Revision

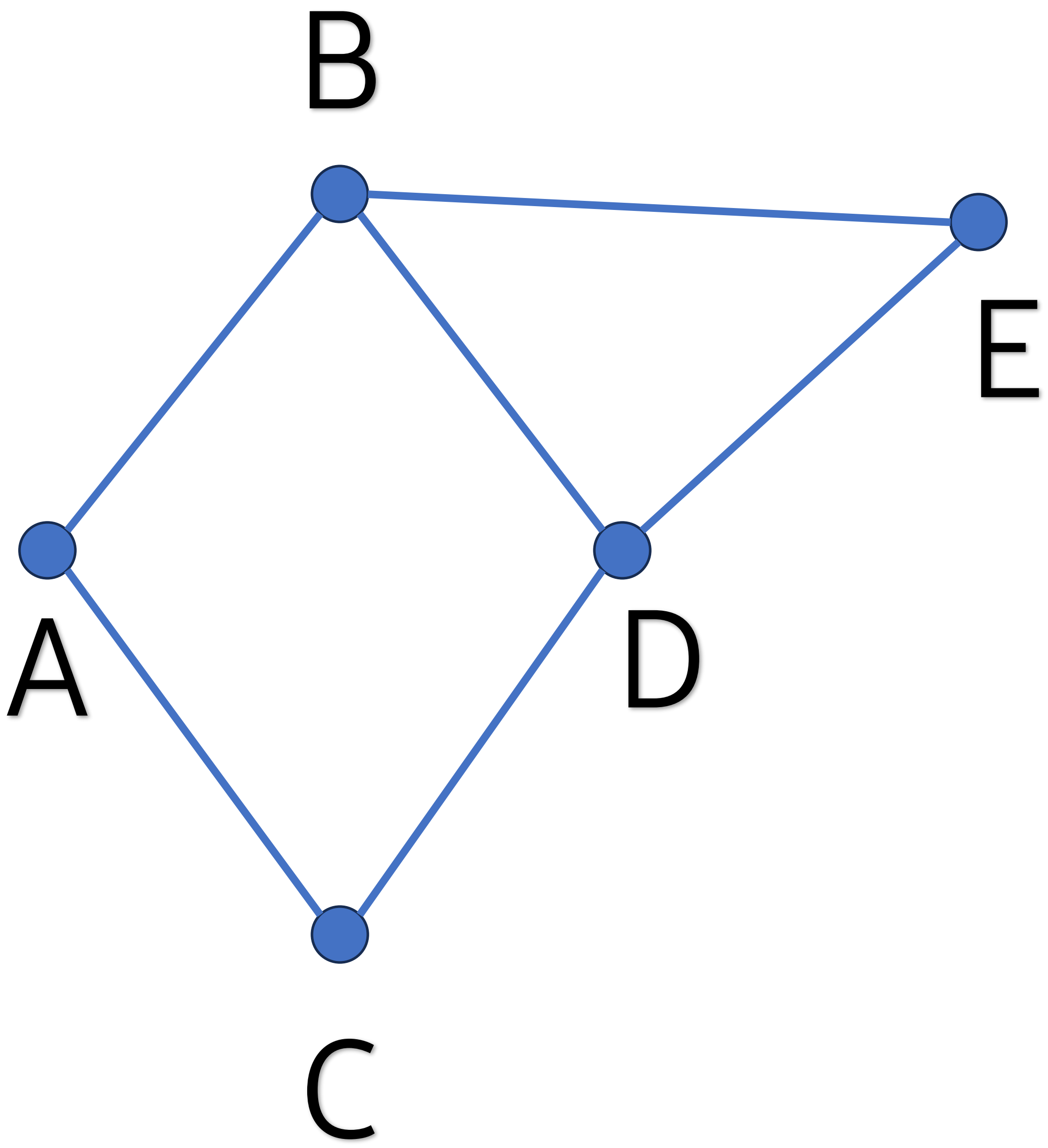
- BFS, DFS on paper: Sequence of tuples.
- Fleury: Algorithm for Euler path finding.
- Dijkstra: Shortest path finding.

Day 10: Minimum spanning tree search

Lecturer: Msc. Minh Tan Le

Today’s lesson

I. Negative cycle strategies: Ford-Bellman & Floyd
II. Minimum spanning tree concepts
III. Kruskal: Edge-based finding
IV. Jarnik – Prim: Vertice-base finding



1. Use DFS to find the path from A to E that passes 5 visited vertices.
2. Use Fleury to find the Euler path.

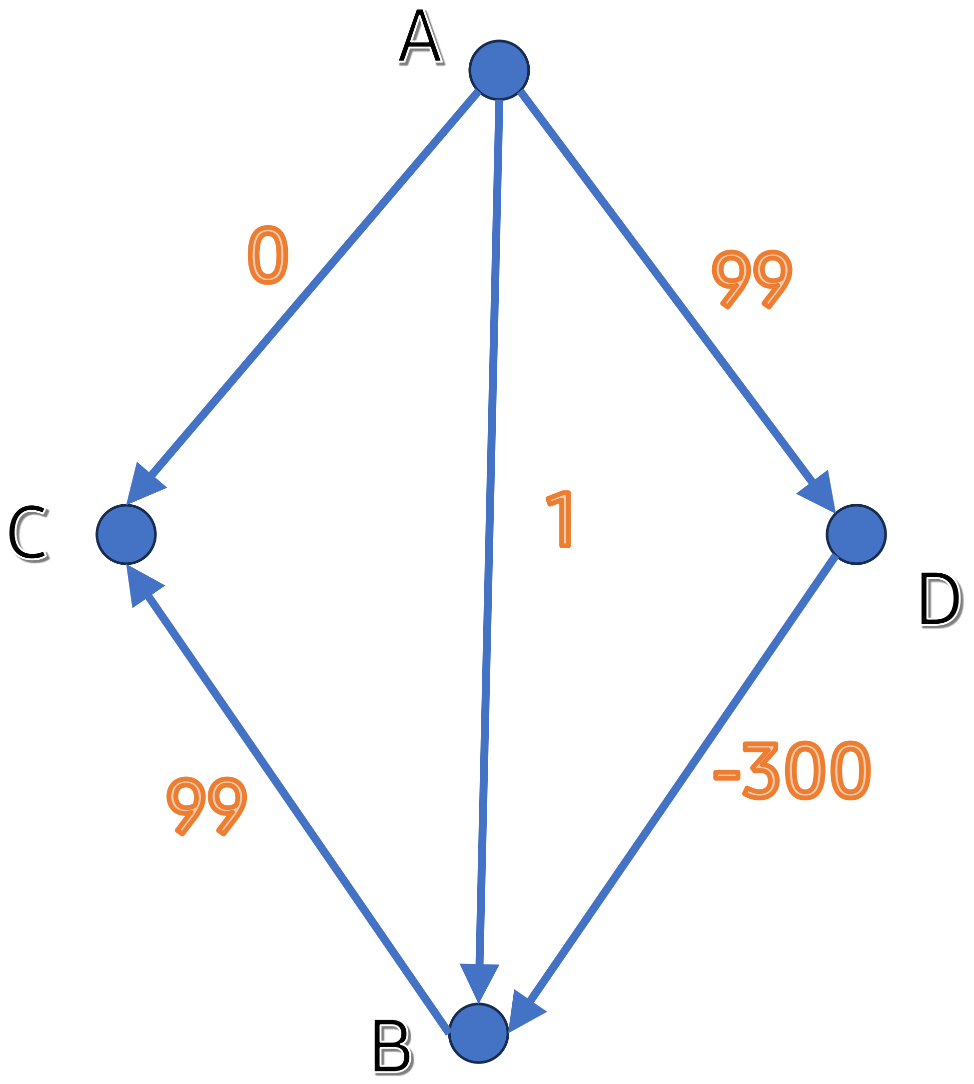
Note: Use tuple to demonstrate.

We need an algorithm that:

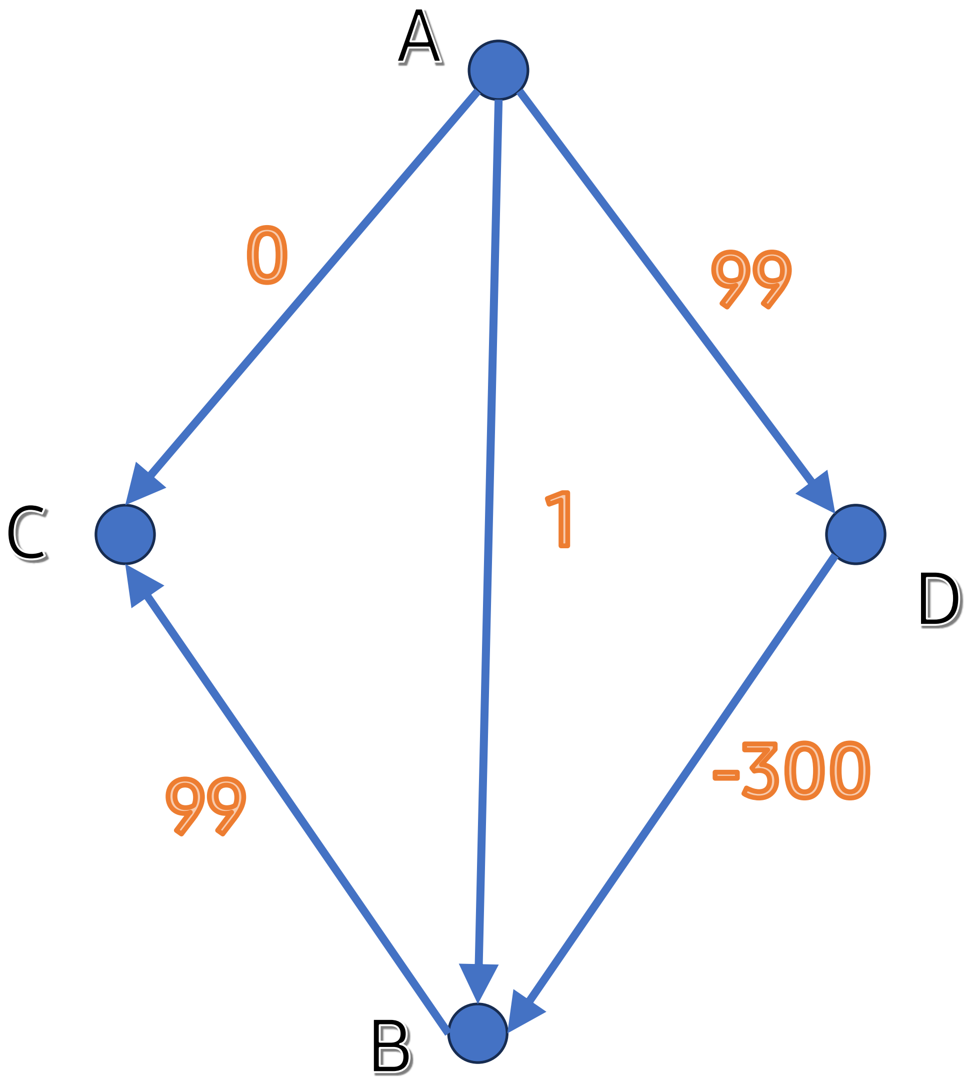
- Detect negative cycle.
- If there's no negative cycle, find the shortest path which may have negative weights.

Ford-Bellman

(Bellman-Ford)



$$D = \{0, \infty, \infty, \infty\}$$
$$P = \{Null, Null, Null, Null\}$$



$D = \{0, \infty, \infty, \infty\}$
$P = \{Null, Null, Null, Null\}$

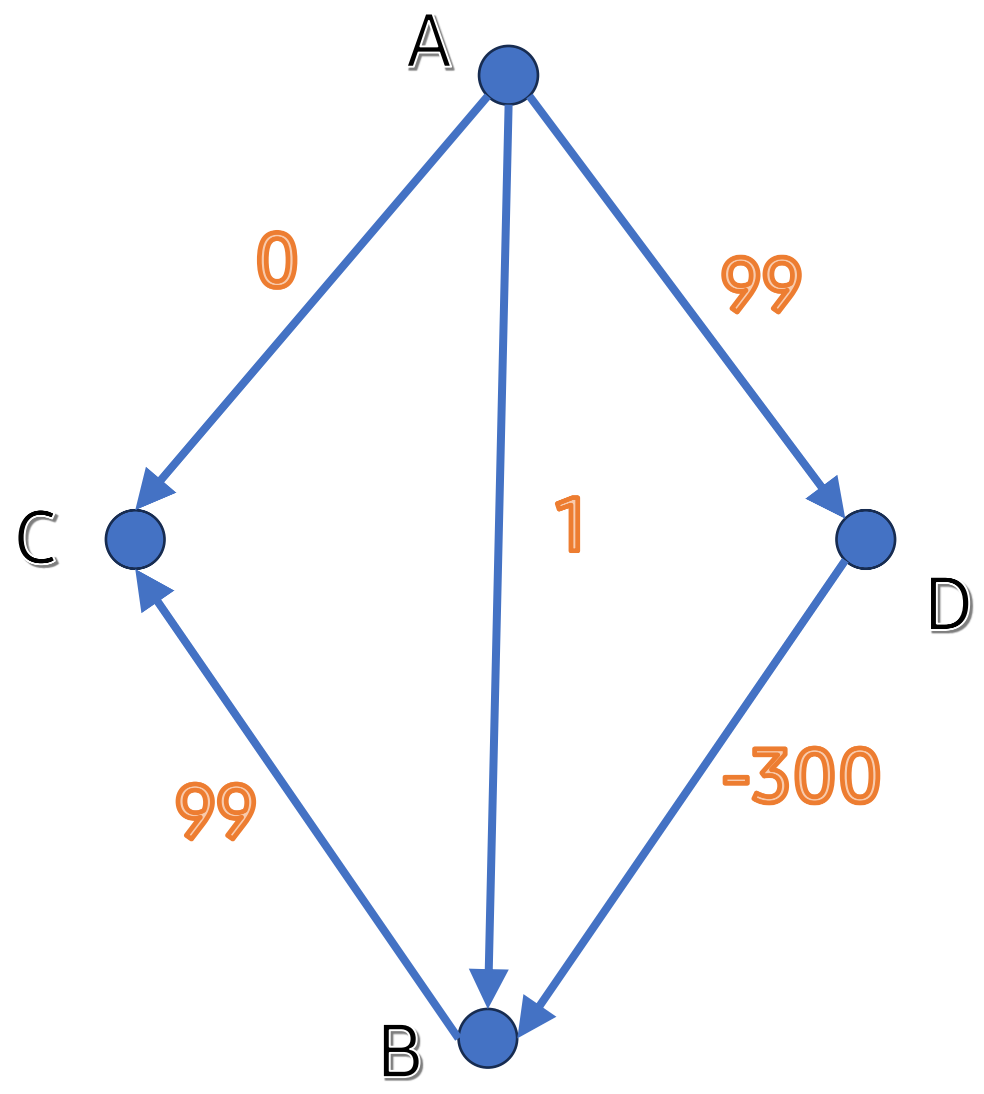
Step 1: For each edge (u, v) with weight w:

if $D[u] + w < D[v]$:

$D[v] = D[u] + w$

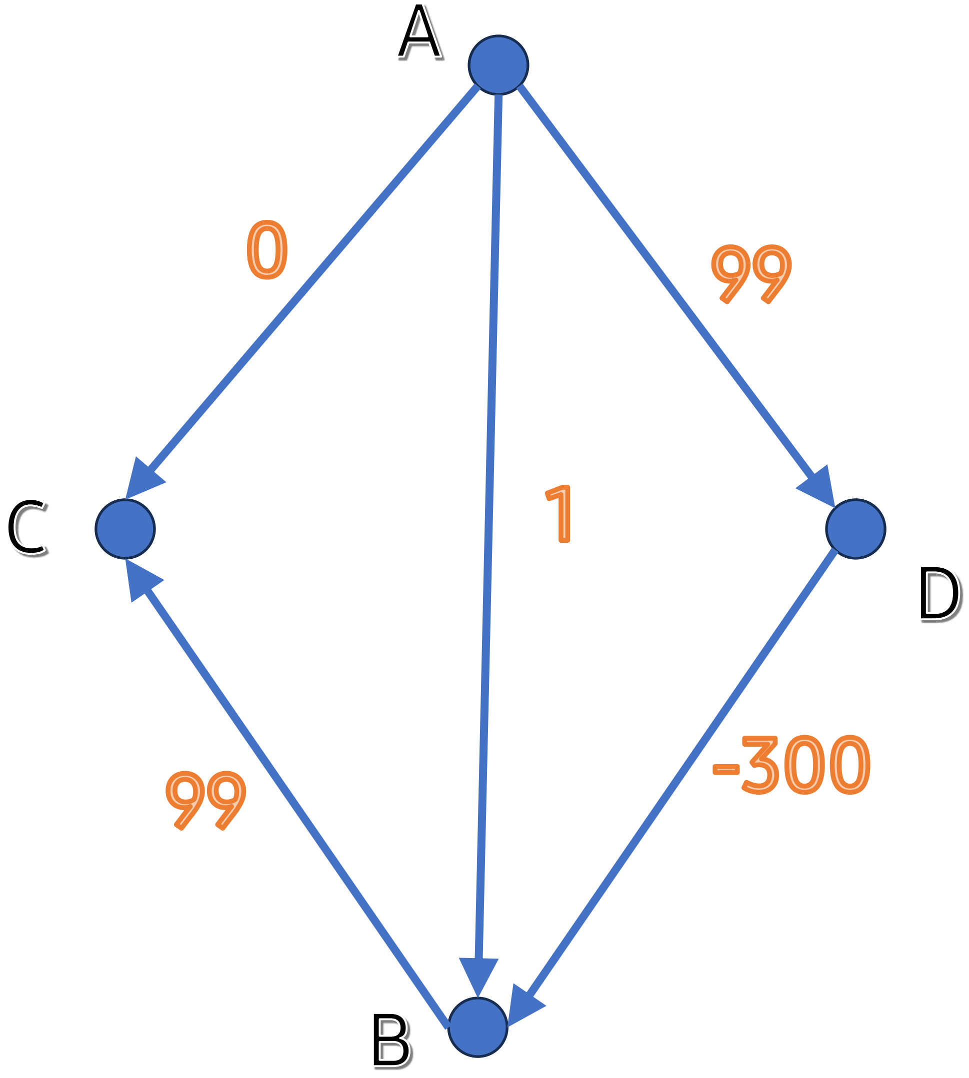
$P[v] = u$

Repeat step 1 $|V| - 2$ more times.



Step	A	B	C	D
0	0	-201	0	99
1	0	-201	-102	99
2	0	-201	-102	99

$$D = \{0, -201, -102, 99\}$$
$$P = \{Null, D, B, A\}$$



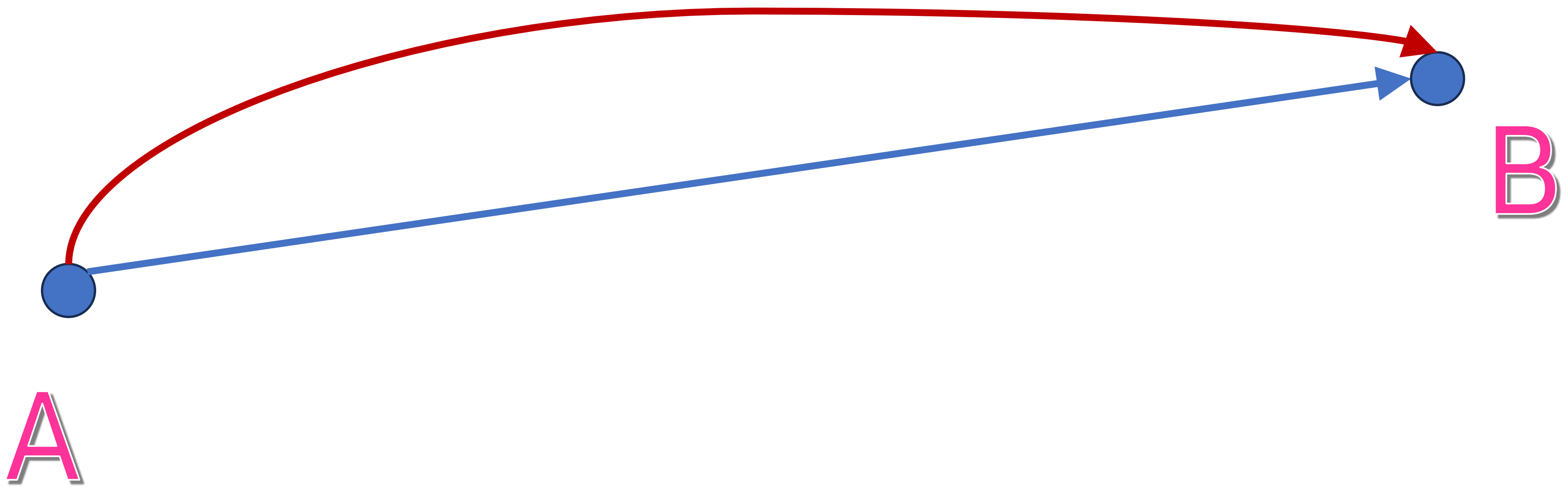
$$D = \{0, -201, -102, 99\}$$
$$P = \{Null, D, B, A\}$$

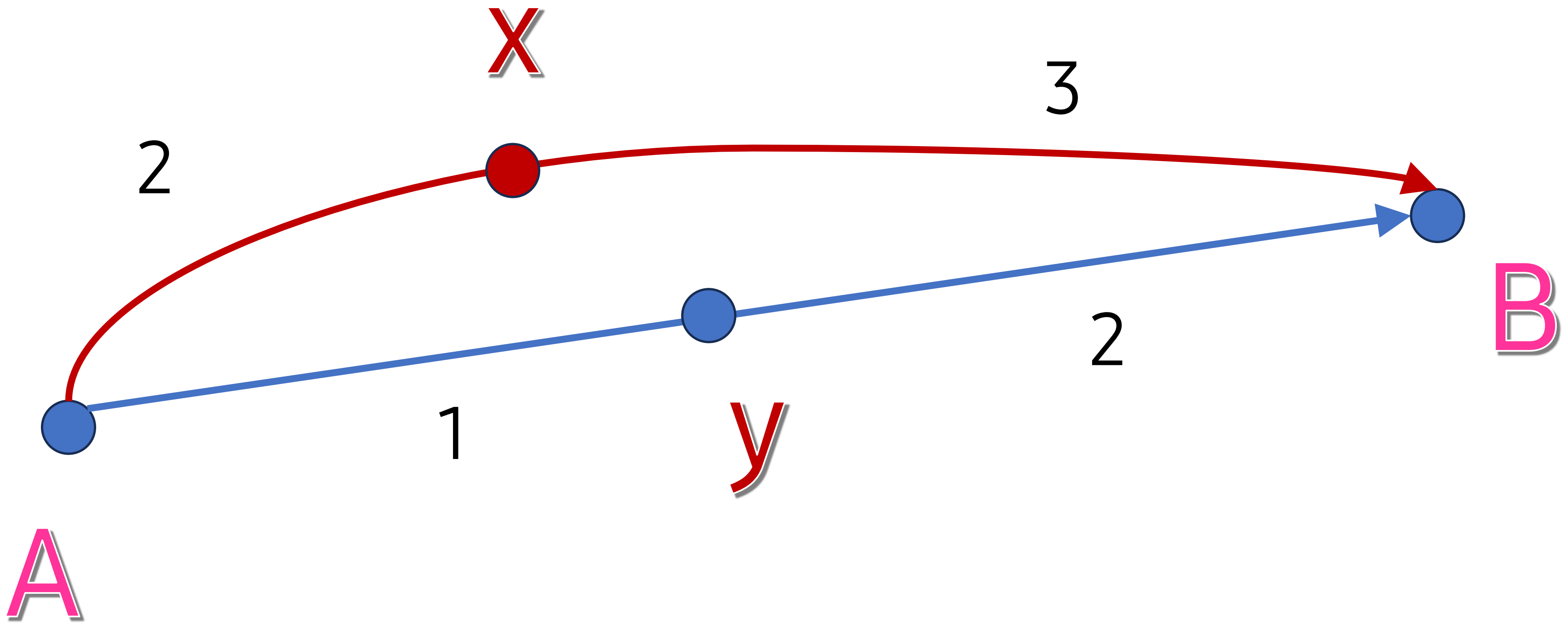
Step 2: For each edge (u, v) with weight w:

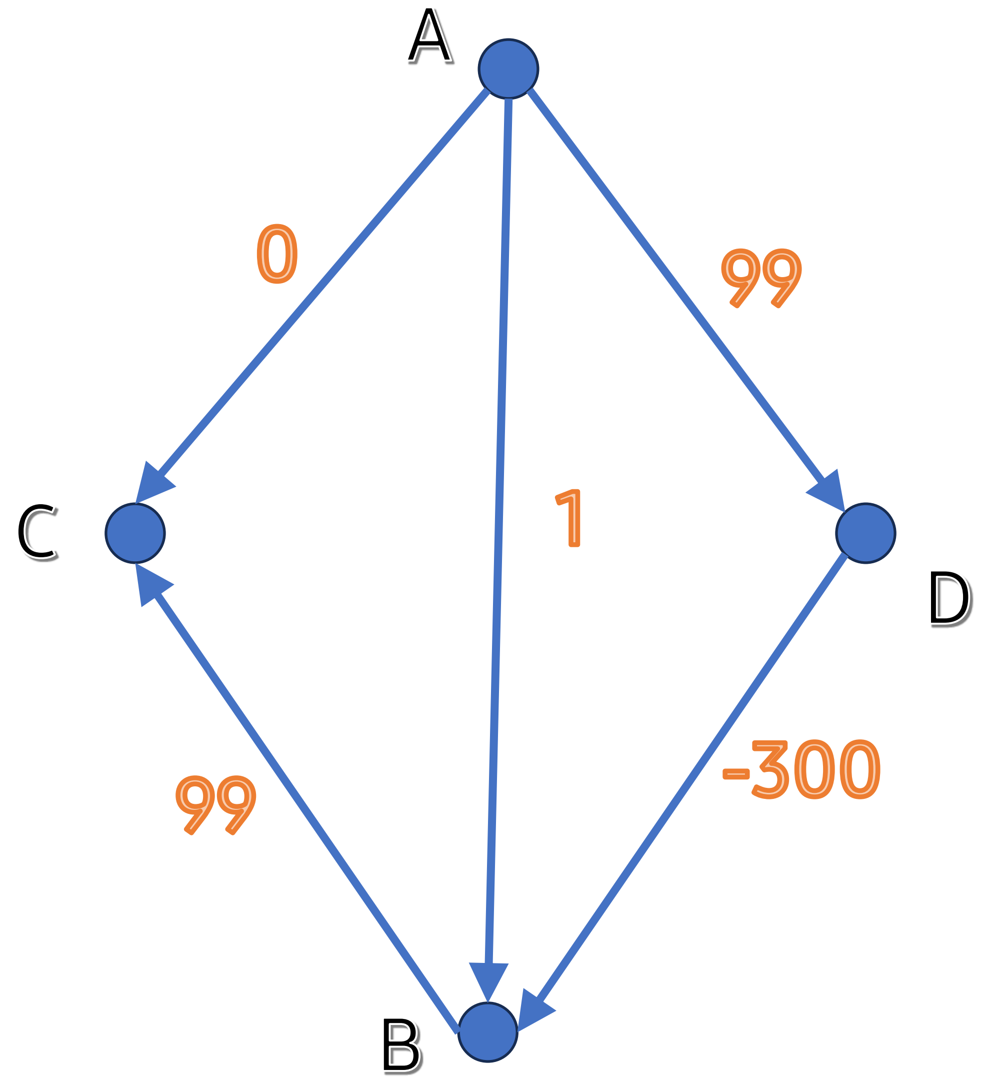
if $D[u] + w < D[v]$:

Error: Graph contains neg. cycle.

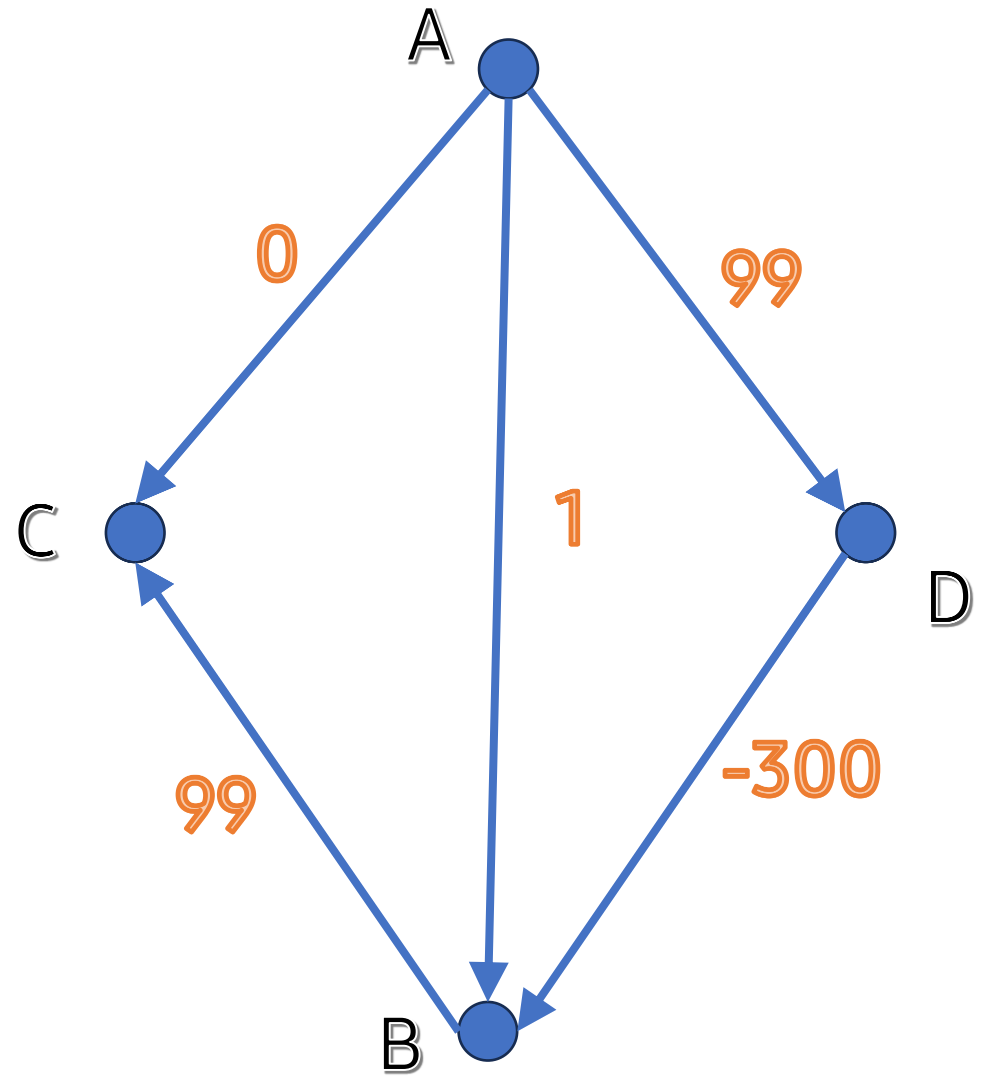
Floyd algorithm


A
B

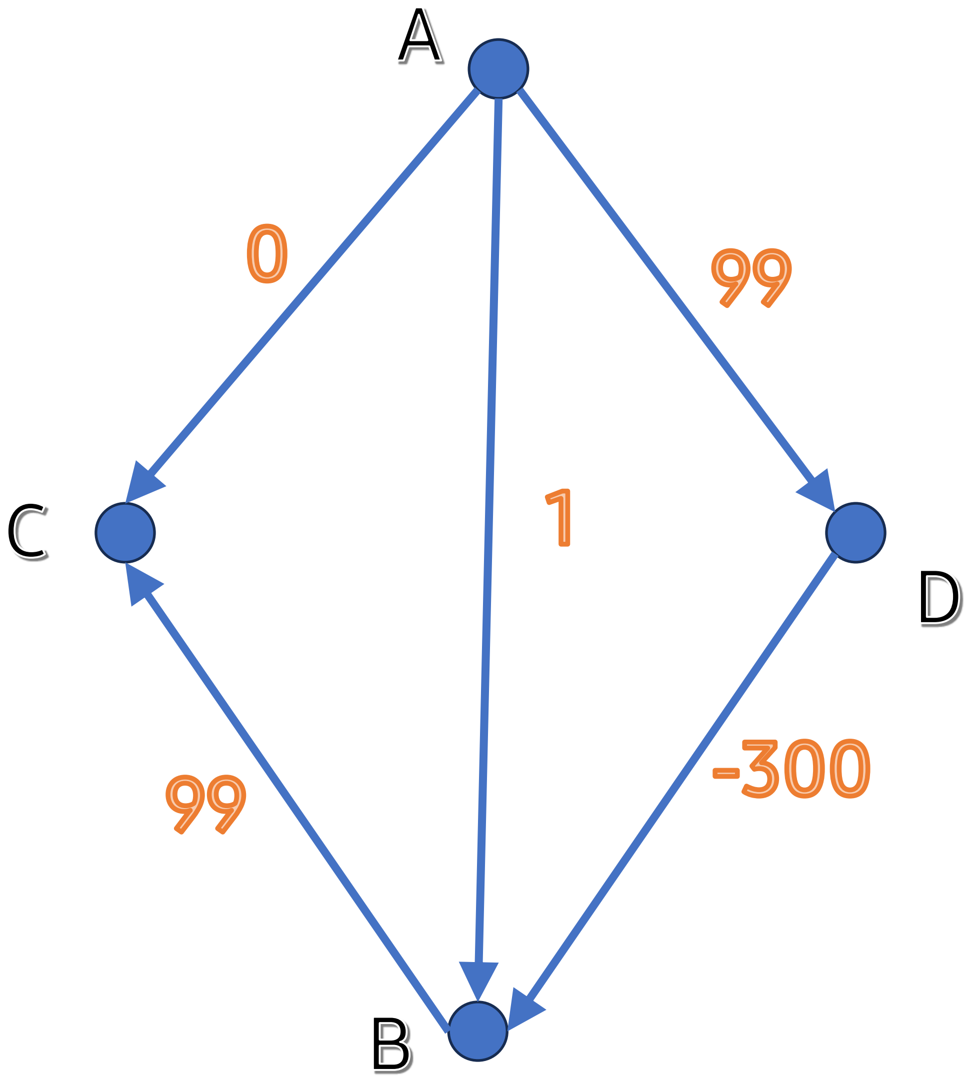

x
3
2
B
2
1
y
A



0	∞	∞	∞
∞	0	∞	∞
∞	∞	0	∞
∞	∞	∞	0



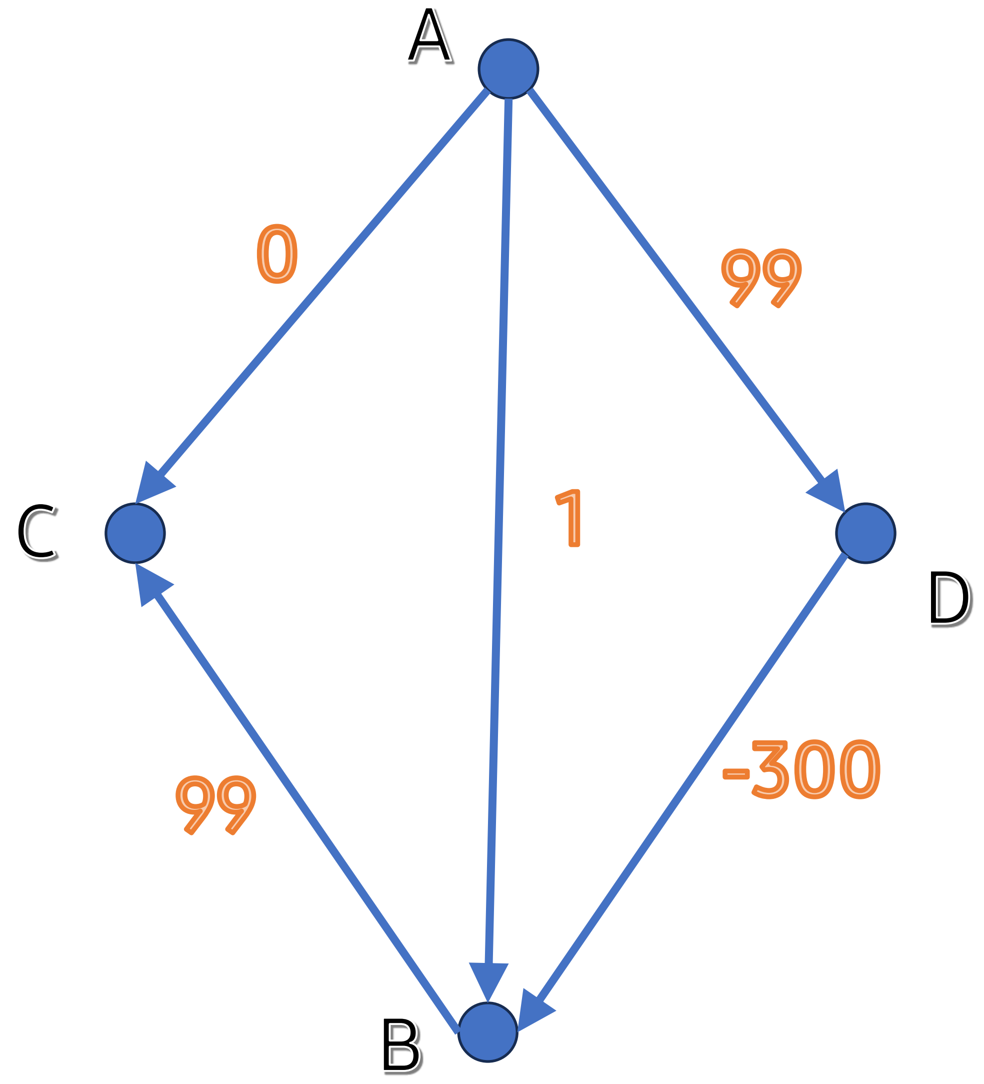
	A	B	C	D
A	0	1	0	99
B	∞	0	99	∞
C	∞	∞	0	∞
D	∞	-300	∞	0



	A	B	C	D
A	0	1	0	99
B	∞	0	99	∞
C	∞	∞	0	∞
D	∞	-300	∞	0

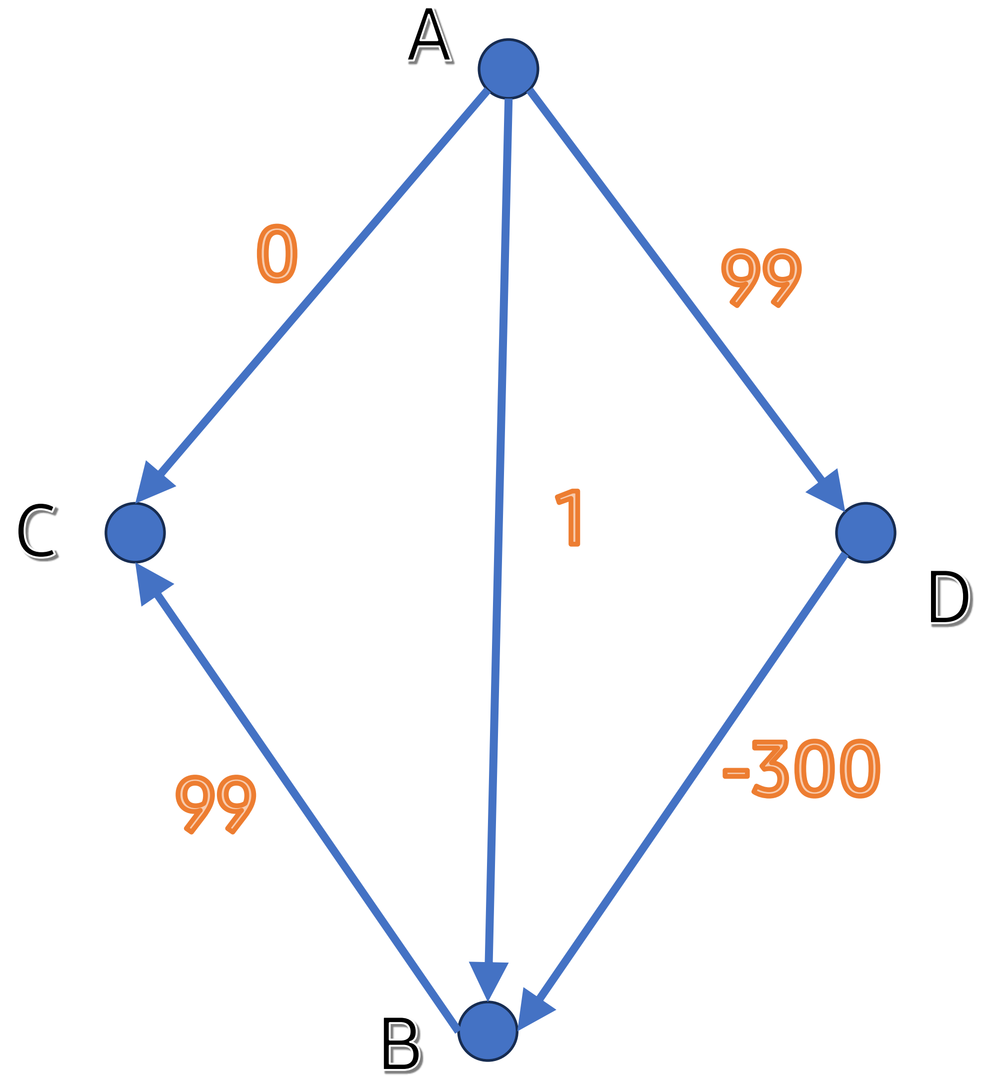
For k from 1 to $|V|$:
  For i from 1 to $|V|$:
    For j from 1 to $|V|$:
      if $D[i][j] > D[i][k] + D[k][j]$:
        $D[i][j] = D[i][k] + D[k][j]$

Exercise 4



	A	B	C	D
A	0	-201	-102	99
B	∞	0	99	∞
C	∞	∞	0	∞
D	∞	-300	-201	0

Exercise 4



0	1	0	99
∞	0	99	∞
∞	∞	0	∞
∞	-300	∞	0

D

A	A	A	A
B	B	B	B
C	C	C	C
D	D	D	D

P

For k from 1 to $|V|$:
  For i from 1 to $|V|$:
    For j from 1 to $|V|$:
      if $D[i][j] > D[i][k] + D[k][j]$:
        $D[i][j] = D[i][k] + D[k][j]$
        $P[i][j] = P[k][j]$

Exercise 4

Negative cycle detection

- If $D[i][i] < 0$

  The graph contains negative cycle.

II. Concepts

Spanning tree

Spanning forest

Min. spanning tree

Min. spanning forest

A spanning tree of graph G is a _ _ _ _ (4) which include all the _ _ _ _ _ _ _ _ (8) of G.

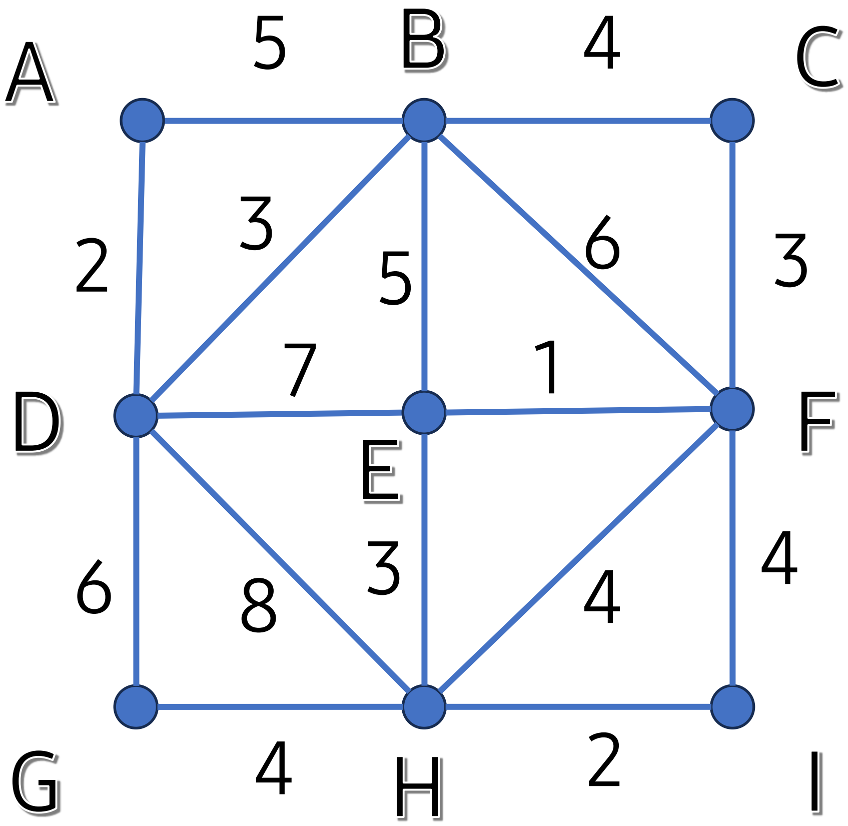
A spanning forest of graph G is a _ _ _ _ _ _ (6) that consists of multiple disjoint _ _ _ _ _ _ _ _ (8) trees.

A min-spanning ... is a spanning ... that has the minimum

_ _ _ _ _ _ (6).

Why min-spanning tree anyway?

Exercise #5



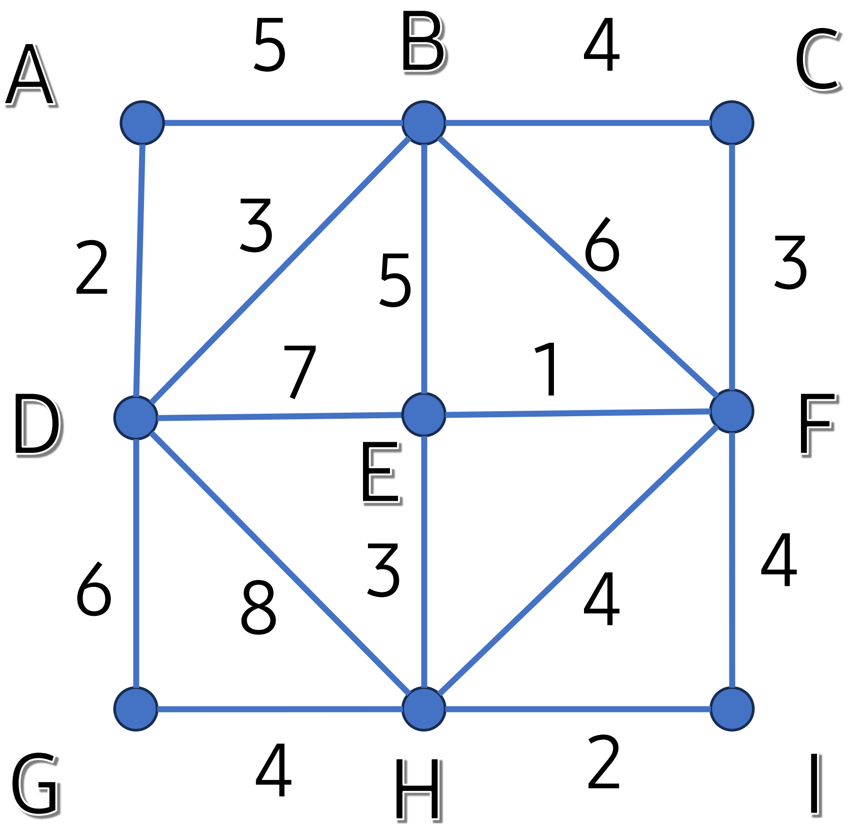
Find the min-spanning tree in this graph?

Remember: All vertices must be met

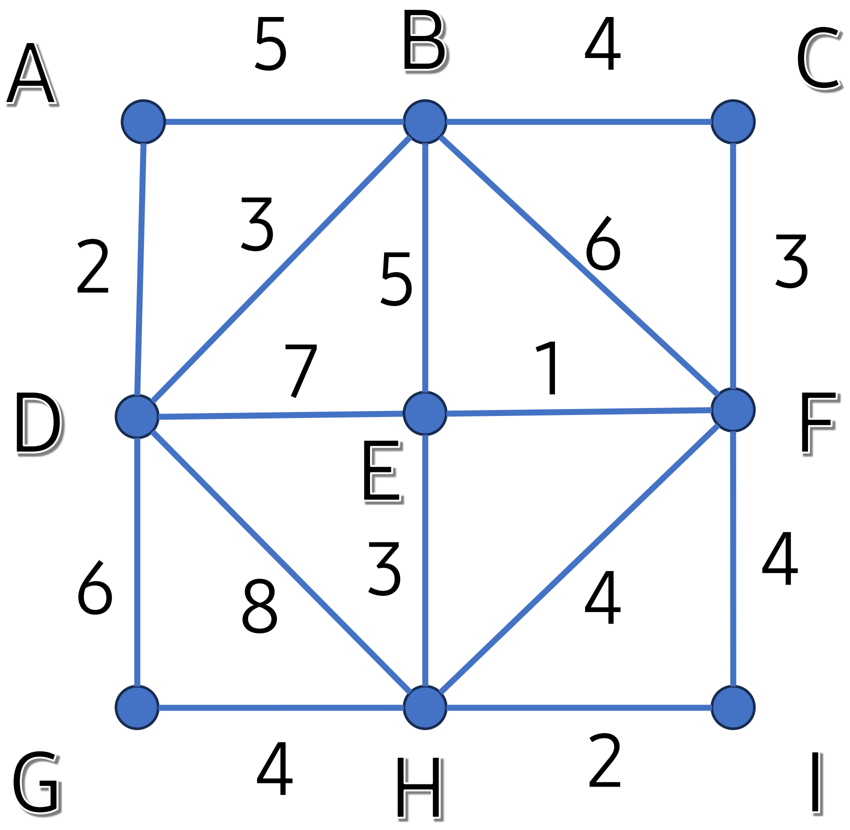
Kruskal

Jarnik-Prim

III. Kruskal



$$F = \{(E, F), (A, D), \dots\}$$
$$R = \{\}$$



$$F = \{(E,F),(A,D),\dots\}$$
$$R = \{\}$$

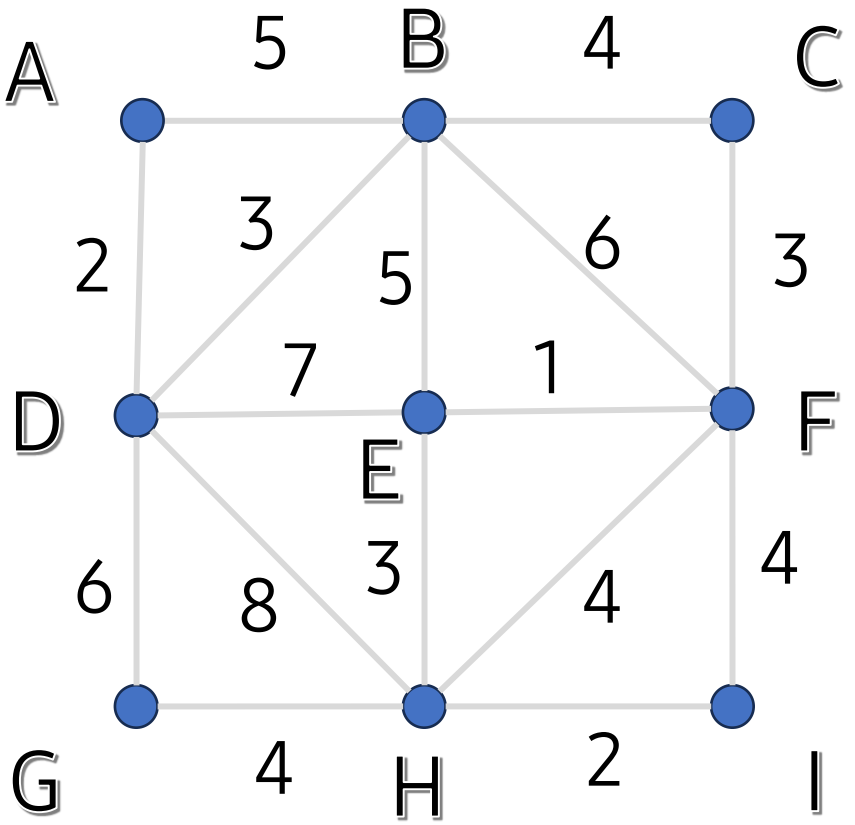
Step 1: Get $e = F[0]$, then remove e from F.

Step 2: Check if $\{e\} \cup R$ has cycle? If it's true, move to step 1.

Step 3: Add e to R.

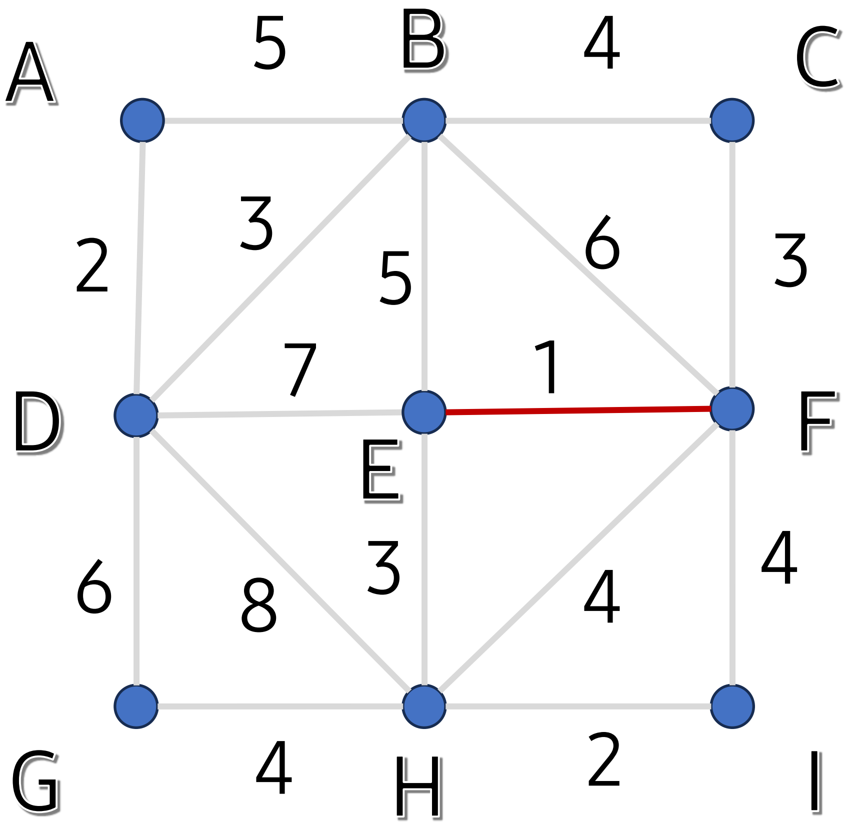
Step 4: If all vertices were met, stop. Else, go to step 1.

$$\{(E,F),(A,D),(H,I),(B,D),(C,F),(E,H),(B,C),(F,I),$$
$$(F,H),(H,G),(A,B),(B,E),(B,F),(D,G),(D,E),(D,H)\}$$

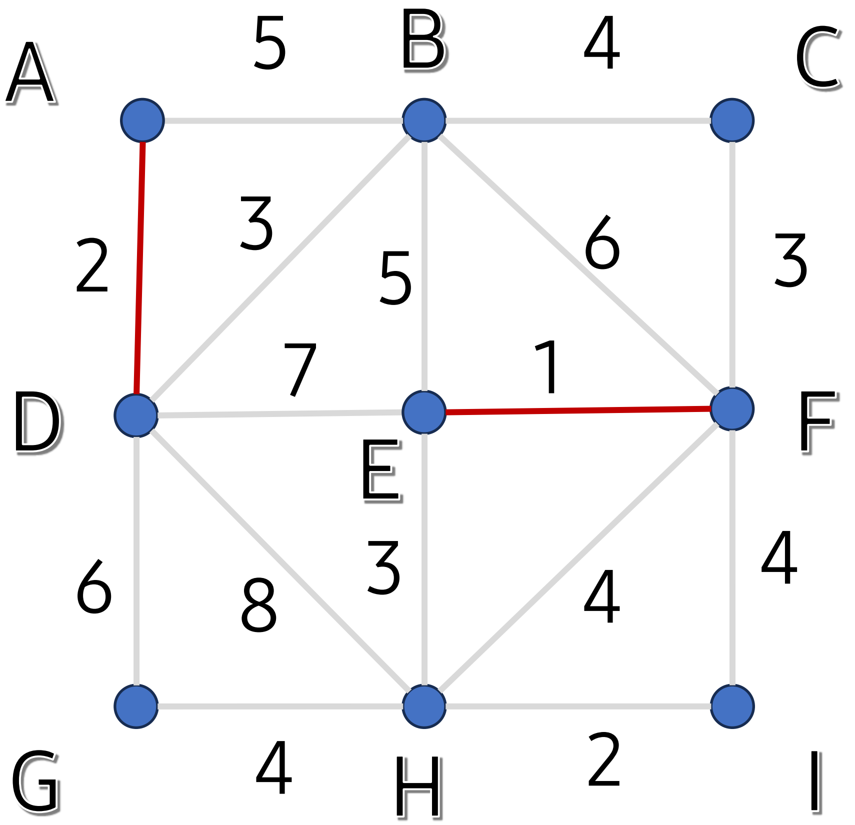


$\{\cancel{(E,F)}, (A,D), (H,I), (B,D), (C,F), (E,H), (B,C), (F,I),$

$(F,H), (H,G), (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$

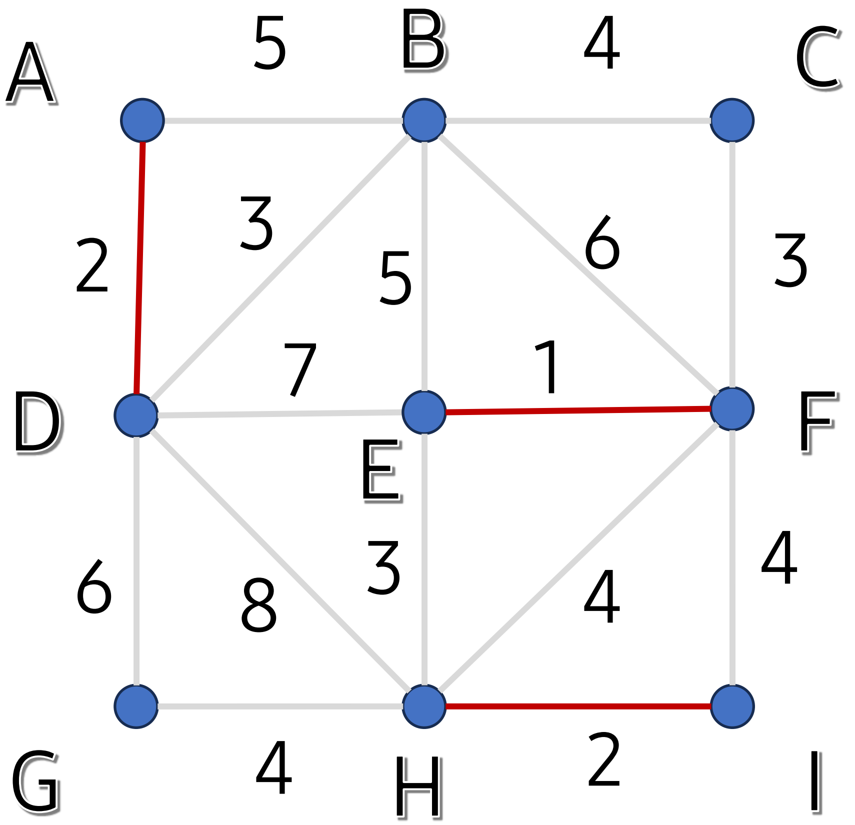


$$\{\cancel{(E,F)}, \cancel{(A,D)}, (H,I), (B,D), (C,F), (E,H), (B,C), (F,I),$$
$$(F,H), (H,G), (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$$



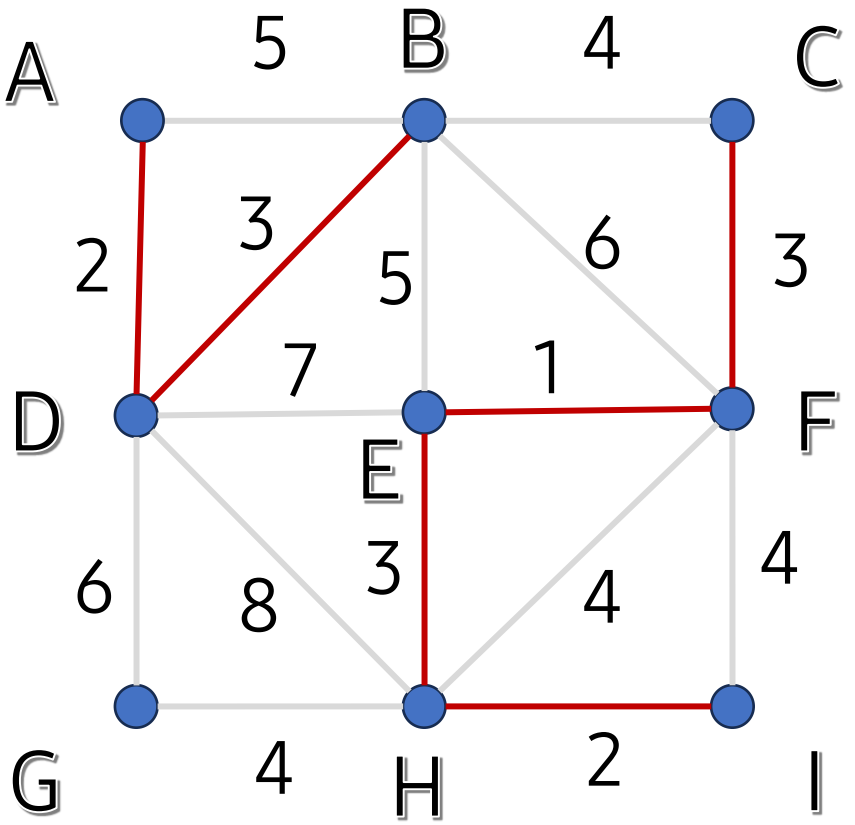
$$\{\cancel{(E,F)},\ \cancel{(A,D)},\ \cancel{(H,I)},\ (B,D),(C,F),(E,H),(B,C),(F,I),$$

$$(F,H),(H,G),(A,B),(B,E),(B,F),(D,G),(D,E),(D,H)\}$$

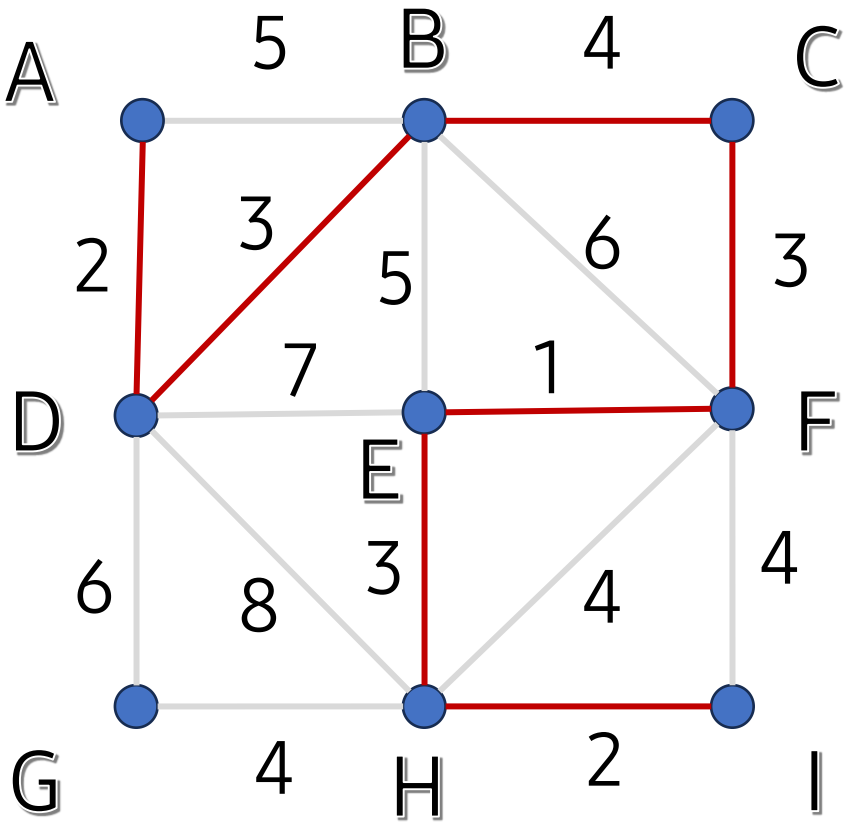


$\{\cancel{(E,F)}, \cancel{(A,D)}, \cancel{(H,I)}, \cancel{(B,D)}, \cancel{(C,F)}, \cancel{(E,H)}, (B,C), (F,I),$

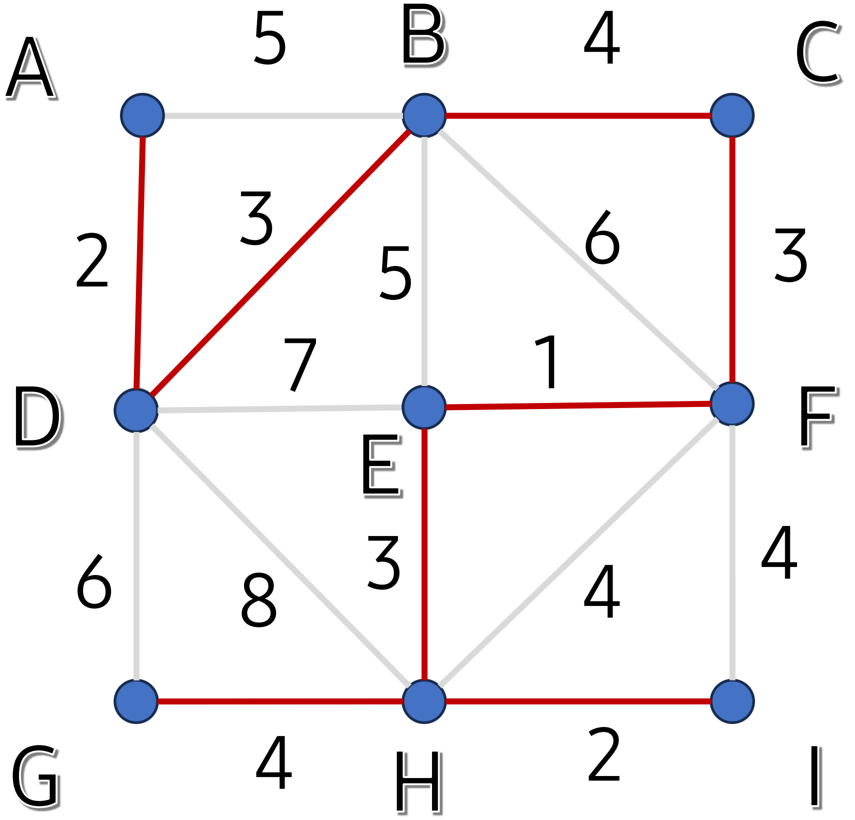
$(F,H), (H,G), (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$



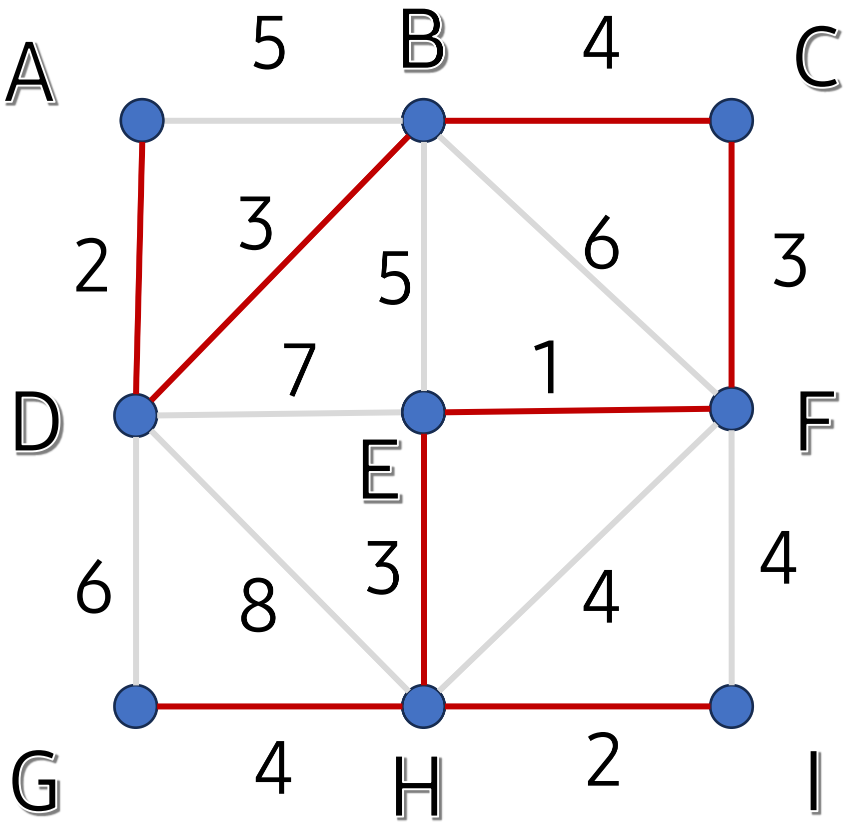
$$\{\cancel{(E,F)}, \cancel{(A,D)}, \cancel{(H,I)}, \cancel{(B,D)}, \cancel{(C,F)}, \cancel{(E,H)}, \cancel{(B,C)}, \cancel{(F,I)},$$
$$\cancel{(F,H)}, (H,G), (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$$



$$\{\cancel{(E,F)},\cancel{(A,D)},\cancel{(H,I)},\cancel{(B,D)},\cancel{(C,F)},\cancel{(E,H)},\cancel{(B,C)},\cancel{(F,I)},$$
$$\cancel{(F,H)},\cancel{(H,G)},(A,B),(B,E),(B,F),(D,G),(D,E),(D,H)\}$$

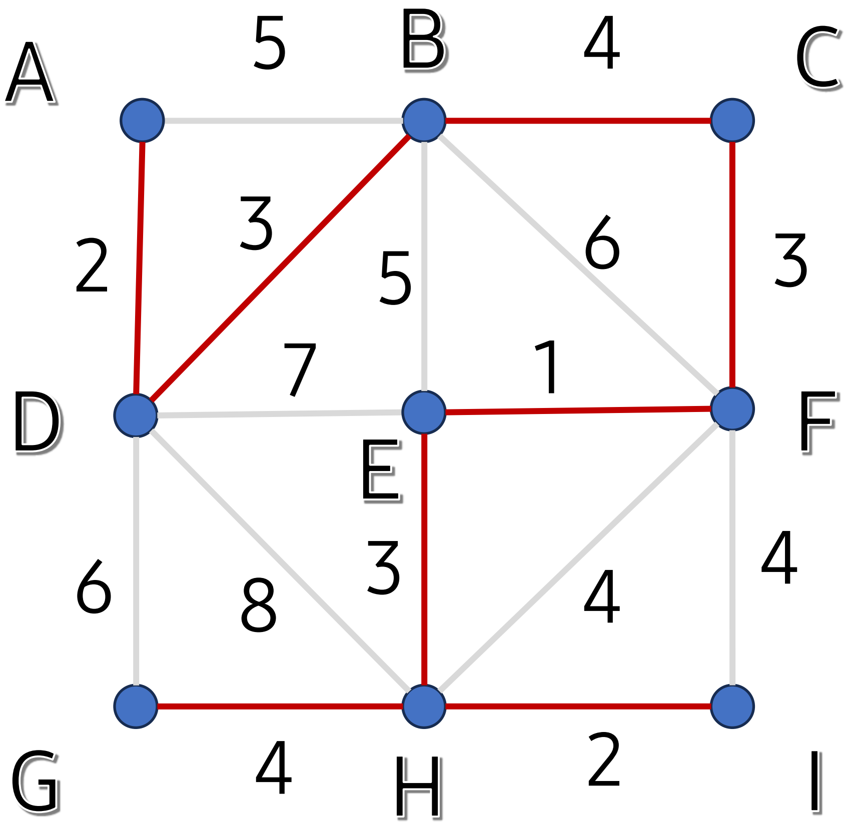


$\{\cancel{(E,F)}, \cancel{(A,D)}, \cancel{(H,I)}, \cancel{(B,D)}, \cancel{(C,F)}, \cancel{(E,H)}, \cancel{(B,C)}, \cancel{(F,I)},$
$\cancel{(F,H)}, \cancel{(H,G)}, (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$



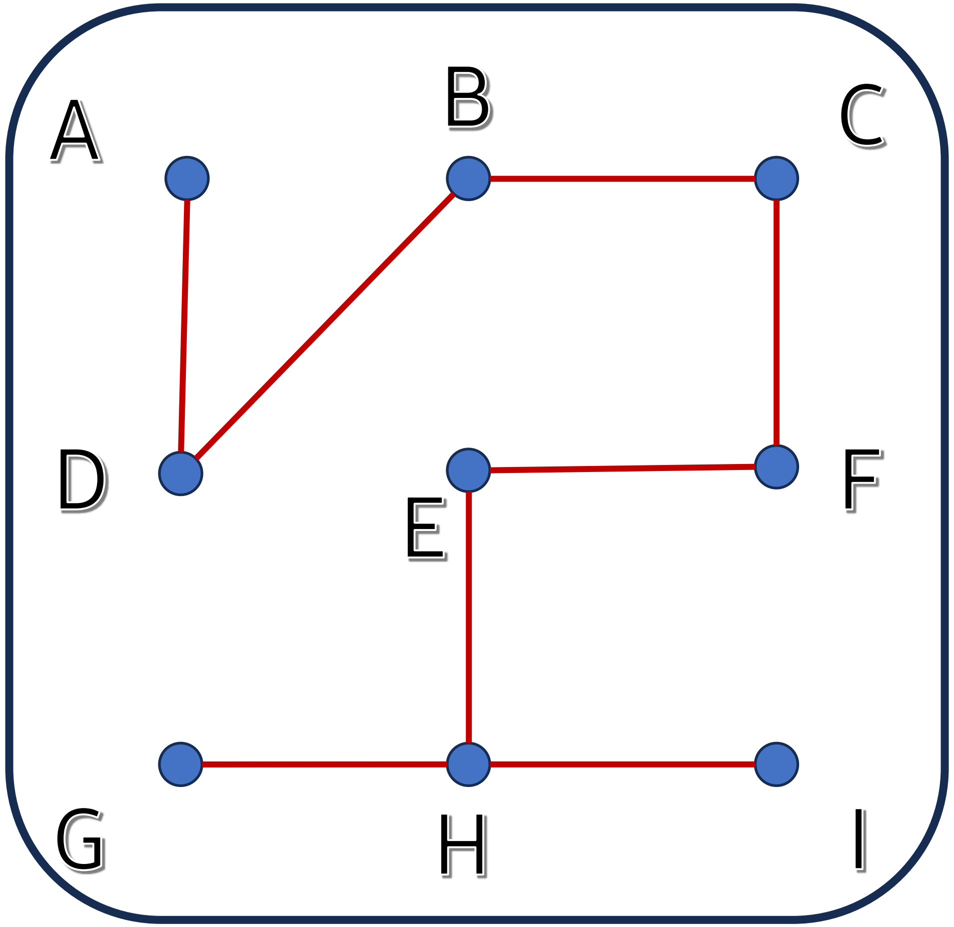
Step	Edge	Weight
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
	Sum:	

$\{\cancel{(E,F)}, \cancel{(A,D)}, \cancel{(H,I)}, \cancel{(B,D)}, \cancel{(C,F)}, \cancel{(E,H)}, \cancel{(B,C)}, \cancel{(F,I)},$
$\cancel{(F,H)}, \cancel{(H,G)}, (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$



Step	Edge	Weight
0	$\{E,F\}$	1
1	$\{A,D\}$	2
2	$\{H,I\}$	2
3	$\{B,D\}$	3
4	$\{C,F\}$	3
5	$\{E,H\}$	3
6	$\{B,C\}$	4
7	$\{F,I\}$	
8	$\{F,H\}$	
9	$\{H,G\}$	4
	Sum:	22

$\{\cancel{(E,F)}, \cancel{(A,D)}, \cancel{(H,I)}, \cancel{(B,D)}, \cancel{(C,F)}, \cancel{(E,H)}, \cancel{(B,C)}, \cancel{(F,I)},$
$\cancel{(F,H)}, \cancel{(H,G)}, (A,B), (B,E), (B,F), (D,G), (D,E), (D,H)\}$



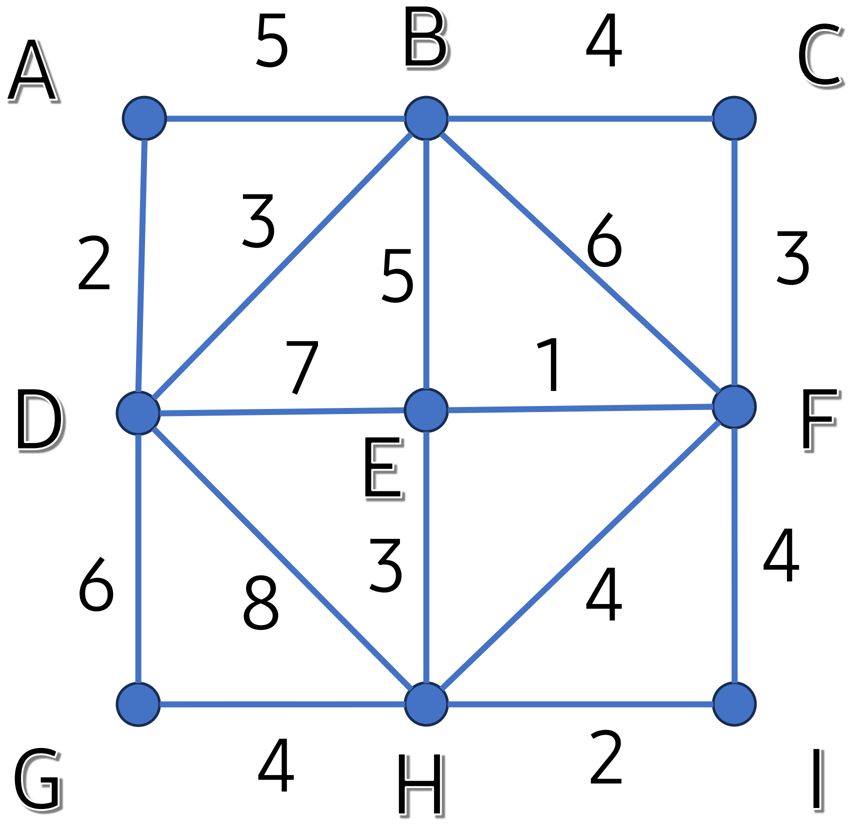
2

1

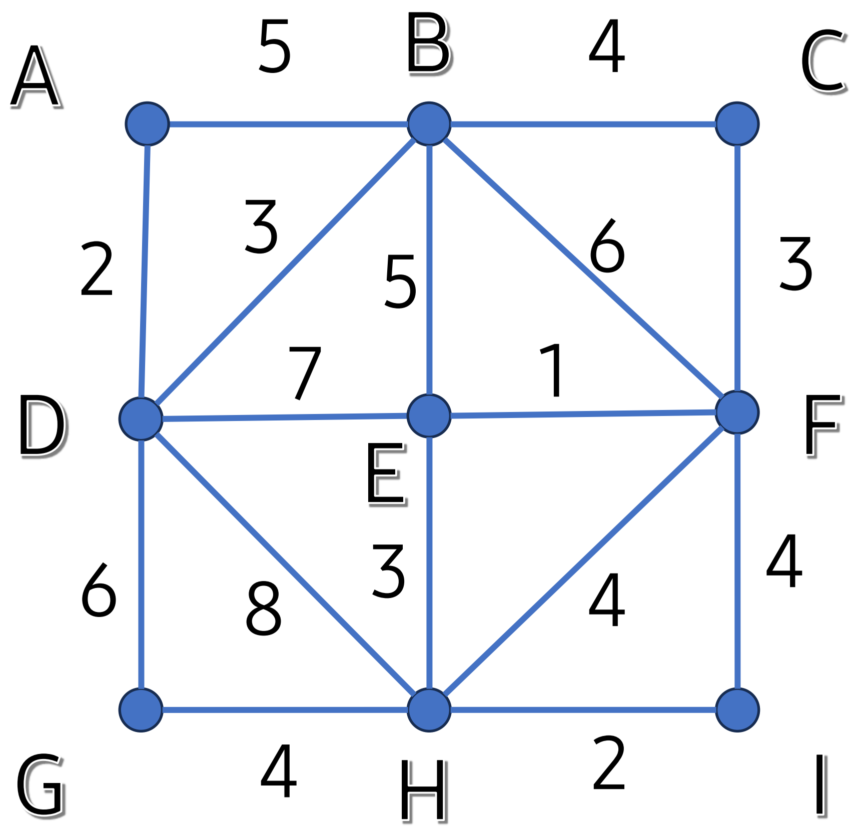
Step	Edge	Weight
0	$\{E,F\}$	1
1	$\{A,D\}$	2
2	$\{H,I\}$	2
3	$\{B,D\}$	3
4	$\{C,F\}$	3
5	$\{E,H\}$	3
6	$\{B,C\}$	4
7	$\{F,I\}$	
8	$\{F,H\}$	
9	$\{H,G\}$	4
	Sum:	22

Does Kruskal use edges or vertices?

IV. Jarnik - Prim



$$W = \{A\}$$
$$R = \{\}$$

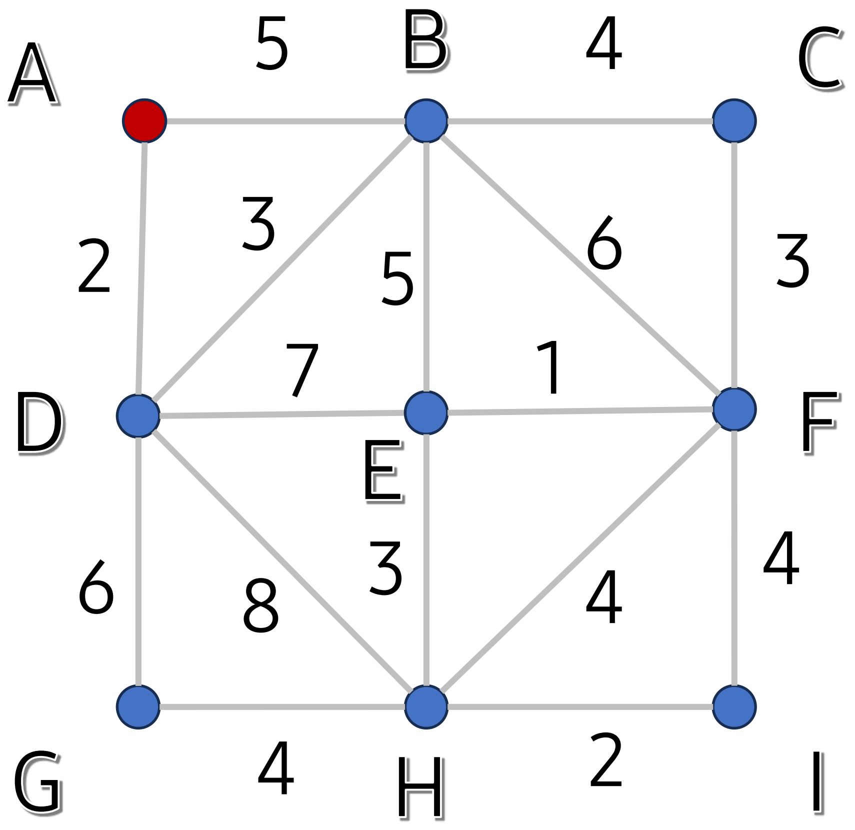


$$W = \{A\}$$
$$R = \{\}$$

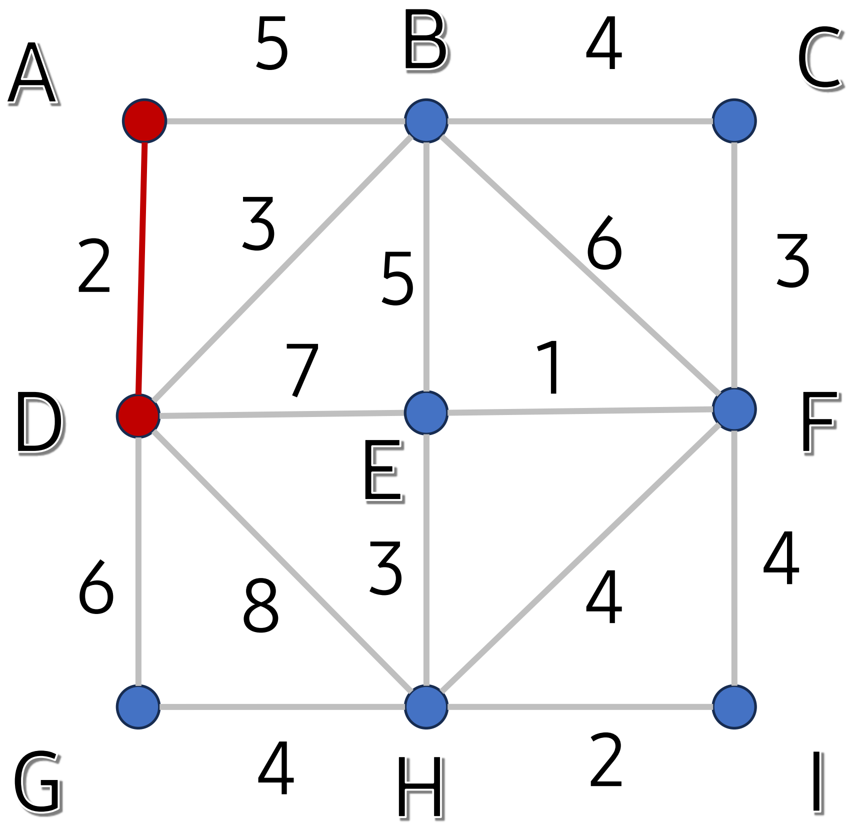
Step 1: Find $u \in W$, $v \notin W$ so that $w_{u,v}$ is minimum.

Step 2: If $\{u, v\}$ exists, add v to W, $\{u, v\}$ to R. Else, stop.

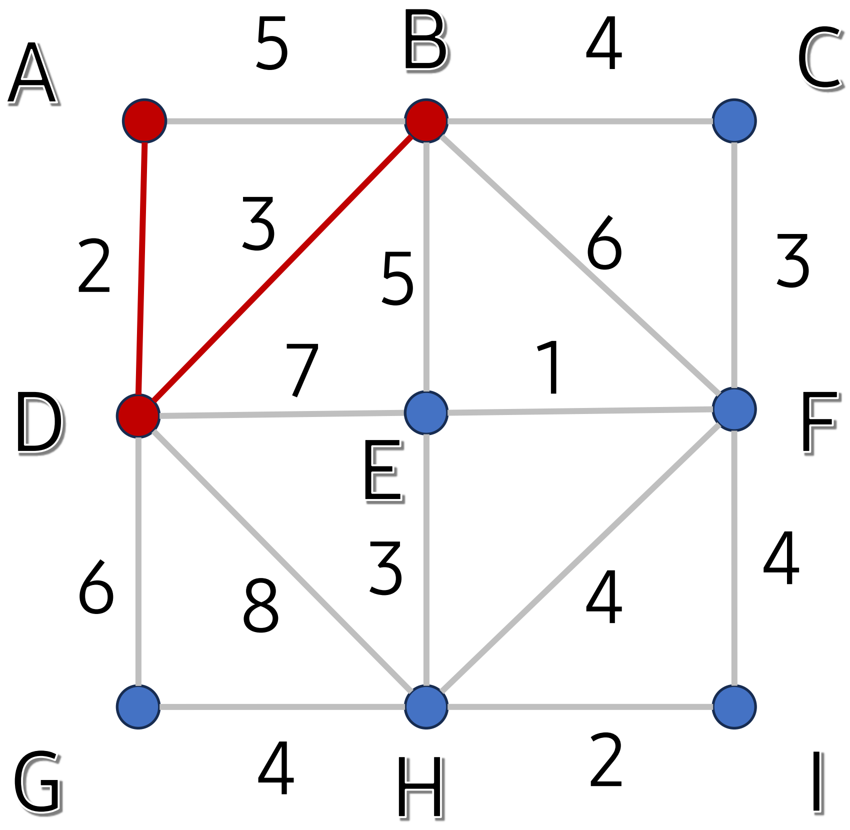
Step 3: Go to step 1.



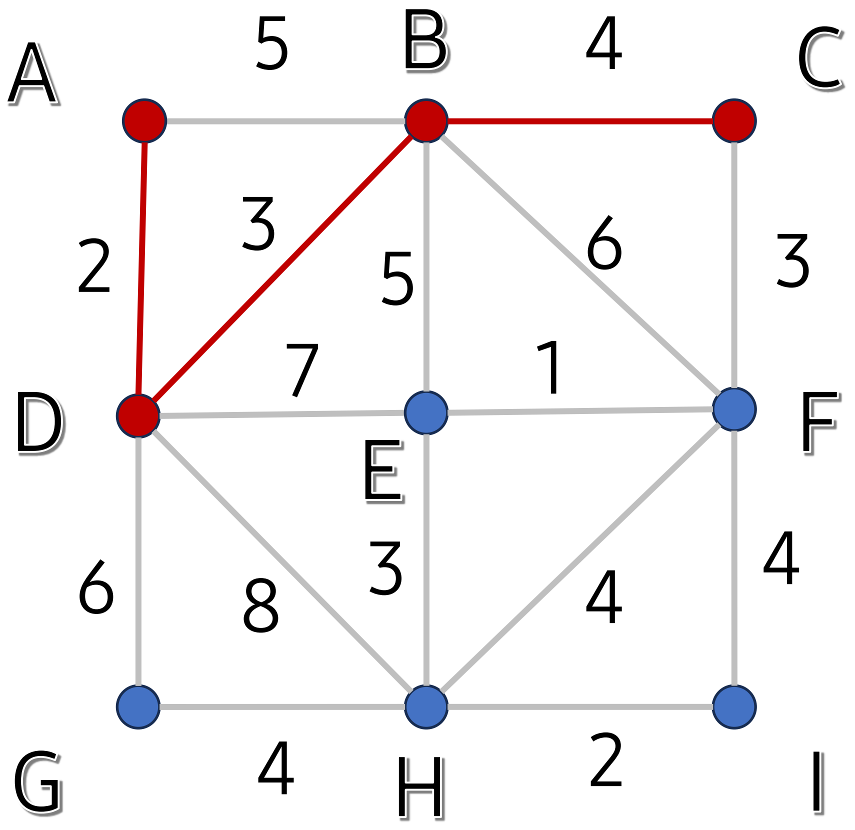
$$W = \{A\}$$
$$R = \{\}$$



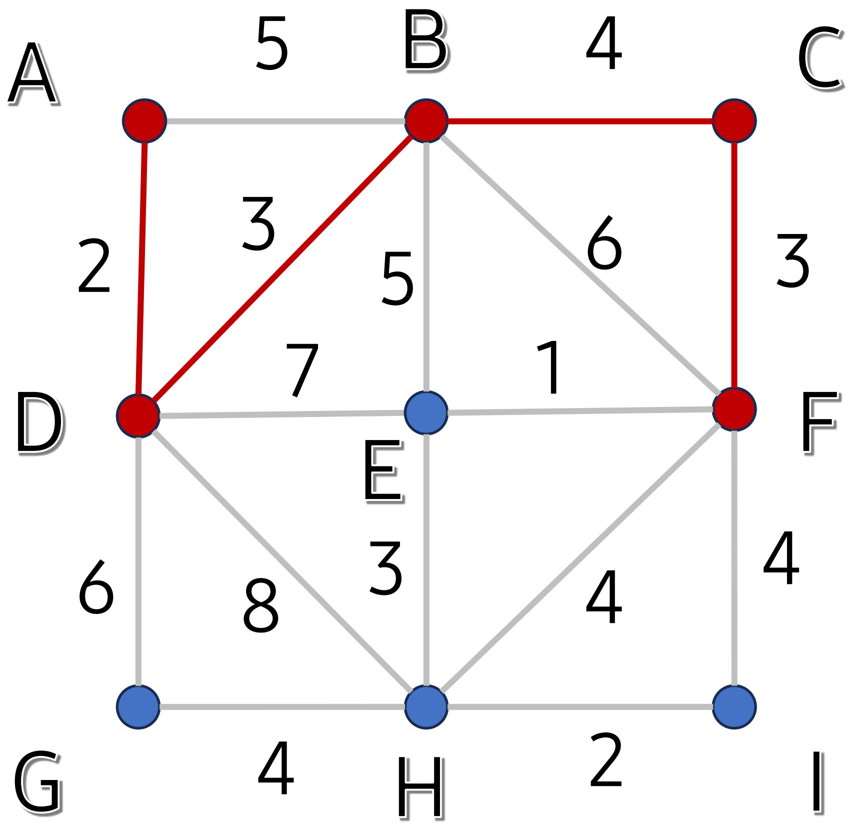
$$W = \{A, D\}$$
$$R = \{\{A, D\}\}$$



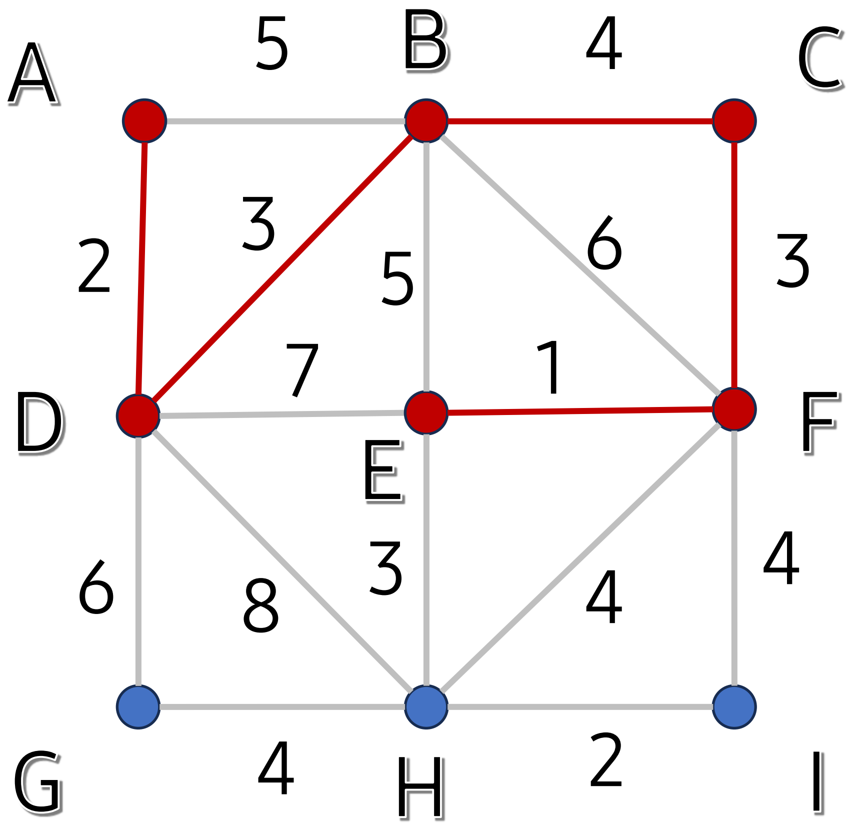
$$W = \{A, D, B\}$$
$$R = \{\{A, D\}, \{D, B\}\}$$



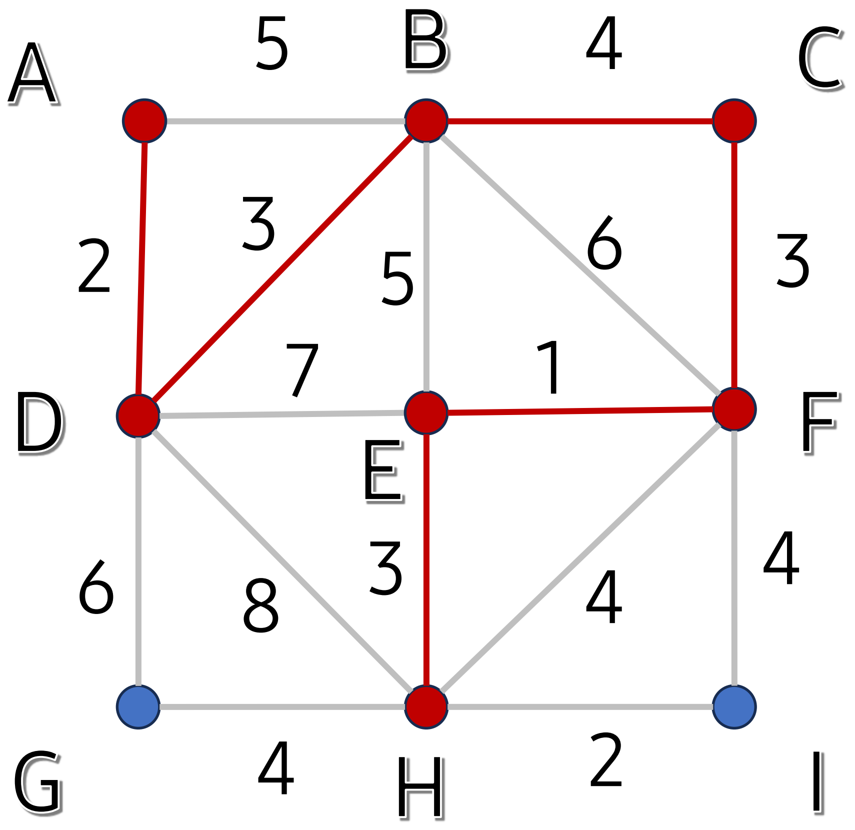
$$W = \{A, D, B, C\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}\}$$



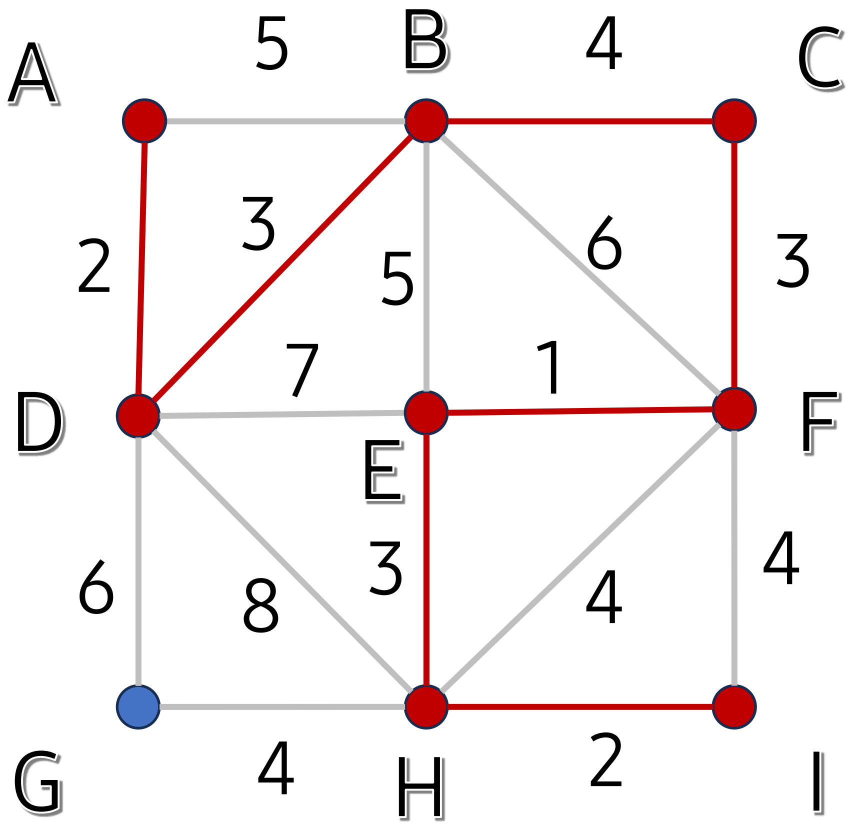
$$W = \{A, D, B, C, F\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}\}$$



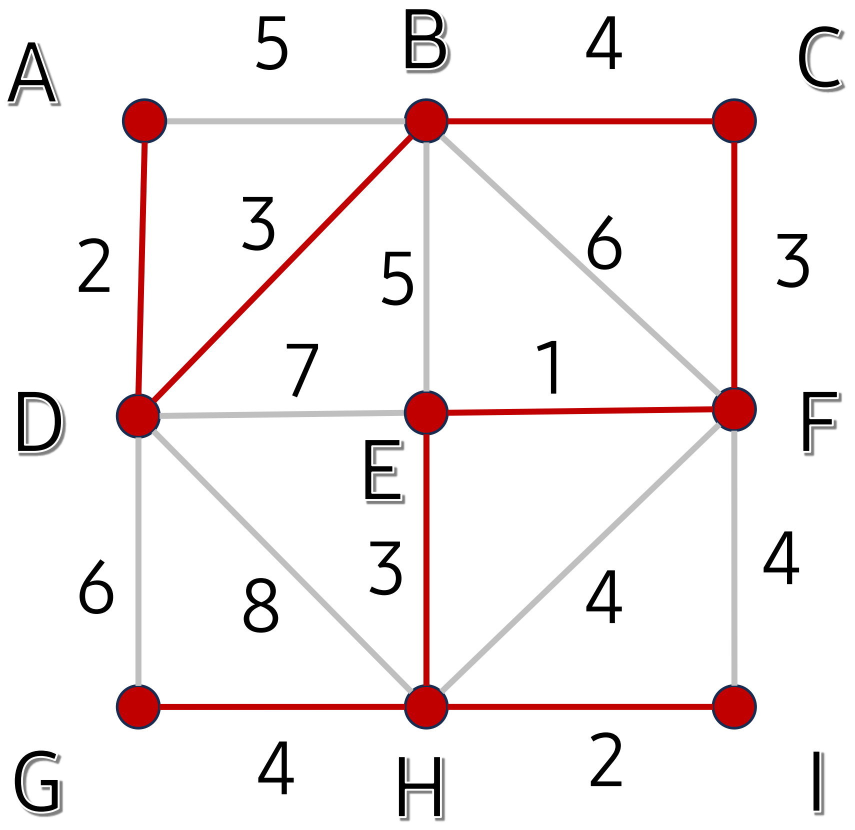
$$W = \{A, D, B, C, F, E\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\},$$
$$\{F, E\}\}$$



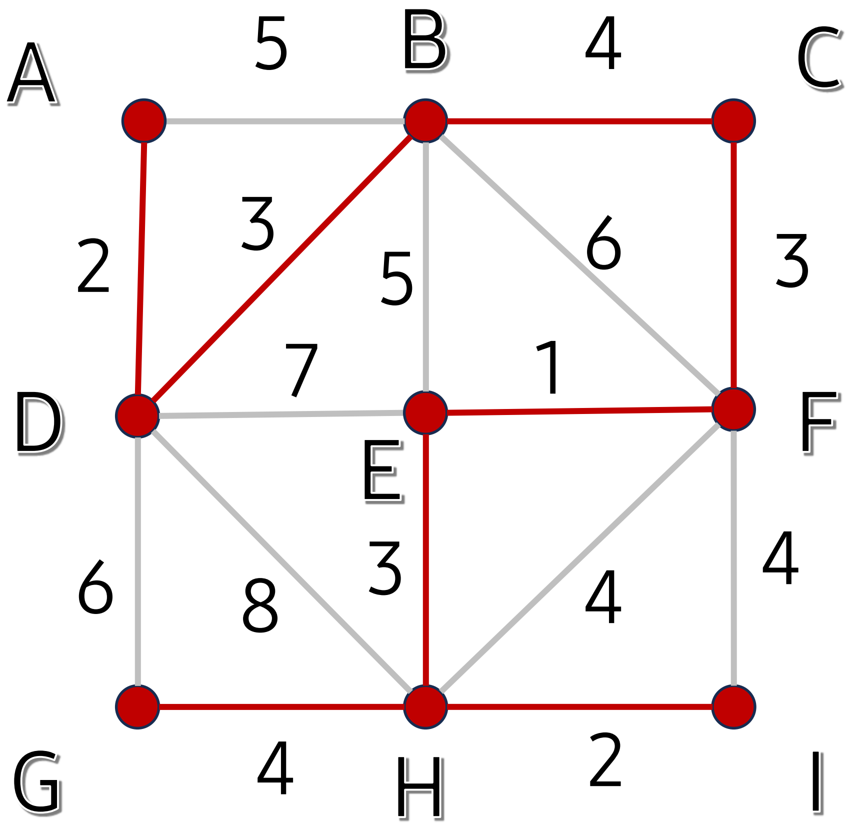
$$W = \{A, D, B, C, F, E, H\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}, \{F, E\}, \{E, H\}\}$$



$$W = \{A, D, B, C, F, E, H, I\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\},$$
$$\{F, E\}, \{E, H\}, \{H, I\}\}$$

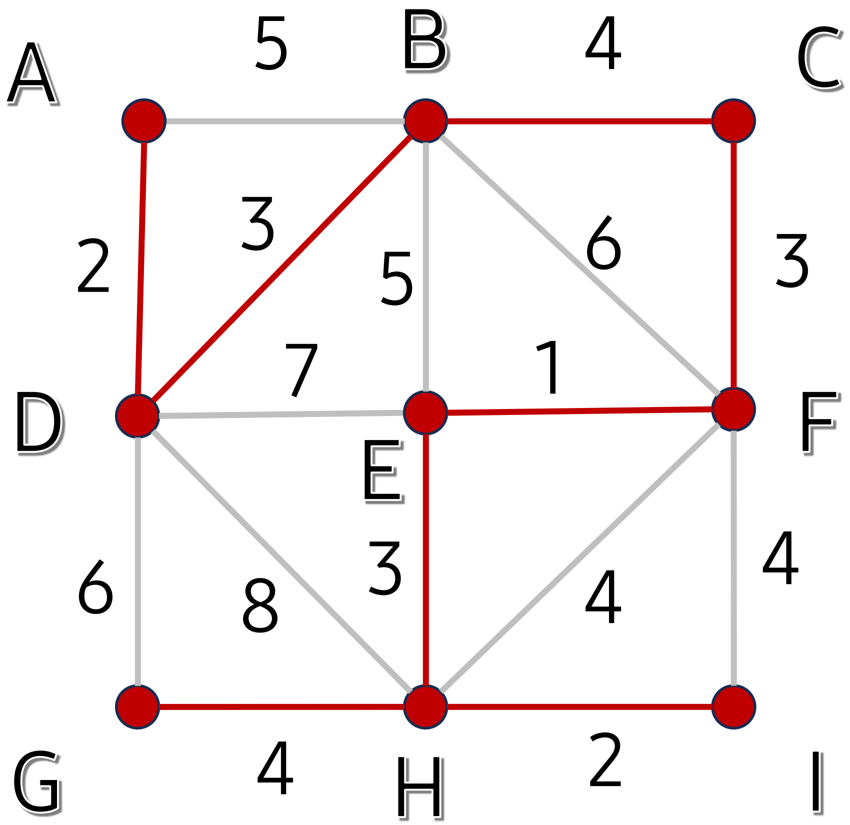


$$W = \{A, D, B, C, F, E, H, I, G\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}, \{F, E\}, \{E, H\}, \{H, I\}, \{H, G\}\}$$



$$W = \{A, D, B, C, F, E, H, I, G\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}, \{F, E\}, \{E, H\}, \{H, I\}, \{H, G\}\}$$

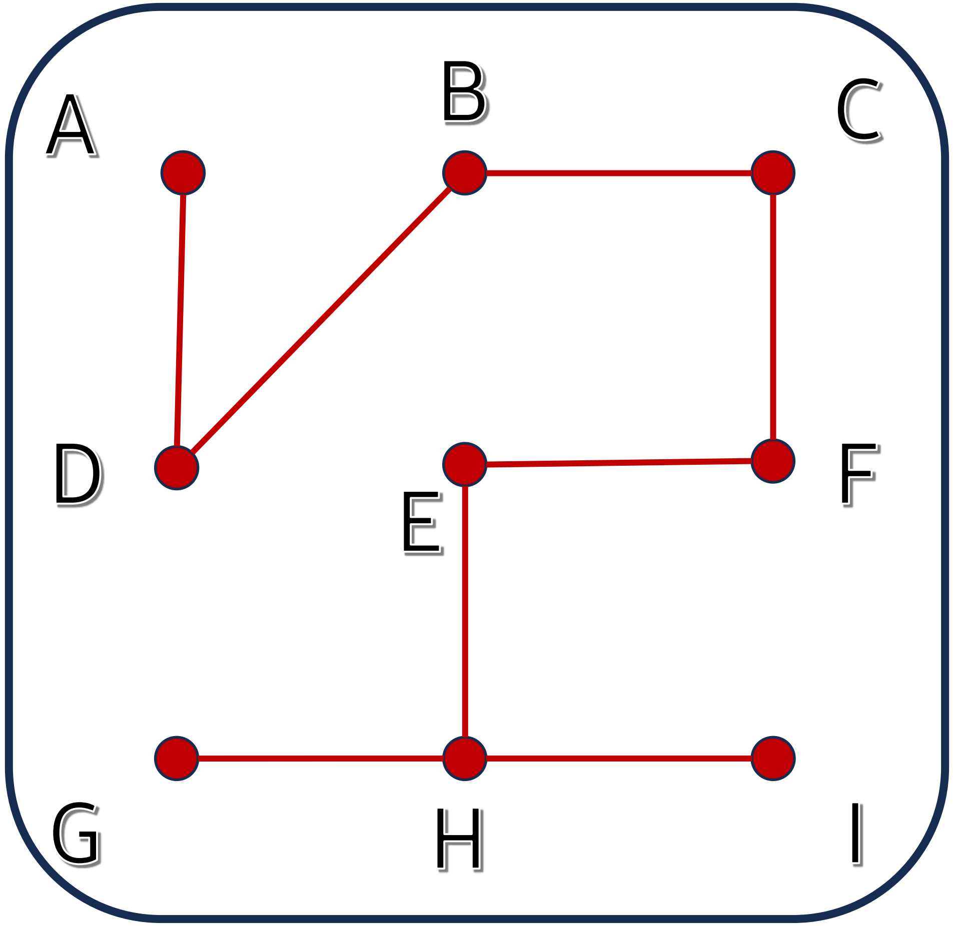
Step	Edge	Weight
0		
1		
2		
3		
4		
5		
6		
7		
	Sum:	



$$W = \{A, D, B, C, F, E, H, I, G\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}, \{F, E\}, \{E, H\}, \{H, I\}, \{H, G\}\}$$

Step	Edge	Weight
0	$\{A, D\}$	2
1	$\{D, B\}$	3
2	$\{B, C\}$	4
3	$\{C, F\}$	3
4	$\{F, E\}$	1
5	$\{E, H\}$	3
6	$\{H, I\}$	2
7	$\{H, G\}$	4
	Sum:	22

2



$$W = \{A, D, B, C, F, E, H, I, G\}$$
$$R = \{\{A, D\}, \{D, B\}, \{B, C\}, \{C, F\}, \{F, E\}, \{E, H\}, \{H, I\}, \{H, G\}\}$$

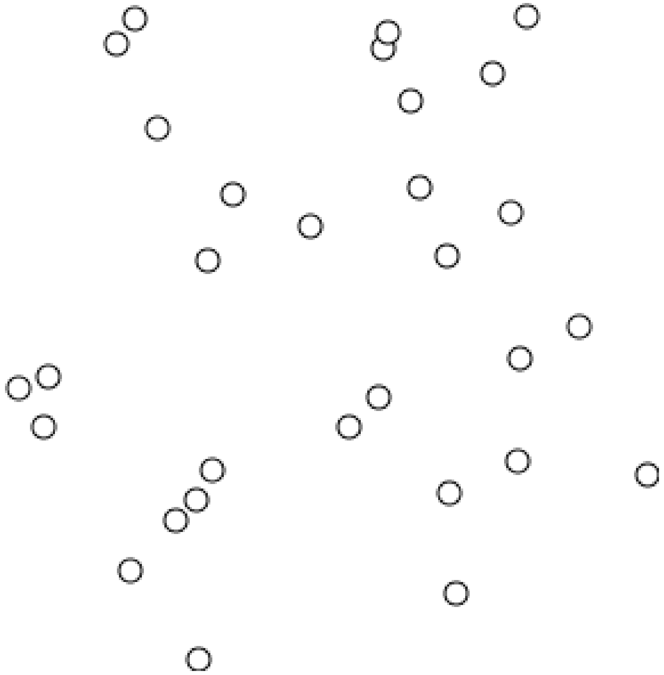
1

Step	Edge	Weight
0	$\{A, D\}$	2
1	$\{D, B\}$	3
2	$\{B, C\}$	4
3	$\{C, F\}$	3
4	$\{F, E\}$	1
5	$\{E, H\}$	3
6	$\{H, I\}$	2
7	$\{H, G\}$	4
	Sum:	22

Kruskal

✔ Better with sparse graph

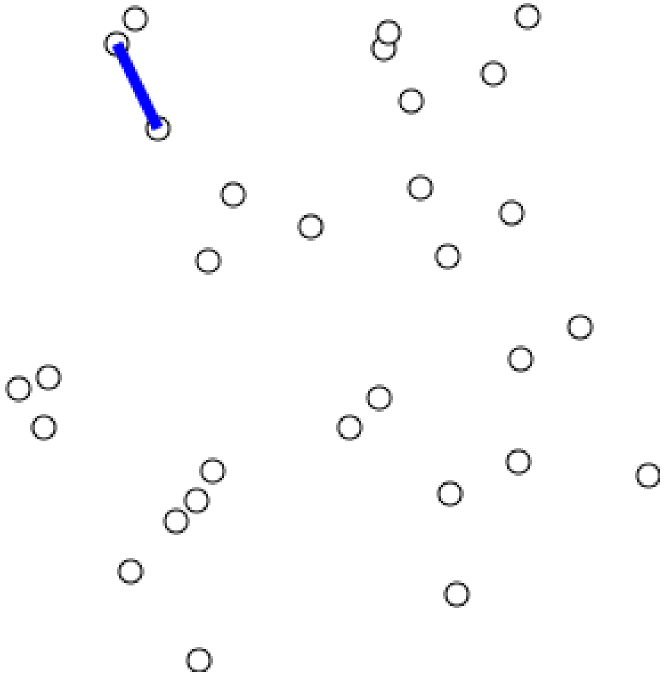
$$O(|E| log|E|)$$



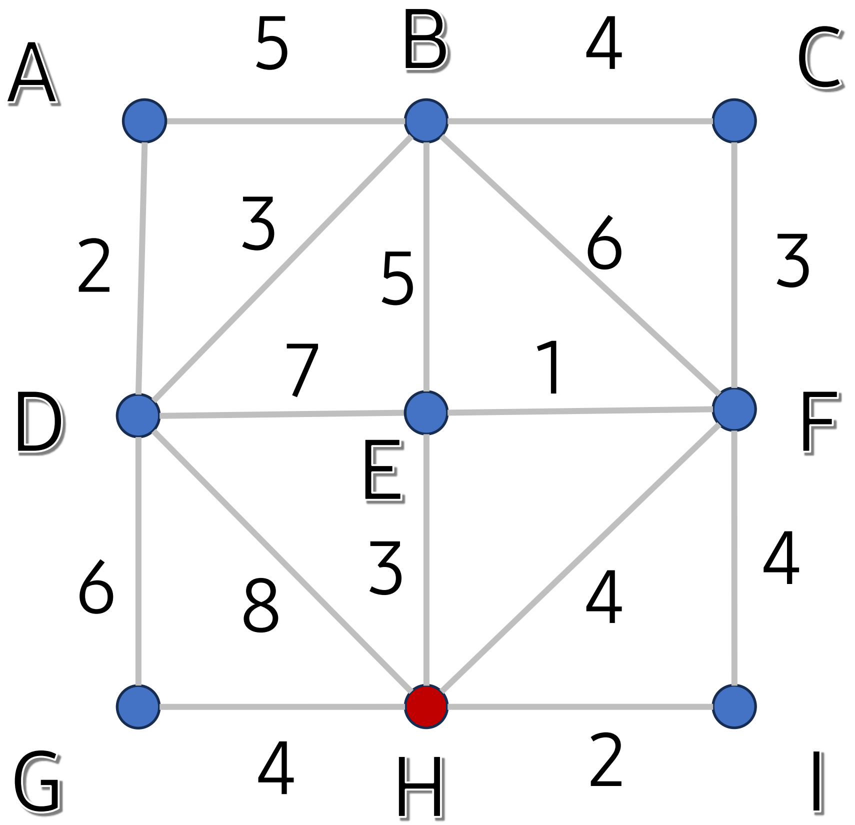
Jarnik-Prim

✔ Better with dense graph

$$O(|V|^2)$$



Exercise #6: Jarnik – Prim



$$W = \{\mathrm{H}\}$$
$$R = \{\}$$

No.	Algorithms	How to demonstrate on paper?
1	DFS	Sequence of tuple: (A, B), (B, C),... (day 7, 8)
2	BFS	Sequence of tuple: (A, B), (B, C),... (day 7, 8)
3	Fleury	Sequence of tuple: (A, B), (B, C),... (day 8)
4	Dijkstra	Progress table (day 9, slide 23)
5	Ford-Bellman	Progress table (day 9, slide 33)
6	Floyd	Matrix (day 9, slide 41)
7	Kruskal	Progress table, tree (day 10, slide 20)
8	Jarnik - Prim	Progress table, tree (day 10, slide 35)

Homework

- Implement:
  1. Dijkstra
     - Print out the progress table
  2. Ford-Bellman
     - Print out the progress table
  3. Floyd
     - Print out the matrix