Compilation Principle
编 译 原 理

第2讲：词法分析(2)

张献伟

xianweiz.github.io

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Review Questions

Q1: input and output of lexical analysis?

character stream → tokens

Q2: how to denote a token?

<class, lexeme>

Q3: atomic and compound REs?

atomic: ε, {a}
compound: R1|R2, R1R2, R1*

Q4: (+|-)?([0-9])*(0|2|4|6|8)
even numbers

Q5: RE of identifiers in C language?

(_letter)(_letter|digit)*

Alphabet Operations[字母表运算]

- Product[乘积]: $\Sigma_1 \Sigma_2 = \{ab \mid a \in \Sigma_1, b \in \Sigma_2\}$
  - E.g., {0, 1}{a, b} = {0a, 0b, 1a, 1b}

- Power[幂]: $\Sigma^n = \Sigma^{n-1} \Sigma$, $n \geq 1$; $\Sigma^0 = \{\varepsilon\}$
  - Set of strings of length n
  - $\{0, 1\}^3 = \{0, 1\}\{0, 1\}\{0, 1\} = \{000, 001, 010, 011, 100, 101, 110, 111\}$

- Positive Closure[正闭包]: $\Sigma^+ = \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \ldots$
  - {a, b, c}+ = {a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, aab, ...}

- Kleene Closure[闭包]: $\Sigma^* = \Sigma^0 \cup \Sigma^+$

Regular Expressions

- Atomic[原子]
  - ε is a RE: $L(\varepsilon) = \{\varepsilon\}$
  - If $a \in \Sigma$, then a is a RE: $L(a) = \{a\}$

- Compound[组合]
  - If both r and s are REs, corr. to languages L(r) and L(s), then：
  - $r|s$ is a RE: $L(r|s) = L(r) \cup L(s)$
  - rs is a RE: $L(rs) = L(r)L(s)$
  - r^* is a RE: $L(r^*) = (L(r))^*$
  - (r) is a RE: $L((r)) = L(r)$

Different REs of the Same Language

- $(a|b)^* = ?$
  - $L((a|b)^*) = (L(a|b))^* = (L(a) \cup L(b))^* = (\{a\} \cup \{b\})^* = \{a, b\}^*$
  - $= \{a, b\}^0 + \{a, b\}^1 + \{a, b\}^2 + \ldots$
  - $= \{\varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$

- $(a^*b^*)^* = ?$
  - $L((a^*b^*)^*) = (L(a^*b^*))^* = (L(a^*)L(b^*))^*$
  - $= L(\{\varepsilon, a, aa, \ldots\}\{\varepsilon, b, bb, \ldots\})^*$
  - $= L(\{\varepsilon, a, b, aa, ab, bb, \ldots\})^*$
  - $= \varepsilon + \{\varepsilon, a, b, aa, ab, bb, \ldots\} + \{\varepsilon, a, b, aa, ab, bb, \ldots\}^2 + \{\varepsilon, a, b, aa, ab, bb, \ldots\}^3 + \ldots$

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Lexical Specification of a Language

- S0: write a regex for the lexemes of each token class
  - Numbers = digit+
  - Keywords = ‘if’ + ‘else’ + …
  - Identifiers = letter(letter + digit)*
- S1: construct R, matching all lexemes for all tokens
  - R = numbers + keywords + identifiers + … = R1 + R2 + R3 + …
- S2: let input be $x_q \ldots x_n$, for $1 \leq i \leq n$, check $x_1 \ldots x_i \in L(R)$
- S3: if successful, then we know $x_1 \ldots x_i \in L(R_j)$ for some j
- S4: remove $x_1 \ldots x_i$ from input and go to step S2

Lexical Specification of a Language

- How much input is used?
  - $x_1 \ldots x_i \in L(R)$, $x_1 \ldots x_j \in L(R)$, $i \neq j$
  - Which one do we want? (e.g., '==' or '=')
  - <u>Maximal match</u>: always choose the longer one
- Which token is used if more than one matches?
  - $x_1 \ldots x_i \in L(R)$ where $R = R_1 + R_2 + \ldots + R_n$
  - $x_1 \ldots x_i \in L(R_m)$, $x_1 \ldots x_i \in L(R_n)$, $m \neq n$
  - E.g., keywords = 'if', identifier = letter(letter+digit)*
  - Keyword has higher priority
  - <u>Rule of thumb</u>: choose the one listed first
- What if no rule matches?
  - $x_1 \ldots x_i \notin L(R) \rightarrow$ Error

Summary: RE

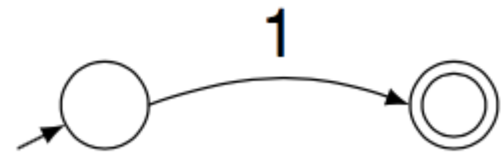
- We have learnt how to specify tokens for lexical analysis
  - Regular expressions
  - Concise notations for the string patterns
- Used in lexical analysis with some extensions
  - To resolve ambiguities
  - To handle errors
- REs is only a language specification
  - An implementation is still needed
  - Next: to construct a token recognizer for languages given by regular expressions – by using **finite automata**

Implementation of Lexical Analyzer

- How do we go from specification to implementation?
  - RE → finite automata
- Solution 1: to implement using a tool — Lex (for C), Flex (for C++), Jlex (for java)
  - Programmer specifies tokens using REs
  - The tool generates the source code from the given REs
    - The Lex tool essentially does the following translation: REs (Specification) ⇒ FAs (Implementation)
- Solution 2: to write the code yourself
  - More freedom; even tokens not expressible through REs
  - But difficult to verify; not self-documenting; not portable; usually not efficient
  - Generally not encouraged

Transition Diagram[转换图]

- REs → transition diagrams
  - By hand
  - Automatic



- Node: state
  - Each state represents a condition that may occur in the process
  - Initial state (Start): only one, circle marked with ‘start →’
  - Final state (Accepting): may have multiple, double circle

- Edge: directed, labeled with symbol(s)
  - From one state to another on the input

Finite Automata[有穷自动机]

- **Regular Expression** = specification
- **Finite Automata** = implementation

- Automaton (pl. automata): a machine or program
- Finite automaton (FA): a program with a finite number of states

- Finite Automata are similar to transition diagrams
  - they have states and labelled edges
  - there are one unique start state and one or more than one final states

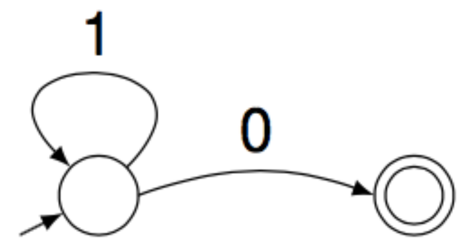
FA: Language

- An FA is a program for classifying strings (accept, reject)
  - In other words, a program for recognizing a language
  - The Lex tool essentially does the following translation: REs (Specification) ⇒ FAs (Implementation)
  - For a given string ‘x’, if there is transition sequence for ‘x’ to move from start state to certain accepting state, then we say ‘x’ is accepted by the FA

- Language of FA = set of strings accepted by that FA
  - L(FA) ≡ L(RE)

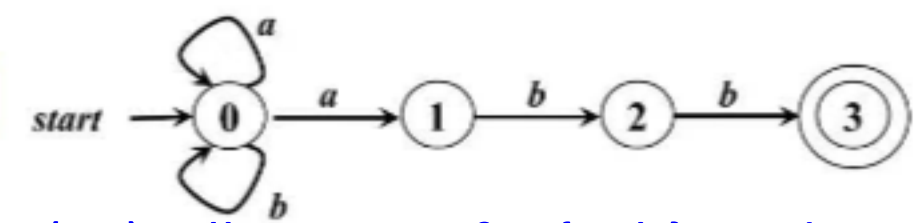
Example

- Are the following strings acceptable?
  - 0 ✓
  - 1 ✗
  - 11110 ✓
  - 11101 ✗
  - 11100 ✗
  - 1111110 ✓



- What language does the state graph recognize? ∑ = {0, 1}

  Any number of ‘1’s followed by a single 0



L(FA): all strings of ∑ {a, b}, ending with ‘abb’

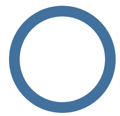
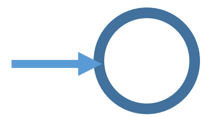
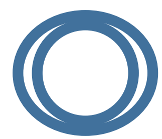
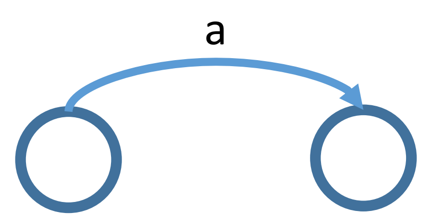
L(RE) = (a|b)*abb

DFA and NFA

- Deterministic Finite Automata (**DFA**): the machine can exist in only one state at any given time
  - One transition per input per state
  - No ε-moves
  - Takes only one path through the state graph

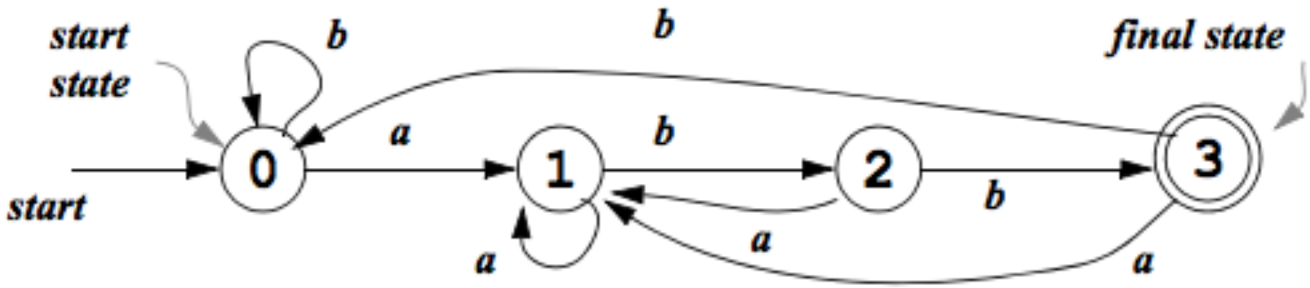
- Nondeterministic Finite Automata (**NFA**): the machine can exist in multiple states at the same time
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves
  - Can choose which path to take
    - An NFA accepts if some of these paths lead to accepting state at the end of input

State Graph

- 5 components （Σ, S, n, F, δ）
  - An input alphabet Σ
  - A set of states S 
  - A start state $n \in S$ 
  - A set of accepting states $F \subseteq S$ 
  - A set of transitions δ: $S_a \xrightarrow{\text{input}} S_b$ 

Example: DFA

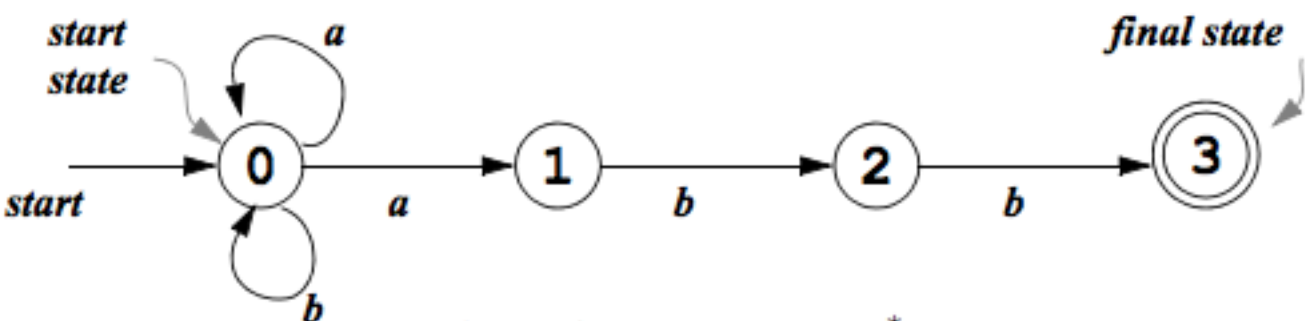
- There is **only one** possible sequence of moves --- either lead to a final state and accept or the input string is rejected
  - Input string: aabb
  - Successful sequence: $0 \xrightarrow{a} 1 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3$



A DFA accepts (a|b)*abb

Example: NFA

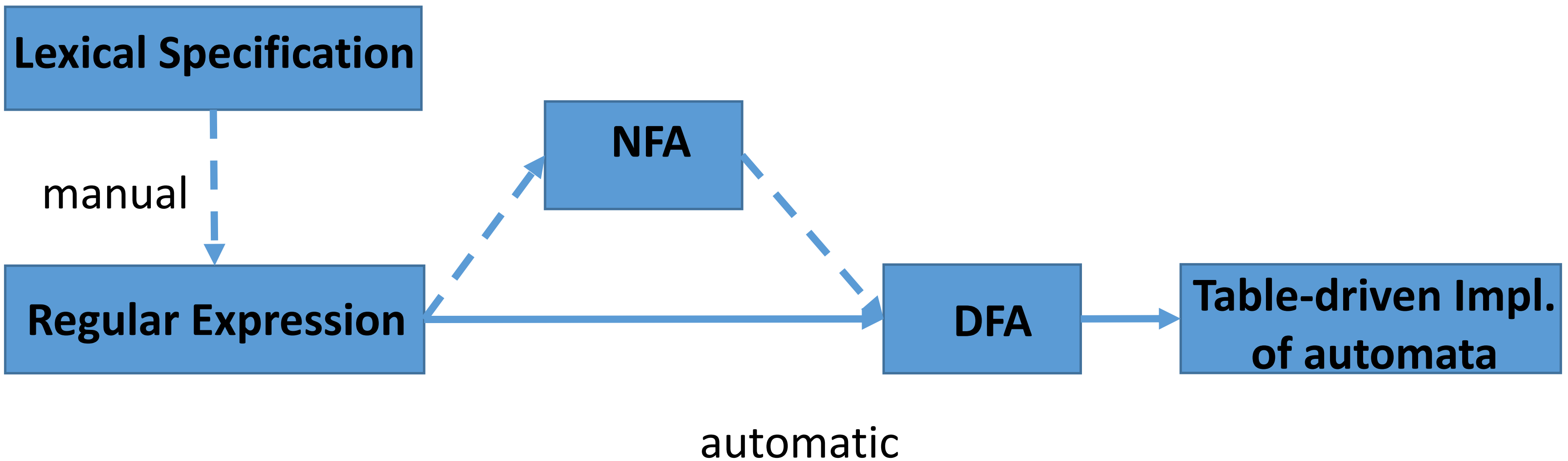
- There are **many possible** moves --- to accept a string, we only need one sequence of moves that lead to a final state
  - Input string: aabb
  - Successful sequence: $0 \xrightarrow{a} 0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3$
  - Unsuccessful sequence: $0 \xrightarrow{a} 0 \xrightarrow{a} 0 \xrightarrow{b} 0 \xrightarrow{b} 0$



An NFA accepts (a|b)*abb

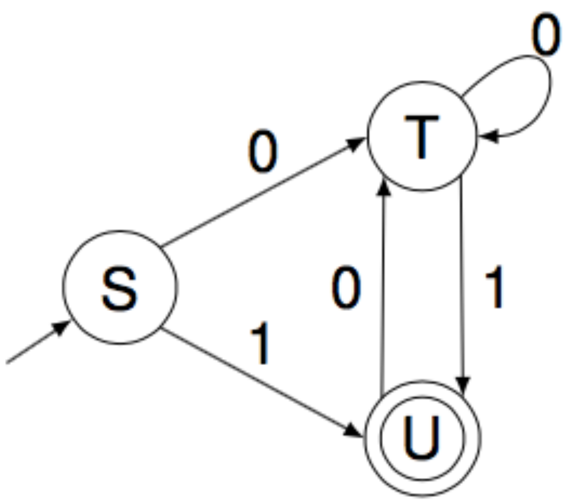
Conversion Flow

- Outline: RE → NFA → DFA → Table-driven Implementation
  - Converting DFAs to table-driven implementations
  - Converting REs to NFAs
  - Converting NFAs to DFAs



DFA → Table

- FA can also be represented using transition table



alphabet →

state ↓

	0	1
S	T	U
T	T	U
U	T	x

```
Table-driven Code:
DFA() {
    state = "S";
    while (!done) {
        ch = fetch_input();
        state = Table[state][ch];
        if (state == "x")
            print("reject");
    }
    if (state ∈ F)
        printf("accept");
    else
        printf("reject");
}
```

Q: which is/are accepted?

111

000

001

19

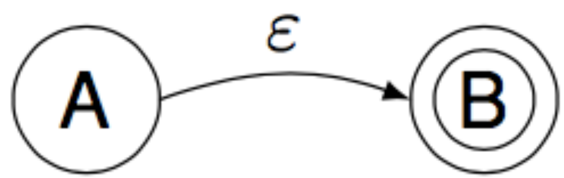
Discussion

- Implementation is efficient
  - Table can be automatically generated
  - Need finite memory O(S x ∑)
    - Size of transition table
  - Need finite time O(input length)
    - Number of state transitions

- Pros and cons of table:
  - Pro: can easily find the transitions on a given state and input
  - Con: takes a lot of space, when the input alphabet is large, yet most states do not have any moves on most of the input symbols

RE → NFA

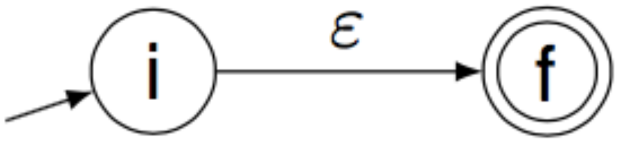
- NFA can have ε-moves
  - Edges labelled with ε
  - move from state A to state B without reading any input



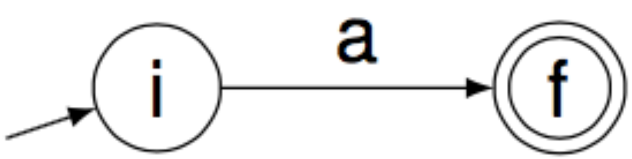
- M-Y-T algorithm to convert any RE to an NFA that defines the same language
  - Input: RE r over alphabet ∑
  - Output: NFA accepting L(r)

RE → NFA (cont.)

- Step 1: processing atomic REs
  - ε expression

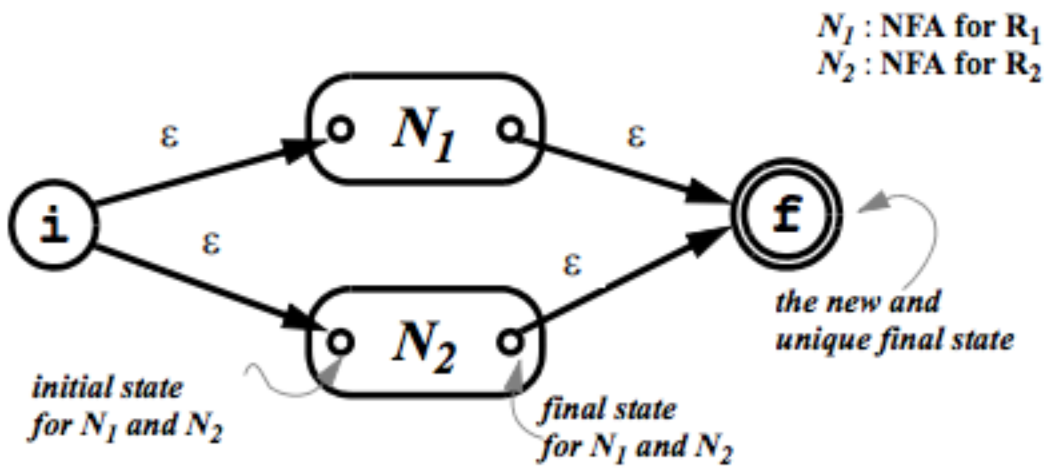


    - i is new state, the start state of NFA
    - f is another new sate, the accepting state of NFA
  - Single character RE a

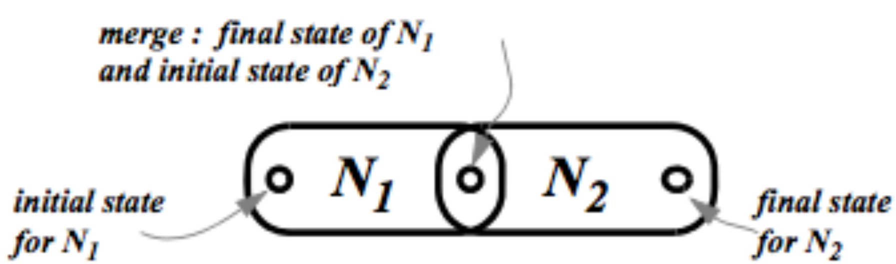


RE → NFA (cont.)

- Step 2: processing compound REs
  - R = R1 | R2

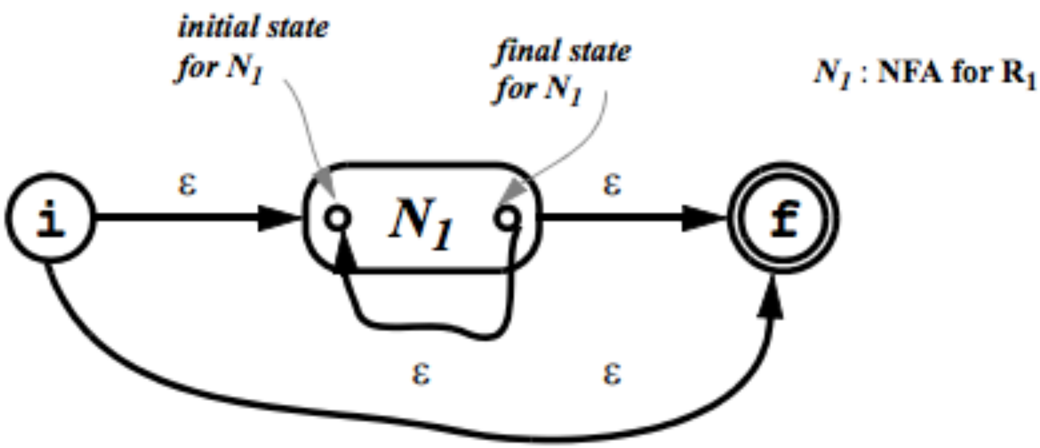


  - R = R1R2



RE → NFA (cont.)

- Step 2: processing compound REs
  - R = R1*

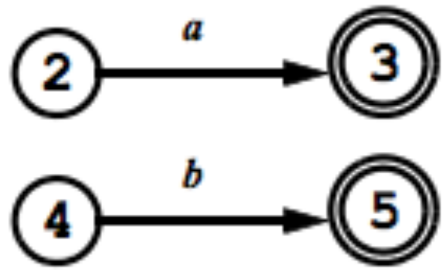


Example

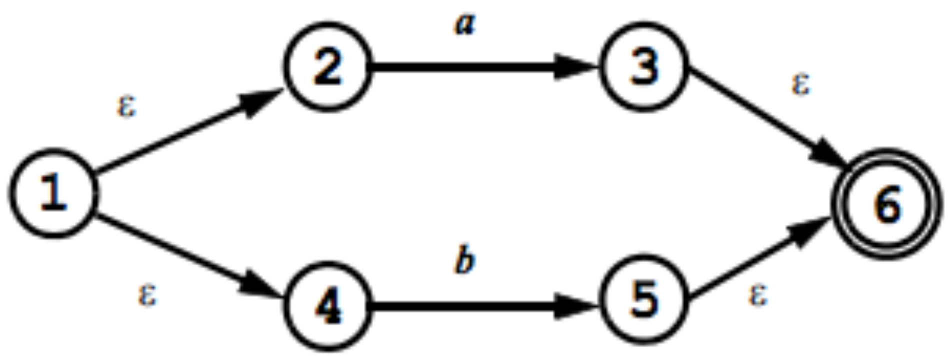
- Convert “(a|b)*abb” to NFA

a (in a|b) ===>

b (in a|b) ===>

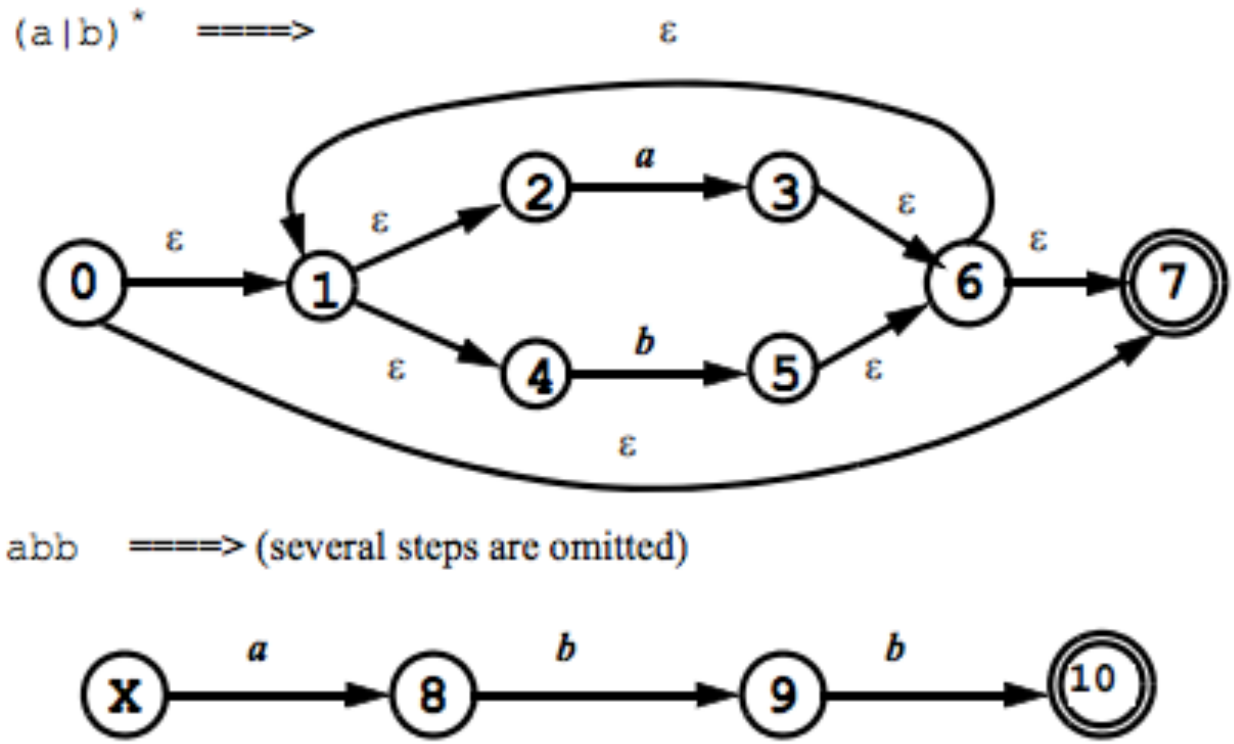


a|b ====>



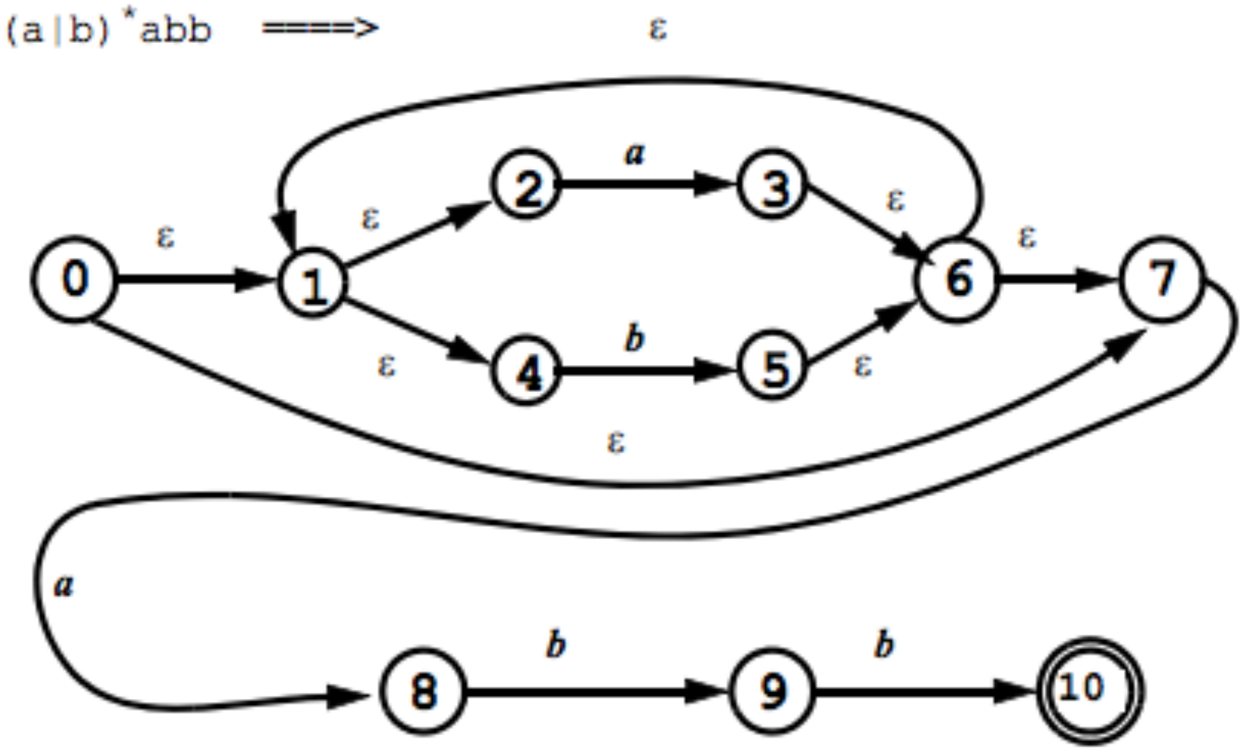
Example (cont.)

- Convert "(a|b)*abb" to NFA



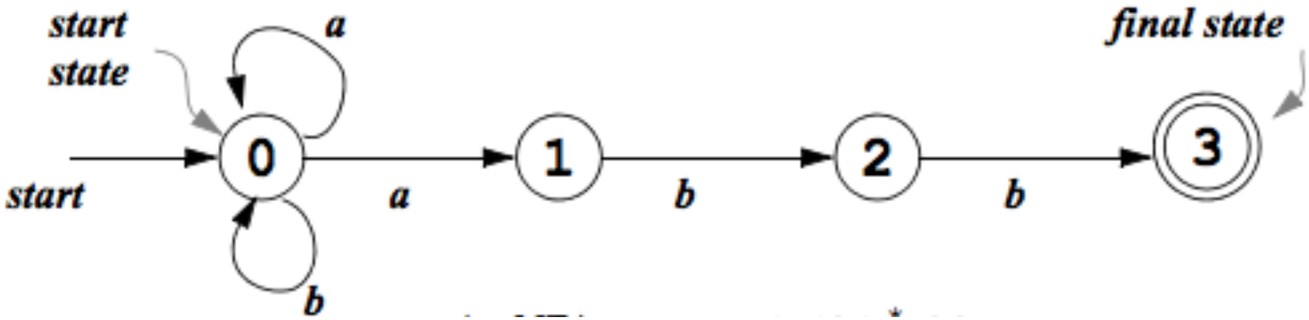
Example (cont.)

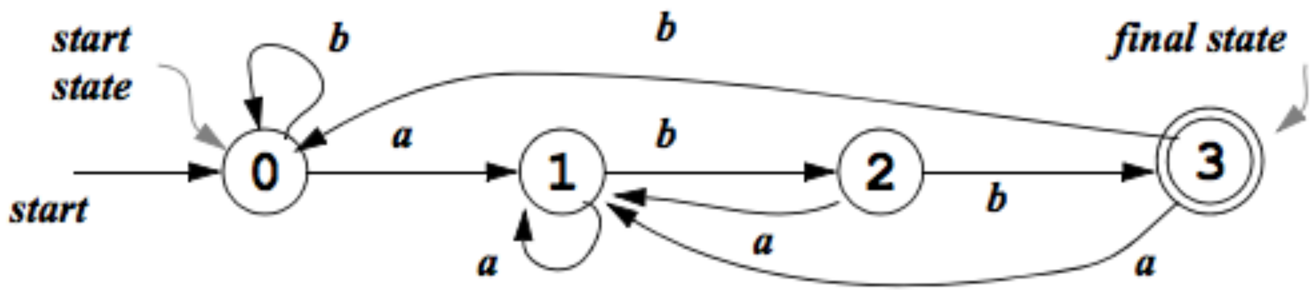
- Convert “(a|b)*abb” to NFA



NFA → DFA: Same

- NFA and DFA are equvalent





NFA → DFA: Theory

- Question: is L(NFA) ⊆ L(DFA)
  - Otherwise, conversion would be futile
- Theorem: L(NFA) ≡ L(DFA)
  - Both recognize regular languages L(RE)
  - Will show L(NFA) ⊆ L(DFA) by construction (NFA → DFA)
  - Since L(DFA) ⊆ L(NFA), L(NFA) ≡ L(DFA)
- Resulting DFA consumes more memory than NFA
  - Potentially larger transition table as shown later
- But DFAs are faster to execute
  - For DFAs, number of transitions == length of input
  - For NFAs, number of potential transitions can be larger
- NFA → DFA conversion is done because the speed of DFA far outweigh its extra memory consumption