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Compilation Principle
编 译 原 理

第5讲：语法分析(2)

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Review Questions (1)

- Why don't we keep using RE in syntax parsing?
  RE is not powerful enough, it cannot express nested structures
- Formal definition of Grammar?
  (T, N, s, σ): T – terminals; N – non-terminals, s – start, σ – productions
- What is CFG?
  Context free grammar
- Language classification based on grammar rules?
  0 - unrestricted, 1 - context sensitive, 2 - context free, 3 - regular
- What is derivation? What is parse tree?
  Derivation is the sequence of applying production rules.
  Parse tree is a graphical representation of the derivation.

Review Questions (2)

- What is leftmost derivation?

  Always replace the leftmost non-terminal in each derivation step.

- Grammar G: E→E*E | E+E | (E) | id

  T = {*, +, (,), id}

  N = {E}

  s = E

  σ = E→E*E | E+E | (E) | id

- Is id + E * E an sentence of grammar G?

  NO. It is a sentential form (句型), as E is non-terminal symbol.

- Is id + id * id an sentence of grammar G?

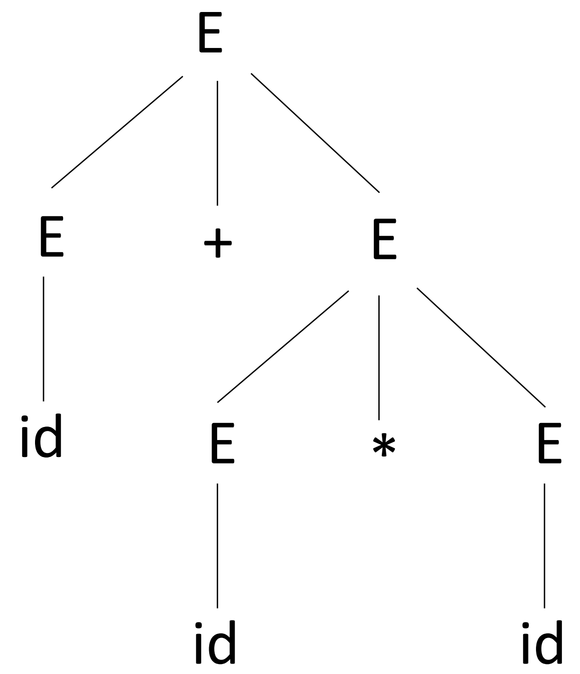
  YES. It can be derived using the production rules.

Derivation Example

- Grammar: E→E*E | E+E | (E) | id

- Leftmost derivation

  E ⇒ E + E
  ⇒ id + E
  ⇒ id + E * E
  ⇒ id + id * E
  ⇒ id + id * id

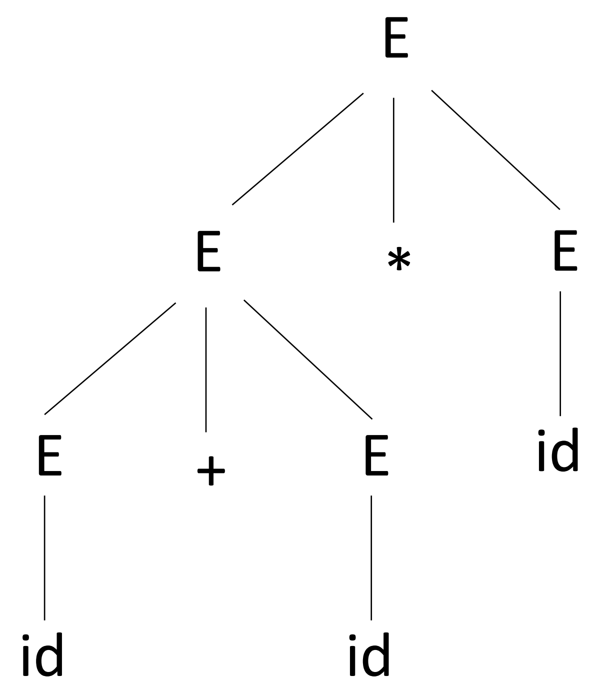


Derivation Example (cont.)

- Grammar E→E*E | E+E | (E) | id

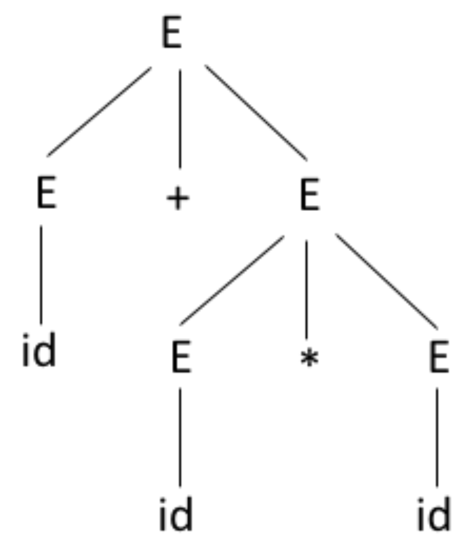
- Leftmost derivation

$$
\begin{aligned}
E &\Rightarrow E * E \\
&\Rightarrow E + E * E \\
&\Rightarrow id + E * E \\
&\Rightarrow id + id * E \\
&\Rightarrow id + id * id
\end{aligned}
$$

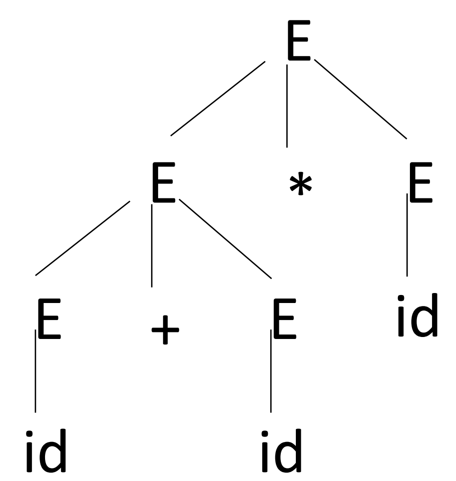


Derivation Example (cont.)

- Two distinct leftmost derivations for the sentence id + id * id
  - Above: id + (id * id)
  - Below: (id + id) * id



- How to evaluate a + b * c ?
  - a + (b * c) ?
  - (a + b) * c ?



- Grammar E→E*E | E+E | (E) | id is ambiguous

Ambiguity[二义性]

- grammar G is ambiguous if
  - It produces more than one parse tree some sentence
  - i.e., there exist a string *str* ∈ L(G) such that
  - more than one parse tree derives *str*

    ≡ more than one leftmost derivation derives *str*

    ≡ more than one rightmost derivation derives *str*
- Unambiguous grammars are preferred for most parsers
  - If not, we cannot uniquely determine which parse tree to select for a sentence
  - In minor cases, it is convenient to use carefully chosen ambiguous grammars, together with disambiguating rules that "throw away" undesirable parse trees, leaving only one tree for each sentence

Ambiguity (cont.)

- Ambiguity is the property of the grammar, not the language
  - Just because G is ambiguous, does not mean L(G) is inherently ambiguous
  - A G′ can exist where G′ is unambiguous and L(G′) ≡ L(G)

- Impossible to convert ambiguous to unambiguous grammar automatically
  - It is (often) possible to rewrite grammar to remove ambiguity
  - Or, use ambiguous grammar, along with disambiguating rules

Precedence and Associativity

- Two characteristics of operators that determine the evaluation order of sub-expressions w/o brackets
- Operator **precedence** [优先级]
  - Determines which operator is performed first in an expression with more than one operators with different precedence.
  - 10 + 20 * 30
- Operator **associativity** [结合性]
  - Is used when two operators of same precedence appear in an expression. Associativity can be either Left to Right or Right to Left.
  - 100 / 10 * 10 -> (100 / 10) * 10 //left associativity
  - a = b = 1 -> a = (b = 1) //right associativity

How to Remove Ambiguity

- Step I: **Specify precedence** [指定优先级]
  - Build precedence into grammar by recursion on a different non-terminal for each precedence level. Insight:
    - Lower precedence — tend to be higher in tree (close to root)
    - Higher precedence — tend to be lower in tree (far from root)
  - Place lower precedence non-terminals higher up in the tree

- Rewrite E→E*E | E+E | (E) | id to:

  E → E + E | T　　//lowest precedence +
  T → T * T | F　　//middle precedence *
  F → (E) | id　　//highest precedence ()

How to Remove Ambiguity (cont.)

- Step II: **Specify associativity** [指定结合性]
  - Allow recursion only on either left or right non-terminal
    - Left associative — recursion on left non-terminal
    - Right associative — recursion on right non-terminal
- Even after step 1, ambiguous due to associativity
  - E → E + E .... ; allows both left/right associativity
- Rewrite:

  E → E + E | T //lowest precedence +
  T → T * T | F //middle precedence *
  F → (E) | id //highest precedence ()

  to

  E → E + T | T // + is left-associative
  T → T * F | F // * is left-associative
  F → (E) | id

The Example

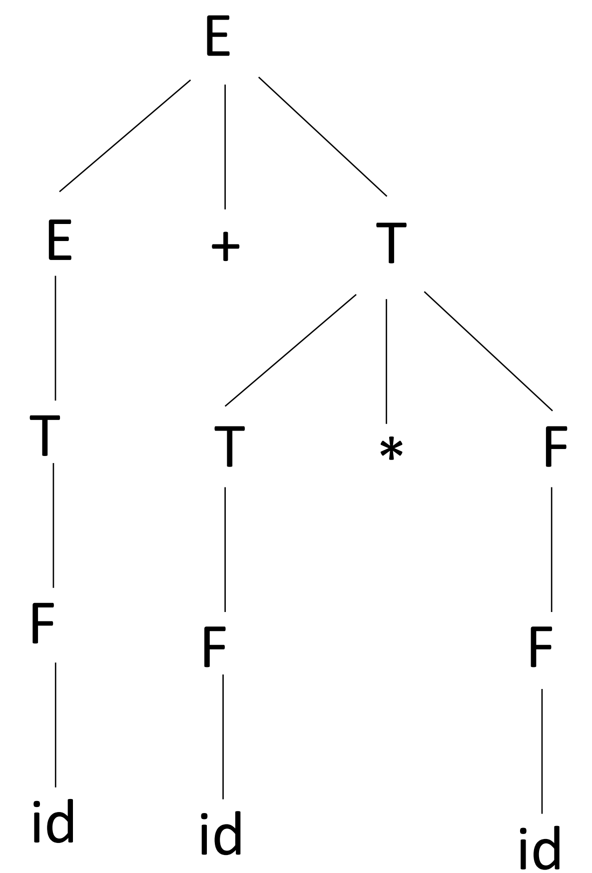
- Grammar E→E*E | E+E | (E) | id was ambiguous
  - Rewrite to

    E → E + T | T

    T → T * F | F

    F → (E) | id
- The id + id * id has only one parse tree now

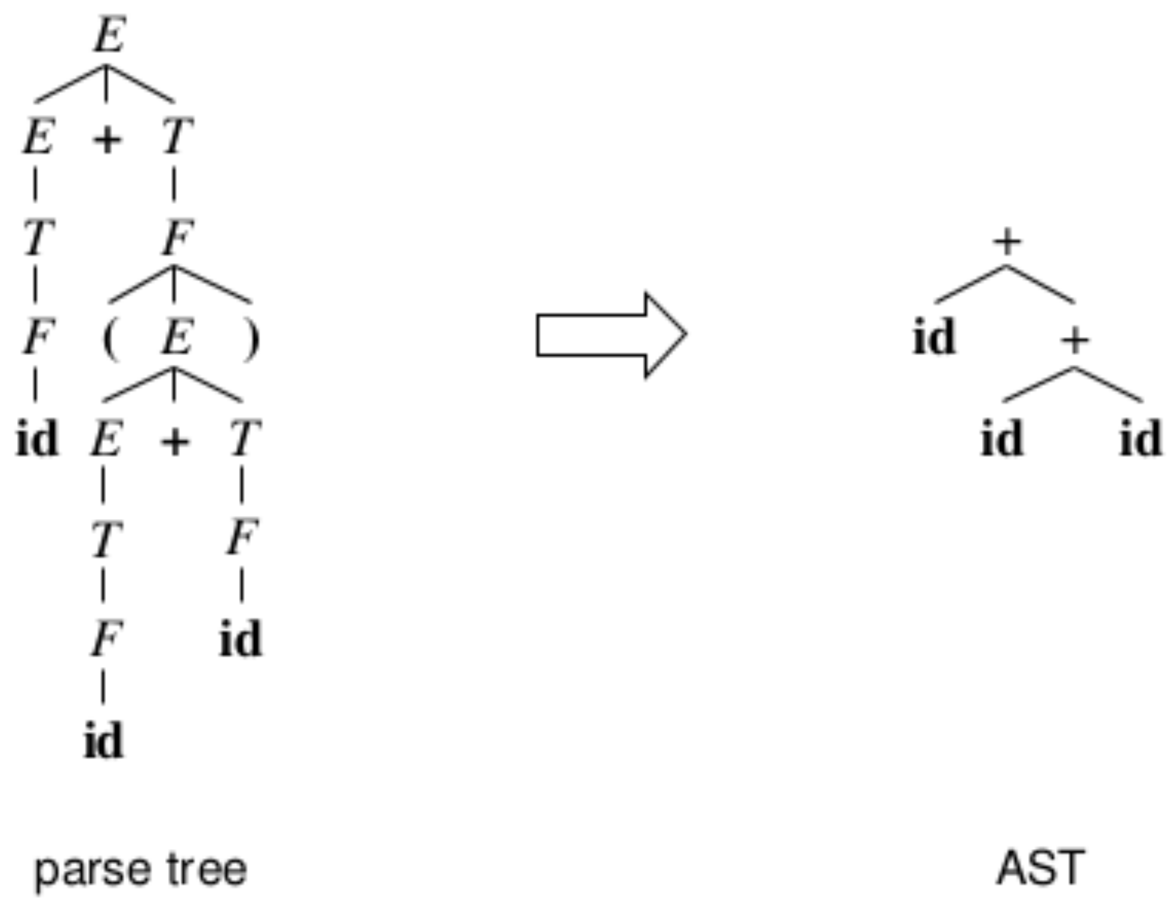
$$
\begin{aligned}
E &\Rightarrow E + T \\
&\Rightarrow T + T \\
&\Rightarrow F + T \\
&\Rightarrow id + T * F \\
&\Rightarrow id + F * F \\
&\Rightarrow id + id * F \\
&\Rightarrow id + id * id
\end{aligned}
$$



Grammar → Parser

- What exactly is **parsing**, or syntax analysis?
  - To process an input string for a given grammar,
  - and compose the derivation if the string is in the language
  - Two subtasks
    - determine if string can be derived from grammar or not
    - build a representation of derivation and pass to next phase
- What is the best representation of the derivation?
  - Can be a parse tree or an abstract syntax tree
- An abstract syntax tree is
  - an abbreviated representation of a parse tree
  - drops some details without compromising meaning
    - some terminal symbols that no longer contribute to semantics are dropped (e.g. parentheses)
    - internal nodes may contain terminal symbols

Example: Abstract Syntax Tree



Summary

- Compilers specify program structure using CFG
  - Most programming languages are not context free
  - Context sensitive analysis can easily be separated out to semantic analysis phase

- A parser uses CFG to
  - ... answer if an input str ∈ L(G)
  - ... and build a parse tree
  - ... or build an AST instead
  - ... and pass it to the rest of compiler

Parser Types

- Most compilers use either **top-down** or **bottom-up** parsers

- Top-down parsing[自顶向下分析]
  - Starts from root and expands into leaves
    - Tries to expand start symbol to input string
    - Finds leftmost derivation[最左推导]
  - In each step
    - Which non-terminal to replace?
    - Which production of the non-terminal to use?
  - Parser code structure closely mimics grammar
    - Amenable to implementation by hand
    - Automated tools exist to convert to code (e.g. ANTLR)

Parser Types (cont.)

- Top-down parsing[自顶向下分析]
  - Starts from root and expands into leaves

- Bottom-up parser[自底向上分析]
  - Starts at leaves and builds up to root
    - Tries to reduce the input string to the start symbol
    - Finds reverse order of the rightmost derivation[最右推导的逆 → 最左归约, 也称为**规范归约**]
  - Parser code structure nothing like grammar
    - Very difficult to implement by hand
    - Automated tools exist to convert to code (e.g. Yacc, Bison)
    - LL ⊂ LR (Bottom-up works for a larger class of grammars)

Example

- Consider a CFG grammar G

  S→AB A→aC B→bD D→d C→c

- This language has only one sentence: L(G) = {acbd}

Top-down (Leftmost Derivation)

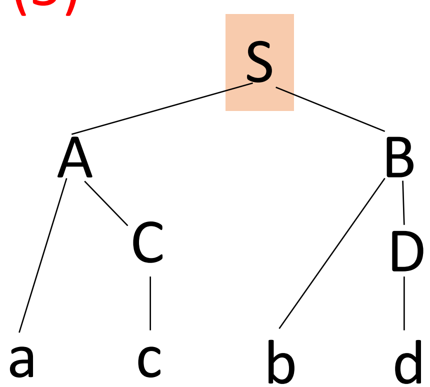
$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)



Bottom-up (reverse of rightmost derivation)

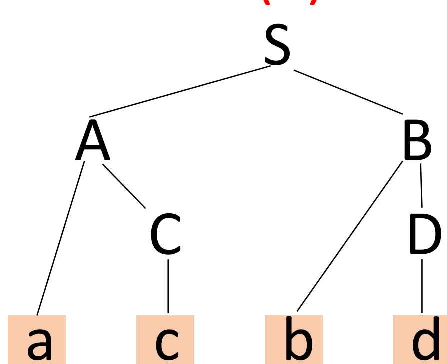
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbD$ (2)

$\Rightarrow acbd$ (1)



Top-down Parsers

- **Recursive descent parser** (RDP, 递归下降分析) with backtracking[回溯]
  - Implemented using recursive calls to functions that implement the expansion of each non-terminal
  - Goes through all possible expansions by trial-and-error until match with input; backtracks when mismatch detected
  - Simple to implement, but may take exponential time

- **Predictive parser**[预测分析]
  - Recursive descent parser with prediction (no backtracking)
  - Predict next rule by looking ahead *k* number of symbols
  - Restrictions on the grammar to avoid backtracking
    - Only works for a class of grammars called LL(k)

RDP with Backtracking

- **Approach**: for a non-terminal in the derivation, productions are tried in some order until
  - A production is found that generates a portion of the input, or
  - No production is found that generates a portion of the input, in which case backtrack to previous non-terminal

- Terminals of the derivation are compared against input
  - Match — advance input, continue parsing
  - Mismatch — backtrack, or fail

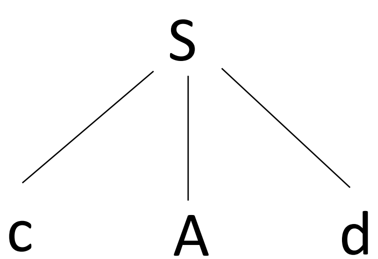
- Parsing fails if no derivation generates the entire input

Recursive Decent Example

- Consider the grammar

  S → cAd   A → ab | a

- To construct a parse tree top-down for input string *w*=cad
  - Begin with a tree consisting of a single node labeled *S*
  - The input pointer pointing to *c*, the first symbol of *w*
  - *S* has only one production, so we use it to expand *S* and obtain the tree

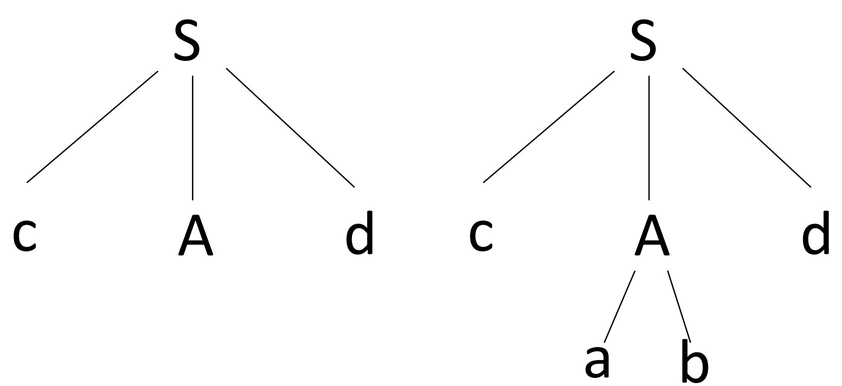


Recursive Decent Example (cont.)

- Consider the grammar

  S → cAd   A → ab | a

- To construct a parse tree top-down for input string *w*=cad
  - The leftmost leaf, labeled *c*, matches the first symbol of *w*
    - So we advance the input pointer to *a* (i.e., the 2^{nd} symbol of *w*) and consider the next leaf *A*
  - Next, expand *A* using *A* → *ab*
    - Have a match for the 2^{nd} input symbol, *a*, so advance the input pointer to *d*, the 3^{rd} input symbol

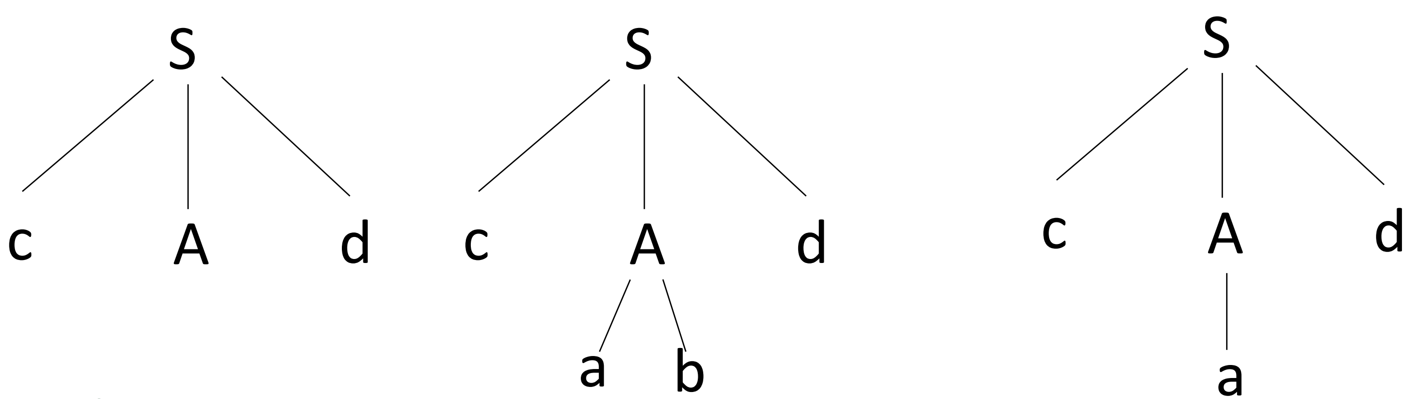


Recursive Decent Example (cont.)

- Consider the grammar

  $S \rightarrow cAd \quad A \rightarrow ab \mid a$

- To construct a parse tree top-down for input string *w*=cad
  - *b* does not match *d*, report failure and go back to *A*
    - See whether there is another alternative for *A* that has not been tried
    - In going back to *A*, we must reset the input pointer as well
  - Leaf a matches the 2nd symbol of *w*, and leaf *d* matches the 3rd
  - We have produced a parse tree for *w*, we halt and announce successful completion of parsing



Left Recursion Problem

- Recursive descent doesn't work with left recursion
  - Right recursion is okay
- Why is left recursion[左递归] a problem?
  - For left recursive grammar

    A→Ab|c
  - We may repeatedly choose to apply A b

    $A \Rightarrow A\ b \Rightarrow A\ b\ b \ldots$
  - Sentence can grow indefinitely w/o consuming input
  - How do you know when to stop recursion and choose *c*?

- Rewrite the grammar so that it is right recursive
  - Which expresses the same language

Left Recursion

- A grammar is left recursive if
  - It has a nonterminal *A* such that there is a derivation $A \Rightarrow^{+} A\alpha$ for some string α
- Recursion types [直接和间接左递归]
  - **Immediate left recursion**, where there is a production $A \rightarrow A\alpha$
  - Non-immediate: left recursion involving derivation of 2+ steps

    $S \rightarrow Aa \mid b$

    $A \rightarrow Sd \mid \varepsilon$
  - $S \Rightarrow Aa \Rightarrow Sda$

- Algorithm to systematically eliminates left recursion from a grammar

Remove Left Recursion

- Grammar: $A \rightarrow A\alpha \mid \beta$ ($\alpha \neq \beta$, β doesn't start with A)

  $A \Rightarrow A\alpha$

  $\Rightarrow A\alpha\alpha$

  …

  $\Rightarrow A\alpha\ldots\alpha\alpha$

  $\Rightarrow \beta\alpha\ldots\alpha\alpha$

  r= $\beta\alpha^*$

- Rewrite to:

  $A \rightarrow \beta A'$ //begins with β (A' is a new non-terminal)

  $A' \rightarrow \alpha A' \mid \varepsilon$ //A' is to produce a sequence of α

  $\Rightarrow \alpha\alpha A'$

  …

  $\Rightarrow \alpha\ldots\alpha A' \Rightarrow \alpha\ldots\alpha$

Remove Left Recursion (cont.)

- Grammar:

  $A \rightarrow A\alpha \mid \beta$

  to

  $A \rightarrow \beta A'$

  $A' \rightarrow \alpha A' \mid \varepsilon$

- $E \rightarrow E \underbrace{+\,T}_{\alpha} \mid \underbrace{T}_{\beta}$ ⇨ $E \rightarrow TE'$, $E' \rightarrow +TE' \mid \varepsilon$

- $T \rightarrow T \underbrace{*\,F}_{\alpha} \mid \underbrace{F}_{\beta}$ ⇨ $T \rightarrow FT'$, $T' \rightarrow *FT' \mid \varepsilon$

- $F \rightarrow (E) \mid id$ ⇨ $F \rightarrow (E) \mid id$

Summary of Recursive Descent

- **Recursive descent** is a simple and general parsing strategy
  - Left-recursion must be eliminated first
    - Can be eliminated automatically using some algorithm
  - L(Recursive_Descent) ≡ L(CFG) ≡ CFL

- However it is not popular because of **backtracking**
  - Backtracking requires re-parsing the same string
  - Which is inefficient (can take exponential time)
  - Also undoing semantic actions may be difficult
    - E.g. removing already added nodes in parse tree

- Real world parsers do no (or minimal) backtracking ...
  - At the cost of restricting the class of grammar