AI(Fall 2020) - Assignment 3

Planning and Uncertainty

1. Shakey the robot wakes up in a dark room r_1, there is a light bulb but the switch is turned off. After inspection, she finds out there are another two rooms r_2, r_3 next to this one, where r_2 contains a box $b1$ and r_3 contains a box b_2. To light up the bulb, box b_1 must be in room r_1 and b_2 in r_2. The domain of objects in this problem is thus, $\{shakey, r_1, r_2, r_3, b_1, b_2\}$. The robot Shakey can achieve her goal using the following 3 actions:

   - $walkTo(loc_1, loc_2)$: Shakey walks from location loc_1 to loc_2 where the two locations are adjacent.

   - $push(box, loc_1, loc_2)$: Shakey pushes the box from loc_1 to loc_2 where the two locations are adjacent.

   - $turnOn$: Shakey turns on the light bulb in room r_1, only if the boxes are in their correct positions.

   The descriptions would require the two fluents and one predicate:

   - $lightOn(s)$: in situation s, the light bulb in room r_1 is on.

   - $at(obj, loc, s)$: in situation s, the object obj (box or shakey) is in the location loc (r_1, r_2 or r_3).

   - $adj(l_1, l_2)$: the two locations are adjacent.

   Using the situation calculus, help Shakey to see the light.

   (a) An object cannot be in two different locations in the same situation. Formalize this axiom. (**Hint: quantify over situation s, object o, and two locations l_1 and l_2.**)

   (b) Write a sentence describing the initial situation S_0; Write a sentence of the form $\exists s.\phi(s)$ that asserts the existence of the final goal situation using the fluents. (you are not required to provide the adjacency of the rooms)

   (c) Write the precondition and effect axioms for the actions.

   (d) Write the goal situation σ as a ground situation term, using as few actions as possible. (**Hint: Review the definition of a term; The do(a, s) function is your friend in situation calculus.**)
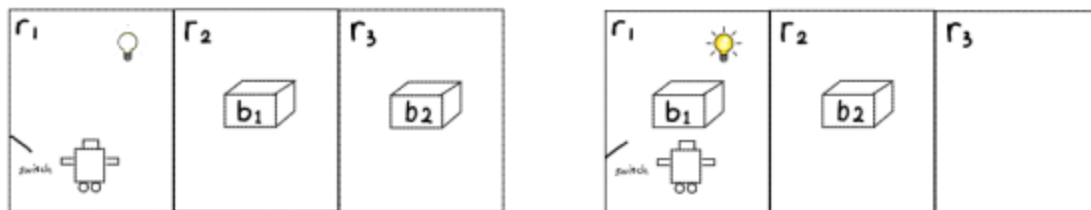

Figure 1: (left) the initial situation; (right) the goal situation

Answer:

1. (a) $\forall s \, \forall o \, \forall l_1 \, \forall l_2 \, \neg(at(o, l_1, s) \wedge at(o, l_2, s))$

(b) S_0: $at(shakey, Y_1, S_0) \wedge at(b_1, Y_2, S_0) \wedge at(b_2, Y_3, S_0)$

final goal: $\exists s \, (lightOn(s) \wedge at(b_1, Y_1, s) \wedge at(b_2, Y_2, s) \wedge at(shakey, Y_1, s))$

(c) $WalkTo(loc1, loc2)$:

pre: $at(shakey, loc1, S) \wedge adj(loc1, loc2)$

Effect: $at(shakey, loc2, do(WalkTo(loc1, loc2), S)) \wedge$
$\neg at(shakey, loc1, do(WalkTo(loc1, loc2), S))$

$Push(box, loc1, loc2)$:

pre: $at(box, loc1, S) \wedge at(shakey, loc1, S) \wedge adj(loc1, loc2)$

Effect: $\neg at(box, loc1, do(Push(box, loc1, loc2), S)) \wedge$
$\neg at(shakey, loc1, do(Push(box, loc1, loc2), S)) \wedge$
$at(box, loc2, do(Push(box, loc1, loc2), S)) \wedge$
$at(shakey, loc2, do(Push(box, loc1, loc2), S))$

$turnOn$:

pre: $at(shakey, Y_1, S) \wedge at(b_1, Y_1, S) \wedge at(b_2, Y_2, S)$

Effect: $lightOn(do(turnOn, S))$

(d) $6 = do(turnOn, do(Push(b_1, Y_2, Y_1), do(Push(b_2, Y_3, Y_2),$
$do(WalkTo(Y_2, Y_3), do(WalkTo(Y_1, Y_2), S_0)))))$

即 shakey 从 $Y_1 \to Y_2 \to Y_3$, 然后推 b_2 到 Y_2, 再推 b_1 到 Y_1, 此时再点亮灯泡即可.

2. Consider the tower of Hanoi problem. There are 3 disks d_1, d_2, d_3 (where d_i is smaller than d_j for $i < j$) and 3 pegs p_1, p_2, p_3. At first the 3 disks are stacked orderly in the first peg, and you are asked to move the entire stack to the third one. There are three predicates:

- $clear(x)$: either peg x is clear or there is no disk on top of disk x.
- $on(x, y)$: disk x is on disk y, or disk x is on peg y.
- $smaller(x, y)$: disk x is smaller than disk y, or x is a disk and y is a peg.

Suppose we have an enhanced robotic arm that is also able to move two consecutive disks in a single action:

- $move(x, a, b)$: move disk x from the peg/disk a to the peg/disk b.
- $moveTwo(x, y, a, b)$: move a consecutive stack of two disks x, y from the peg/disk a to the peg/disk b.

With this enhancement, complete the following tasks:

(a) Write the STRIPS representation of actions, the initial KB and the goal.

(b) Use reachability analysis to compute the heuristic value for the initial state. Draw the state and action layers. For each call of CountActions, indicate the values of G, G_P, G_N and A.

Note: Assume that the axioms of all *smaller* relations of the disks and pegs are given, thus there is no need to specify these relations in the initial KB, goal and the state layers.
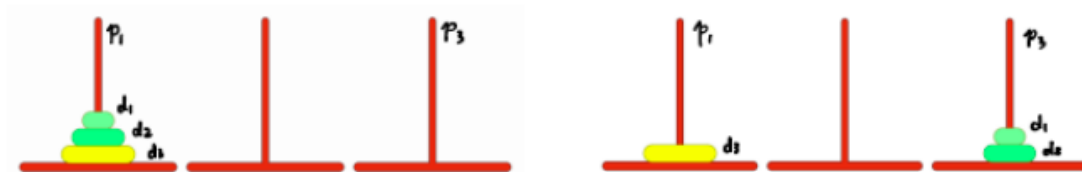

Figure 2: The result (right) of action $moveTwo(d_1, d_2, d_3, p3)$

Answer:

2.(a)   move (x, a, b):

Pre: $\{clear(x), clear(b), on(x,a), smaller(x,b)\}$

Add: $\{clear(a), on(x,b)\}$

Del: $\{clear(b), on(x,a)\}$

moveTwo (x, y, a, b):

Pre: $\{clear(x), clear(b), on(x,y), on(y,a), smaller(y,b)\}$

Add: $\{clear(a), on(y,b)\}$

Del: $\{clear(b), on(y,a)\}$

KB: $\{clear(P_2), clear(P_3), on(d_1, d_2), on(d_2, d_3), on(d_3, P_1), clear(d_1)\}$

Goal: $\{on(d_1, d_2), on(d_2, d_3), on(d_3, P_3), clear(P_2), clear(d_1), clear(P_1)\}$

(b) $S_0 = \{clear(P_2), clear(P_3), on(d_1, d_2), on(d_2, d_3), on(d_3, P_1), clear(d_1)\}$

$G = \{on(d_1, d_2), on(d_2, d_3), on(d_3, P_3), clear(P_2), clear(d_1), clear(P_1)\}$

接下来根据课件里的如下内容得到state and action layers:

- Start with the initial state S_0.

- Alternate between state and action layers.

- Find all actions whose preconditions are contained in S_0.

- These actions comprise the first action layer A_0.

- The next state layer $S_1 = S_0$ U all facts added by actions in A_0.

- Continue this process

Reachability analysis

- In general,
  - A_i: set of actions not in A_{i-1} whose preconditions are in S_i
  - $S_i = S_{i-1}$ U the add lists of all actions in A_i
- Intuitively
  - the actions at level A_i are the actions that could be executed at the i-th step of some plan, and
  - the facts in level S_i are the facts that could be made true within a plan of length i
- Some of the actions/facts have this property. But not all!
- We continue until
  - the goal G is contained in the state layer, or
  - the state layer no longer changes (reaches fix point).

$A_0 = \{$ move(d_1, d_2, P_2), move(d_1, d_2, P_3), moveTwo(d_1, d_2, d_3, P_2),
moveTwo$(d_1, d_2, d_3, P_3)\}$

$S_1 = \{$ clear(d_1), clear(P_2), clear(P_3), clear(d_2), clear(d_3), on(d_1, d_2), on(d_2, d_3),
on(d_3, P_1), on(d_1, P_2), on(d_1, P_3), on(d_2, P_2), on$(d_2, P_3)\}$

$A_1 = \{$ move(d_1, d_2, d_3), move(d_1, P_2, P_3), move(d_1, P_2, d_2), move(d_1, P_2, d_3),

move(d_1, P_3, P_2), move(d_1, P_3, d_2), move(d_1, P_3, d_3), move(d_2, d_3, P_2),
move(d_2, d_3, P_3), move(d_2, P_2, P_3), move(d_2, P_2, d_3), move(d_2, P_3, P_2),
move(d_2, P_3, d_3), move(d_3, P_1, P_2), move(d_3, P_1, P_3), moveTwo(d_1, d_2, P_2, P_3),
moveTwo(d_1, d_2, P_2, d_3), moveTwo(d_1, d_2, P_3, P_2), moveTwo(d_1, d_2, P_3, d_3),
moveTwo(d_2, d_3, P_1, P_2), moveTwo$(d_2, d_3, P_1, P_3)\}$

$S_2 = \{$ clear(d_1), clear(P_2), clear(P_3), clear(d_2), clear(d_3), clear(d_1),
on(d_1, d_2), on(d_2, d_3), on(d_3, P_1), on(d_1, P_2), on(d_1, P_3), on(d_2, P_2),
on(d_2, P_3), on(d_1, d_3), on(d_3, P_2), on$(d_3, P_3)\}$

想法和前 CountActions:

CountActions (G, S_2):

$G_P = \{clear(d_1), clear(P_2), on(d_1, d_2), on(d_2, d_3)\}$

$G_N = \{clear(P_1), on(d_3, P_3)\}$

$A = \{moveTwo(d_2, d_3, P_1, P_3)\}$

$G_1 = \{clear(P_2), clear(d_1), on(d_1, d_2), on(d_2, d_3), on(d_3, P_1), clear(P_3), clear(d_2)\}$

return $1 + CountActions(G_1, S_1)$

CountActions (G_1, S_1):

$G_P = \{clear(P_2), clear(d_1), clear(P_3), on(d_1, d_2), on(d_2, d_3), on(d_3, P_1)\}$

$G_N = \{clear(d_2)\}$

$A = \{move(d_1, d_2, P_2)\}$

$G_0 = \{clear(P_2), clear(d_1), clear(P_3), on(d_1, d_2), on(d_2, d_3), on(d_3, P_1)\}$

return $1 + CountActions(G_0, S_0)$ 至此已结束

故为 $0 + 1 + 1 = 2$

3. Consider the following example: An addition to games is a possible cause of a student getting a low score in the final exam and it could also lead to lack of physical exercises. In turn, either the low score or the lack of exercises could render a student unpopular among classmates. Besides, the low score in the final exam might attribute to the rejection of scholarship application.

   (a) Represent these casual links in a belief network. Let a stand for addition to games, b for lack of exercises, c for low score in the final exam, d for being unpopular among classmates, and e for rejection of scholarship application.

   (b) Give an example of an independence assumption that is implicit in this network.

   (c) Suppose the following probabilities are given:

$$
\begin{aligned}
P(a) &= 0.2 \\
P(b|a) &= 0.7 & P(b|\neg a) &= 0.2 \\
P(c|a) &= 0.2 & P(c|\neg a) &= 0.05 \\
P(e|c) &= 0.7 & P(e|\neg c) &= 0.6 \\
P(d|b, c) &= 0.8 & P(d|\neg b, c) &= 0.7 \\
P(d|b, \neg c) &= 0.7 & P(d|\neg b, \neg c) &= 0.05
\end{aligned}
$$

   and assume that it is also given that some student is popular among classmates but has no scholarship. Calculate joint probabilities for the eight remaining possibilities (that is according to whether a, b, and c are true or false)

   (d) According to the numbers given, the a priori probability that the student is addicted to games is 0.2. Given that the student has no scholarship but is popular among classmates, are we now more or less inclined to believe that the student has game addiction? Explain.

Answer:

3.(a) 如图:
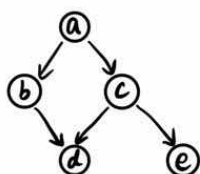
(b) 给定 b 和 c 的前提下，d 与 a 独立

(c) 关求联合概率：(代数过程省略)

$P(a,b,c,\neg d,e) = P(a)P(b|a)P(c|a)P(\neg d|b,c)P(e|c) = 0.00392$

$P(a,b,\neg c,\neg d,e) = P(a)P(b|a)P(\neg c|a)P(\neg d|b,\neg c)P(e|\neg c) = 0.02016$

$P(a,\neg b,c,\neg d,e) = P(a)P(\neg b|a)P(c|a)P(\neg d|\neg b,c)P(e|c) = 0.00252$

$P(a,\neg b,\neg c,\neg d,e) = P(a)P(\neg b|a)P(\neg c|a)P(\neg d|\neg b,\neg c)P(e|\neg c) = 0.02736$

$P(\neg a,b,c,\neg d,e) = P(\neg a)P(b|\neg a)P(c|\neg a)P(\neg d|b,c)P(e|c) = 0.00112$

$P(\neg a,b,\neg c,\neg d,e) = P(\neg a)P(b|\neg a)P(\neg c|\neg a)P(\neg d|b,\neg c)P(e|\neg c) = 0.02736$

$P(\neg a,\neg b,c,\neg d,e) = P(\neg a)P(\neg b|\neg a)P(c|\neg a)P(\neg d|\neg b,c)P(e|c) = 0.00672$

$P(\neg a,\neg b,\neg c,\neg d,e) = P(\neg a)P(\neg b|\neg a)P(\neg c|\neg a)P(\neg d|\neg b,\neg c)P(e|\neg c) = 0.34656$

将上述 8 项相加: $P(\neg d,e) = 0.43572$

则 $P(a,b,c|\neg d,e) = \dfrac{P(a,b,c,\neg d,e)}{P(\neg d,e)} = 0.00900$

同理 $P(a,b,\neg c|\neg d,e) = 0.04627$

$P(a,\neg b,c|\neg d,e) = 0.00578$

$P(a,\neg b,\neg c|\neg d,e) = 0.06279$

$P(\neg a,b,c|\neg d,e) = 0.00257$

$P(\neg a,b,\neg c|\neg d,e) = 0.06279$

$P(\neg a,\neg b,c|\neg d,e) = 0.01542$

$P(\neg a,\neg b,\neg c|\neg d,e) = 0.79537$

(d) 题期要求计算 $P(a|\neg d,e)$, $P(a|\neg d,e) = \dfrac{P(a,\neg d,e)}{P(\neg d,e)} = \dfrac{\sum\limits_{b,c} P(a,b,c,\neg d,e)}{0.43572} = \dfrac{0.05396}{0.43572} = 0.124 < 0.2 = P(a)$

∴ 不倾向于该学生有网瘾.

4. Consider the following belief network

$$
\begin{array}{llll}
P(a) & = & 0.8 & \quad P(d|b) & = & 0.1 \\
P(b) & = & 0.2 & \quad P(d|\neg b) & = & 0.8 \\
P(c|a,b) & = & 0.2 & \quad P(e|c) & = & 0.8 \\
P(c|a,\neg b) & = & 0.7 & \quad P(e|\neg c) & = & 0.1 \\
P(c|\neg a,b) & = & 0.8 & \quad P(f|c) & = & 0.2 \\
P(c|\neg a,\neg b) & = & 0.4 & \quad P(f|\neg c) & = & 0.8
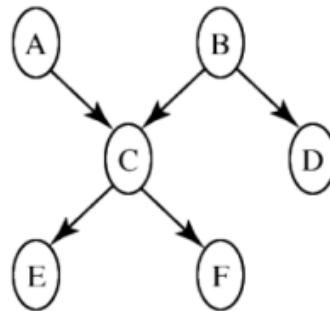\end{array}
$$

Figure 3: BN

(a) Compute $P(e)$ using VE. You should first prune irrelevant variables. Show the factors that are created for a given elimination ordering.

(b) Supposed you want to compute $P(e|\neg f)$ using VE. How much of the previous computation can be reused? Show the factors that are different from those in part (a).

Answer:

4(a) 无关变量有 d, f; 相关变量有 a, b, c, e. 设 A, B, C, E 的 CPT 分别为 $f_1(A)$, $f_2(B)$, $f_3(A,B,C)$, $f_4(C,E)$, 按 A, B, C 的顺序消除.

$$f_5(B,C) = \sum_a f_1(a) f_3(a,B,C)$$

$$f_6(C) = \sum_b f_2(b) f_5(b,C)$$

$$f_7(E) = \sum_c f_6(c) f_4(c,E)$$

C	b	0.32
c	¬b	0.64
¬C	b	0.68
¬C	¬b	0.36

C	0.576
¬C	0.424

e	0.5032
¬e	0.4968

故 $P(e) = 0.5032$

(b) 无关变量 d, 相关变量有 a, b, c, e, f.

$f_1(A)$, $f_2(B)$, $f_3(A,B,C)$、$f_5(B,C)$、$f_6(C)$ 都可以复用. 不同因子为 $f_9(C)$, $f_{10}(C,E)$, $f_{11}(E)$.

令 F 为 f, 则 $f_9(C) = f_8(C, \neg f)$

C	0.8
¬C	0.2

同(a), 得到 $f_5(B,C)$ 和 $f_6(C)$

消除 C: $f_{10}(C,E) = f_4(C,E) \times f_6(C) \times f_9(C)$

C	e	0.36864
c	¬e	0.09216
¬C	e	0.00848
¬C	¬e	0.07632

$$f_{11}(E) = \sum_c f_{10}(C,E)$$

e	0.37712
¬e	0.16848

故 $P(e|\neg f) = \dfrac{0.37712}{0.37712 + 0.16848} = 0.69120$