Lab 4: Channel Coding: Making Communication Reliable
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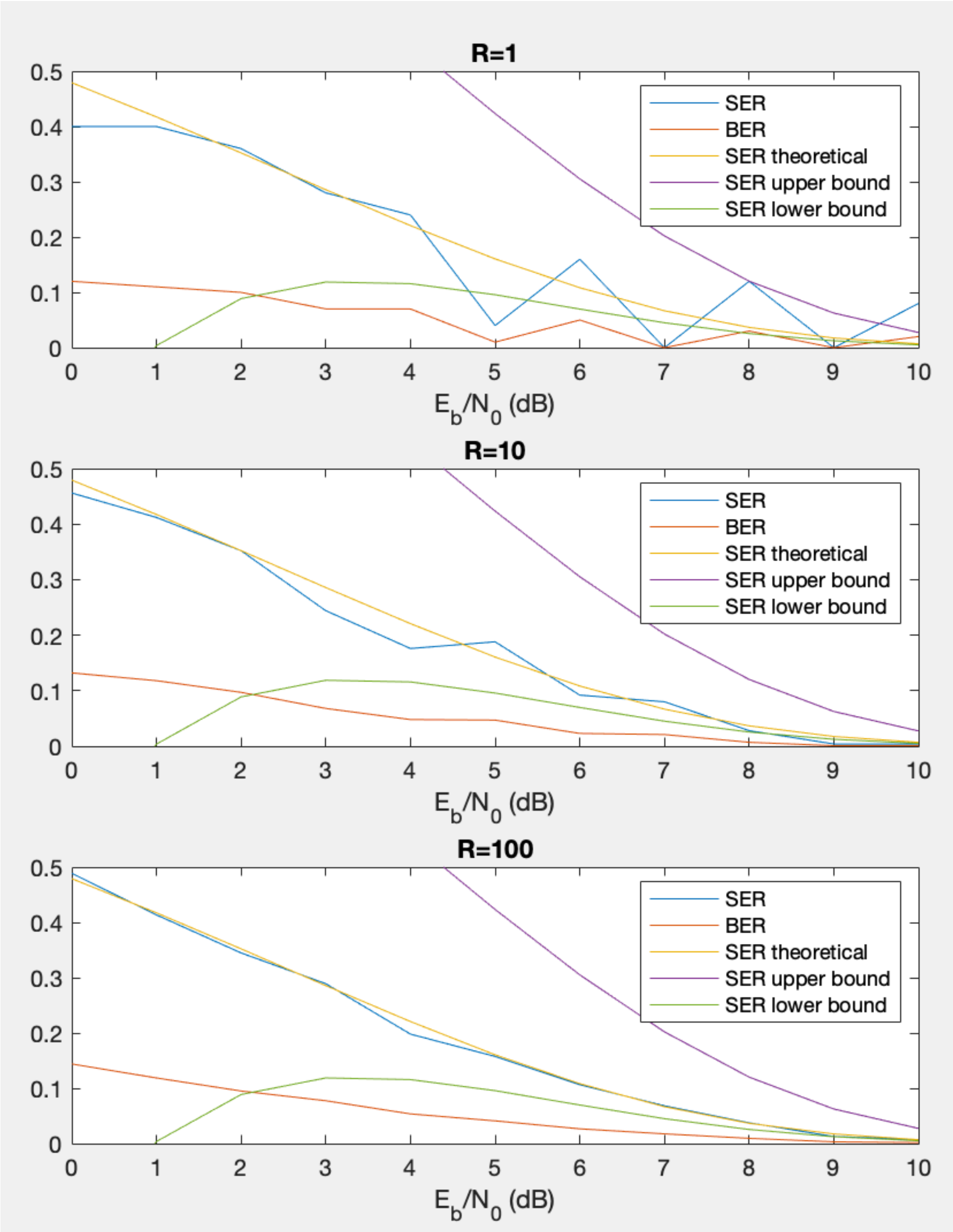
All the codes for this report is stored in https://github.com/Andy19961017/Communication_System_Lab/tree/master/Lab4

Problem 1 is done by src_1.m. Problem 4 is done by src_4.m

1.
Plot for (a) and (c):
To reduce computation time, I used n=100 an R={1,10,100} instead.



(a)

First of all, when R increases, the curve of SER and BER fluctuate less due to the law of large number.

Secondly, both SER and BER decrease when $\frac{E_b}{N_0}$ increases.

Lastly, SER is always 3~4 times the value of BER. This is because for gray mapping (used in this case), the probability of having two bit errors within one symbol is quite low. Thus, having one symbol error ‘usually’ implies one bit error. Since the sequence length of bits is 4 times the length of symbols (for 16-QAM), SER is about 4 times of BER.

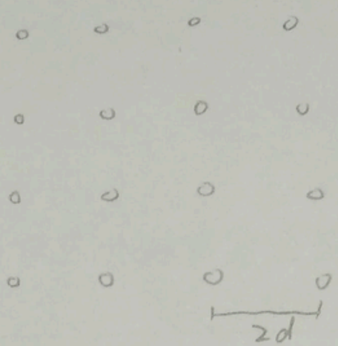
(b)

Let $\frac{E_b}{N_0} = k$

$E_s = \frac{2(16-1)d^2}{3} = 10d^2$

$E_b = \frac{E_s}{4} = \frac{5}{2}d^2$

$k = \frac{5d^2}{2N_0}$, $N_0 = \frac{5d^2}{2k}$



Let $Q\left(\frac{d}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{4k}{5}}\right) = Q$

$$SER = \underbrace{\frac{4}{16}\left(Q+Q-Q^2\right)}_{\text{corner}} + \underbrace{\frac{8}{16}\left(3Q-2Q^2\right)}_{\text{edge}} + \underbrace{\frac{4}{16}\left(2Q+2Q-4Q^2\right)}_{\text{interior}}$$

$$= 3Q - \frac{9}{4}Q^2$$

theoretical

Since $\left(1-\frac{1}{x^2}\right)\frac{1}{x\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right) \le Q(x) \le \frac{1}{2}\exp\left(-\frac{1}{2}x^2\right)$

$$\left(1-\frac{5}{4k}\right)\sqrt{\frac{5}{8k\pi}}\exp\left(-\frac{2k}{5}\right) \le Q\left(\sqrt{\frac{4k}{5}}\right) \le \frac{1}{2}\exp\left(-\frac{2k}{5}\right)$$

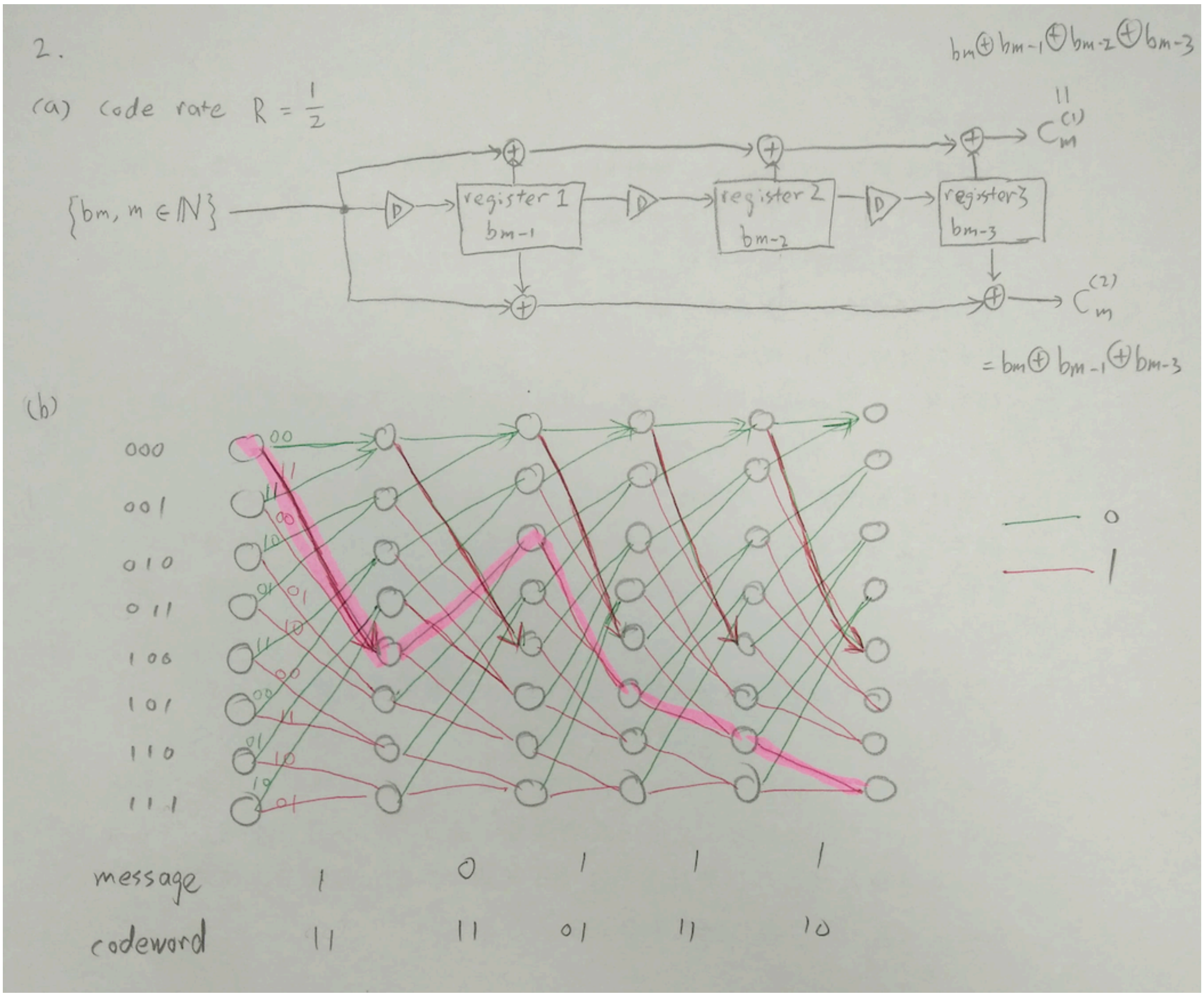
upper bound: $SER = 3Q - \frac{9}{4}Q^2 \le \frac{3}{2}\exp\left(-\frac{2k}{5}\right)$

lower bound: $SER = 3Q - \frac{9}{4}Q^2 \ge Q \ge \left(1-\frac{5}{4k}\right)\sqrt{\frac{5}{8k\pi}}\exp\left(-\frac{2k}{5}\right)$

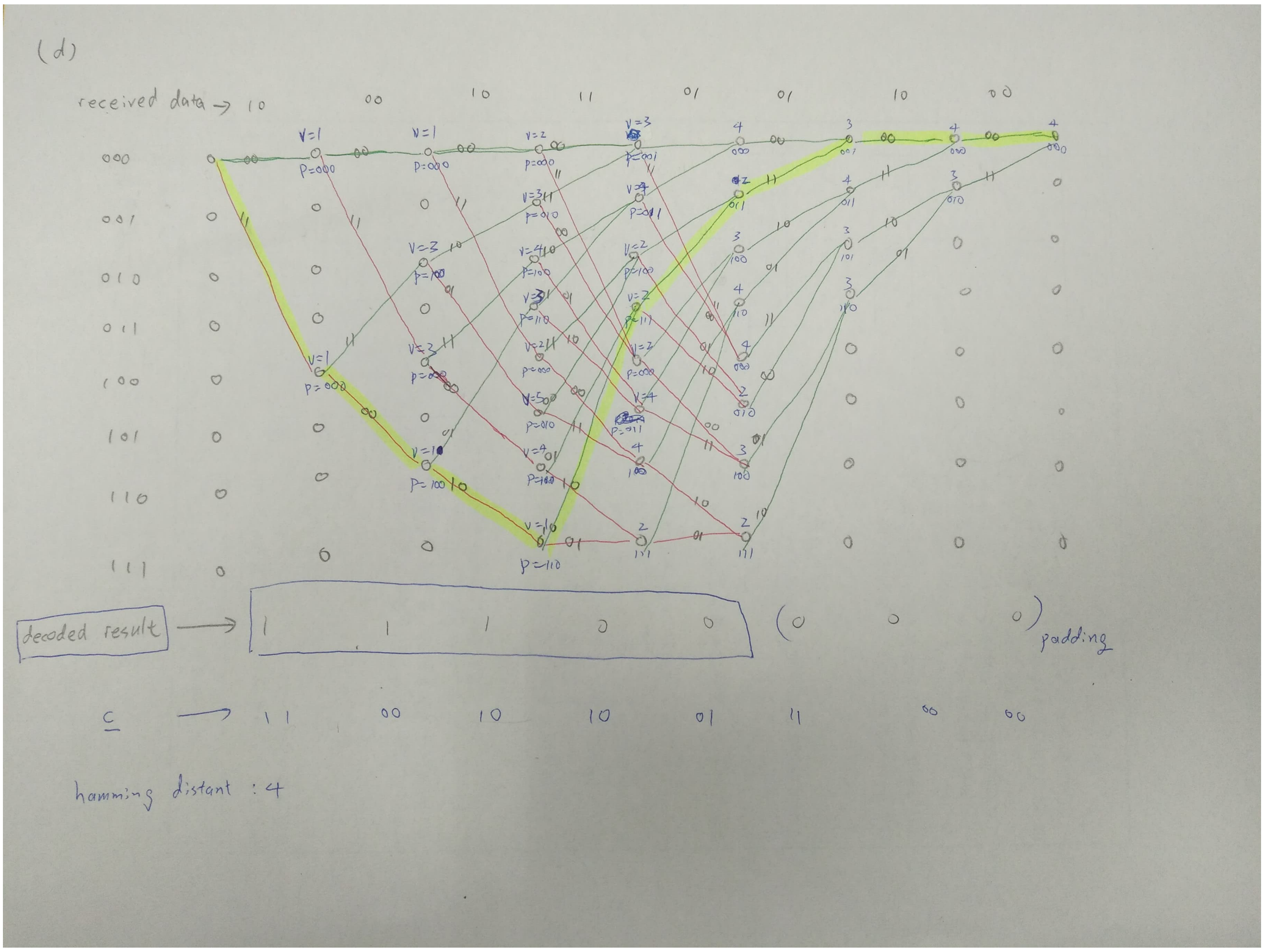
(for $Q \in [0,1]$)

(c)
Since the ‘exact’ value for q-function is used in this case (done by solving equation in matlab), the theoretical SER is very accurate. For large R, the experimental value almost perfectly fit the theoretical value.
The upper and lower bounds also successfully bound the experimental value for large R.

2.



(c)
The function is defined in convolutional_enc.m.

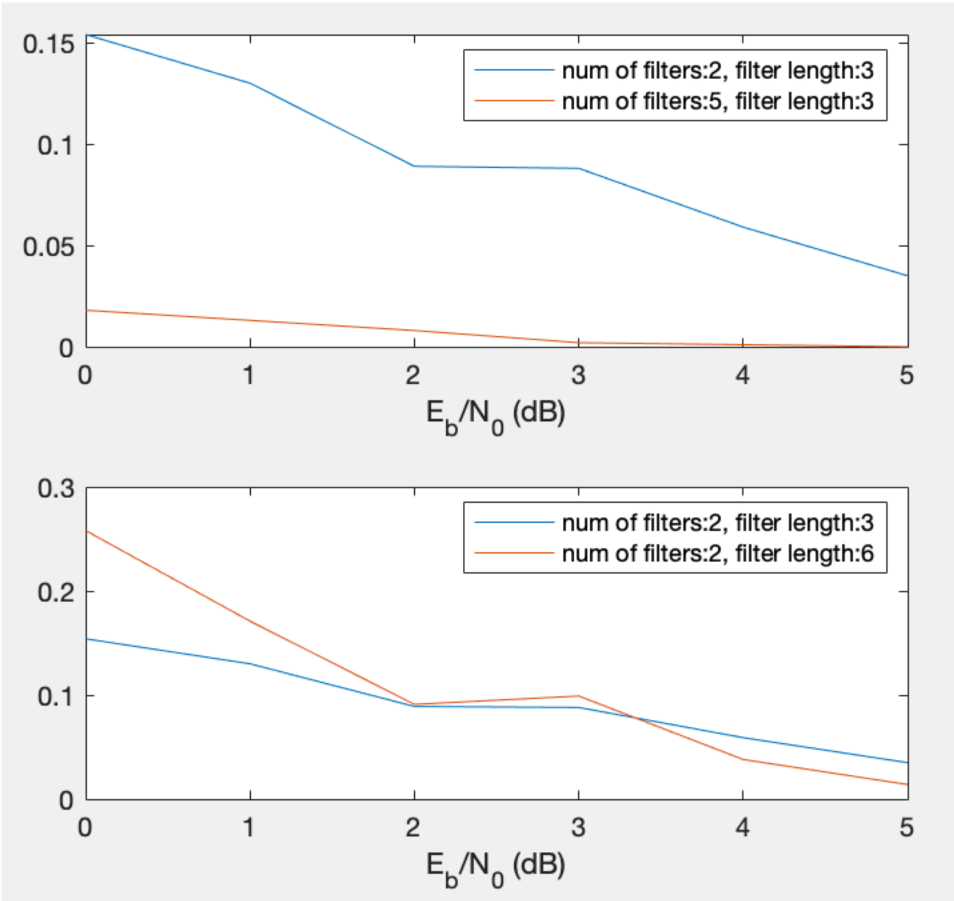


(e)
The function is defined in convolutional_dec.m.

3.

Number of quantizer	The music sound almost the same for 4+ bits quantizer. For number of bits less than 2, the signal becomes noisy still identifiable.
Constellation	Has no effect on the sound under noiseless environment. Under noisy environment, gray mapping usually preserves better sound quality.
Huffman coding	Has no effect on the sound under noiseless environment.
Error correction code	Has no effect on the sound under noiseless environment.
SNR	The sound becomes noisy under low SNR.

4.**Effect of filter length and filter number of convolutional code on error rate(bonus)**
When we implement convolutional code, the more filter we use, the smaller error rate we get with data rate being the trade-off. Also, zero error rate is reached only if the data sending rate R is smaller than the channel capacity and filter length goes to infinity.
Here, I plot the error rate under different SNR for different filter parameters(number of filter and length of filter). Each time, I randomly generate 500 bits to encode, to add noise, and to decode. The filter is also randomly generated with each bit in the filter being iid Ber(0.5).
In the first plot, we can see significant improvement on BER when we increase the number of filters. In the second plot, we can see that increasing the filter length have no significant benefit.



Reference

https://www.mathworks.com/matlabcentral/fileexchange/25859-convolutional-encoder-decoder-of-rate-1-n-codes