# Day 1 Discrete maths: The first chapter

Lecturer: Msc. Le Minh Tan

#### Introduction

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# So what's the plan?

- Progress score: Weekly homework + 2 tests (each x2)
- Final score: Exam
- Preparation: Laptop (VS C++, Python >= 3.10), e. socket
- Please check out the subject curriculum on UTEx.

## **Textbooks**

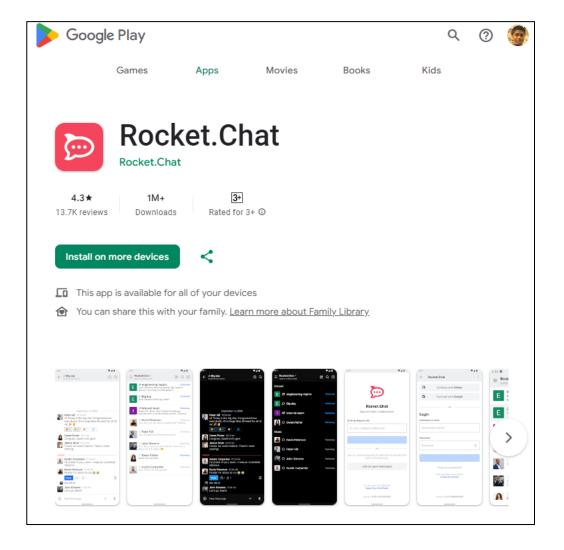
- Nguyễn Hữu Anh "Toán rời rạc"
- Đặng Trường Sơn & Lê Văn Vinh et al. "Giáo trình lý thuyết đồ thị"
- Nguyễn Thành Sơn & Đặng Trường Sơn et al. "Giáo trình toán rời rạc và lý thuyết đồ thị"
- Kenneth H. Rosen & Kamala Krithivasan "Discrete Mathematics and Its Applications"
- Adrian Bondy & U.S.R. Murty "Graph theory"

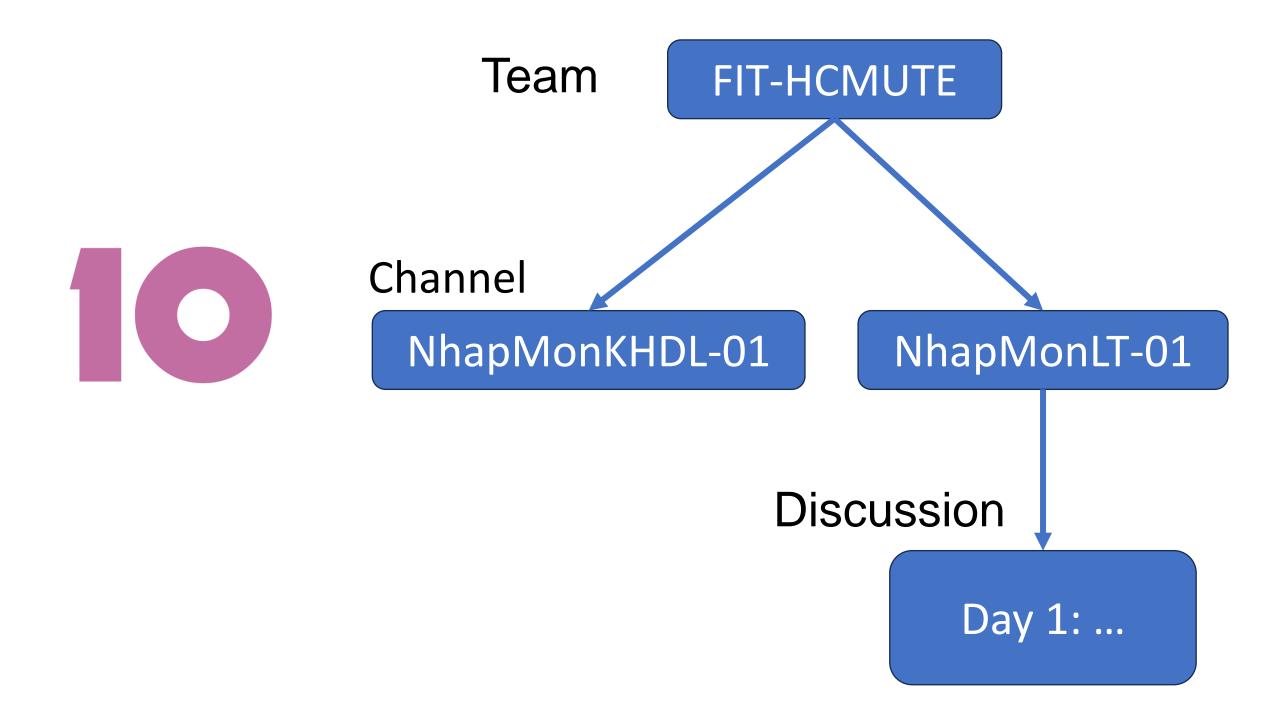
#### How we work

- By chatting platform:
  - Website: https://chat.cntt.io
  - Android/iOS: Rocket.Chat
  - Desktop:

https://github.com/RocketChat/Roc

ket.Chat.Electron/releases





# You all need a group!

Yes, each needs to be in a group for some activities.

#### • Steps:

- 1. Gather 2-3 members.
- 2. Vote for a leader.
- 3. Post the list of members (ids + full names), leader id to discussion.

#### • Rules:

- 1. No multiple groups per student.
- 2. The smaller size, the better.
- 3. Deadline: Before class ends.
- 4. Those with no group need to contact me in one week.



#### Outline

- I. Discrete mathematics
- II. Propositional logic: Concepts
- III. Logical equivalences and rules of inference

# I. Discrete mathematics



A B
C D

C

A

B

Discrete objects

## Discrete maths problems

- 1. How many ways are there to choose a valid password on a computer system?
- 2. What is the probability of winning a lottery?
- 3. Is there a link between two computers in a network?
- 4. How can I identify spam e-mail messages?

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Kenneth H. Rosen & Kamala Krithivasan "Discrete Mathematics and Its Applications"

## Discrete maths problems

- 1. How many ways are there to choose a valid password on a computer system? **Quantity**
- 2. What is the probability of winning a lottery? **Events**
- 3. Is there a link between two computers in a network? Yes/No
- 4. How can I identify spam e-mail messages? Spam/Not spam

Discrete mathematics is used when ever <u>objects are counted</u>, when <u>relationships between finite</u> (or countable) sets are studied, and when <u>processes involving a finite number of steps</u> are analyzed.

## II. Propositional logic: Concepts

- A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.
- Some easy examples:
  - Joe Biden is the president of the United States. ✓
  - 1 + 1 = 2 **✓**
  - What time is it? X
  - Read this carefully. X
  - x + 1 = 2 X

- "In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two other sides"
- "If a > c then a + b > c"
- "Every even integer greater than 2 can be written as the sum of two primes"

- 1. What are the above statements about?
- 2. Can you prove them?
- 3. Is conjecture a proposition?

In discrete maths, we don't work with sentences, but variables and symbols:

Order	Symbol	Feature	Expression
1	¬, −	NOT (Negation)	$\neg p$ , $ar{p}$
2	Λ	AND/but	$p \wedge q$
3	V	OR (inclusive or)	$p \lor q$
4	⊕, <u>∨</u>	XOR (exclusive or)	$p \oplus q$
5	$\rightarrow$	Conditional statement/implication	$p \rightarrow q$
6	$\leftrightarrow$	Biconditional statement/bi-implication	$p \leftrightarrow q$
	≣,⇔	Equivalence	$p \equiv q$

- Let p, q, r, s are the following propositions:
  - I'm sick.
  - I finished my homework.
  - I am going to stay home.
  - I will attend today's class.
- 1. How can you express these in sentence?
  - ¬p
  - $p \wedge q$
  - $q \rightarrow r$
- 2. Is there anything illogical?

- A proposition always has a true value: True or false.
- Truth table helps determine the truth of compound propositions.

p	q	$\neg q$	$p \wedge q$
Т	Т	F	Т
Т	F	Т	F
F	Т	F	F
F	F	Т	F

- 1/0 can also be interpreted as T/F.
- True table can be useful as a proof.

Exercise: Proving the below equivalence is always true

$$p \oplus q \equiv (p \lor q) \land \neg (p \land q)$$

Tips: A truth table could help.

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Tips: A truth table could help.

$oldsymbol{p}$	q	$p{\oplus}q$	$r:=p\lor q$	$s:=p\wedge q$	$r \wedge \neg s$
1	0	1	1	0	1
0	1	1	1	0	1
0	0	0	0	0	0
1	1	0	1	1	0

## III. Logical equivalences and rules of inference

No.	Law	Expression
1	Double negation Multi-negation	$\neg \neg p \equiv p$ $\neg \neg \neg \neg p \equiv p$
2	De Morgan	$\neg(p \land q) \equiv \neg p \lor \neg q$ $\neg(p \lor q) \equiv \neg p \land \neg q$
3	Commutative	$p \land q \equiv q \land p$ $p \lor q \equiv q \lor p$
4	Associative	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
5	Distributive	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

No.	Law	Expression
6	Idempotent	$p \land p \equiv p$ $p \lor p \equiv p$
7	Identity	$\begin{array}{c} p \wedge 1 \equiv p \\ p \vee 0 \equiv p \end{array}$
8	Negation	$p \land \neg p \equiv 0$ $p \lor \neg p \equiv 1$
9	Domination	$p \land 0 \equiv 0$ $p \lor 1 \equiv 1$
10	Absorption	$p \land (p \lor q) \equiv p$ $p \lor (p \land q) \equiv p$

- End of logical equivalences -

No.	Law	Expression
1	Substitution	$p \to q \equiv \neg p \lor q$
2	Modus Ponens	$(p \land (p \to q)) \to q$
3	Modus Tollens	$(\neg q \land (p \to q)) \to \neg p$
4	Hypothetical syllogism	$((p \to q) \land (q \to r)) \to (p \to r)$
5	Disjuctive syllogism	$((p \lor q) \land \neg p) \to q$
6	Addition	$p \to (p \lor q)$
7	Simplification	$(p \land q) \rightarrow p$
8	Resolution	$((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)$
9	Contradictory	$[(p_1 \land p_2 \land \dots \land p_n) \to q] \equiv [(p_1 \land p_2 \land \dots p_n \land \neg q) \to 0]$
10	Conditionally proving	$((p \to r) \land (q \to r)) \to ((p \lor q) \to r)$

#### - rules of inference -

# Bài tập tại lớp

Xác định suy diễn sau là đúng hay sai:

- Nếu Toàn tới lớp kịp lúc, bạn ấy sẽ không bị đánh vắng.
- Nếu Toàn tới lớp kịp lúc, bạn ấy sẽ hiểu bài hơn.
- Vậy, nếu Toàn tới lớp kịp lúc, bạn ấy sẽ không bị đánh vắng và sẽ hiểu bài hơn.

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

## Homework

- Homework: 27, 28, 29, 30, 31/80 (2pts each)
- Work in group
- Deadline: 1 week
- At least one student per group must submit a docx file containing the answers. Copying from other group is strictly prohibited.