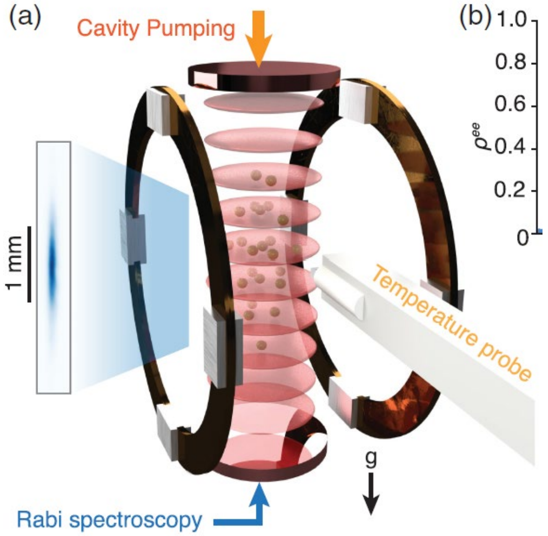
The are many magic tricks here
Forbidden transitions
Optical (lattice) traps
Magic wavelength trapping

Optical frequency combs
Ultra-narrow-linewidth lasers

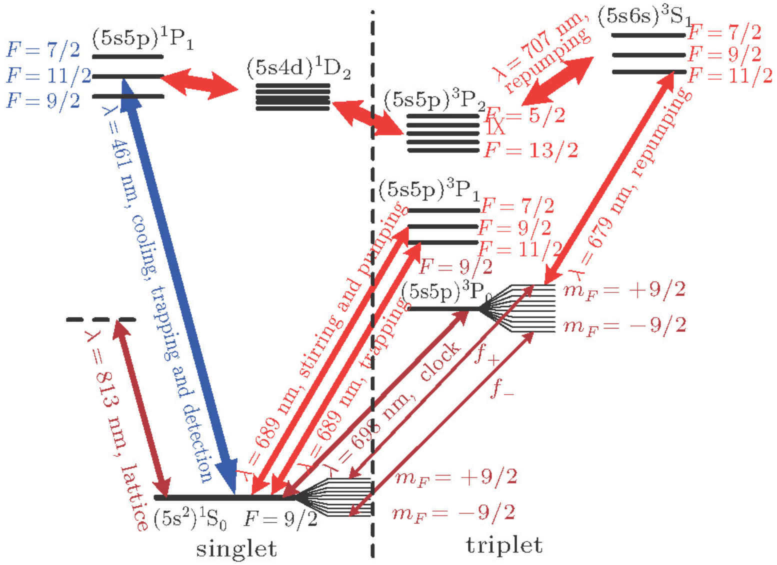
Understanding your system,
e.g. blackbody radiation



A strontium lattice clock



Clock with 8×10^{-19} Systematic Uncertainty

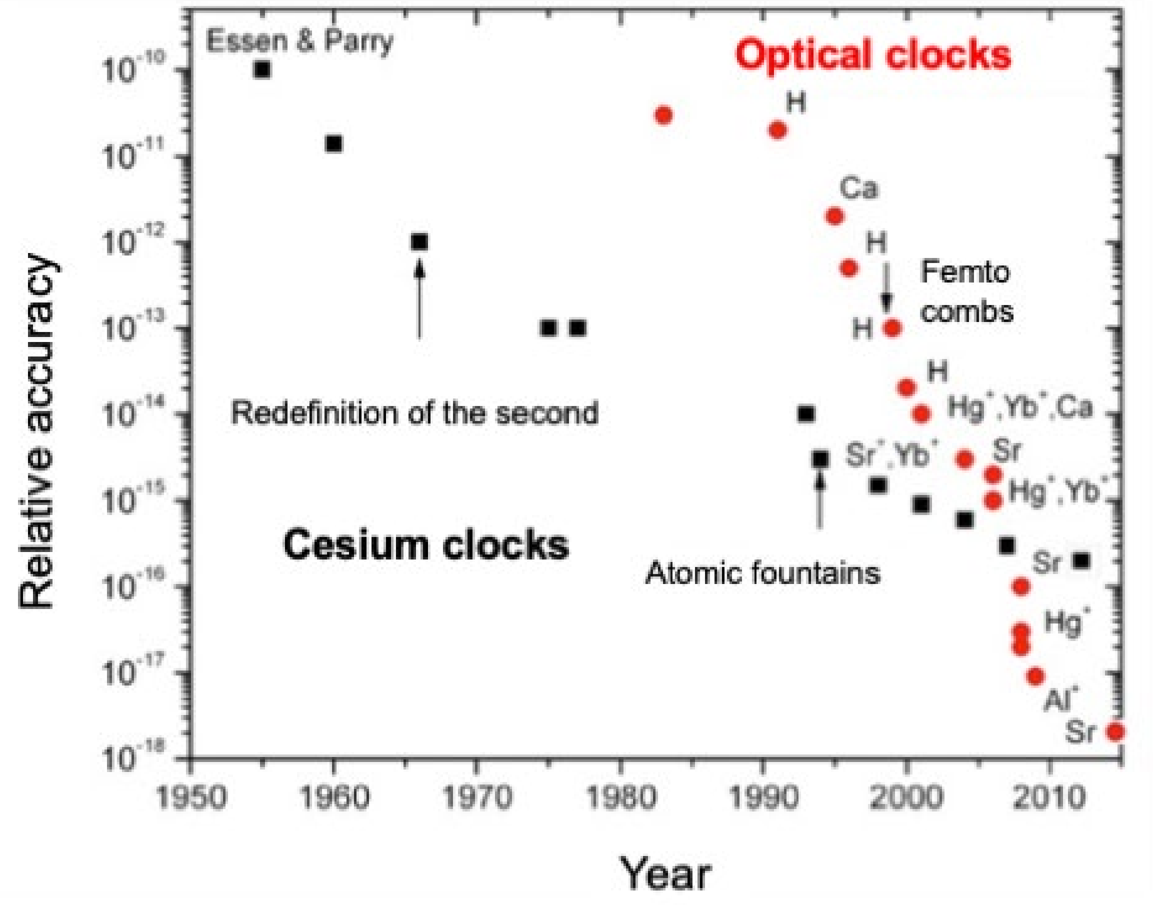


Energy-level diagram for Sr

But… why?

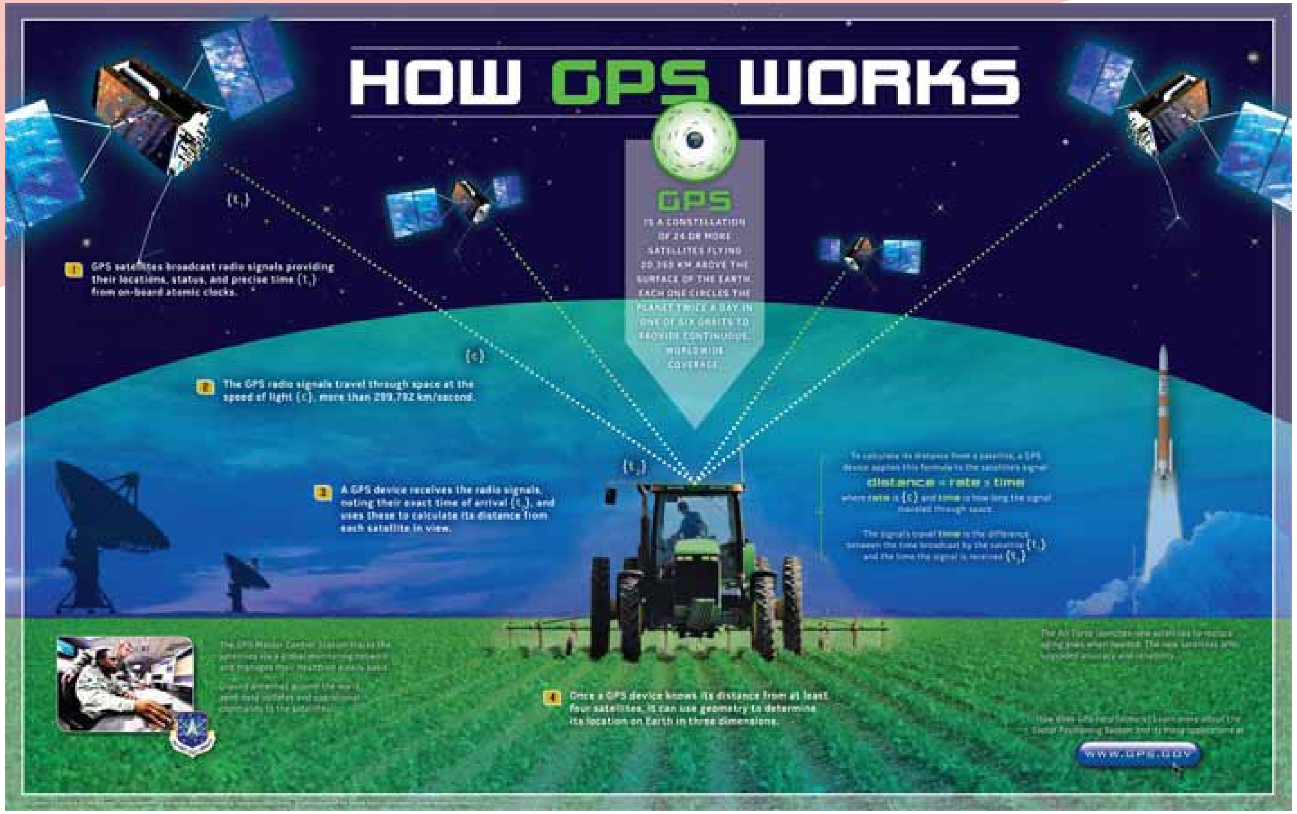
“This precision [greater than 1 part in 10^{15}] allows sufficient control of a quantum bit (qubit) to enable complex quantum computations, permits measurement of a quantum state to “image” molecules with unprecedented sensitivity in a biosensor, and enables you to find your position to centimetre accuracy using atomic clock signals relayed via GPS satellites”

Frequency standards underpin commonplace technology (communication networks, GPS, etc.) and modern science (quantum computing, NMR/MRI, radioastronomy, etc.)



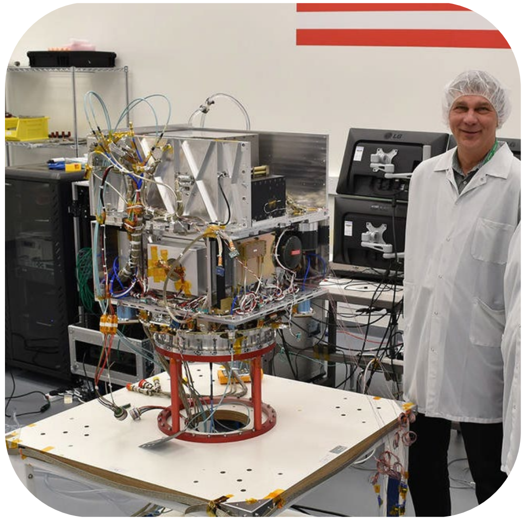
The accuracy of clocks versus time

Well positioned

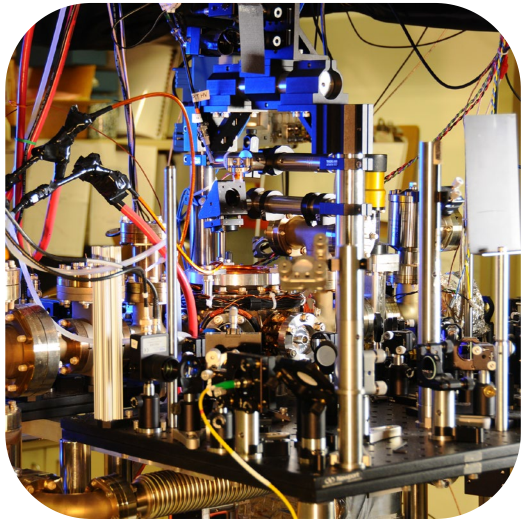


GPS propaganda from [GPS.gov]

A GPS satellite



An (okay) atomic clock



A (very good) atomic clock

Ground station

Info on space-based atomic clocks (NASA article): [How an Atomic Clock Will Get Humans to Mars on Time]

Gravitation redshift

A gravitational field will cause a redshift, which in turn means that there will be a frequency difference as a function of height:

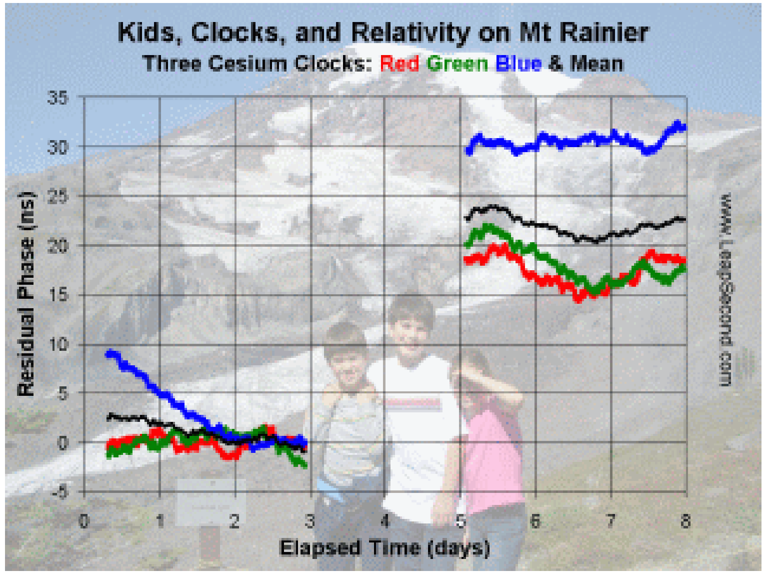
$$\frac{\Delta f}{f} \approx \frac{g\Delta h}{c^2}$$

2005: a clock enthusiast took (microwave) Cs clocks on holiday with Δh = 1645 m for a few days. Should accumulate 22 ns, measured 23 ns.

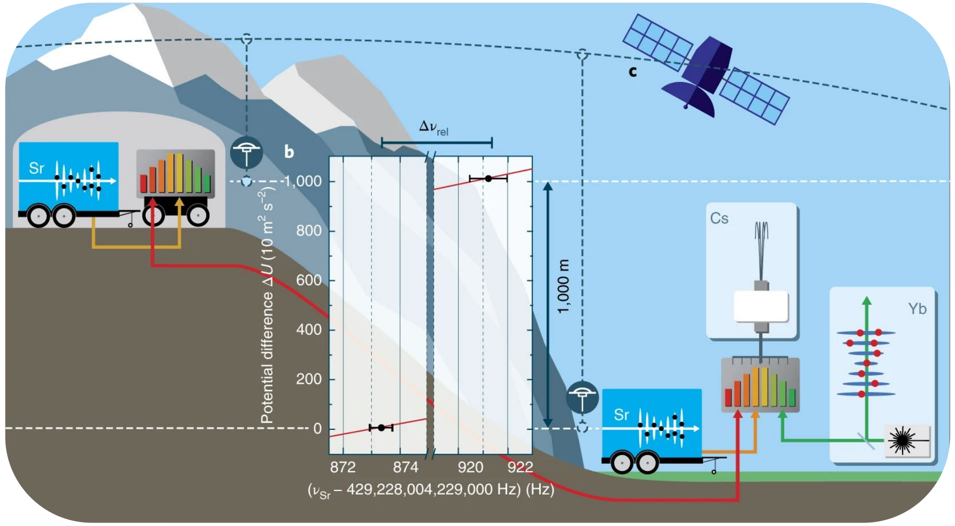
> *"Instead of fanciful stories of rocket ships and twins, the kids got a hands-on introduction to general relativity with real clocks and a family road trip. Furthermore, by being at high altitude for the weekend, we experienced more time together, relatively speaking. It was the best extra 22 nanoseconds I've ever spent with the kids."*

Has also been performed with modern optical clocks

Clock stability allows for precision tests of physics



Clocks, Kids, and General Relativity on Mt Rainier



Geodesy and metrology with a transportable optical clock

In the wild

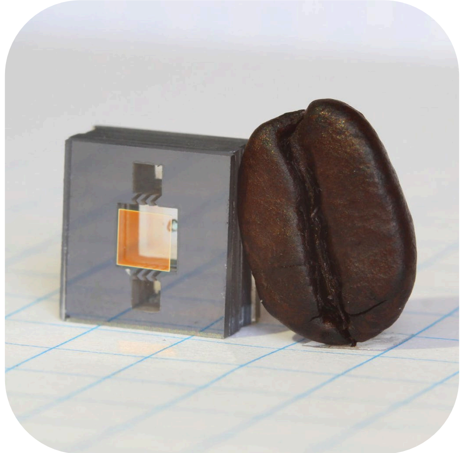
What about clocks outside the lab?

Hydrogen maser

- Stabilities on the order of 1×10^{-12}

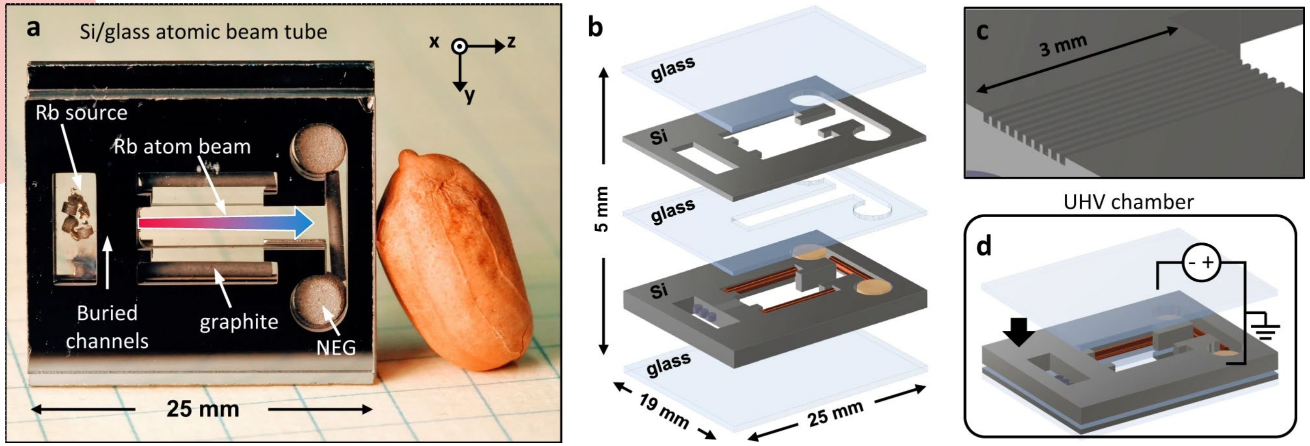
Optical “chip” clocks:

- Stabilities on the order of 1×10^{-9}

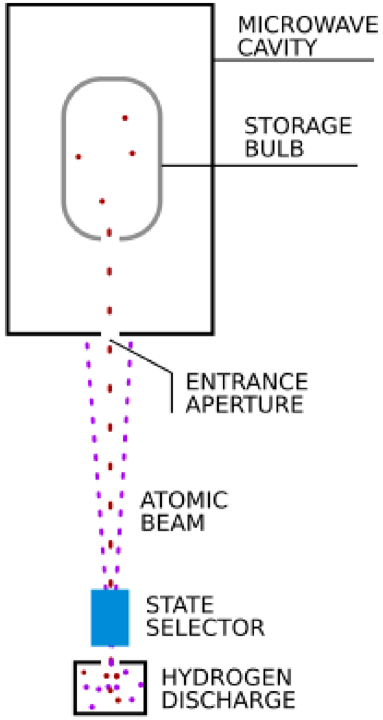


Another chip clock

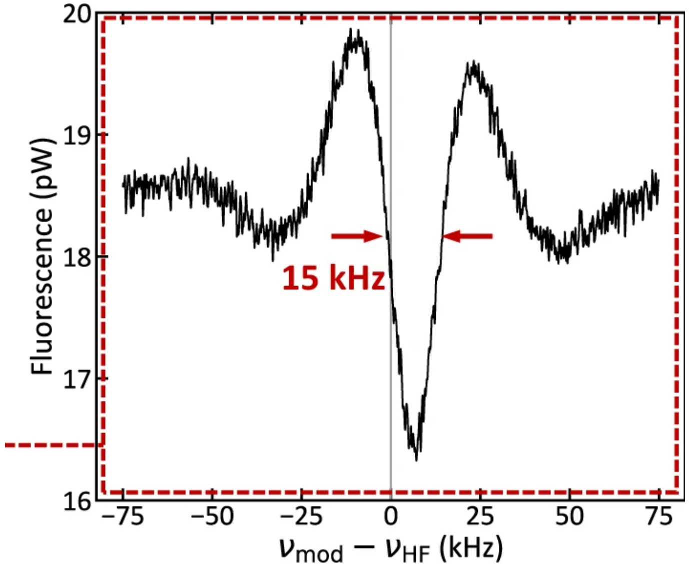
A hydrogen maser



A chip-scale atomic beam clock



A hydrogen maser schematic



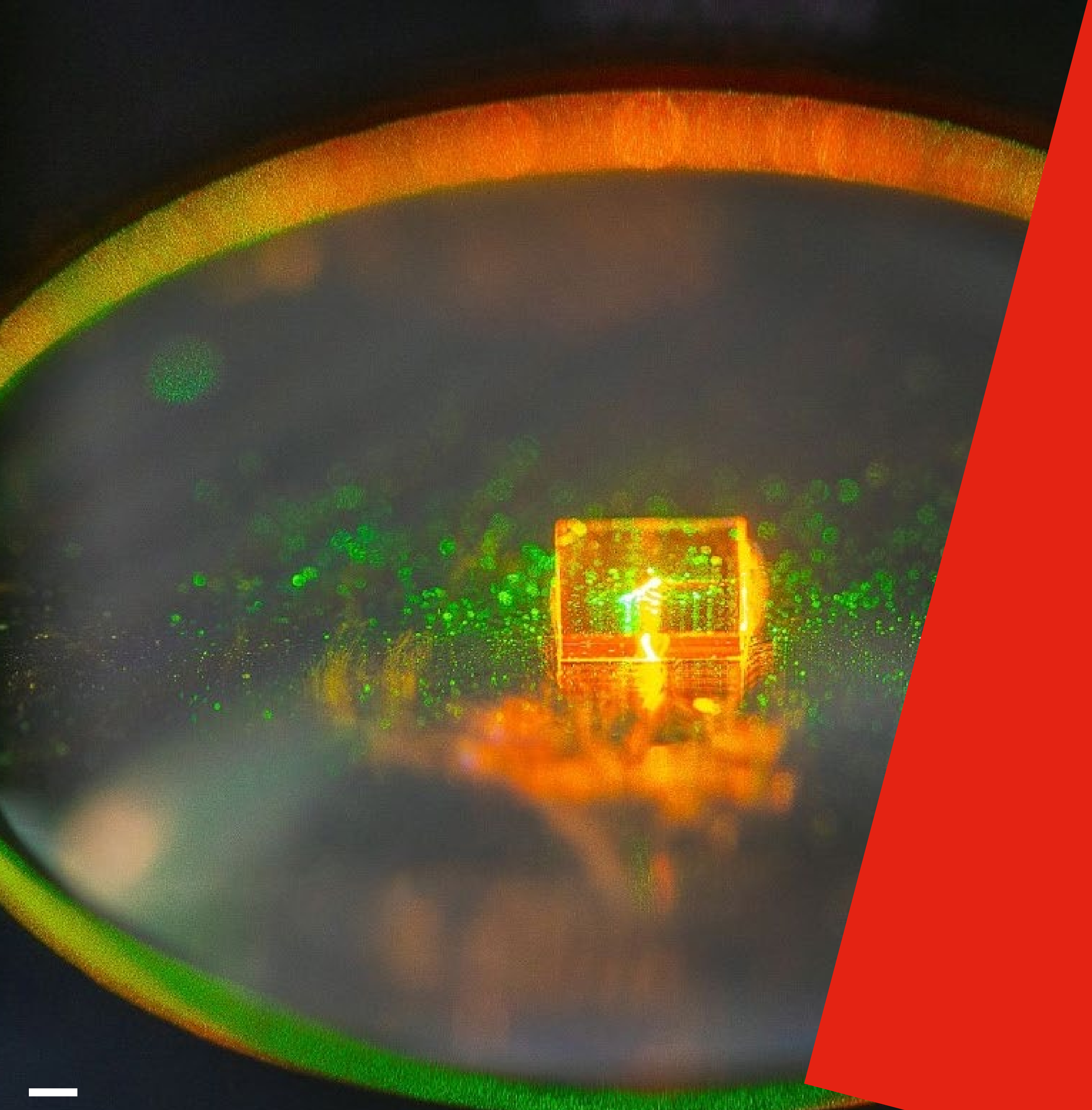
Ramsey fringes* from a chip clock

Summary

Problems

S

- Quantum states
    - The Rabi method for frequency measurement is fine, but limited by linewidth and experimental factors
    - The Ramsey method allows interferometry to be performed on atomic states
    - Bloch sphere allows for visualisation of the evolution of the system, notably precession of the superposition state around the z −axis at a rate of Δ
    - Fitting the Ramsey interference fringes provides an accurate measurement of ω_0
    - Clock stability can be improved by decreasing $\delta\omega$ (e.g. Ramsey), or increasing ω (optical transition)



Quantum sensors image on the atomic scale with unprecedented sensitivity

Quantum control and sensing

Foot Ch. 7 // Steck Ch. 4, 5

Learning outcomes

Week 5, lecture 2

Foot §13.1 – 8.5 // Steck §13.1 – 8.5 :

Time evolution of the two-level atom

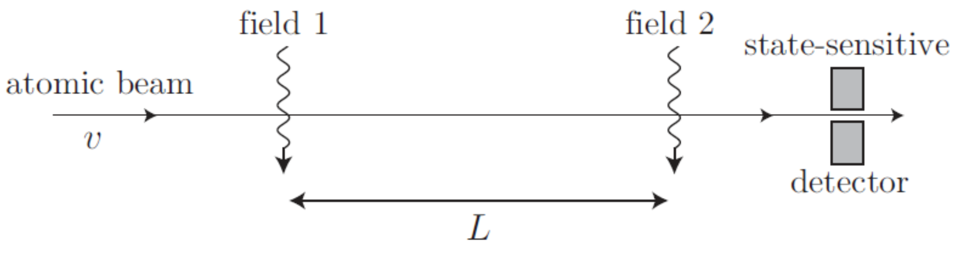
- The two-level atom
  - How do we evolve the density matrix in time?
  - The Rabi frequency
  - Solving for the population of an atomic system coupled to a light field
- The optical Bloch equations
  - How do these relate to the Einstein rate equations
  - Steady-state solution and predications

Atomic state interference

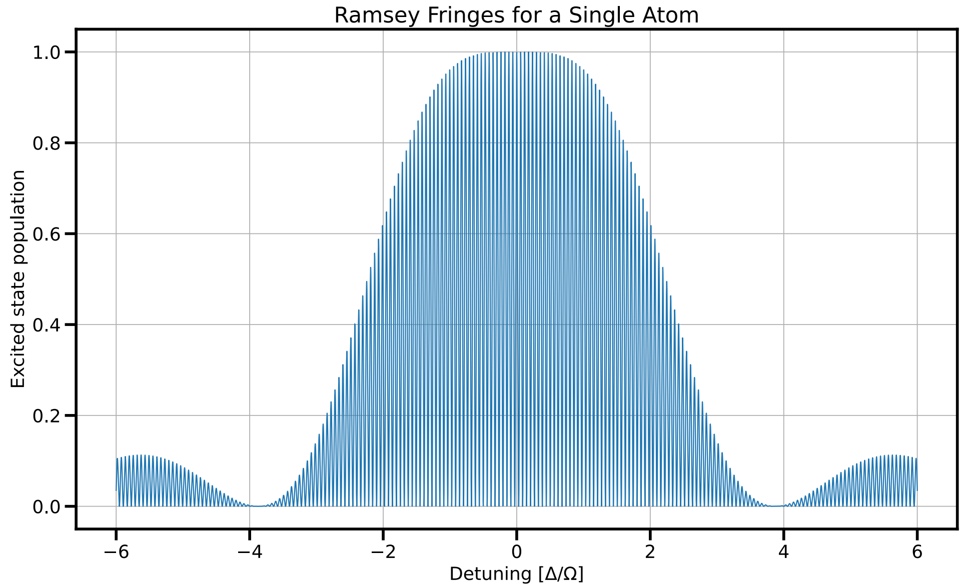
We saw that setting up a $\pi/2 - \pi/2$ pulse sequence, Ramsey interference was observed

Each state in the system acquires phase at a different rate, and this leads to interference between states

Work really well when all atoms experience the same Hamiltonian, but works much less well when *dephasing* occurs



Ramsey interference schematic



Ramsey interference fringes

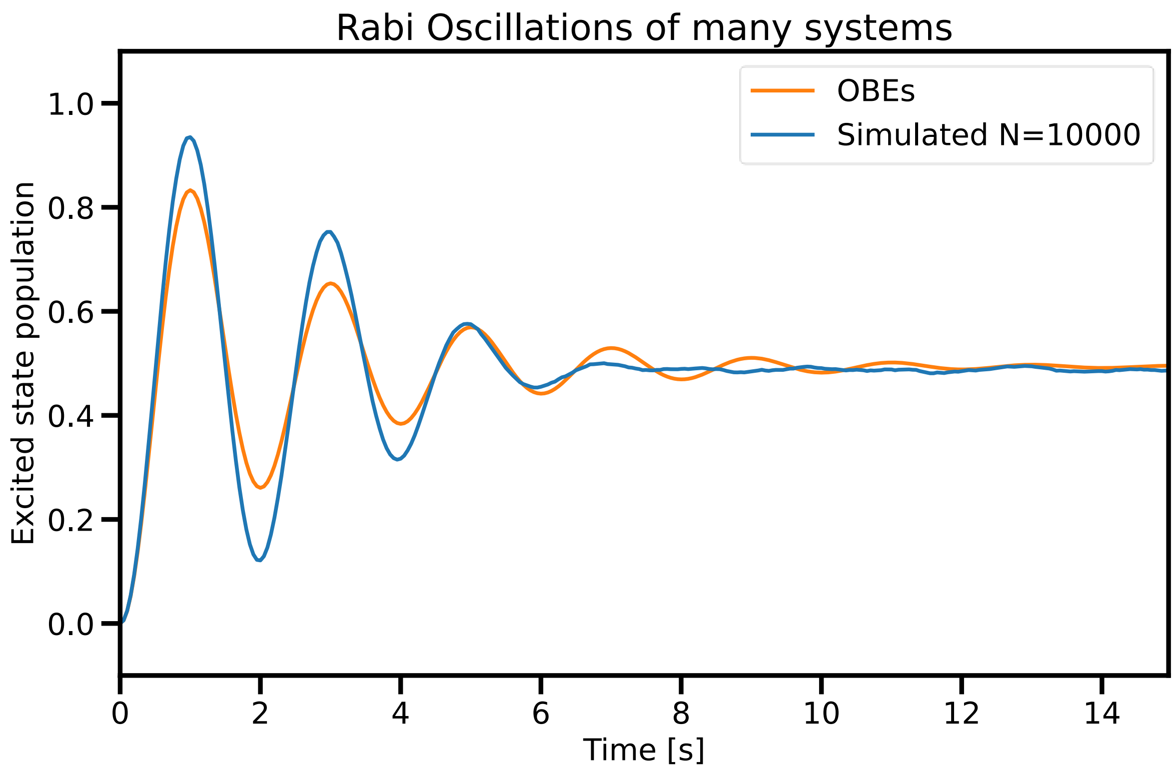
Dephasing

We saw dephasing in the context of Rabi oscillations: spontaneous decay results in atoms in the ensemble "leaving" synchronicity

Inhomogeneous broadening, that is, where atoms in an ensemble have different resonant frequencies, e.g.

- Stark or Zeeman shift
- Doppler broadening
- Local environmental effects

quickly leads to a dephasing of states, such that no macroscopic interference can be observed



Dephasing in an ensemble of atoms undergoing Rabi oscillations

Inhomogeneous state evolution

Let us perform the Ramsey sequence:

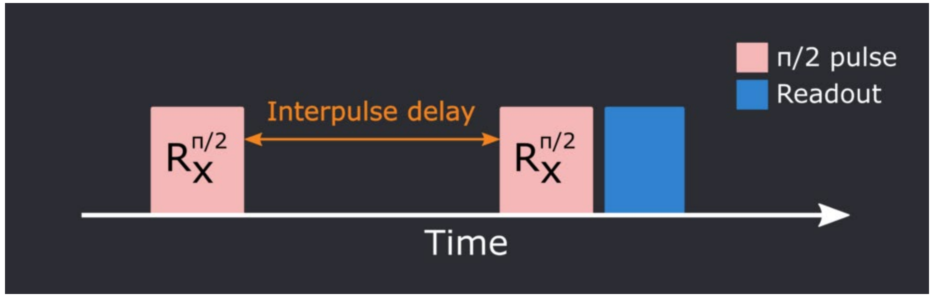
$\pi/2$-pulse $\rightarrow$ free evolution for time T

$\rightarrow \pi/2$-pulse

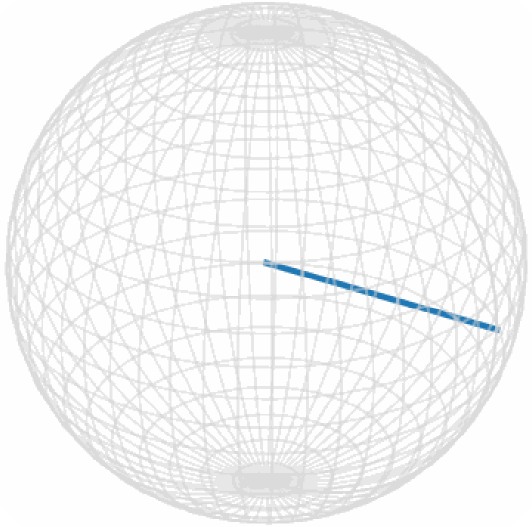
but with an ensemble of atoms which have slightly different $\Delta E = \omega_0$

$\Rightarrow$ The accumulated phase difference between the states will be different for each atom

$\Rightarrow$ The rate of precession on the Bloch sphere will be different



Ramsey pulse sequence

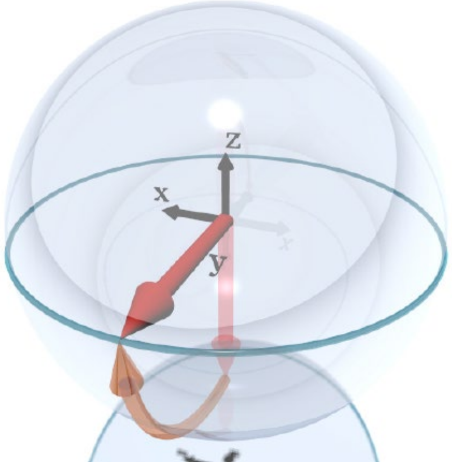


Precession of the Bloch vector

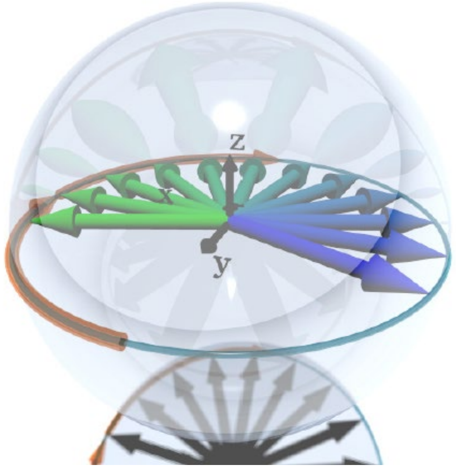
Ramsey with inhomogeneity

Let's begin in the ground state ($\rho_{11} = 1$)

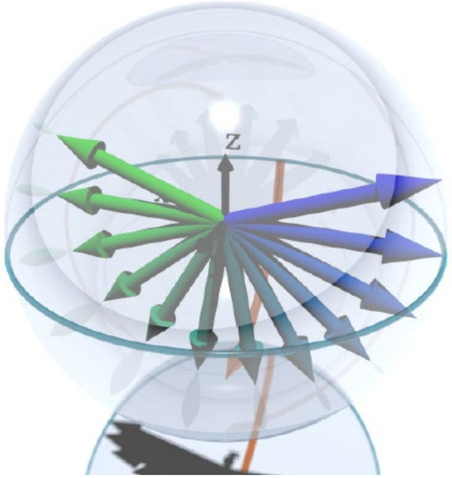
$\Omega' \tau = \pi/2$



T



$\Omega' \tau = \pi/2$



Atoms are place in a superposition

Atoms undergo precession at different rates

Only a fraction of atoms are in the ground state

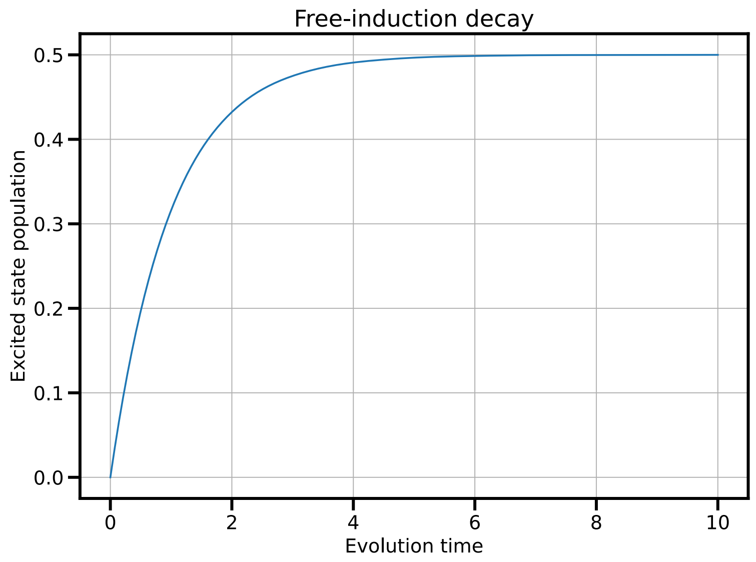
In the limit of large dephasing, atoms are spread around uniformly the Bloch sphere $\Rightarrow \rho_{22} = 1/2$ for all evolution times T

The damping of the ensemble-averaged dipole moment due to dephasing is called the *free-induction decay*

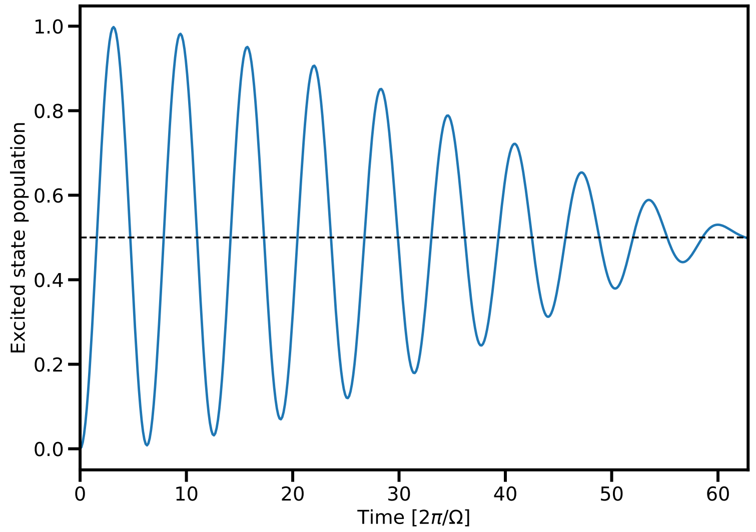
Fringes lost?

The excited state population looks distinctly less impressive: if my Bloch vectors are all out of phase, how will I ever do quantum experiments, which depend critically on phase!?

Remember that this dephasing is due to inhomogeneity, which goes *all the way down*, e.g. uniformity of applied field, static field, time-variance of applied field, static field, temperature, exotic atoms



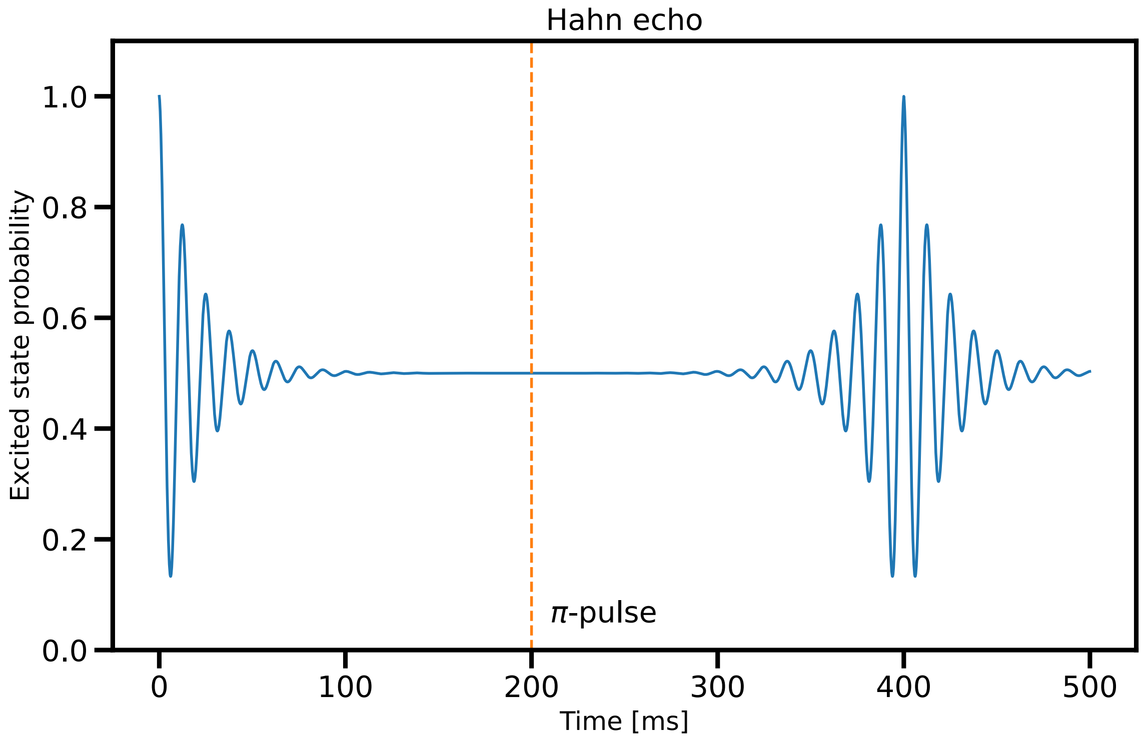
Free induction decay



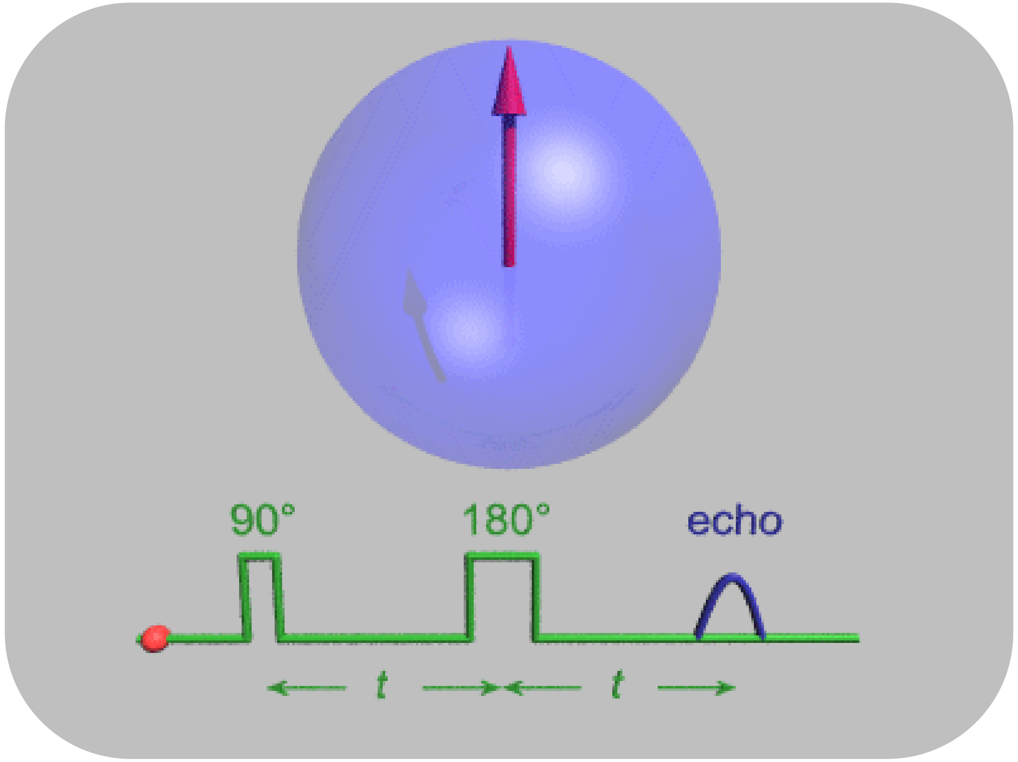
Dephasing due to a variation in Ω

The spin echo

By applying an extra π-pulse, the *Ramsey sequence* becomes the *Hahn echo sequence*: $\pi/2 \rightarrow T \rightarrow \pi \rightarrow \pi/2$, and we can still do quantum things despite dephasing!



The excited state probability in a Hahn echo sequence



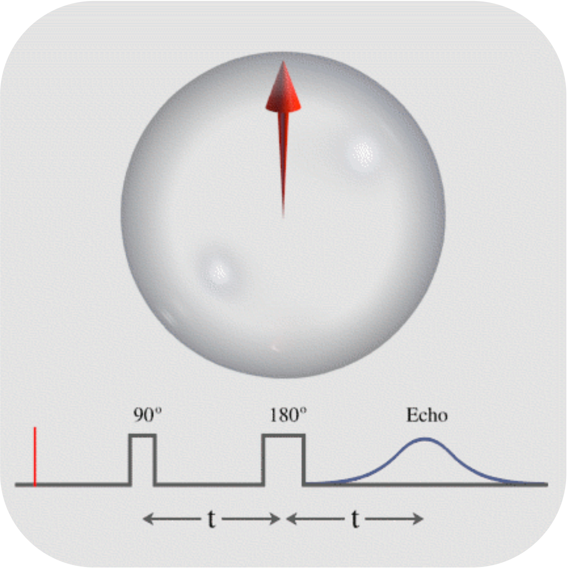
A gif tells 1000 words

Why does this work?

Atoms undergo precession around the equator at a constant rate Δ (in the rotating wave approximation)

Inhomogeneities result in slightly different rates of precession, but the dephasing does not destroy the coherence – the phase relationship – between atoms

The π-pulse flips the equatorial distribution of Bloch vectors, and the "fast" ones – those which accumulate phase due to a larger Δ – are now *behind* the "slow" ones, so will catch up after another free evolution time, T

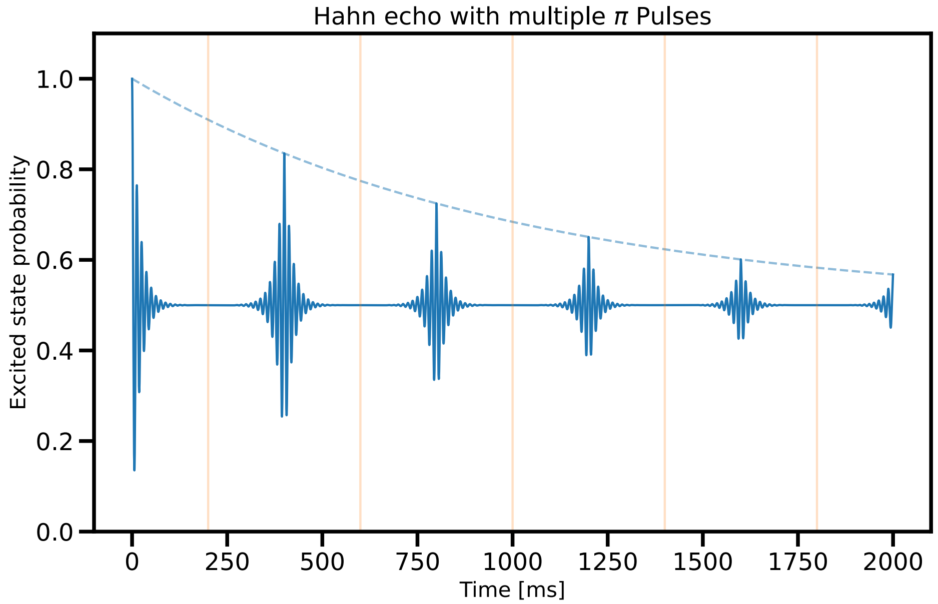


I have an idea

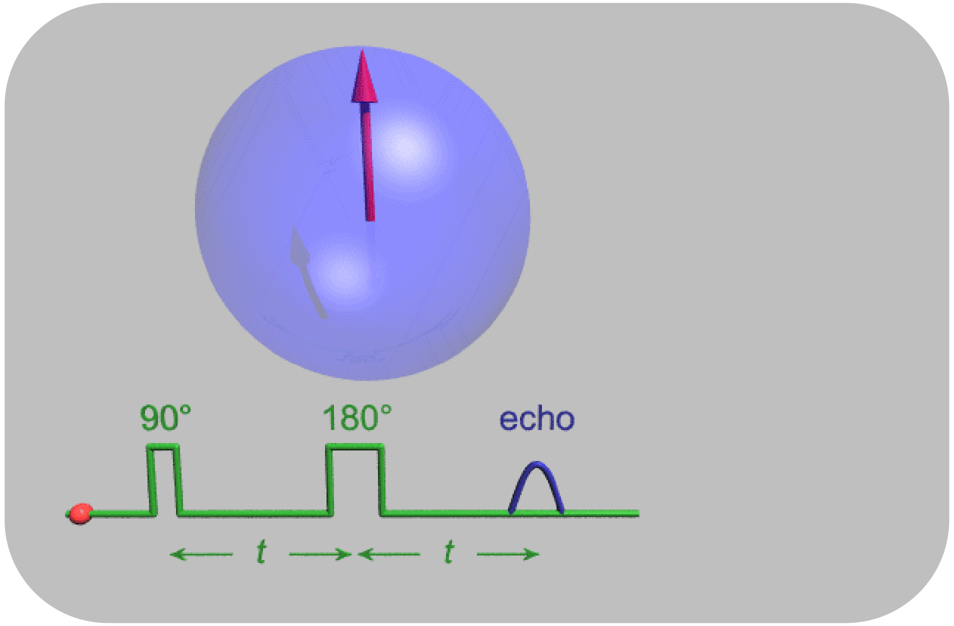
If the echo worked once, will it work again?

The amplitude of the echo signal decays, and this really is the loss of coherence, and is due to interactions with the environment which destroy the phase relationship between atoms. From the optical Bloch equations, this is characterised by the decay rate $\gamma_\perp = 1/T_2$. This decay rate is called the *spin-spin relaxation* time.

The free induction decays with the characteristic time T_2^*



A multiple Hahn echo sequence



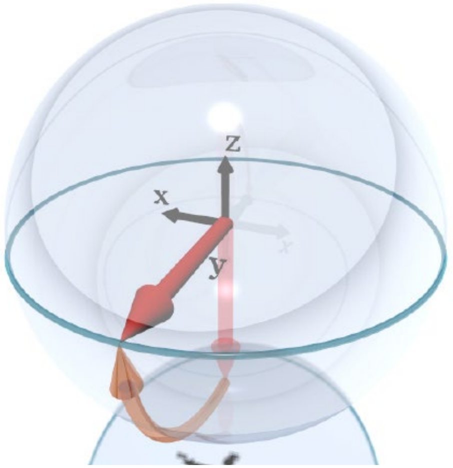
A gif still tells 1000 words

A relaxing time

We have encountered the idea numerous times that decay has two flavours:

The spontaneous decay, characterised by $\dot{\rho}_{22} = -\Gamma\rho_{22}$

Loss of coherence, characterised by $\dot{\sigma}_{12} = -\gamma_{\perp}\sigma_{12}$



The names make sense with the Bloch sphere

$T_1 = 1/\Gamma$	is called the *spin-lattice relaxation* time or the *longitudinal relaxation* time	These are properties of the system
$T_2 = 1/\gamma_{\perp}$	is called the *spin-spin relaxation* time or the *transverse relaxation* time	

T_2^*	Does not have an associated decay rate	Property of the local environment

The Zeeman effect

Consider a spin-1/2 system in a magnetic field $\boldsymbol{B}$

The interaction component of the Hamiltonian is given by

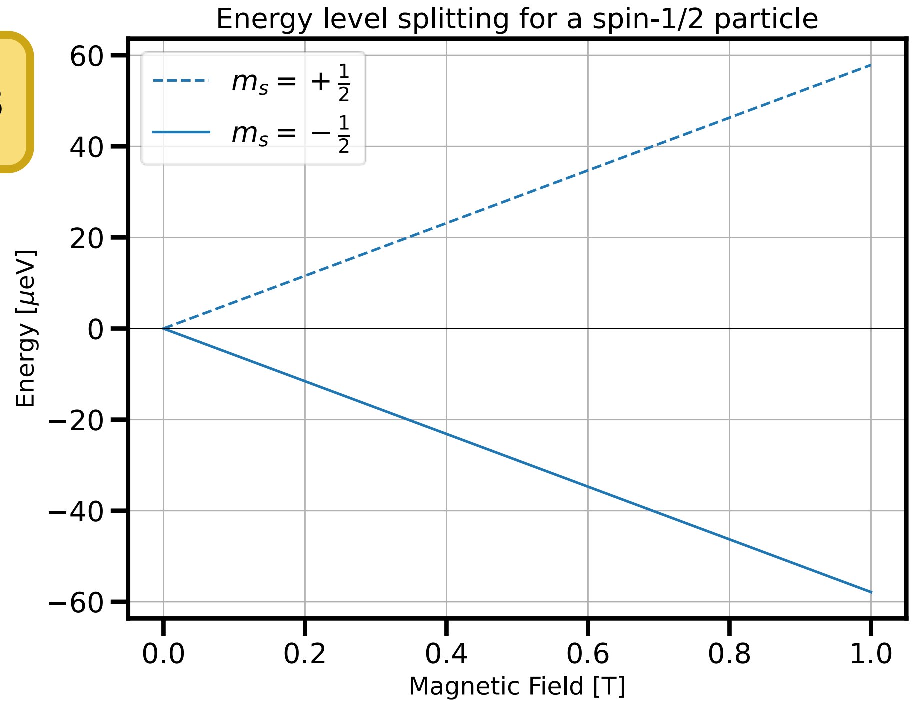
$$H' = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

See tutorial 2, problem 3

where $\boldsymbol{\mu}$ is the magnetic dipole moment. We consider a static magnetic field oriented along the z-axis, so $-\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu_z B_0$

In the absence of a magnetic field, the states $|S_z; \pm 1/2\rangle \equiv |\pm\rangle$ are degenerate, but this is lifted with a non-zero magnetic field (i.e. the Zeeman shift)

We have seen (and solved) for similar Hamiltonains



Energy-level splitting for a spin-1/2 particle in an external magnetic field

Magnetic resonance

The magnetic dipole moment is

$$\boldsymbol{\mu} = \gamma \boldsymbol{S}$$

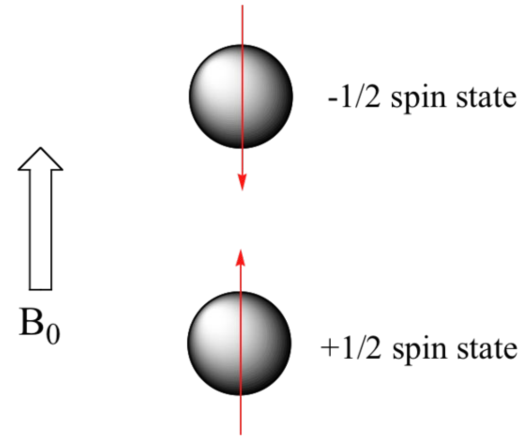
where $\gamma = \frac{q}{2m}$ is the gyromagnetic ratio, so for a spin-1/2 particle:

$$\Rightarrow \mu_z = \gamma \frac{\hbar}{2}$$

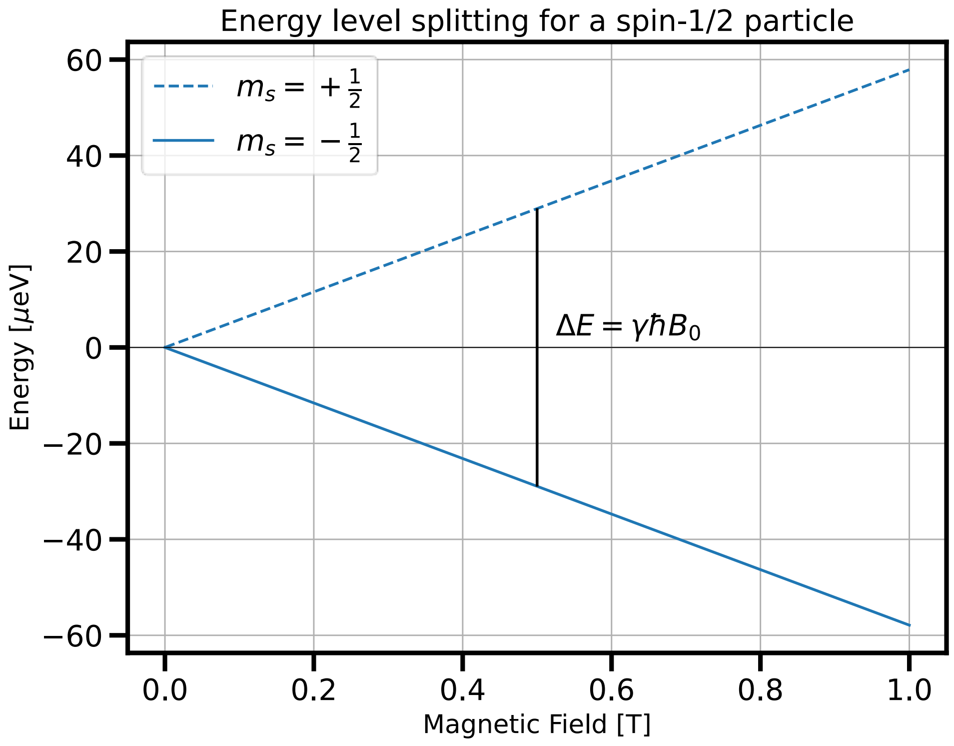
Hence the energy is

$$E = -\mu_z B_0 = -\gamma \frac{\hbar}{2} B_0$$

$$\Rightarrow \Delta E = \gamma \hbar B_0$$



Spin-1/2 particles in a magnetic field



Zeeman splitting for $S = 1/2$

Expectations?

Compute the expectation values $\langle S_x \rangle, \langle S_y \rangle$ and $\langle S_z \rangle$

$$\langle S_z \rangle = \mathrm{tr}(S_z \rho) = \mathrm{tr}\left(\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\right)$$

$$= \mathrm{tr}\left(\frac{\hbar}{2}\begin{pmatrix}\rho_{11} & \rho_{12}\\ -\rho_{21} & -\rho_{22}\end{pmatrix}\right) = \frac{\hbar}{2}(\rho_{11} - \rho_{22})$$

$$\langle S_x \rangle = \mathrm{tr}(S_x \rho) = \mathrm{tr}\left(\frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\right)$$

$$= \frac{\hbar}{2}(\rho_{21} + \rho_{12})$$

$$\langle S_y \rangle = \mathrm{tr}(S_y \rho) = \mathrm{tr}\left(\frac{\hbar}{2}\begin{pmatrix}0 & i\\ -i & 0\end{pmatrix}\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\right)$$

$$= \frac{i\hbar}{2}(\rho_{21} + \rho_{12})$$

The density matrix evolves via the *Liouville-von Neumann equation*:

$$i\hbar\frac{d\rho}{dt} = -[\rho, H]$$

$$\dot{\rho} = \frac{i}{\hbar}\left[\begin{pmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{pmatrix}\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\right]$$

$$= \frac{i}{\hbar}\left(\begin{pmatrix}E_1\rho_{11} & E_2\rho_{12}\\ E_1\rho_{21} & E_2\rho_{22}\end{pmatrix} - \begin{pmatrix}E_1\rho_{11} & E_1\rho_{12}\\ E_2\rho_{21} & E_2\rho_{22}\end{pmatrix}\right)$$

$$= \begin{pmatrix}0 & (E_2 - E_1)\rho_{12}\\ (E_1 - E_2)\rho_{21} & 0\end{pmatrix} = -\frac{i\Delta E}{\hbar}\begin{pmatrix}0 & -\rho_{12}\\ \rho_{21} & 0\end{pmatrix}$$

$$\Rightarrow \dot{\rho}_{12} = \exp(i\Delta E t/\hbar)\rho_{12}, \qquad \dot{\rho}_{21} = \exp(-i\Delta E t/\hbar)\rho_{21}$$

Which gives

$$\langle S_x(t) \rangle = \hbar\rho_{21}(0)\cos(\Delta E t/\hbar)$$

$$\boxed{= \hbar\rho_{21}(0)\cos(\gamma B_0 t)}$$

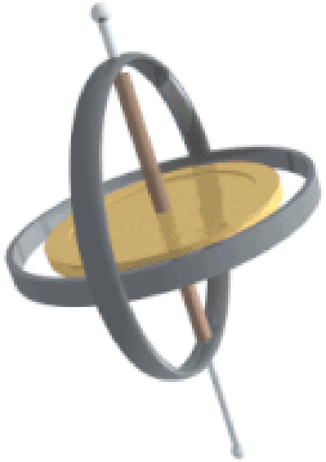
Precession and magnetisation

The spin vector undergoes precession around the magnetic (z) axis with frequency $\omega_0 = \gamma B_0$, known as the *Larmor frequency*.

Classically, we can think of the precession arising from torque that is applied due to the magnetic moment $(\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B})$ much like a gyroscope, but the quantum precession arises from the evolution of the density matrix, and the rate of phase accumulation for different energy eigenstates

The magnetisation $\boldsymbol{M}$ of the system is the total magnetic dipole moment (per unit volume), and is proportional to $\langle \hat{S}_z \rangle$

Joseph Larmor in 1920



Classical precession

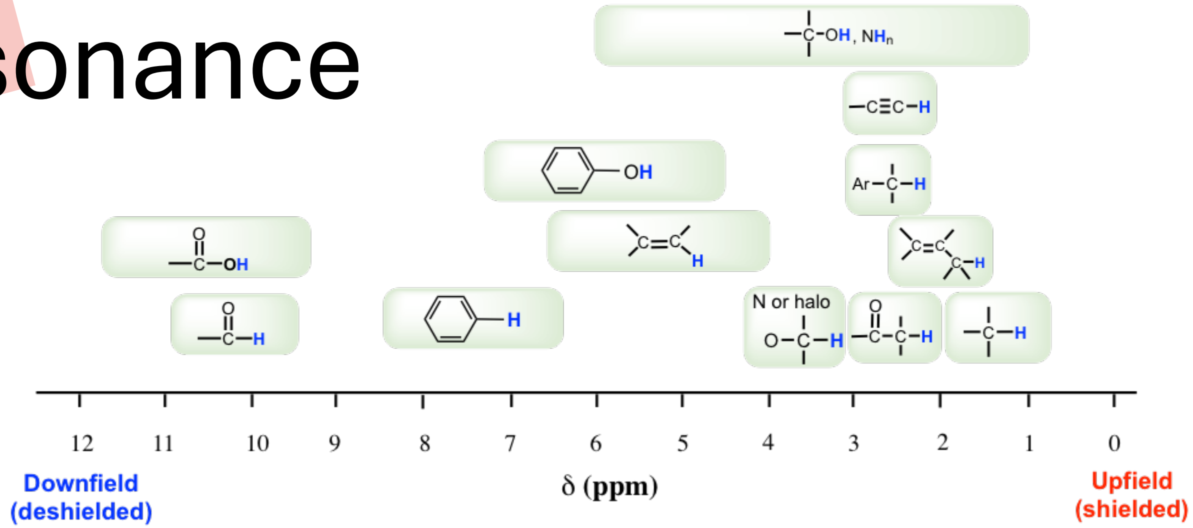
Nuclear magnetic resonance

If we consider a (non-zero) nuclear spin system, for example, spin-1/2 nuclei

^{1}H, ^{3}H, ^{13}C, ^{15}N, …

and put them in an external magnetic field, we have the system that we have just described.

Is such a system useful?



Chemical shifts of common configurations

Molecules experience a shift in resonance peak depending on atomic arrangement (isomers, conformation)

- Shielding/exposure of nucleus due to electronegativity
- Anisotropy due to π-bonded

The Dependence of a Nuclear Magnetic Resonance Frequency upon Chemical Compound*

W. G. Proctor and F. C. Yu

Department of Physics, Stanford University, Stanford, California

January 18, 1950

Can identify molecular structure!

The Dependence of a Nuclear Magnetic Resonance Frequency upon Chemical Compound (1950)

NMR

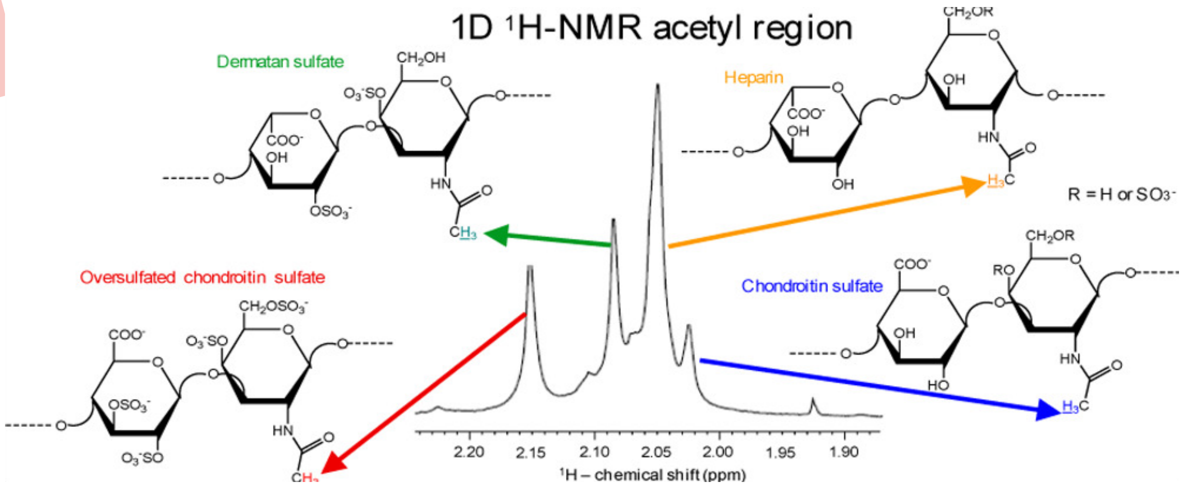
UTAS has 400 MHZ and 600 MHz NMR systems in chemistry

NMR systems usually given their frequency rather than field strength, but we know

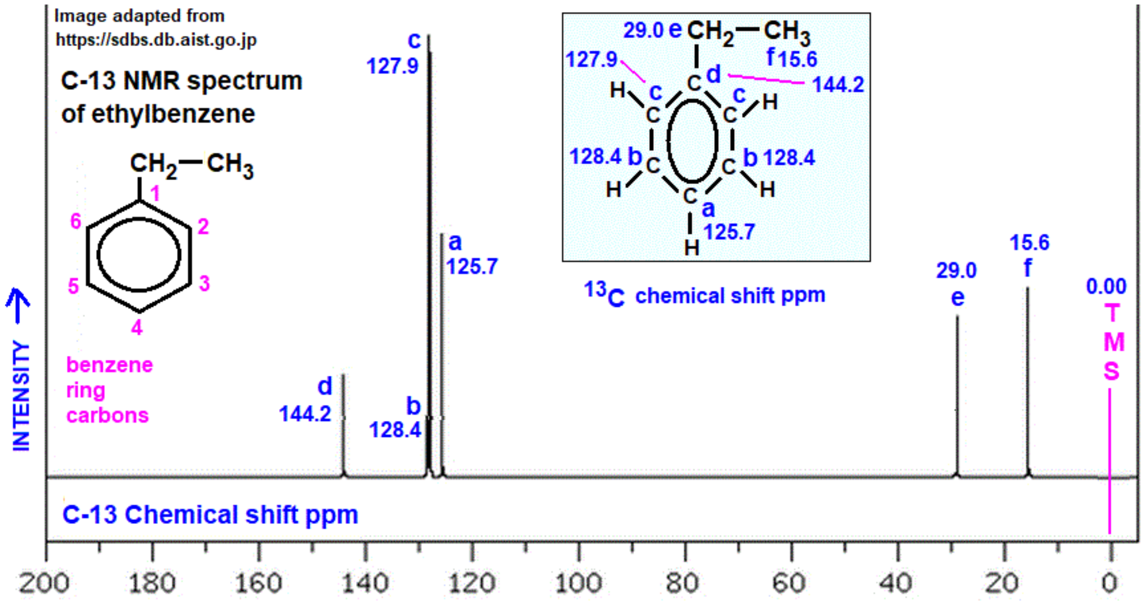
$$f = \gamma B_0$$

$$\Rightarrow B_0 = 600/42.577 = 14.1\ T$$

They are some big magnets!



Chemical shifts for H-1



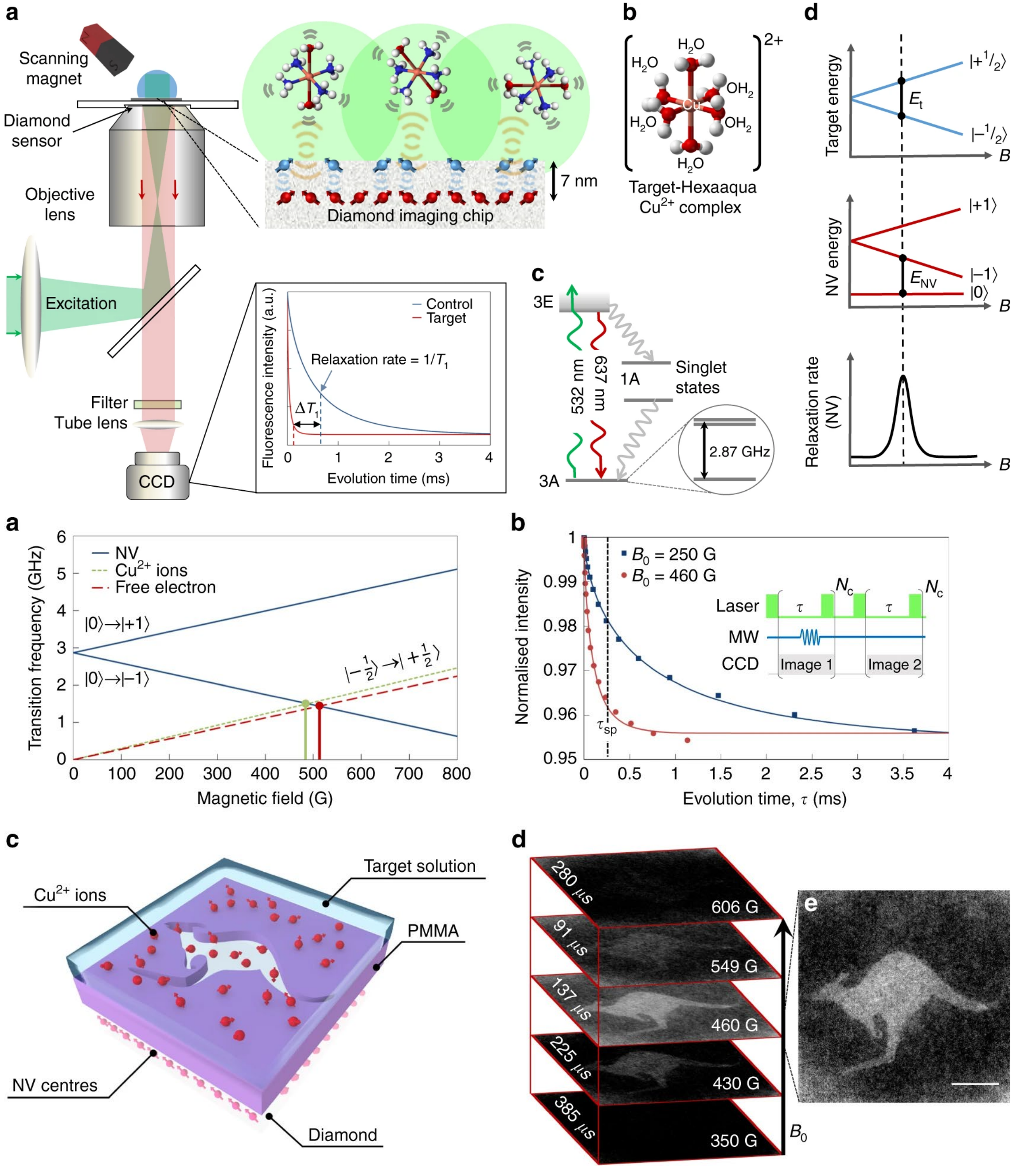
Chemical shifts for C-13

Imaging with NMR

Beyond measuring the energy ΔE, we can use coherent manipulation of the spins to control the population

We can use the loss of coherence (T_2 and T_2^*) and the loss of excited state population (T_1) to map the surrounding environment, and the system's interaction with it

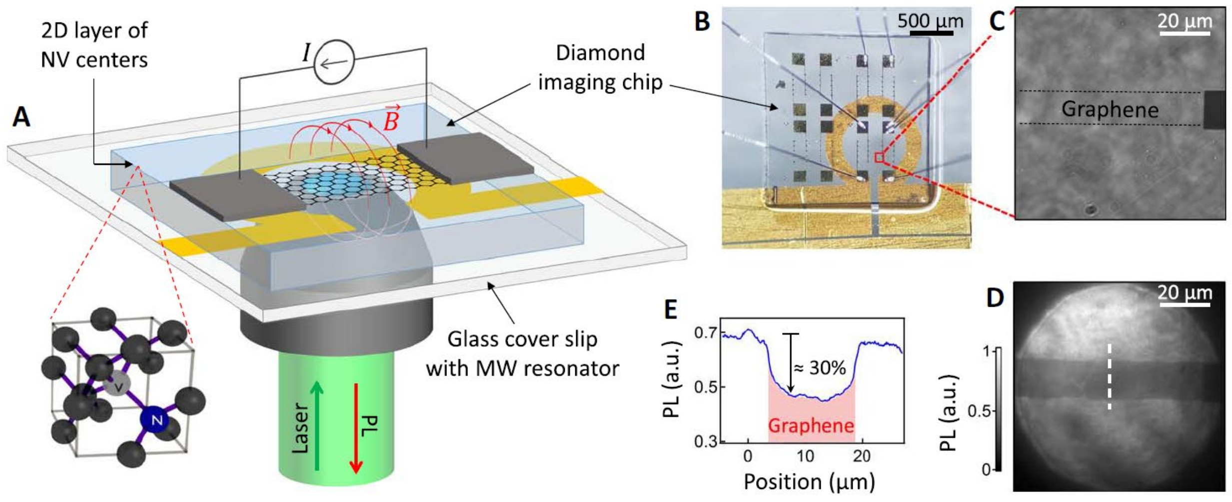
e.g. presence of something which degrades spin coherence $\Rightarrow$ smaller T_2



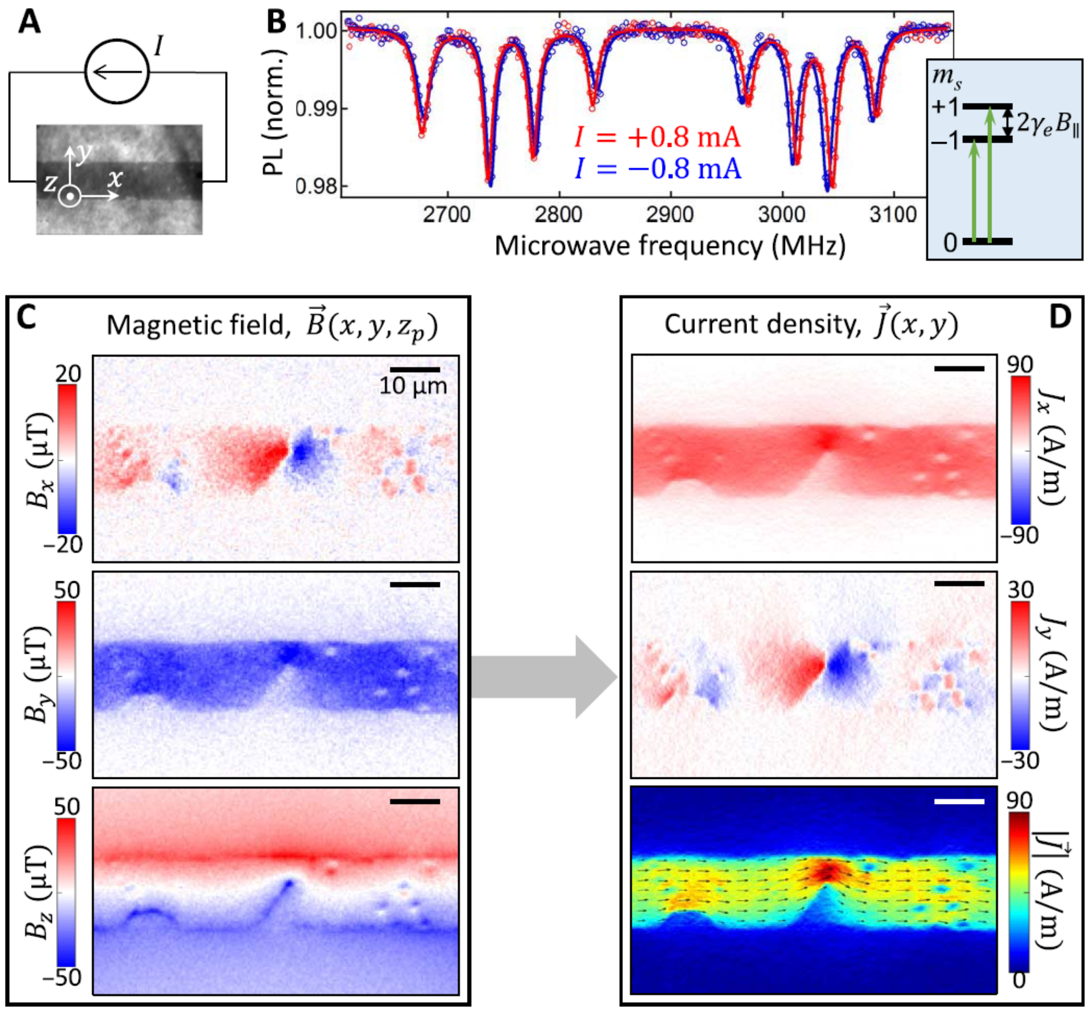
Electron paramagnetic resonance microscopy using spins in diamond under ambient conditions

Quantum imaging

Coherent manipulation of systems and understanding of decoherence mechanisms allows novel forms of imaging down to the atomic scale



Experimental setup for NMR imaging



Magnetic field imaging, and calculated current densities

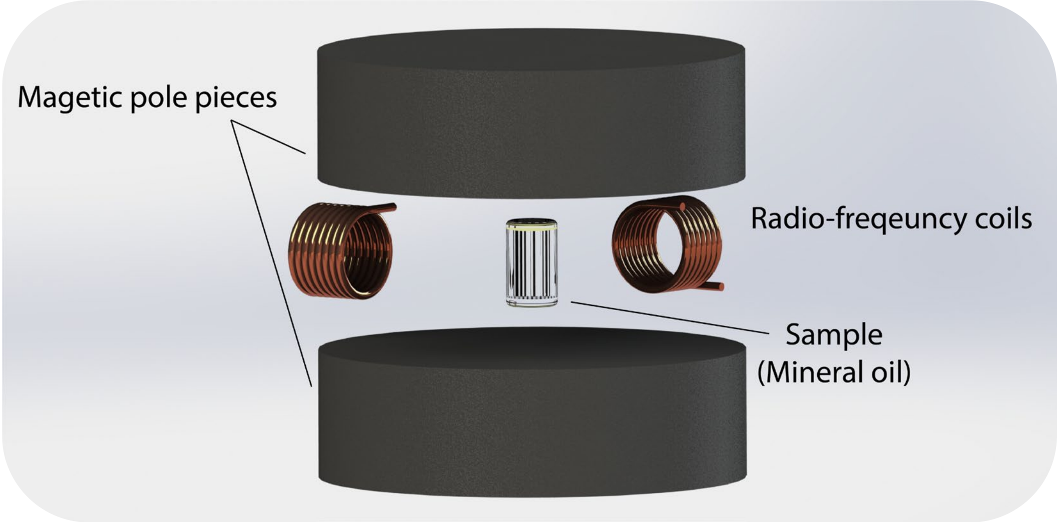
[Quantum imaging of current flow in graphene](#)

NMR imaging?

Have an ensemble in a magnetic field, and use coils to induce transitions, and measure the magnetisation in the $x - y$ plane

By introducing a magnetic field gradient along one direction, there will now be a correlation with spin position and energy splitting, and thus Larmor frequency

This means our measured signal will be a mess, with many single frequency signals superimposed. How ever can we extract the information?



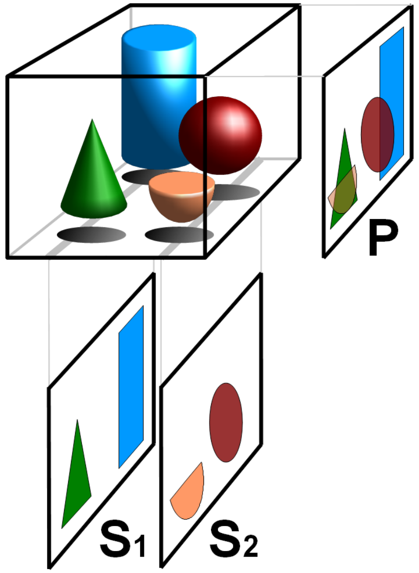
A schematic of an NMR apparatus

The NMR experiment in the lab

NMRI

With a field gradient, we now have 1D spatial resolution!

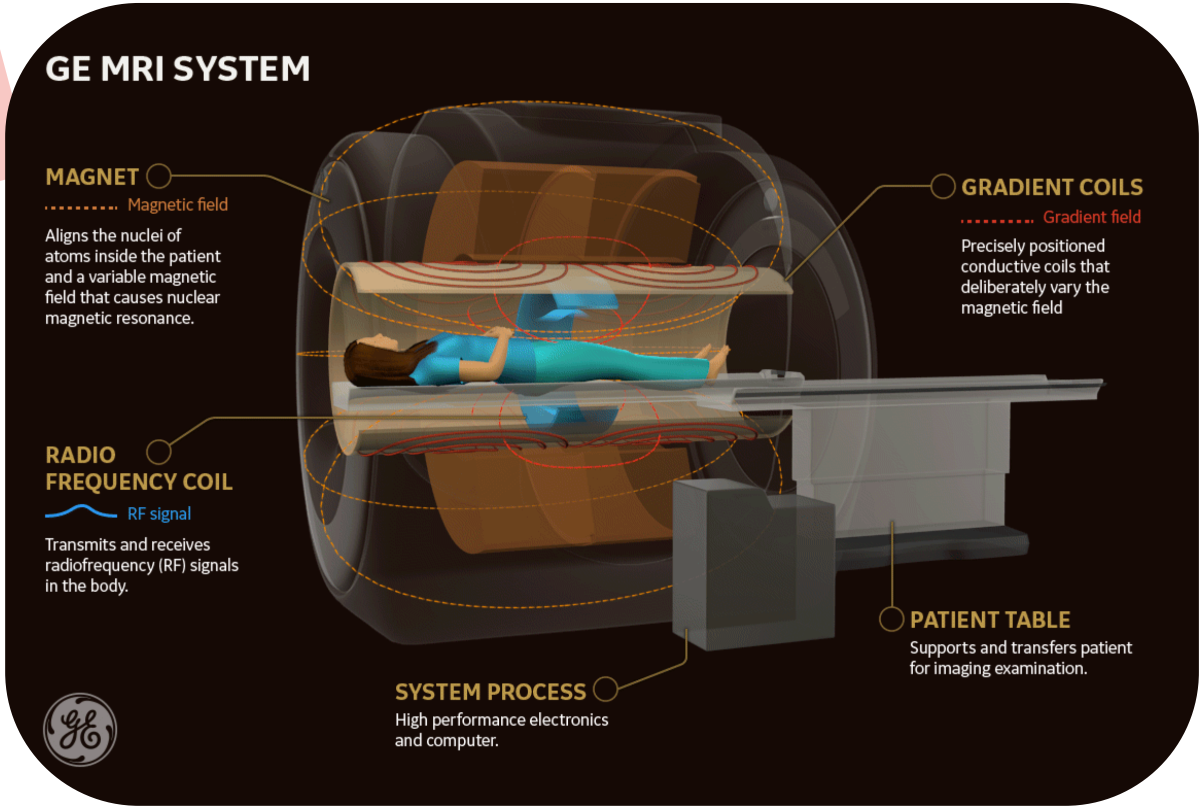
We can add more gradients, and with a dash of the secret ingredient (tomography) we can produce 3D resolved maps of



- Larmor frequency
- T_1
- T_2
- T_2^*
- Spin density

⇒ We need to understand these!

Tomography is the secret sauce!



Shilling for GE

Tissue Type	T1 (ms)	T2 (ms)
Bone	0.001–1	0.001–1
Muscle	460–650	26–34
Fat	180–300	47–63
Body fluid	1,000–2,000	150–480
White matter	220–350	80–120
Gray matter	340–610	90–130

Table of decay times for H-1 for body tissues

MRI contrast

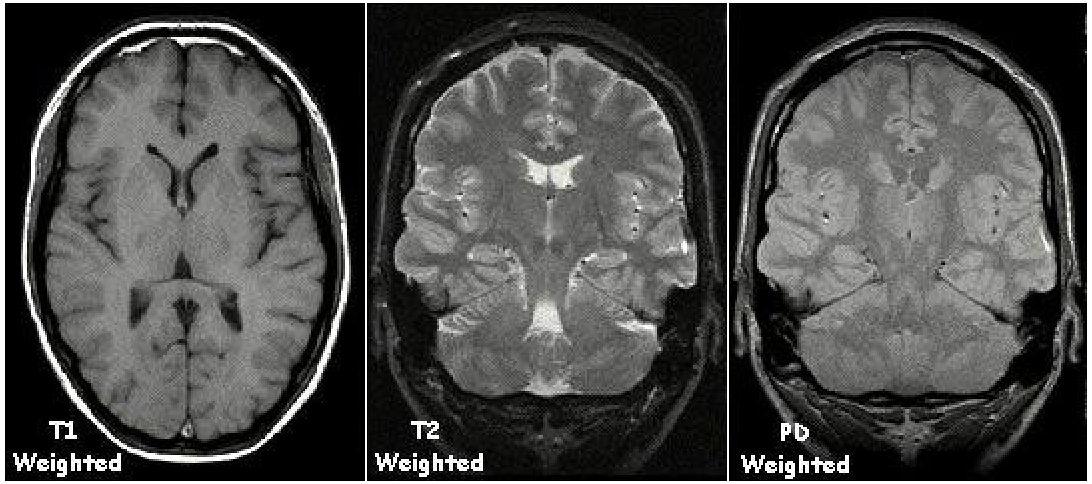
T_1 times reflect the spontaneous decay rate, which is altered by the decay pathways available (e.g. phonon emission)

T_2 times indicate the stochastic interaction between the surrounding environment (spin-spin), so is *longer* in fluids

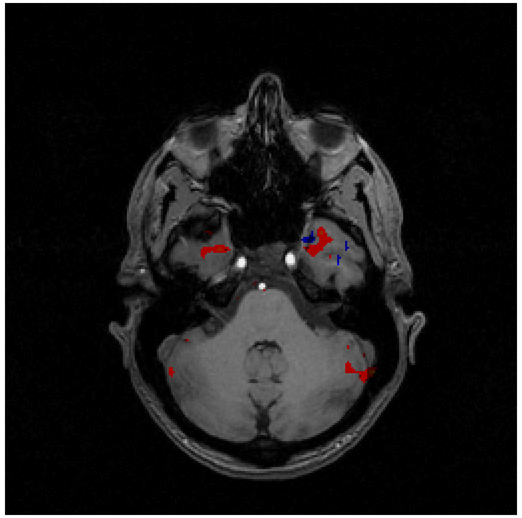
T_2^* times indicate the local magnetic environment and thus are good for differentiating paramagnetic components

Proton density mapping is exactly what you would expect: the density of H-1

Understanding brings power!



Different contrast mechanisms available with MRI



Functional MRI (fMRI) uses the magnetic differences between oxyhaemoglobin and deoxyhaemoglobin to map brain activity

UNIVERSITY of TASMANIA

Summary

- Decay mechanisms
  - Dephasing of atoms does not destroy the coherence between states
  - Atoms can be rephased using coherent manipulation of the atomic population, e.g. Hahn spin echo
  - The decay rates T_1, T_2, and T_2^* tell us different things about the system and its interaction with the environment

- Magnetic resonance
  - A resonance and transitions between magnetic substates can be induced in a uniform magnetic field subjected to magnetic dipole radiation
  - The application of magnetic field gradients correlates the Larmor frequency with position, enabling magnetic resonance imaging

Problems