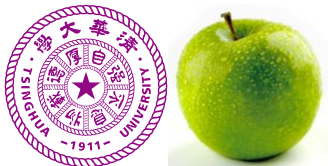


# Segment Tree

$$n + e$$

Tsinghua University

2016 年 7 月 16 日



# Introduction

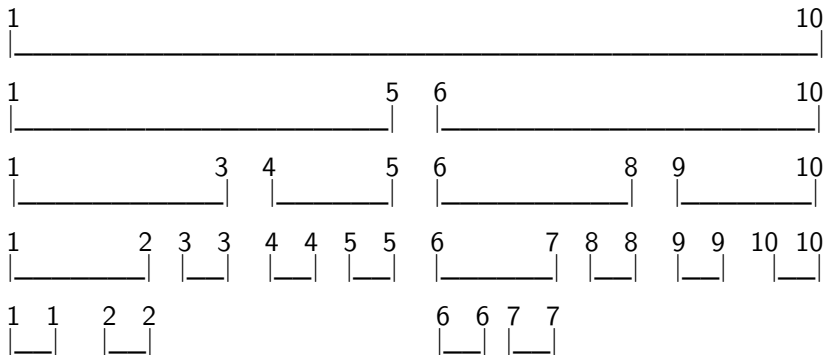
- 给你一段长度为  $N$  的序列  $A[]$ , 求:
  - ① 单点修改, 单点查询;
  - ② 单点修改, 区间查询;
  - ③ 区间修改, 单点查询;
  - ④ 区间修改, 区间查询;
- $N \leq 100000$
- 修改操作包括但不限于  $+$   $C$ ,  $*$   $C$ , 强制  $= C$ , .....
- 查询操作包括但不限于  $\max$ ,  $\min$ ,  $\text{sum}$ , .....

## ② Application



## 举个栗子

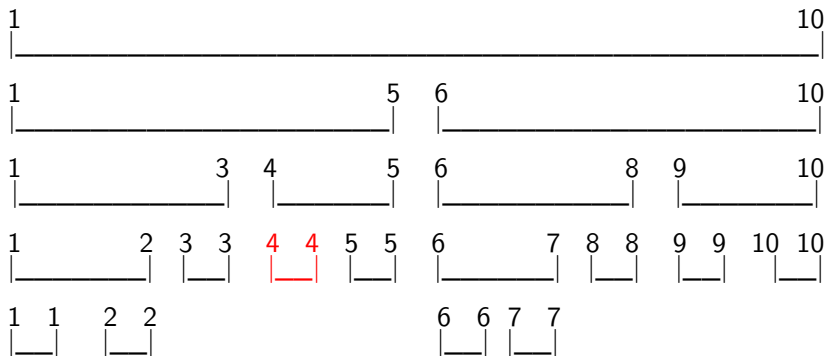
假设  $N=10$



- ```
void Build_Tree(int o, int l, int r) {
    if (l == r) { sum[o] = a[l]; return;}
    int mid = l + r >> 1;
    Build_Tree(o << 1, l, mid);
    Build_Tree(o << 1 | 1, mid + 1, r);
    sum[o] = sum[o << 1] + sum[o << 1 | 1];
}
```

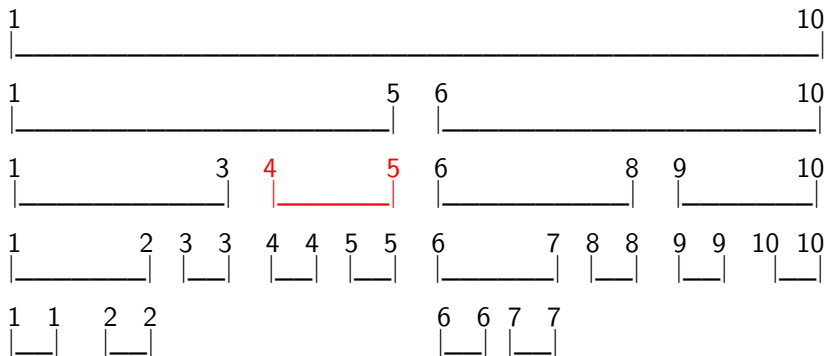


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1 10

1 5 6 10

1 3 4 5 6 8 9 10

1 2 3 3 4 4 5 5 6 7 8 8 9 9 10 10

1 1 2 2 6 6 7 7

- 易知单点修改复杂度为  $\log N$
- 单点查询类似

```
void Update(int o, int l, int r) { // A[x] = y
    if (l == r) { sum[o] = y; return; }
    int mid = l + r >> 1;
    if (x <= mid) Update(o << 1, l, mid);
    else Update(o << 1 | 1, mid + 1, r);
    sum[o] = sum[o << 1] + sum[o << 1 | 1];
}
```

## ① Introduction

长啥样?

修改

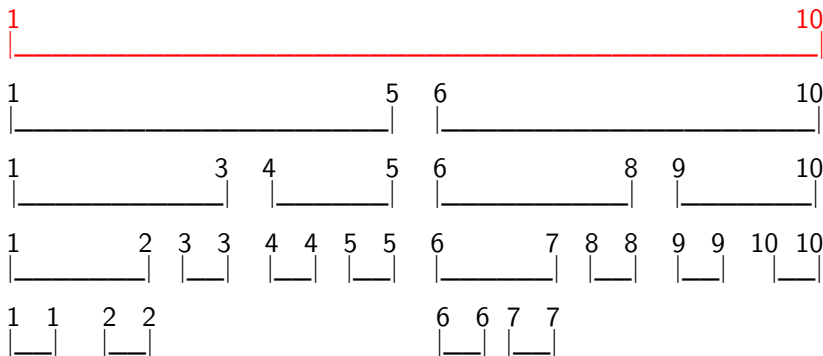
**查询**

Lazy tag

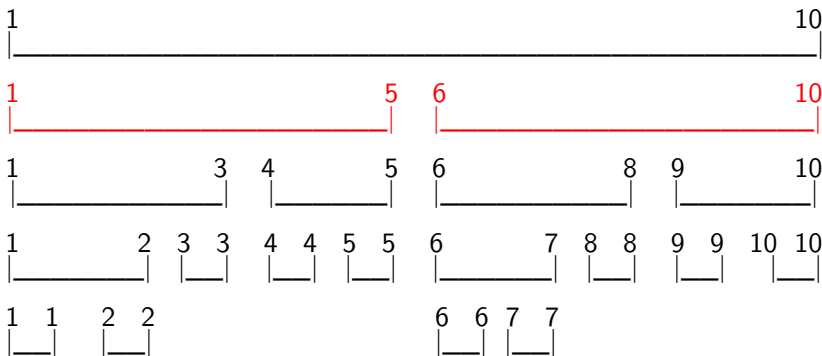
代码

## ② Application

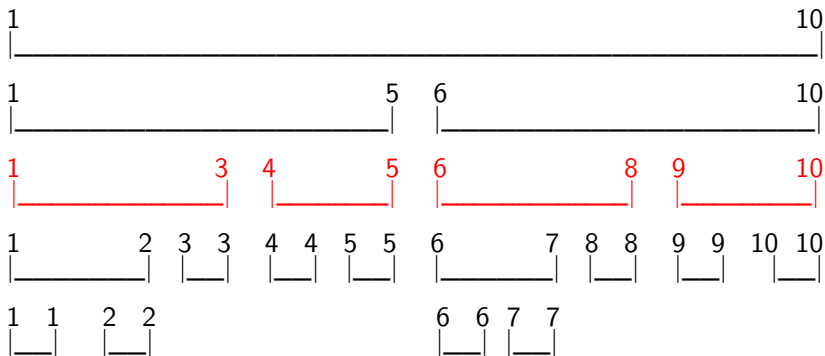
查询 A[2..9]



查询 A[2..9]

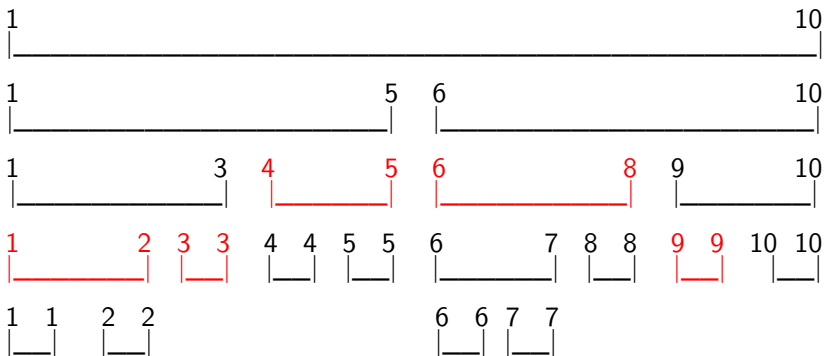


查询  $A[2..9]$

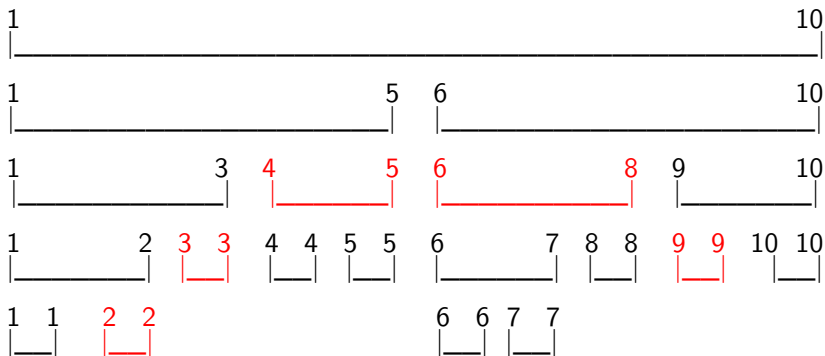




1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 26



查询  $A[2..9]$



- Q: 为什么不直接查  $[2,9]$ ? A: 因为没有  $[2,9]$  这段区间啊 ...
- 易知能够通过访问不超过  $2 \cdot \log N$  个线段树上的区间来获得任意区间  $[l,r]$  的答案

```
void Query(int o, int l, int r) { // A[x..y]
    if (x <= l && r <= y) { ans += sum[o]; return; }
    int mid = l + r >> 1;
    if (x <= mid) Query(o << 1, l, mid);
    if (mid < y) Query(o << 1 | 1, mid + 1, r);
}
```

- 区间修改类似

- Q: 为什么不直接查  $[2,9]$ ? A: 因为没有  $[2,9]$  这段区间啊 ...
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    if (mid < y) Query(o << 1 | 1, mid + 1, r);
}
```

- 区间修改类似吗?

## 1 Introduction

长啥样?

修改

查询

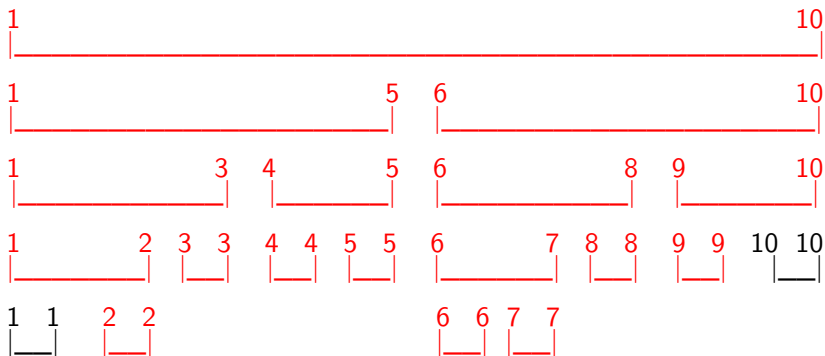
Lazy tag

代码

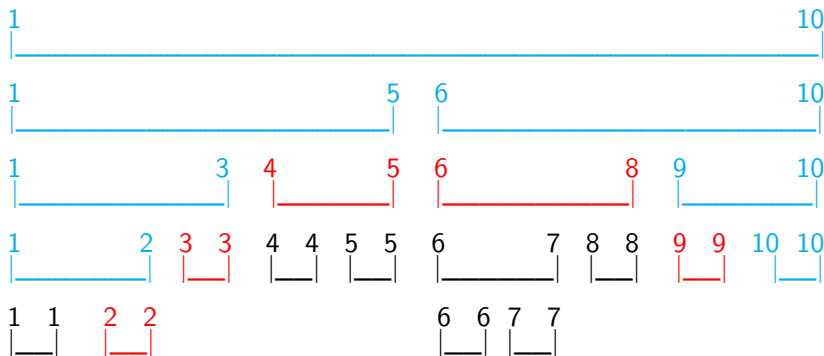
## 2 Application

- Lazy-Tag 记录的是每一个线段树节点的变化值
- 当这部分区间的一致性被破坏时, 就将这个变化值传递给子区间
- 每个节点存一个 Tag 值, 表示这个区间进行的变化
- 每当访问到某一个节点时, Tag 下传

如果把  $A[2..9]$  每个数都  $+C$ , 那么真正要修改的节点大概这么多



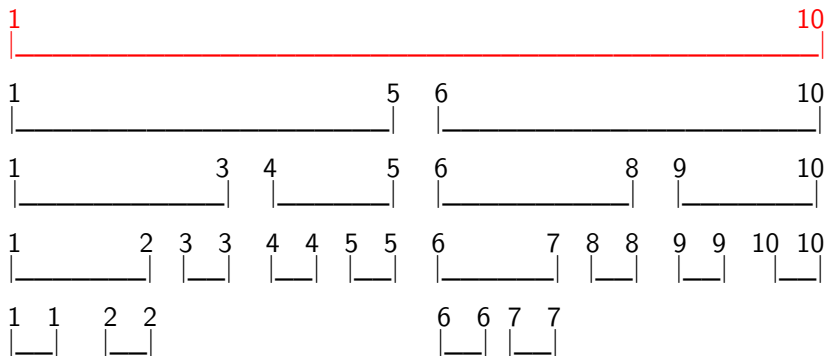
使用 Lazy Tag 以后, 情况是这样的:



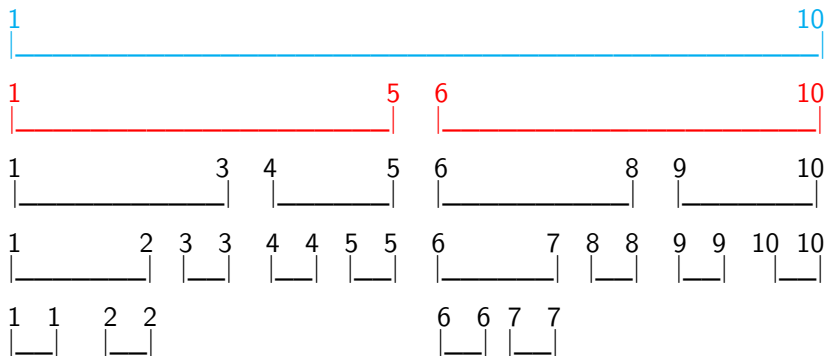


- 蓝色是要把信息及时维护的节点, 红色是本次区间修改操作  
Lazy Tag 下传停止的位置.

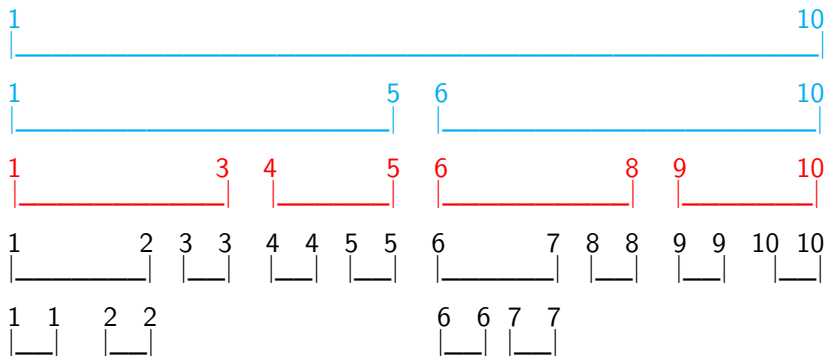
修改 A[2..9]



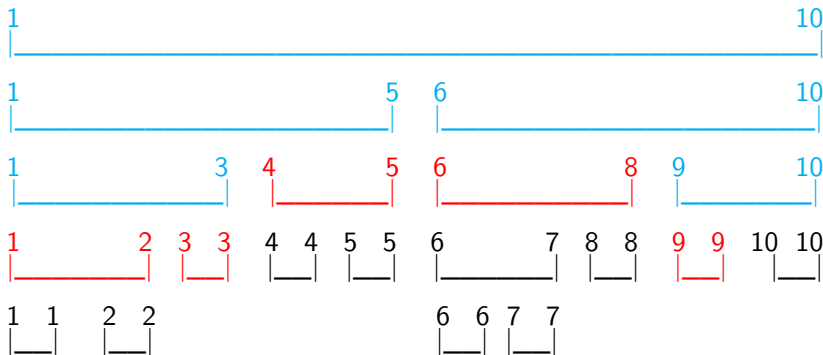
修改 A[2..9]



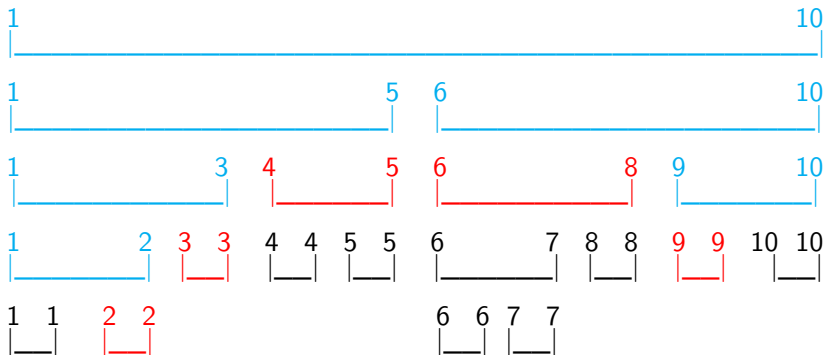
修改 A[2..9]



修改 A[2..9]

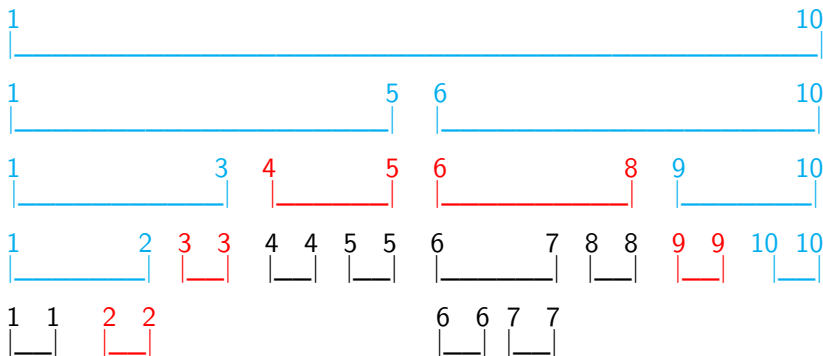


修改 A[2..9]



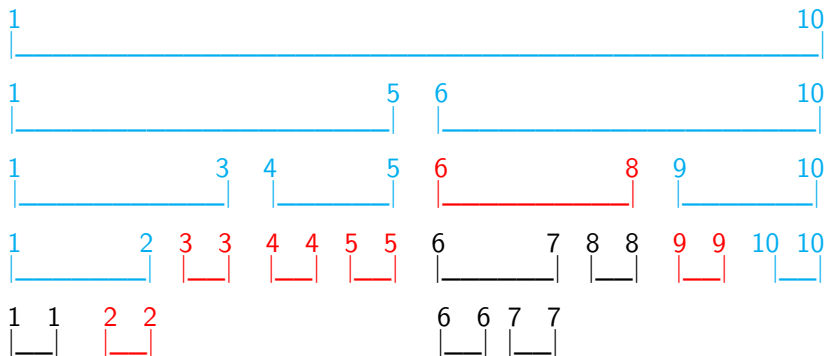
- 由于每一行最多只有两个蓝色区间和两个红色区间, 因此线段树区间修改的自带常数为 4.
- zkw 线段树: 满二叉树, 靠蓝色区间维护上去

要查询  $A[5]$

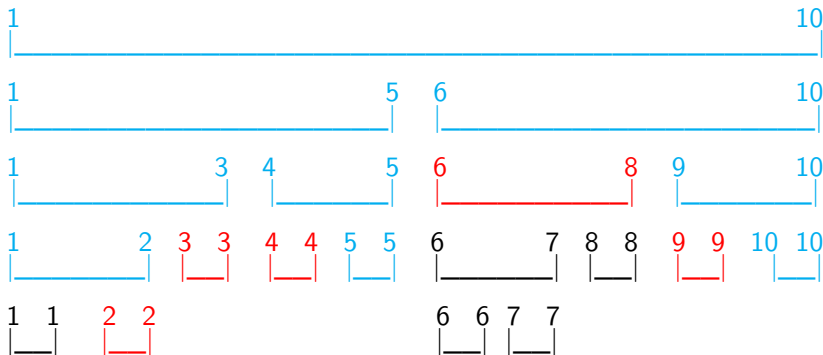




要查询  $A[5]$



要查询 A[5]



- 多个 Lazy Tag 咋办?
- 考虑打标记运算的优先级, 优先级高的先下传.

## 1 Introduction

长啥样?

修改

查询

Lazy tag

代码

## 2 Application

- 这里
- 线段树里面每个节点都要记录统计量和 Lazy Tag (修改量)
- 建议写相关的函数都传 3 个参: (int o, int l, int r)
- 网络上说线段数要开 4 倍空间, 实际上如果没写挂的话, 正常只要这样开:  
先把 N 补成 2 的幂次, 再  $\times 2$  即可

1000

1000

- 统计量可合并
- 修改量可合并
- 通过修改量可直接修改统计量





接下来的题目中如果 N 和 M 没说明范围, 默认 10w

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

请优化以下代码:

```
void operation1 (int l, int r, int c) {
    for (int i = l; i <= r; i++) a[i] = (a[i] + c) % p;
}

int operation2 (int l, int r) {
    int cnt = 0;
    for (int i = l; i < r; i++) cnt += a[i] > a[i + 1];
    return cnt;
}
```

- 差分
- 单点修改, 区间查询小于 0 的数的个数
- 不会做! 求大神指点迷津

## 1 Introduction

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

- 对于一个序列维护以下操作:
  - ① 修改某个  $a[i]$  的值
  - ② 输入  $l, r$ , 选出区间  $[l, r]$  中的所有  $a[i]$ , 问他们两两之差的和是多少, 差的平方和是多少.
- 答案  $\text{mod } 10^9 + 7$  输出

- 第一问答案是 0 hhhh
- 第二问拆式子, 发现要维护区间和、区间平方和

## 1 Introduction

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9



- 1853

- ① 询问  $A[l..r]$  中最大的数
- ② 删除  $A[x]$ , 并且  $x+1$  以后的元素整体前移
- ③ 在末尾增加一个数

- 用 Splay 也是可以的我并没有意见
- 用线段树记录每个位置的数是否存在, 存在则标为 1, 不存在则标为 0
- 当然相应的 l,r 也要修改
- 转成线段树中第 k 小的数是哪个, 在线段树上二分

```
if (k <= sum[o]) Query(o << 1, l, mid);
else k -= sum[o], Query(o << 1 | 1, mid + 1, r);
```

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

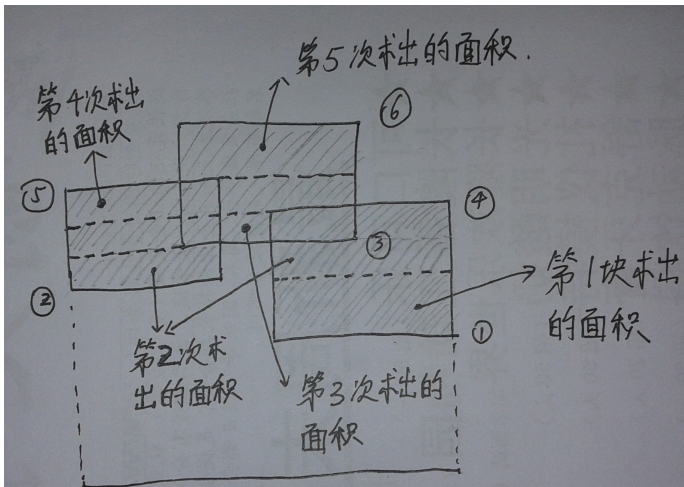
### 例题 9

- 二维平面上给出 N 个矩形, 求它们覆盖的总面积
- 1820

## 例题 4

## Solution

- 扫描线 灰不似窝的图!!! 窝的字木有喇么丑!!!



## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

- 给出  $A[]$ , 要求支持:
  - $A[x]=y$
  - 询问  $A[x..y]$  的最大子段和
- 1647

- 都是套路.
- 一段答案的存在方式只有三种: 完全被左区间包含 / 完全被右区间包含 / 跨过左右区间分界点
- 只要合并答案就好
- 维护 `sum` && 从左/右端点开始的最大子段和, 答案可顺便维护



## 1 Introduction

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

### 例题 6

- 对长度为  $n$  的数列进行  $m$  次操作, 操作为:
  - $a[l..r]$  每一项都加一个常数  $C$ , 其中  $0 \leq C \leq 10^{11}$
  - 求  $F[a[l]] + F[a[l+1]] + \dots + F[a[r]] \bmod 10000$  的余数
- 其中  $F[i]$  表示斐波那契数列. 即  $F[0] = F[1] = 1$ ,  
 $F[n+2] = F[n+1] + F[n]$ .

- 对于每个位置, 保存  $F[a[i]]$  和  $F[a[i]+1]$
- 矩阵乘法具有分配律, Lazy Tag 只要记录幂次即可
- 预处理出循环节, 就不用每次都来矩阵快速幂了

## 1 Introduction

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

### 例题 7

- 对长度为  $n$  的数列进行  $m$  次操作, 操作为:
  - 对  $i \in [l, r]$  执行:  $a[i] = a[i]^2$
  - 求  $\sum_{i=l}^r a[i] \bmod p$  的余数,  $p$  在程序开始运行时给出

- 找循环节
- 进了循环节后, 打上在循环节整体移动的 Lazy Tag
- 如果还没进, **暴力**递归修改, 根据均摊复杂度的那套理论, 这一部分复杂度不会超过  $O(N\log N)$

## 1 Introduction

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

### 例题 8

- 有一个  $2*n$  的点阵, 平行于坐标轴的方向上相邻的点之间可以连边, 维护以下操作:
  - ① 在某相邻两点之间连边
  - ② 删除某条边
  - ③ 询问某两点是否连通
- BZOJ1018



- $$n + e$$

## ② Application

## 什么时候要用线段树?

### 例题 1

### 例题 2

### 例题 3

### 例题 4

### 例题 5

### 例题 6

### 例题 7

### 例题 8

### 例题 9

- 求  $A[]$  的一个最长子序列  $B[]$ , 满足  $B_i - B_{i-1} \leq d$ , 输出长度即可
- 听说  $A$  和  $d$  很大? 2442

- $$n + e$$