Solving Wordle using Information Theory

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1 Task 6

Suppose there are multiple guesses $g_1, g_2, ..., g_k \in W$ with the same minimum value for $\alpha_{g_1} = \alpha_{g_2} = ... = \alpha_{g_k}$. Then, let π be a distribution s.t. $\sum_{i=1}^{k} \pi(g_i) = 1$ and $\pi(g_i) \geq 0$. Note that $\pi(g_j) = 0 \ \forall g_j \in W$ s.t. $g_j \neq g_i$. Additionally, let $\alpha_\phi = \alpha_{g_i}$ for any $i \in [1, .., k]$. Consider:

$$\frac{1}{|W|} \sum_{g \in W} \pi(g)\alpha_g = \frac{1}{|W|} \sum_{i=1}^{k} \pi(g_i)\alpha_\phi = \frac{\alpha_\phi}{|W|} \sum_{i=1}^{k} \pi(g_i)$$

$$= \frac{\alpha_\phi}{|W|}$$

Now, to show that π is optimal, we show that for an arbitrary distribution π^*, $\frac{1}{|W|} \sum_{g \in W} \pi^*(g)\alpha_g \geq \frac{1}{|W|} \sum_{g \in W} \pi(g)\alpha_g$. Let g_j be a guess such that $g_j \neq g_i \ \ \forall i \in [1, ..., k], \forall j \in [1, ..., t]$. Consider:

$$\frac{1}{|W|} \sum_{g \in W} \pi^*(g) = \frac{1}{|W|} (\sum_{i=1}^{k} \pi^*(g_i)\alpha_\phi + \sum_{j=1}^{t} \pi^*(g_j)\alpha_j)$$

But note that:

$$\sum_{j=1}^{t} \pi^*(g_j)\alpha_j = \sum_{j=1}^{t} \pi^*(g_j)(\alpha_\phi + \beta_j)$$

for some $\beta_j \geq 0$, because α_ϕ is minimum and ≥ 0. So:

$$= \sum_{j=1}^{t} \pi^*(g_j)\alpha_\phi + \sum_{j=1}^{t} \pi^*(g_j)\beta_j$$

Plugging this into the earlier expression:

$$\frac{1}{|W|} (\sum_{i=1}^{k} \pi^*(g_i)\alpha_\phi + \sum_{j=1}^{t} \pi^*(g_j)\alpha_j) = \frac{1}{|W|} (\sum_{i=1}^{k} \pi^*(g_i)\alpha_\phi + \sum_{j=1}^{t} \pi^*(g_j)\alpha_\phi + \sum_{j=1}^{t} \pi^*(g_j)\beta_j)$$

$$= \frac{1}{|W|}(\alpha_\phi(\sum_{i=1}^{k}\pi^*(g_i) + \sum_{j=1}^{t}\pi^*(g_j)) + \sum_{j=1}^{t}\pi^*(g_j)\beta_j) = \frac{1}{|W|}(\alpha_\phi + \sum_{j=1}^{t}\pi^*(g_j)\beta_j)$$

Because $\sum_{j=1}^{t}\pi^*(g_j) \geq 0$:

$$\geq \frac{\alpha_\phi}{|W|}$$