



中山大學  
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# Compilation Principle 编译原理

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## 第8讲：语法分析(5)

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# Review Questions (1)

- What are the parts of a table-driven predictive parser?

Input buffer, stack, parse table and a driver

- What are the operations on the stack?

Expand the non-terminal, match the terminal

- How to predict the next production to use?

Next input symbol, current nonterminal being processed

- What does LL(k) mean?

L: scans the input from left to right

L: produces a leftmost derivation

k: using k input symbols of lookahead

- How to build the LL(1) parse table?

Two sets: FIRST, FOLLOW

# Review Questions (2)

- Which one is typically used, LL(0), LL(1), LL(2) ...? Why not others?

LL(1). LL(0) is too weak, LL(k) has a too large table

- Which are the key differences between top-down and bottom-up parsing?

Top-down is based on leftmost derivation;  
bottom-up is the reverse of rightmost derivation.

- What are the key operations of bottom-up parsing?

Shift: pushes a terminal on the stack

Reduce: pops RHS and pushes LHS

# Properties of Bottom-up Parsing

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- Handles always appear at the **top of the stack**
  - Never in middle of stack
  - Justifies use of stack in shift – reduce parsing
- Results in an easily generalized **shift – reduce** strategy
  - If there is no handle at the top of the stack, shift
  - If there is a handle, reduce to the non-terminal
  - Easy to automate the synthesis of the parser using a table
- Can have conflicts
  - If it is legal to either shift or reduce then there is a shift-reduce conflict
  - If there are two legal reductions, then there is a reduce-reduce conflict
  - Most often occur because of ambiguous grammars
    - In rare cases, because of non-ambiguous grammars not amenable to parser

# Types of Bottom Up Parsers

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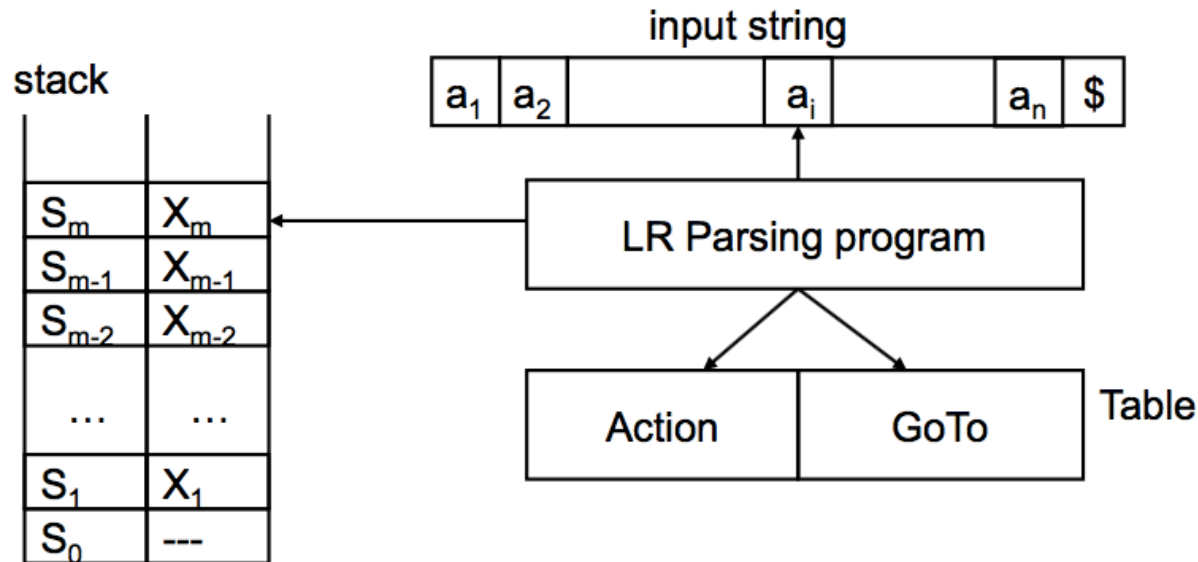
- Types of bottom up parsers
  - Simple precedence parsers
  - Operator precedence parsers
  - Recursive ascent parsers
  - LR family parsers
  - ...
- In this course, we will only discuss **LR family parsers**
  - Efficient, table-driven shift-reduce parsers
  - Most automated tools for bottom-up parsing generate LR family

# LR(k) Parser

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- **LR(k)**: member of LR family of parsers
  - **L**: scan input from left to right
  - **R**: construct a rightmost derivation in reverse
  - **k**: number of input symbols of lookahead to make decisions
    - $k = 0$  or  $1$  are of particular interests, is assumed to be 1 when omitted
- Comparison with LL(k) parser
  - Efficient as LL(k)
    - Linear in time and space to length of input (same as LL(k))
  - Convenient as LL(k)
    - Can generate automatically from grammar – YACC, Bison
  - More complex than LL(k)
    - Harder to debug parser when grammar causes conflicting predictions
  - More powerful than LL(k)
    - Handles more grammars: no left recursion removal, left factoring needed
    - Handles more (and most practical) languages:  $LL(1) \subset LR(1)$

# LR Parser



- The stack holds a sequence of states,  $s_0s_1\dots s_m$  ( $s_m$  is the top)
  - States are to track where we are in a parse
  - Each grammar symbol  $X_i$  is associated with a state  $s_m$
- Contents of stack + input ( $X_1X_2\dots X_ma_i\dots a_n$ ) is a right sentential form
  - If the input string is a member of the language
- Uses  $[S_m, a_i]$  to index into parsing table to determine action

# Parse Table

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- LR parsers use two tables: **action table** and **goto table**
  - The two tables are usually combined
  - Action table specifies entries for terminals
  - Goto table specifies entries for non-terminals
- Action table[动作表]
  - $Action[s, a]$  tells the parser what to do when the state on top of the stack is  $s$  and terminal  $a$  is the next input token
  - Possible actions: **shift, reduce, accept, error**
- Goto table[跳转表]
  - $Goto[s, X]$  indicates the new state to place on top of the stack after a reduction of the non-terminal  $X$  while state  $s$  is on top of the stack



# Possible Actions

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- **Shift**

- Transfer the next input symbol onto the top of the stack

- **Reduce**

- If there's a rule  $A \rightarrow w$ , and if the contents of stack are  $qw$  for some  $q$  ( $q$  may be empty), then we can reduce the stack to  $qA$

- **Accept**

- The special case of reduce: reducing the entire contents of stack to the start symbol with no remaining input
- Last step in a successful parse: have recognized input as a valid sentence

- **Error**

- Cannot reduce, and shifting would create a sequence on the stack that cannot eventually be reduced to the start symbol

# Possible Actions (cont.)

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- Grammar

$S \rightarrow E$

$E \rightarrow T \mid E + T$

$T \rightarrow \text{id} \mid (E)$

- Input: (id + id)

–  $\#(\text{id} + \text{id})\$ \Rightarrow (\text{id}\# + \text{id})\$ \Rightarrow (T\# + \text{id})\$ \Rightarrow (E\# + \text{id})\$ \Rightarrow (E + \text{id}\#)\$ \Rightarrow (E + T\#)\$ \Rightarrow (E\#)\$ \Rightarrow (E)\#\$ \Rightarrow T\#\$ \Rightarrow E\#\$ \Rightarrow S\#\$$

- Input: id+)

–  $\#\text{id}+)\$ \Rightarrow \text{id}\#+)\$ \Rightarrow T\#+)\$ \Rightarrow E\#+)\$ \Rightarrow E + \#)\$ \dots$

# Example: Parse Table

Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String:  $bab$

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

- Table entry:

- $si$ : shifts the input symbol and moves to state  $i$  (i.e., push state on stack)
- $rj$ : reduce by production numbered  $j$
- acc: accept
- blank: error

# Example: Parse Table (cont.)

Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String: **bab**

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state  $\rightarrow$  0 4

b a b

symbol  $\rightarrow$  \$ b

b a b \$

# Example: Parse Table (cont.)

Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String: **bab**

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state → 0 4

symbol → \$ B

a b \$

# Example: Parse Table (cont.)

Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String:  $bab$

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

B

b

a

b

state  $\rightarrow$  0 2 3 4

symbol  $\rightarrow$  \$ B a b

a b \$

# Example: Parse Table (cont.)

Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String:  $bab$

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

$B$                        $B$   
 $|$                            $|$   
 $b$                        $b$   
 $a$

state  $\rightarrow$  0 2 3 4  
 symbol  $\rightarrow$  \$ B a  $\cancel{B}$

\$

# Example: Parse Table (cont.)

Grammar:

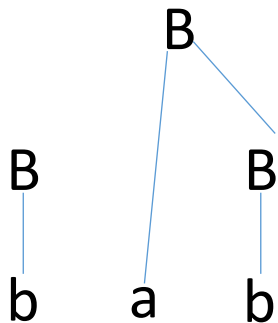
(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String:  $bab$

State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		



state  $\rightarrow$  0 2 3 6  
 symbol  $\rightarrow$  \$ B ~~B~~ B \$



# Example: Parse Table (cont.)

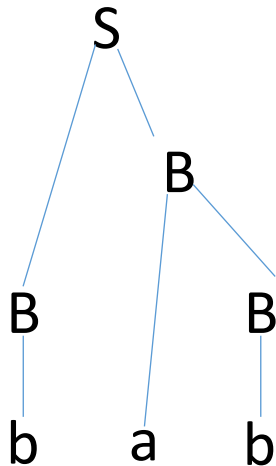
Grammar:

(1)  $S \rightarrow BB$

(2)  $B \rightarrow aB$

(3)  $B \rightarrow b$

String: **bab**



State	ACTION			GOTO	
	a	b	\$	S	B
0	s3	s4		1	2
1			acc		
2	s3	s4			5
3	s3	s4			6
4	r3	r3	r3		
5	r1	r1	r1		
6	r2	r2	r2		

state  $\rightarrow$  0 ~~2~~ 5  
 symbol  $\rightarrow$  \$ ~~B~~ B

\$

# Parser Actions

Initial

$s_0$	
\$	$a_1 a_2 \dots a_n \$$

General

$s_0 s_1 \dots s_m$	
$\$X_1 \dots X_m$	$a_i a_{i+1} \dots a_n \$$

- If  $\text{ACTION}[s_m, a_i] = s_x$ , then do shift

- Pushes  $a_i$  on stack
  - $a_i$  is removed from input
- Enters state  $x$ 
  - i.e., pushes state  $x$  on stack

$s_0 s_1 \dots s_m x$	
$\$X_1 \dots X_m a_i$	$a_{i+1} \dots a_n \$$

# Parser Actions (cont.)

Initial

$s_0$

\$

$a_1 a_2 \dots a_n \$$

General

$s_0 s_1 \dots s_m$

$\$X_1 \dots X_m$

$a_i a_{i+1} \dots a_n \$$

- If  $\text{ACTION}[s_m, a_i] = r_x$ , (i.e., the  $x^{\text{th}}$  production:  $A \rightarrow X_{m-(k-1)} \dots X_m$ ), then do reduce
  - Pops  $k$  symbols from stack
  - Pushes  $A$  on stack
  - No change on input
  - $\text{GOTO}[s_{m-k}, A] = y$ , then

$s_0 s_1 \dots s_{m-k}$

$\$X_1 \dots X_{m-k} A$

$a_i a_{i+1} \dots a_n \$$



$s_0 s_1 \dots s_{m-k} y$

$\$X_1 \dots X_{m-k} A$

$a_i a_{i+1} \dots a_n \$$

# Parser Actions (cont.)

Initial

$s_0$   
 $\$$   $a_1 a_2 \dots a_n \$$

General

$s_0 s_1 \dots s_m$   
 $\$X_1 \dots X_m$   $a_i a_{i+1} \dots a_n \$$

- If  $\text{ACTION}[s_m, a_i] = \text{acc}$ , then parsing is complete
- If  $\text{ACTION}[s_m, a_i] = \langle \text{empty} \rangle$ , then report error and stop

# LR Parsing Program

- **Input:** input string  $\omega$  and parse table with ACTION/GOTO
- **Output:** reduction steps  $\omega$ 's bottom-up parse, or error
- **Initial:**  $s_0$  on the stack,  $\omega\$$  in the input buffer

```
let  $a$  be the first symbol of  $\omega\$$ 
while (1) { /* repeat forever */
    let  $s$  be the state on top of the stack;
    if (ACTION[ $s,a$ ] = shift  $t$ ) {
        push  $t$  onto the stack;
        let  $a$  be the next input symbol;
    } else if (ACTION[ $s,a$ ] = reduce  $A \rightarrow \beta$ ) {
        pop  $|\beta|$  symbols off the stack;
        let state  $t$  now be on top of the stack;
        push GOTO[ $t,A$ ] onto the stack;
        output the production  $A \rightarrow \beta$ ;
    } else if (ACTION[ $s,a$ ] = accept) break; /* parsing is done */
    else call error-recovery routine;
```

```
}
```

# Construct Parse Table

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- Construct parsing table: identify the possible states and arrange the transitions among them
- **LR(0)** parsing
  - Simplest LR parsing, only considers stack to decide shift/reduce
  - Weakest, not used much in practice because of its limitations
- **LR(1)** parsing
  - LR parser that considers next token (lookahead of 1)
  - Compared to LR(0), more complex alg and much bigger table
- **SLR(1)** parsing
  - Simple LR, lookahead from first/follow rules derived from LR(0)
  - Keeps table as small as LR(0)
- **LALR(1)** parsing
  - Lookahead LR(1): fancier lookahead analysis using the same LR(0) automaton as SLR(1)

# Item[项目]

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- An **item** is a production with a “.” somewhere on the RHS
  - Dot indicates extent of RHS already seen in the parsing process
  - The only item for  $X \rightarrow \varepsilon$  is  $X \rightarrow \cdot$
  - Items are often called “**LR(0) items**” (a.k.a., **configuration**)
- The items for  $A \rightarrow XYZ$  are
  - $A \rightarrow \cdot XYZ$ 
    - Indicates that we hope to see a string derivable from XYZ next on input
  - $A \rightarrow X \cdot YZ$ 
    - Indicates that we have just seen on the input a string derivable from X and that we hope next to see a string derivable from YZ
  - $A \rightarrow XY \cdot Z$
  - $A \rightarrow XYZ \cdot$ 
    - Indicates that we have seen the body XYZ and that it may be time to reduce XYZ to A

# State[状态]

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- Example:
  - Suppose we are currently in this position  
 $A \rightarrow X \cdot YZ$
  - We have just recognized X and expect the upcoming input to contain a sequence derivable from YZ (say,  $Y \rightarrow u | w$ )
    - Y is further derivable from either u or w  
 $A \rightarrow X \cdot YZ$   
 $Y \rightarrow \cdot u$   
 $Y \rightarrow \cdot w$
  - The above three items can be placed into a set, called as **configuration set** of the LR parser
- Parsing tables have one **state** corresponding to each set
  - The states can be modeled as a finite automaton where we move from one state to another via transitions marked with a symbol of the CFG



# Augmented Grammar[增广文法]

- We want to start with an item with a dot before the start symbol  $S$  and move to an item with a dot after  $S$ 
  - Represents shifting and reducing an entire sentence of the grammar
  - Thus, we need  $S$  to appear on the right side of a production
  - Only one 'acc' in the table
- Modify the grammar by adding the production  $S' \rightarrow \cdot S$

Grammar:

(1)  $E \rightarrow E + T$   
(2)  $E \rightarrow T$   
(3)  $T \rightarrow T * F$



Augmented grammar:

(0)  $E' \rightarrow E$   
(1)  $E \rightarrow E + T$   
(2)  $E \rightarrow T$   
(3)  $T \rightarrow T * F$

# Example

(0) $S' \rightarrow S$	(1) $S \rightarrow BB$	(2) $B \rightarrow aB$	(3) $B \rightarrow b$
------------------------	------------------------	------------------------	-----------------------

Initial item

$S' \rightarrow .S$

$S \rightarrow .BB$

$S \rightarrow B.B$

$B \rightarrow .aB$

$B \rightarrow a.B$

$B \rightarrow .b$

Reduce item

$S' \rightarrow S.$

$S \rightarrow BB.$

$B \rightarrow aB.$

$B \rightarrow b.$

Accept item

- **Closure:** the action of adding equivalent items to a set
  - Example:  $S' \rightarrow .S$        $S \rightarrow .BB$        $B \rightarrow .aB$        $B \rightarrow .b$
- Intuitively,  $A \rightarrow \alpha.B\beta$  means that we might next see a substring derivable from  $B\beta$  ( $\_sub$ ) as input. The  $\_sub$  will have a prefix derivable from  $B$  by applying one of the  $B$ -productions.
  - Thus, we add items for all the  $B$ -productions, i.e., if  $B \rightarrow \gamma$  is a production, we add  $B \rightarrow .\gamma$  in the closure