Day 3
Graphs for Discrete Maths

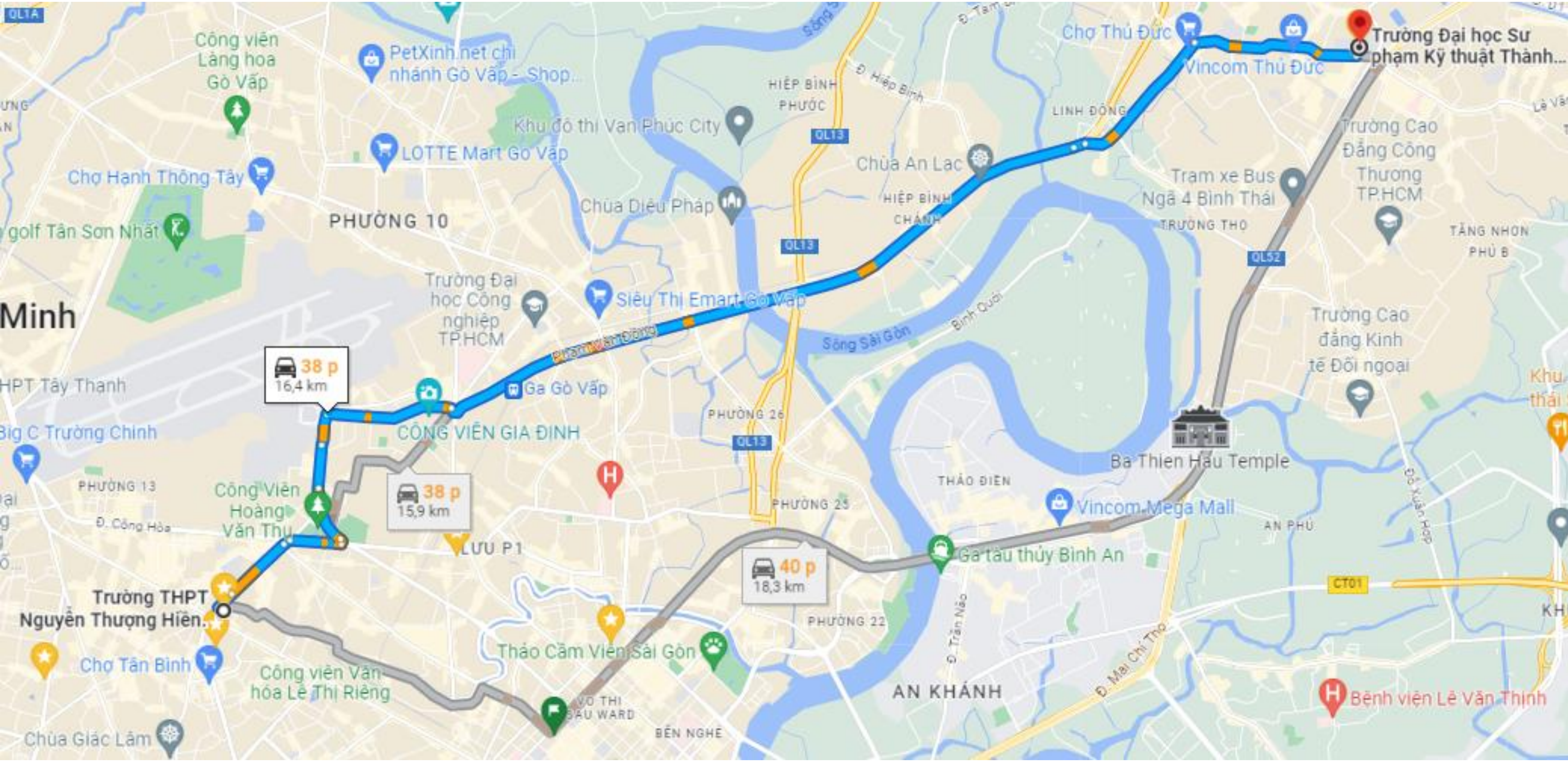
Lecturer: Msc. Minh Tan Le

Overview of day 2

- Proposition calculation
- Relations of sets
  - Binary relation (2 sets)
  - N-ary relation (multiple sets)
- Type of relations
  - Reflexive
  - Symmetric
  - Antisymmetric
  - Transitive
  - Equivalent
- Operators: Union, intersection, minus.

Outline

I. Graph
II. Digraph & Hasse diagram
III. Boolean algebra: The essence
IV. Graph theory: The essence



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Trường THPT Nguyễn Thượng Hiền...
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16,4 km
38 p
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Vincom Thủ Đức
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Chùa An Lạc
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HIỆP BÌNH CHÁNH
Khu đô thị Van Phúc City
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Ba Thien Hau Temple
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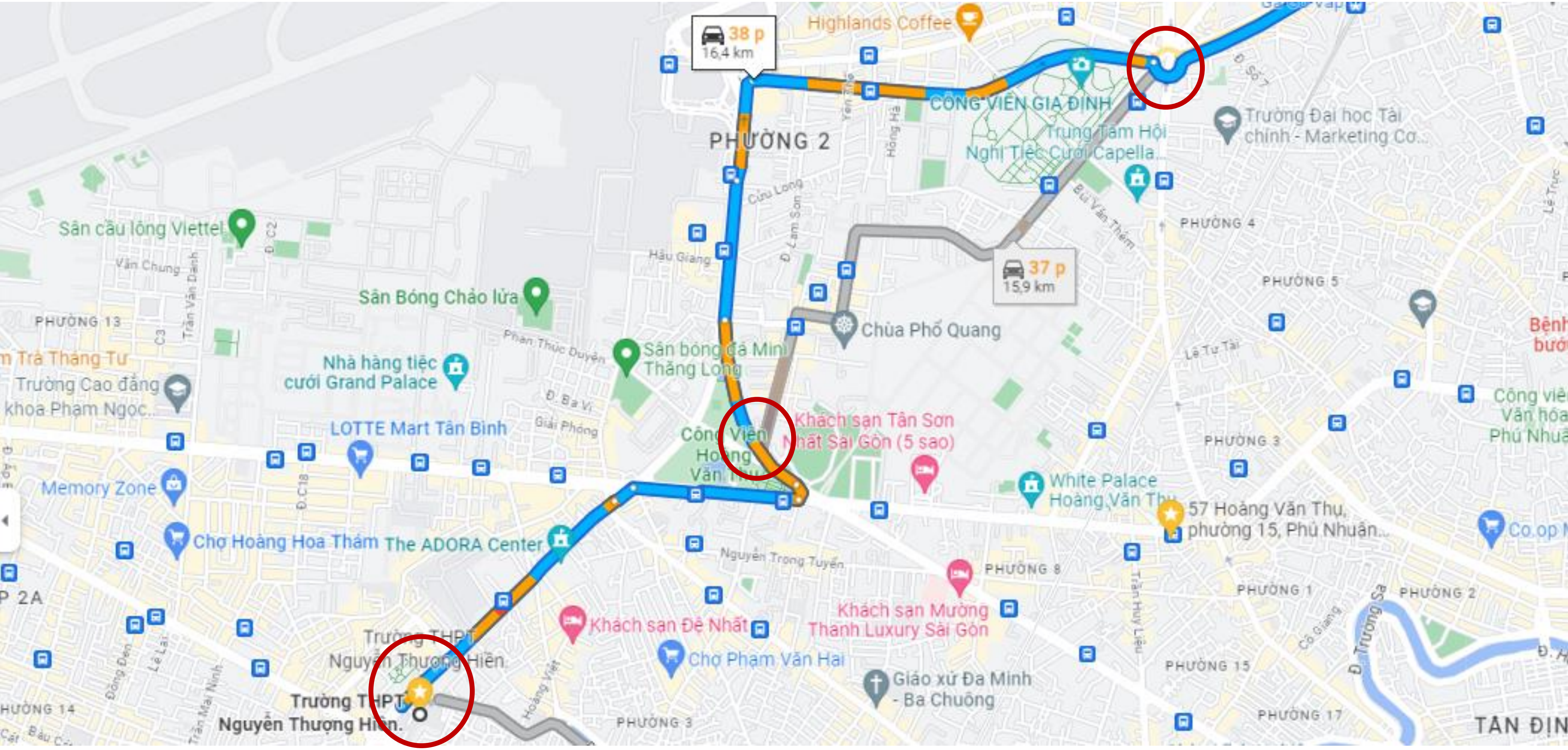
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LOTTE Mart Tân Bình
Khách sạn Tân Sơn Nhất Sài Gòn (5 sao)
Công Viên Hoàng Văn Thụ
Memory Zone
White Palace Hoàng Văn Thụ
57 Hoàng Văn Thụ, phường 15, Phú Nhuận...
Chợ Hoàng Hoa Thám
The ADORA Center
Khách sạn Đệ Nhất
Khách sạn Mường Thanh Luxury Sài Gòn
Chợ Phạm Văn Hai
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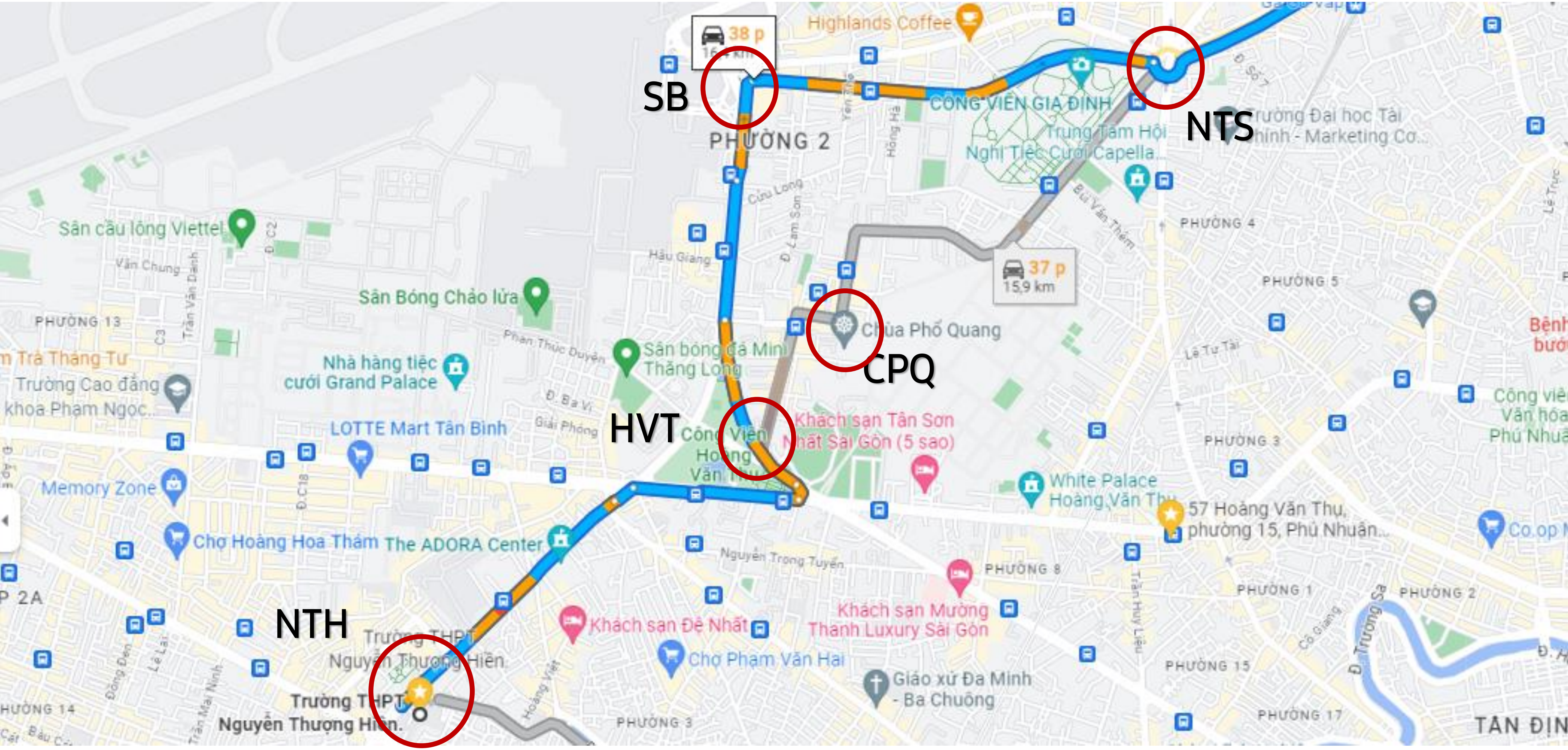
Definition

- A graph consists of a nonempty set of vertices (**nodes**) and a set of edges (**arcs**, links, lines, connections).
- Expression: $G = (V, E)$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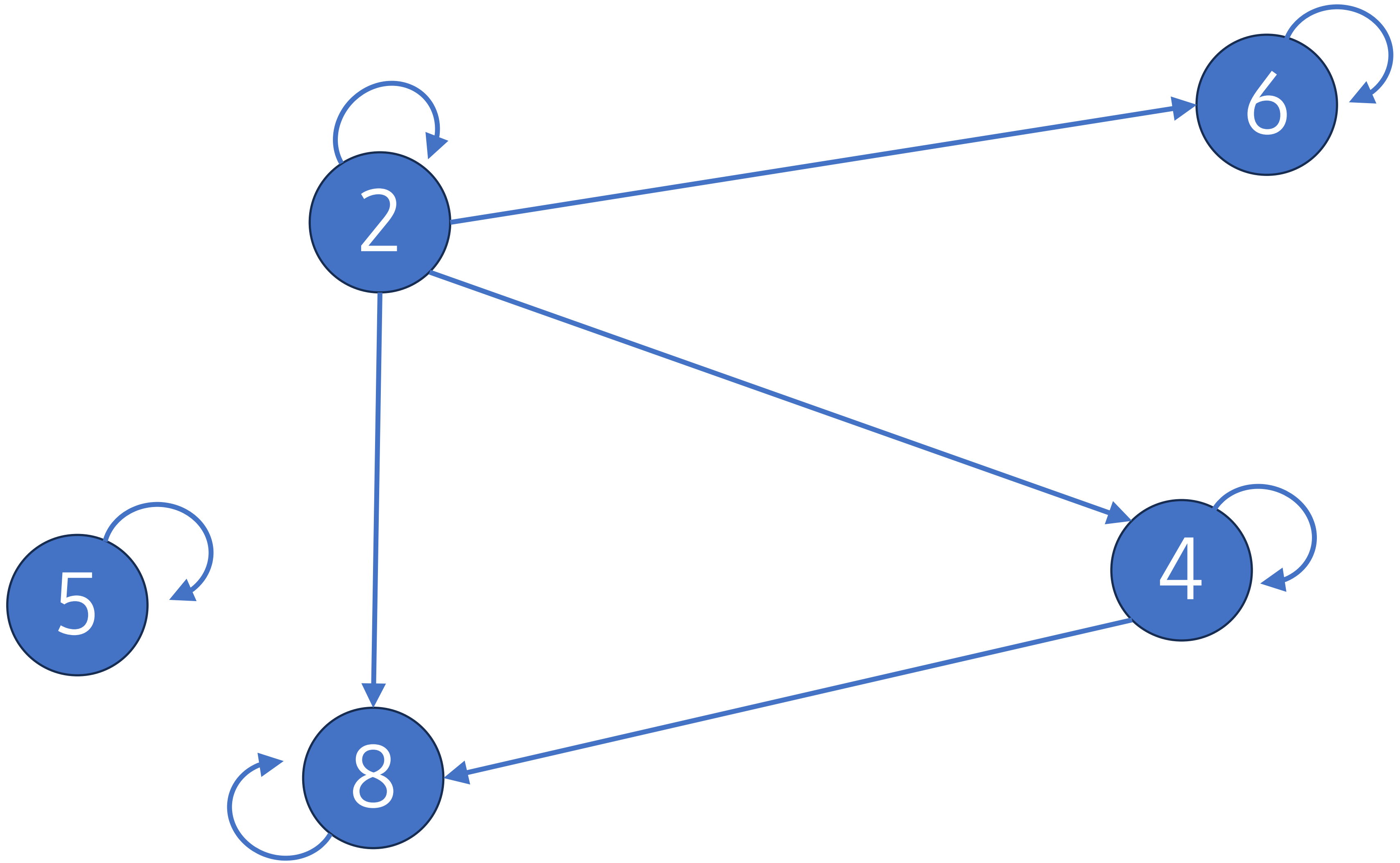
Digraph

- A digraph is a graph, but with directional edges.
  - Ex: $v = (\mathrm{NTH}, \mathrm{HVT})$ represents by a line with arrow towarding HVT from NTH.

$$A = \{2, 4, 5, 6, 8\}$$

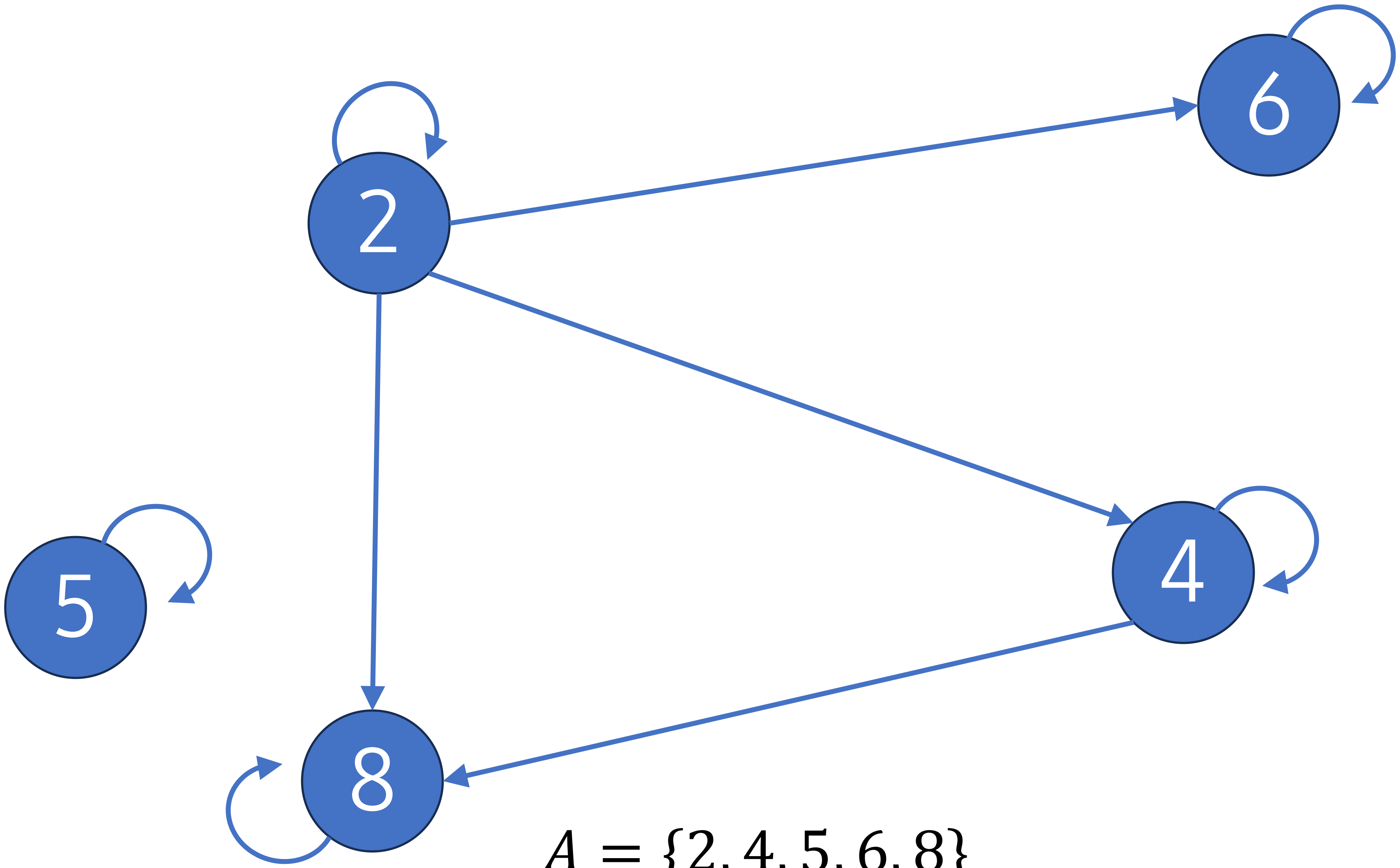
$$R = \{(2,2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8), (5, 5), (6, 6), (8, 8)\}$$

- Each $a \in A$ is a node/**vertex (vertices)**/point.
- Each $r \in R$ is a edge/**arc**.
- The number of edges connected to/from a is the degree of a, or $\deg(a)$.



$$A = \{2, 4, 5, 6, 8\}$$

$$R = \{(2,2), (2,4), (2,6), (2,8), (4,4), (4,8), (5,5), (6,6), (8,8)\}$$



$$A = \{2, 4, 5, 6, 8\}$$

$$R = \{(2,2), (2,4), (2,6), (2,8), (4,4), (4,8), (5,5), (6,6), (8,8)\}$$

R is partial ordering on set A, or A is partially order set (POSET)

Partially order is built on top of algebra

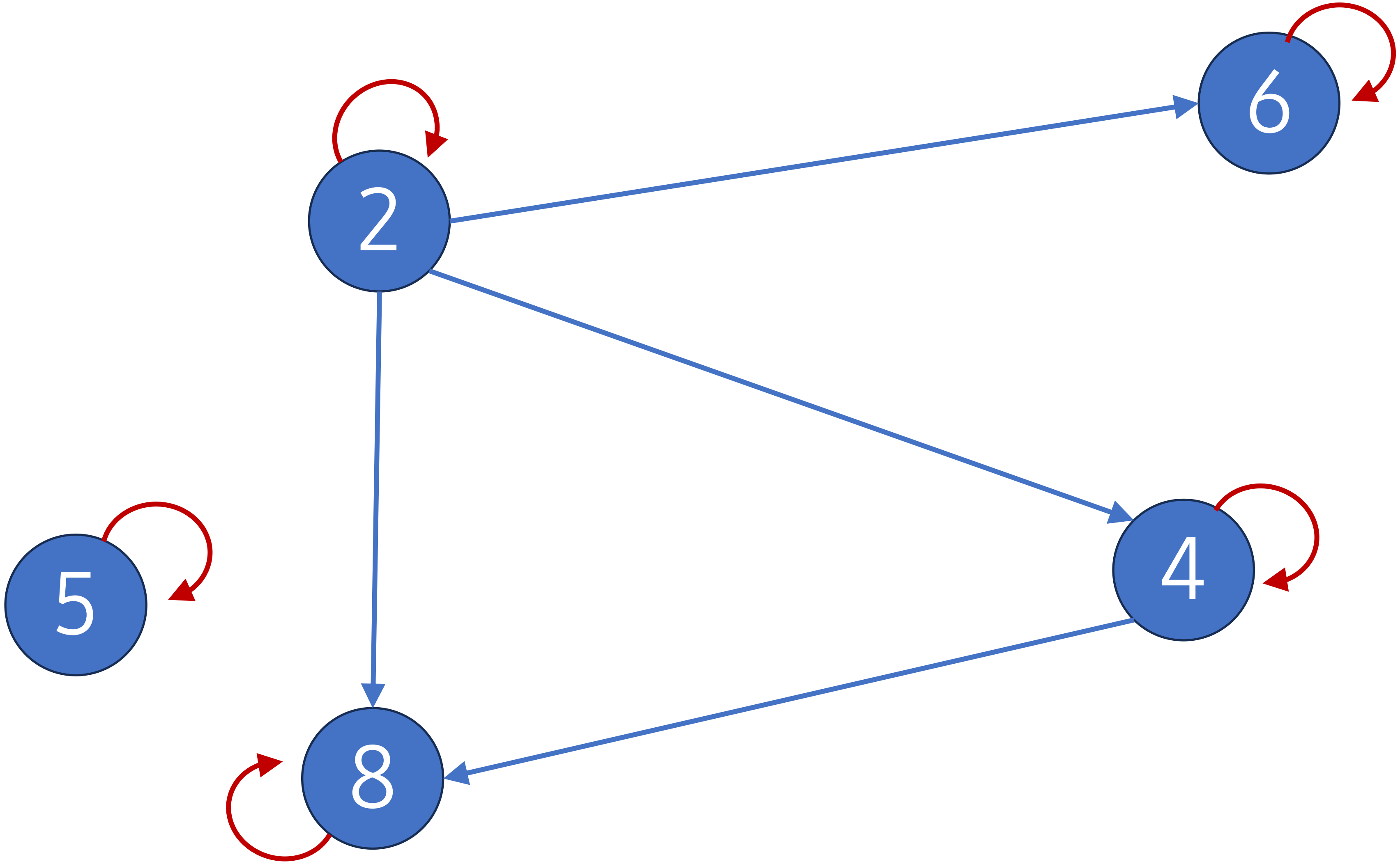
- If there are at least 2 **distinctive** elements, the sequence has **partially order**.
- If a, b, c $(a \neq b, \text{ex:} a < b)$ are real numbers, then:
  - $a = a, b = b, c = c$ is aways true (reflexive).
  - $\mathrm{b} < a$ cannot happen (antisymmetric).
  - If $b < c$, then $a < c$ (transitive).
  - You **can sort** the sequence a, b, c.

How about equivalence?

- If all elements are **equal**, the order is **meaningless** in algebra.
- If a, b, c are equal real numbers, then:
    - $a = a, b = b, c = c$ is aways true (reflexive).
    - $a = b$ and $b = a$ (symmetric).
    - $a = b$, $b = c$, $a = c$ (transitive).
    - You **cannot sort** a, b, c.

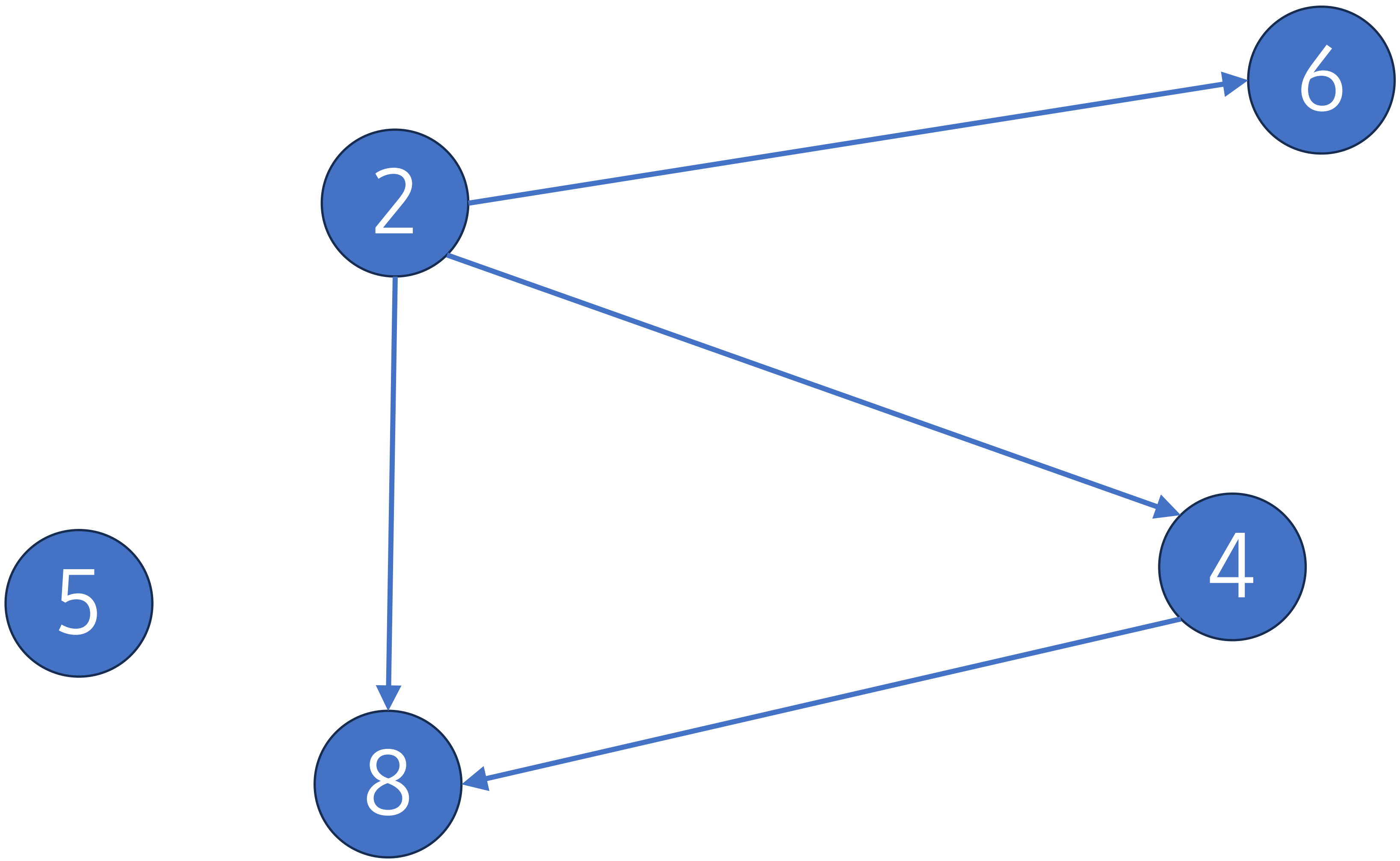
A set is partially order if…

- (By definition) there is at least a pair of element that has order.
- (By proving method) all these conditions are satisfied:
    - Reflexive
    - Antisymmetric
    - Transitive



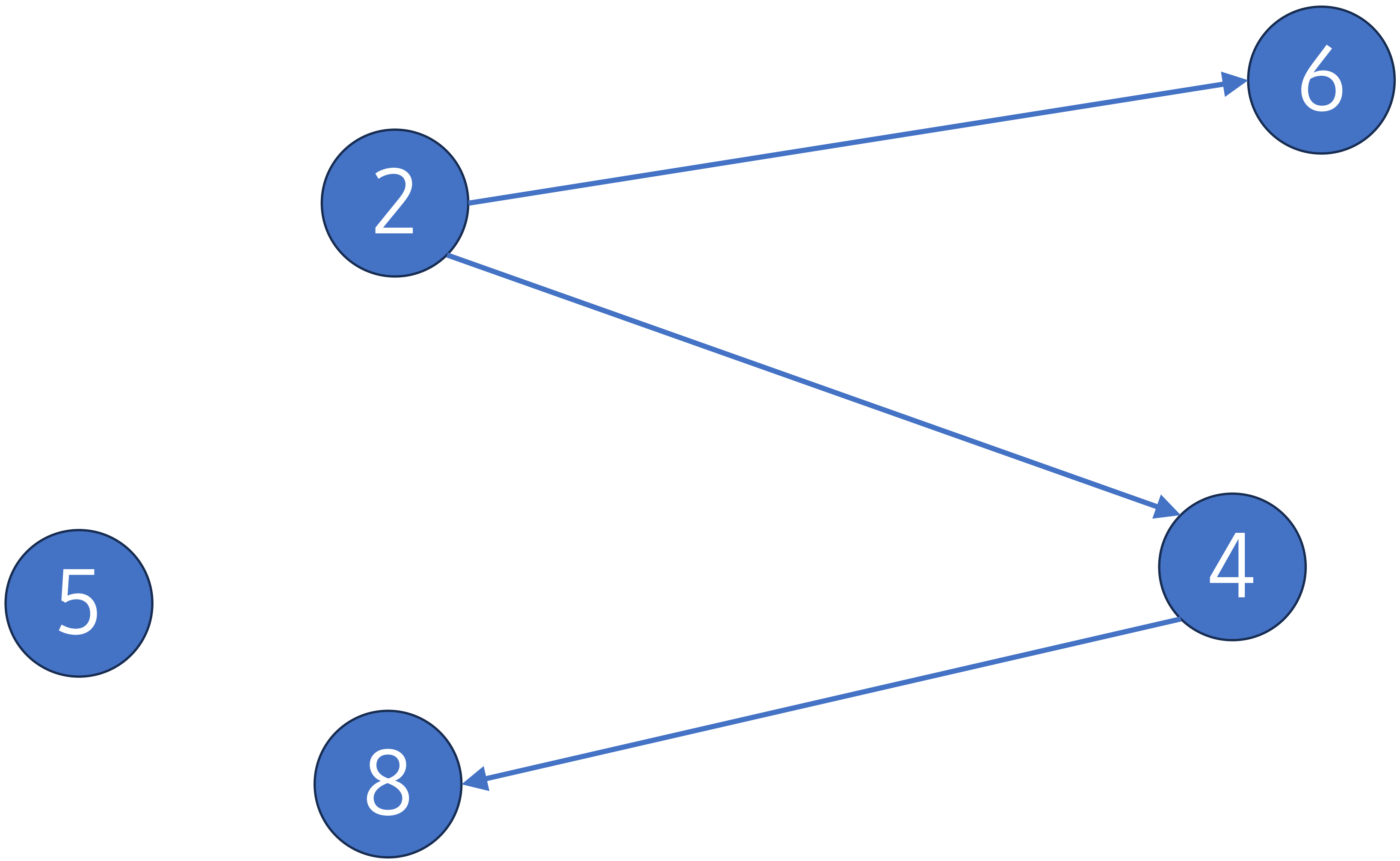
Graph of R

It seems the graph is quite complicated. Do you have any ideas how to simplify?



✓ Remove loops

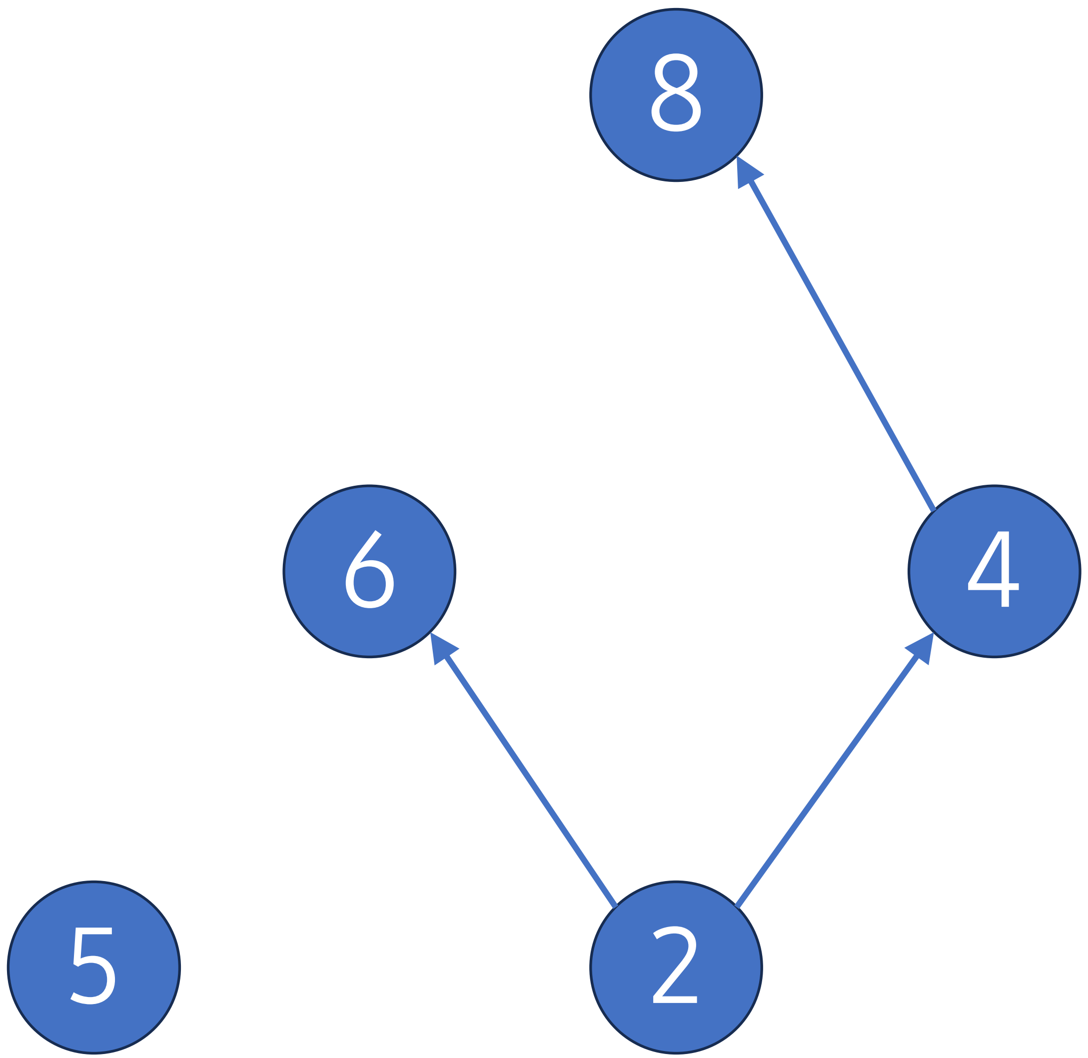
Graph of partially order R



✓ Remove loops

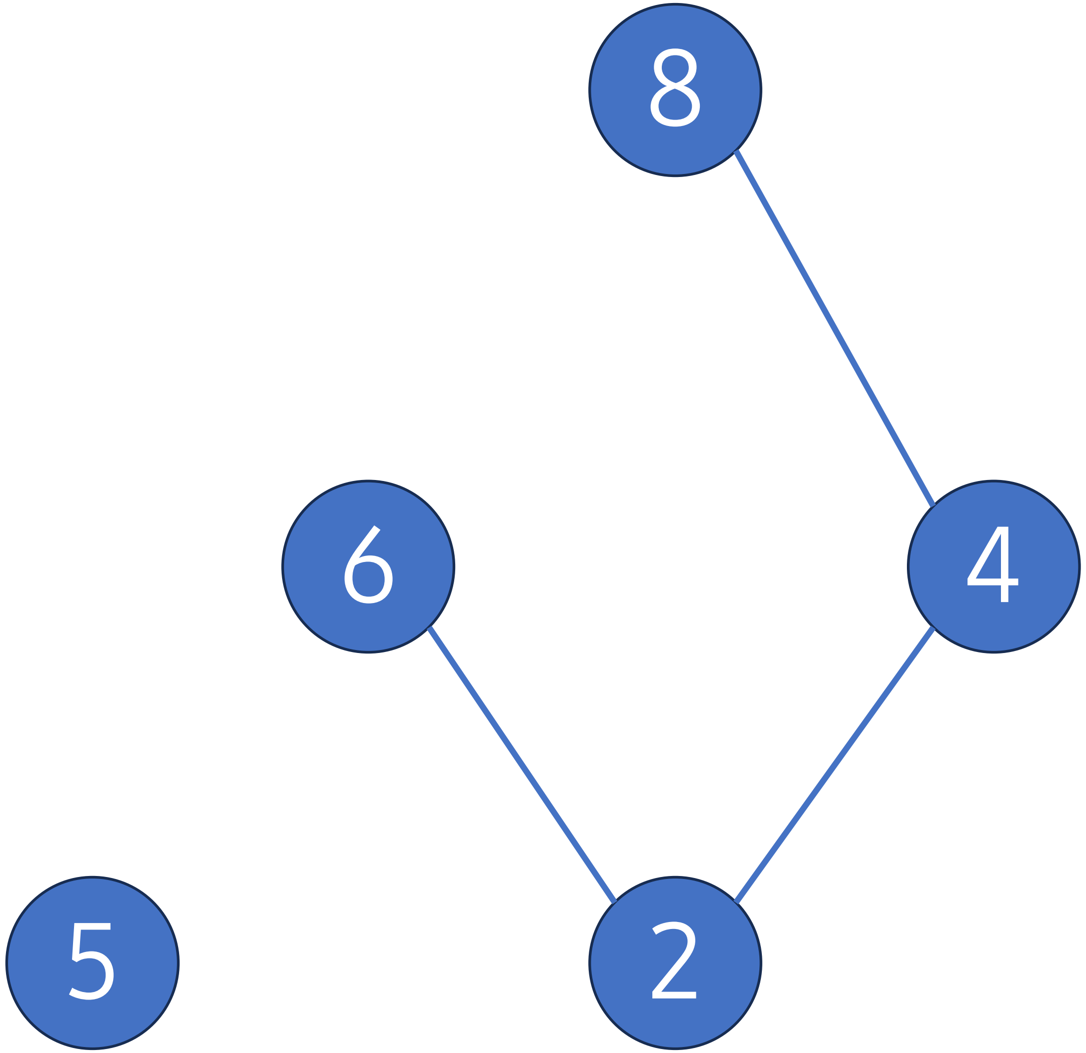
✓ Remove transitive connection

Graph of partially order R



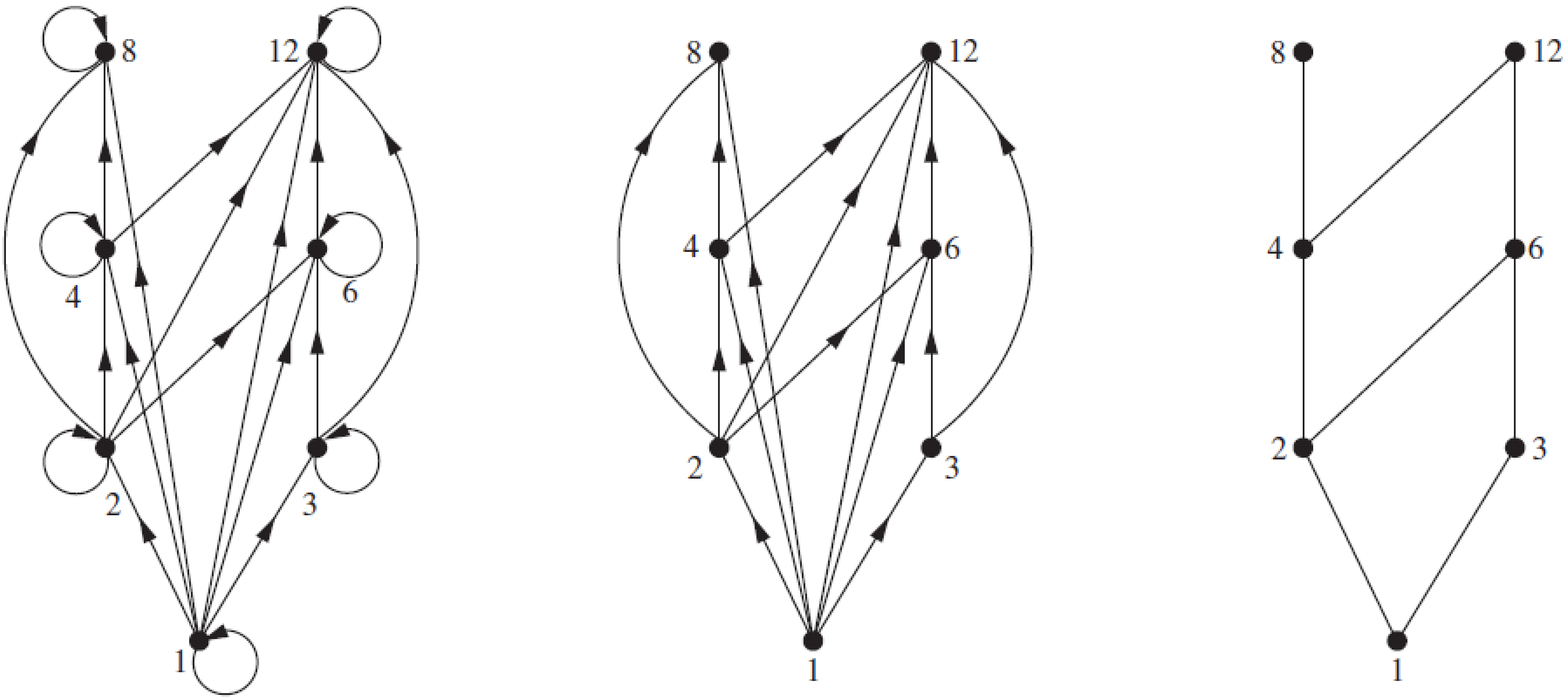
Graph of partially order R

- ✓ Remove loops
- ✓ Remove transitive connection
- ✓ Assuming arrows are pointing upward



Hasse diagram of partially order R

- ✓ Remove loops
- ✓ Remove transitive connection
- ✓ Assuming arrows are pointing upward and remove arrows



An example of Hasse diagram creatation
p. 624, Discrete Mathematics & It's Application – Kenneth H. Rosen

Hasse diagram: A simplified diagram

- Reflexive
  - Remove loops
- Transitive
  - If $a \rightarrow b, b \rightarrow c, a \rightarrow c$, remove $a \rightarrow c$.
- Arrow direction assumption
  - Move nodes, remove arrows by assuming they are pointing upward.

Warning!

- *You can only create Hasse diagram from partially ordering.*
- *You can restore back to the original diagram.*

III. Boolean algebra: The essence

- We don’t have much thing to do with these sets:
  - {A, B, C}
  - {Tam, Tan, Trang}
  - {1, 2, 3}
- But what if there are only 2 discrete values?
  - {False, True}
  - {0, 1}

Boolean algebra

- Operator
- Expression
- Variable
- Function

Some operators in Boolean algebra

Order	Propositional logic	Boolean algebra	Definition
1	$\neg$, $-$	$-$	NOT/Complementation
2	$\wedge$	$\cdot$ (Can be omitted)	AND/Boolean product
3	$\vee$	$+$	OR/Boolean sum

Note: Calculate by order

Some operators in Boolean algebra

Order	Propositional logic	Boolean algebra	Definition
1	¬, −	−	NOT/Complementation
2	∧	· (Can be omitted)	AND/Boolean product
3	∨	+	OR/Boolean sum

$$1 \cdot 0 + \overline{0 + 1}$$

Expression vs. function

$$x \cdot 0 + \overline{0 + y}$$

An expression with variables

$$f(x, y) = x \cdot 0 + \overline{0 + y}$$

A function

Find all values of Boolean function

x	y	$0+y$	$\overline{0+y}$	$x \cdot 0$	$f(x,y)$
1	1	1	0	0	0
1	0	0	1	0	1
0	1	1	0	0	0
0	0	0	1	0	1

$$f(x,y) = x \cdot 0 + \overline{0+y}$$

Part IV. Graph theory: The essence

- Degree, rank, path
- Types of graphs determined by visuals

Degree: The number of edges connected to/from a.

$\deg(v)$: Degree

$\deg^{-}(v)$: Number of edges that are connection to v.

$\deg^{+}(v)$: Number of edges that are connection from v.

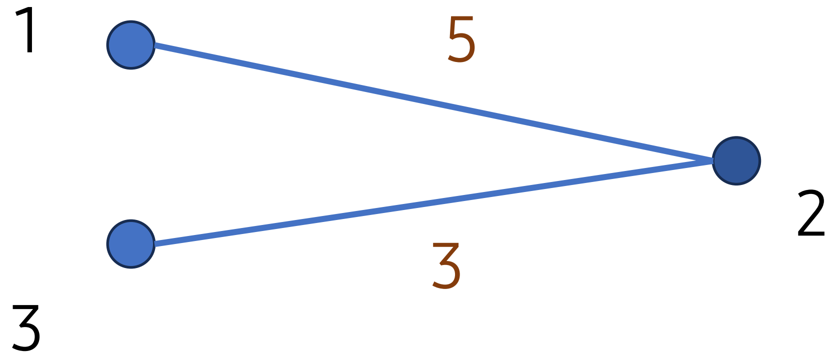
$$\sum_{v \in V} \deg^{-}(v)$$

$$\sum_{v \in V} \deg^{+}(v)$$

Rank

- Rank of an undirected graph is similar to rank of a matrix.
  - Demonstrate a graph into matrix.
  - Count number of linearly independent columns.

Path between two nodes is a string of vertices that are sequentially connected



$$p(v, v') = (v, v_1, v_2, \ldots, v_k, v')$$

$$w(p) = \sum_{i=0}^{k} w(v_i, v_{i+1})$$

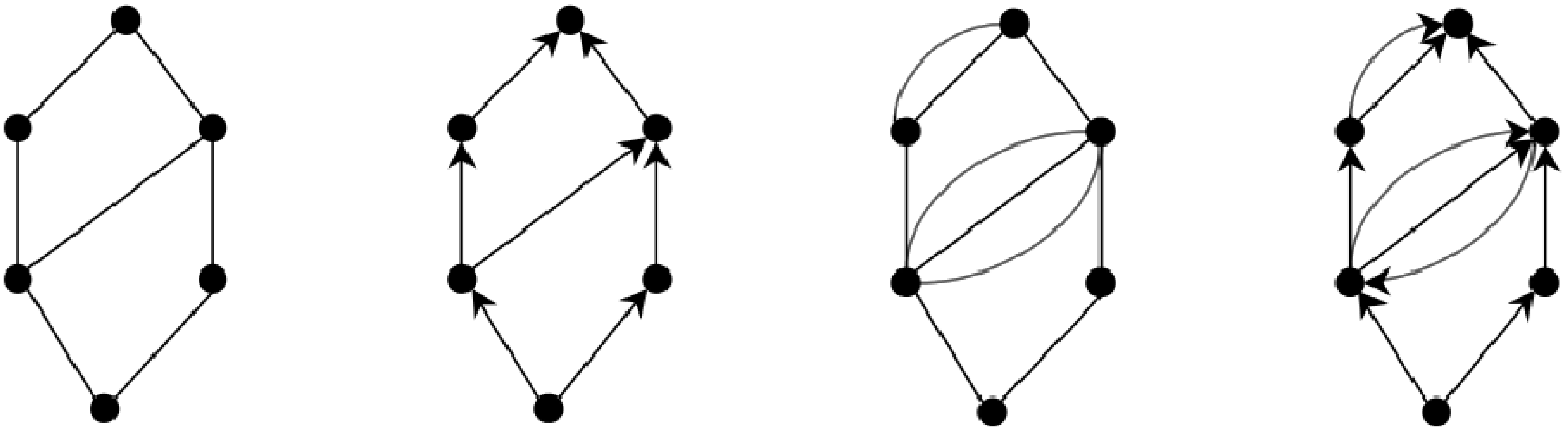
- **Shortest path**: The most efficient path between 2 nodes.

$$d_{w(v,v')} = \min w(p)$$

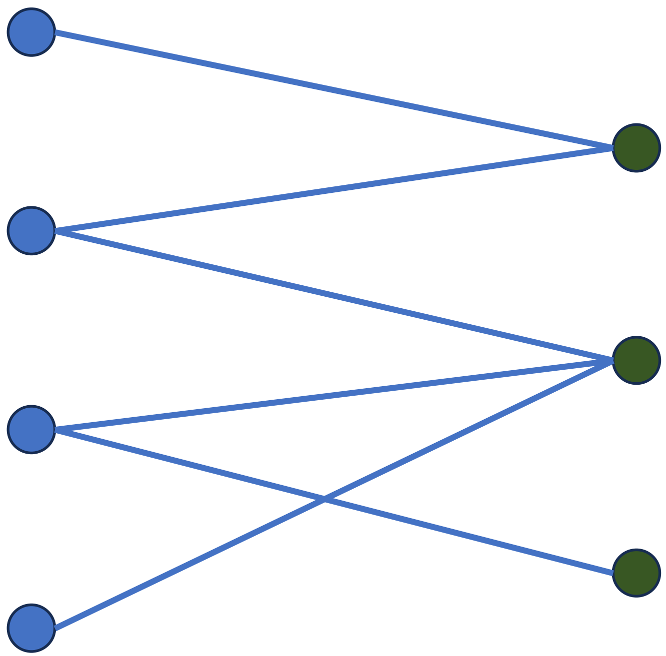
- There are quite many ways to find the shortest path.
- Keyword: *Shortest path problem*.

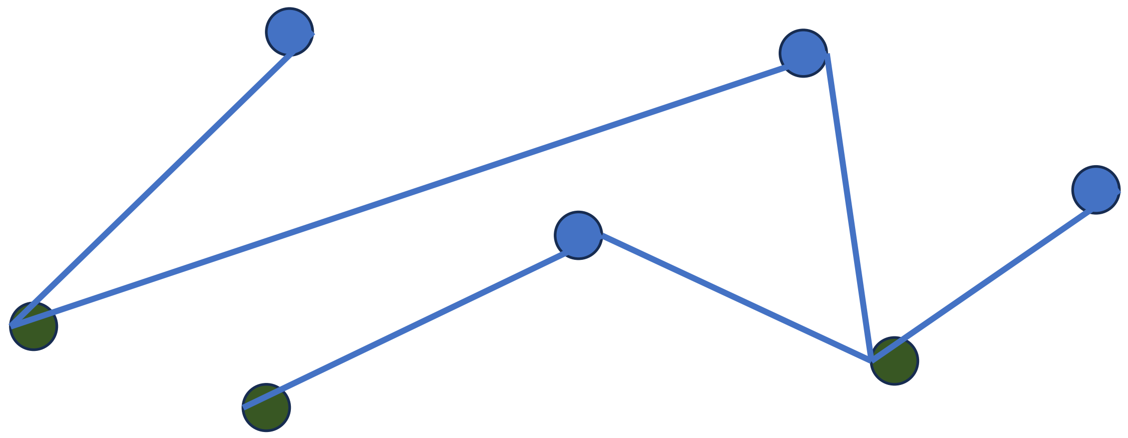
Some popular types

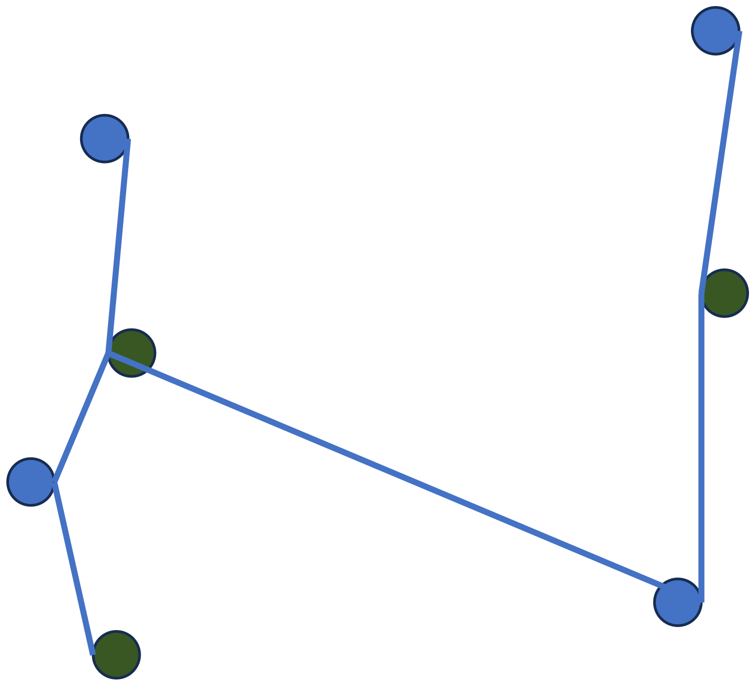
- Digraph (directed graph): Each edge has direction.
- Undirected graph: No direction for all edges.
- Simple graph: At most one edge between two nodes.
- Multigraph: Multiple edges between two nodes are allowed.

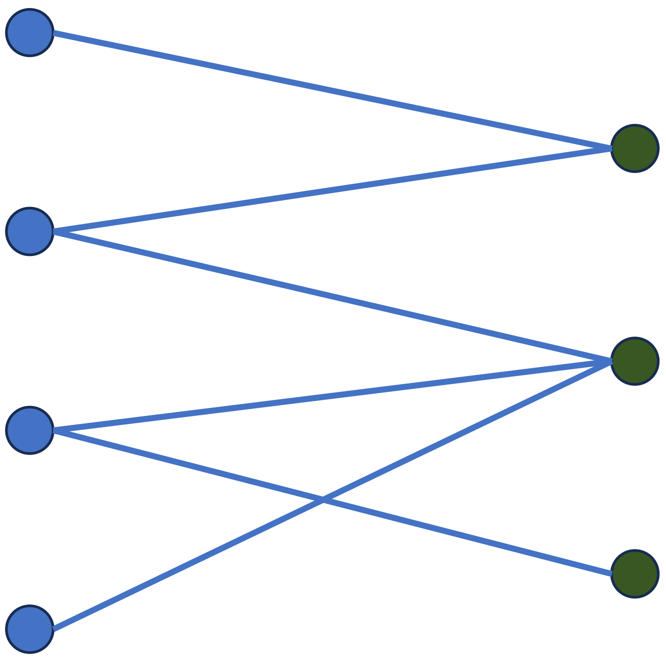
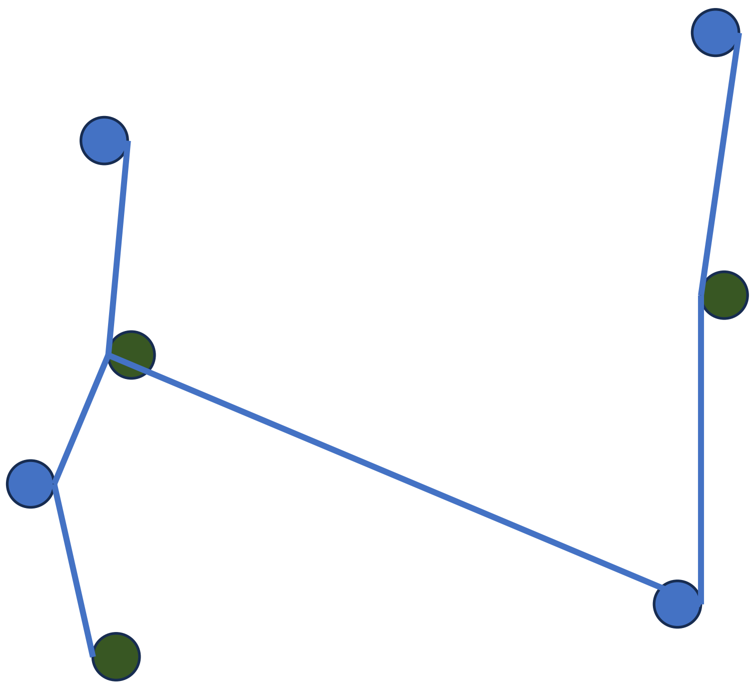


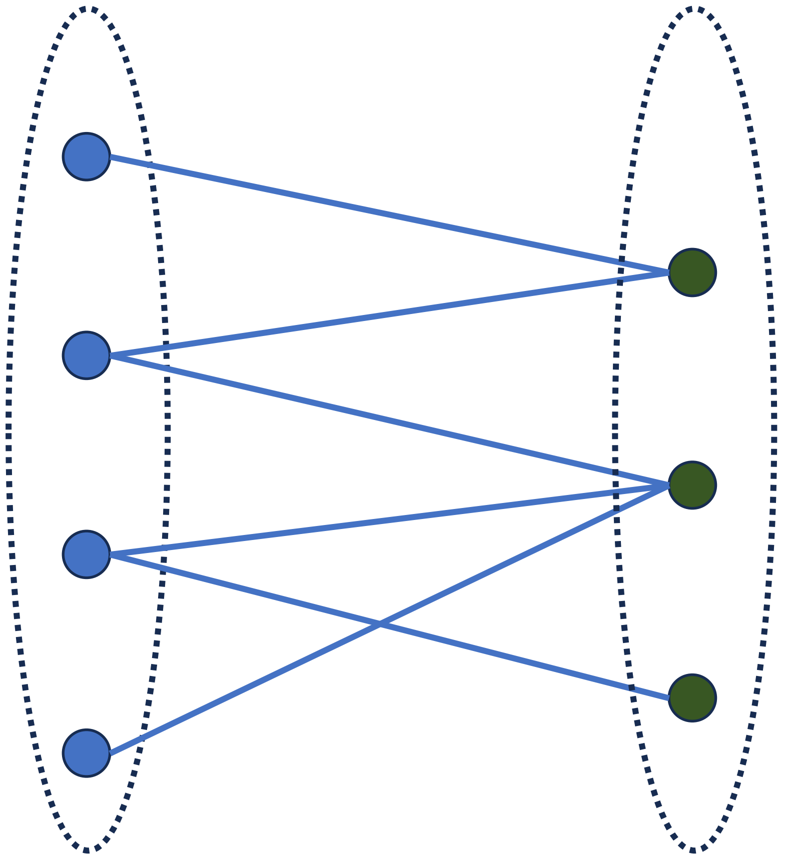
Bipartite graph







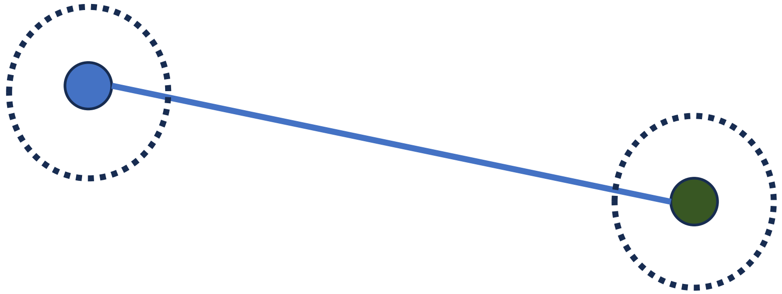





$K_{4,3}$

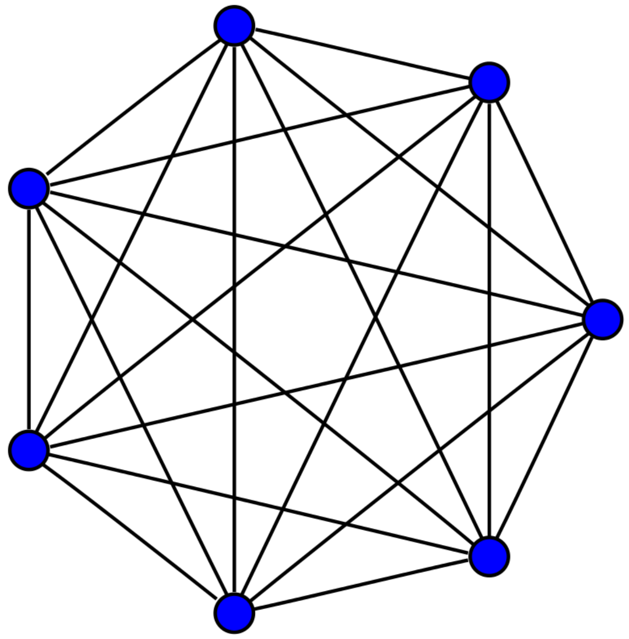
Bipartite graph: The set of vertices can be separated into two that any edge connects nodes of both.

A bipartite graph that has two equal size of set is called a **balanced bipartite graph**.



Complete graph

- An undirected simple graph
- There is one edge between any pair of nodes



A K_7 complete graph

Complete graph properties

- Number of vertices: n
- Number of edges: $C_n^2 = \frac{n(n-1)}{2}$
- All degrees are equal: $n - 1$
- Expression: K_n
- K_1 is also a complete graph.

Complete bipartite graph

- Given s, t as sizes of two sets, we get:

$$K_{s,t}$$

=> The expression is the same as bipartite graph, so we may need to clarify that the graph is also complete.

Exercises/homework

- 22/631 (individual)
- 1, 2, 3, 23/630, 631 (group)

Notes:

- Remember to determine the lacked property.
- Draw initial graph before finding the Hasse diagram.
- Divisibility: $\{(a, b)\}$ which b can be fully divided by a.