Homework4 RRT&Bi-RRT Implementation (Python)

18340013 陈琮昊

Dec. 8, 2020

Contents

| 1 | 实验任务 | 2 |
|---|---------------|----|
| 2 | 算法介绍 | 2 |
| | 2.1 RRT 算法 | 2 |
| | 2.2 Bi-RRT 算法 | 2 |
| 3 | 准备工作 | 3 |
| 4 | 代码实现 | 3 |
| 5 | 实验结果 | 11 |
| 6 | 附录 | 12 |

1 实验任务

实现 RRT 算法和 Bi-RRT(双向 RRT) 算法, 在具有障碍物的地图内使用上述两种算法规划机器人的避障路径. 并进行性能对比与分析.

2 算法介绍

2.1 RRT 算法

RRT(快速扩展随机树) 算法是一种多维空间中有效率的规划方法. 它以一个初始点作为根节点, 通过随机采样增加叶子节点的方式, 生成一个随机扩展树, 当随机树中的叶子节点包含了目标点或进入了目标区域, 便可以在随机树中找到一条由从初始点到目标点的路径.

RRT 的一个弱点是在遇到有狭窄通道的环境时,算法的收敛速度较慢. 因为狭窄通道面积小,被碰到的概率低. 比如下图展示的例子这样,要使用 RRT 通过一个很狭窄的通道,有时 RRT 很快就找到了出路,有时则一直出不去:

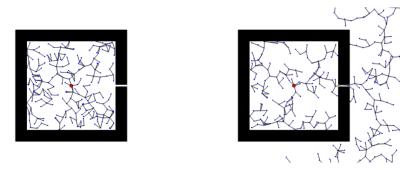


Figure 1. An example

2.2 Bi-RRT 算法

顾名思义,Bi-RRT 算法就是双向的 RRT. 分别从起始位置和目标位置扩展树,直到两棵树的某个节点相遇为止. 很明显,双向扩展的方式要比单向随机扩展的方式要好,因为这样使得树的扩展方向更加明确,因此性能上也会有所提高.

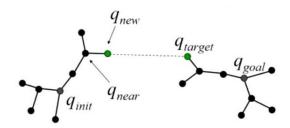


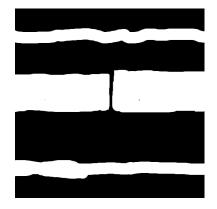
Figure 2. Bi-RRT

很明显,由于其随机性,找到的路径通常不是最优的;但是不管什么机器人类型,不管自由度是多少,不管约束有多复杂都能使用该算法,可以说普适性很强.

3 准备工作

在网上找了三张图,这三张图中黑色的部分是机器人可以行走的地方,白色的部分是不可行走的地方.为了方便,文件命名我直接命名为其难度.很明显可以看到,easy.png 最容易,因为黑色区域较开阔;hard.png 最难,因为它中间只有很窄的一条黑色区域可以穿过.





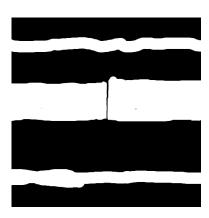


Figure 3. easy.png

Figure 4. medium.png

Figure 5. hard.png

至于显示图形化界面,则是调用了 Python 的 pygame 库,这是我之前无聊时发现的一个库,用它写过一个很小的游戏. 只需 import pygame 即可.

4 代码实现

有 5 个.py 文件,看似很多但每个文件思路很清晰,代码量也不是很多:maths.py 是有关于数学上的一些公式(函数)的实现;rrt_birrt.py 则是实现两个类,分别代表 RRT 算法和 Bi-RRT 算法;img_env.py 则是图片的相关处理;rrt_example.py 则是用 RRT 算法寻找路径;birrt_example.py 同理. 代码部分占据较多空间,实验结果可直接跳至倒数第二页.

maths.py

```
# -*- coding: UTF-8 -*-
#定义一些数学上的函数运算
import numpy as np
# 弧度转角度
def radtodeg(rad):
    return rad*(180.0/np.pi)
# 角度转弧度
def degtorad(deg):
    return deg*(np.pi/180.0)
# 比较两个向量
def combinevector(a, op, b):
    if len(a) != len(b):
```

```
raise ValueError("维度不匹配!")
if op == '+':

\overset{\mathsf{r}}{\operatorname{acc}} = [ \underset{-}{\mathbf{i}} + \underset{-}{\mathbf{j}} \quad \text{for } \underset{-}{\mathbf{i}} , \quad \underset{-}{\mathbf{j}} \quad \text{in } \operatorname{zip}(a, b) ]

       elif op ==
       acc = \begin{bmatrix} i-j & \text{for } i, j & \text{in } zip(a, b) \end{bmatrix}
elif op = '*':
             acc = [i*j for i, j in zip(a, b)]
       elif op == '/':
             acc = [i/float(j) \text{ for } i, j \text{ in } zip(a, b)]
            raise ValueError("不支持该运算!")
       if a.__class__ = tuple:
            return tuple (acc)
       else:
             return acc
# 有关向量的运算
def vectoroperator (a, op, b=1, param=1):
       if op = '+':
acc = [i+b \text{ for } i \text{ in } a]
       elif op = '-':
       \begin{array}{c} \operatorname{acc} = [i - b, \text{ for i in a}] \\ \operatorname{elif op} = [*, *] \end{array}
            acc = [i*b for i in a]
       elif op = 
            \frac{1}{2} \operatorname{acc} = \left[\frac{1}{2} / \operatorname{float}(b) \text{ for i in a}\right]
       elif op ==
                          int
            acc = [int(i) \text{ for } i \text{ in } a]
       elif op = 'round':
            acc = [np.around(i, param) for i in a]
             raise ValueError("不支持的运算类型!")
       if a.\_\_class\_\_ = tuple:
            return tuple (acc)
       else:
             return acc
# 欧氏距离
def vectoreuclidean(a, b):
      if len(a) != len(b):
             raise ValueError("维度不匹配!")
       e = 0
      for i in range(len(a)):

e += pow(a[i] - b[i], 2)
      return np.sqrt(e)
```

rrt_birrt.py

```
# -*- coding: UTF-8 -*-
# rrt 和B-rrt 算法实现文件
import random
import numpy as np
class Node:
    def __init___(self, state):
        self.state = state
        self.parent = None
        self.cost = 0

def distance(s1, s2, metric='euclidean'):
    if len(s1) != len(s2):
        raise ValueError("维度不一致!")
    else:
        if metric == 'euclidean':
        acc = 0
        for i in range(len(s1)):
        acc += pow(s1[i] - s2[i], 2)
        return np.sqrt(acc)
```

```
elif metric = 'manhattan':
             acc = 0
             for i in range (len(s1)):
                 acc += abs(s1[i] - s2[i])
             return acc
         else:
             raise ValueError("不支持的距离类型!")
def randomSampler(a, b):
    return random.uniform(a, b)
#规划器
class MotionPlanner:
           _{\text{init}} (self, env):
    \mathrm{def}
         self.env = None
         self.start = None
         self.goal = None
         if self.is_env_valid(env):
             self.env = env
         else:
             raise ValueError("环境有问题!")
    def is_env_valid(self, env):
         if len(env.state_limits) != 2:
             return False
         else:
                 lim in env.state_limits:
                 if len(lim) != env.state_dim:
                      return False
             return True
    def set_{-}
             _start(self, start):
         if len(start) == self.env.state_dim:
             self.start = start
         else:
             raise ValueError("起点有问题!")
    def set_goal(self, goal):
         if len(goal) == self.env.state_dim:
             self.goal = goal
             raise ValueError("终点有问题!")
    def plan(self, start, goal):
         self.set_start(start)
         self.set_goal(goal)
# RRT实现
class RRT (MotionPlanner):
           _init___(self, env, step_size, growth='connect'):
    def
         MotionPlanner.___init___(self, env)
        self.step_size = step_size
self.growth = growth
self.tree = []
    def start_tree(self, start, goal):
    MotionPlanner.plan(self, start, goal)
         self.tree = [Node(self.start)]
         if self.env.collision(start):
             raise ValueError("起点遇到冲突!")
    def clear tree (self):
         self.tree = []
    def extend tree (self, des state=None, stopAtGoal=False):
         old_len = len(self.tree)
         reachedGoal = False
         if des_state == None:
             des_state = self.random_free_state()
        near_node = self.find_nearest_neighbor(des_state)
         success, next_state, cost = self.env.paraofforward(des_state,
            near_node.state , self.step_size)
         if success:
             next_node = Node(next_state)
```

```
next_node.parent = near_node
         next_node.cost = near_node.cost + cost
self.tree.append(next_node)
         reachedRand = self.env.distance(des state, next state) <
             self.step_size
         reachedGoal = self.env.distance(self.goal, next_state) <
     \begin{array}{ccc} & self.step\_size \\ if & self.growth == \\ \end{array}, \\ \begin{array}{c} connect \\ \end{array}, \\ \vdots \\ \end{array}
         while success and not reachedRand and not (stopAtGoal and
             reachedGoal):
              near_node = next_node
              success, next_state, cost =
                  self.env.paraofforward(des_state, near_node.state,
                  self.step_size)
              if success:
                   next_node = Node(next_state)
                   next_node.parent = near_node
                   next_node.cost = near_node.cost + cost
                   self.tree.append(next_node)
                   reachedRand = self.env.distance(des_state,
                       next_state) < self.step_size
                   reachedGoal = self.env.distance(self.goal,
     next_state) < self.step_size
elif_self.growth == 'extend':
         pass
    else;
     return (stopAtGoal and reachedGoal),
    self.tree[old_len:len(self.tree)]
def grow_tree(self, start, mode='samples', goal=None, samples=0):
     self.start_tree(start)
    new_nodes = [start]
     goalFound = False
     if mode == 'samples':
         for i in range (samples):
     self.extend_tree()
elif mode == 'goal' and goal != None:
while not goalFound:
              goalFound , new_nodes = self.extend_tree(stopAtGoal=True)
     else:
         raise ValueError("无效生成!")
def search_for_goal(self, goal, nodes_list):
     for node in nodes_list:
    if self.env.distance(node.state, goal) < self.step_size:
    return True, node
return False, None
def find_path(self, goal):
     MotionPlanner.set_goal(self, goal)
     if len(self.tree) == 0:
         raise ValueError("树空!")
    best_node = self.find_nearest_neighbor(self.goal)
    found_path = self.path_to_root(best_node)
    path_cost = best_node.cost
return found_path, path_cost
def plan(self, start, goal):
     MotionPlanner.plan(self, start, goal)
     self.grow_tree(start)
    path, cost = self.find path(goal)
    return path, cost
def random_free_state(self):
    while True:
         rand_state = | |
         lim = self.env.state_limits
         for i in range (self.env.state_dim):
              rand\_state += [randomSampler(lim [0][i], lim [1][i])]
```

```
rand state = tuple (rand state)
               if not self.env.collision(rand_state):
                     break
     return rand_state
def find_nearest_neighbor(self, state):
          \min \overline{dist} = \overline{float}(', inf')
          \min \text{-} \text{node} = \text{None}
          for node in self.tree:
               node_dist = distance(state, node.state)
               if node_dist < min_dist:
    min_dist = node_dist</pre>
                    \min_{\text{node}} = \text{node}
          return min_node
     def next_state_action(self, des_state, node):
    min_dist = distance(node.state, des_state)
          sel\_action = None
          next_state = None
          for action in self.env.action_list:
               new_state = self.env.next_state(node.state, action)
               if self.env.collision(new_state):
                    continue
               new_dist = distance(new_state, des_state)
               if new_dist < min_dist:</pre>
                    \min_{\text{dist}} = \text{new\_dist}
                    sel\_action = action \\ next\_state = new\_state
     path_to_root = []
          curr_node = node
          while curr_node != None:
               path_to_root += [curr_node]
               curr_node = curr_node.parent
          return path_to_root
# Bi-RRT实现
class BiRRT(RRT, MotionPlanner):
                   _(self, env, step_size, growth='ce'):
             _{
m init}
          \overline{\text{MotionPlanner.}} __init___(self, env)
          self.step_size = step_size
self.meet_thr = 0.1
if growth == 'ce':
               self.start_growth = 'connect'
          self.goal_growth = 'extend
elif growth == 'cc':
               self.start_growth = 'connect'
               self.goal_growth = 'connect
          elif growth == 'ec'
               self.start_growth = 'extend',
self.goal_growth = 'connect'
          elif growth = 'ee'
               self.start_growth = 'extend'
               self.goal_growth = 'extend
               raise ValueError("双向生成树类型错误!")
          self.rrt_start = RRT(env, self.step_size, self.start_growth)
self.rrt_goal = RRT(env, self.step_size, self.goal_growth)
     def start_tree(self, start, goal):
    MotionPlanner.plan(self, start, goal)
          self.rrt_start.start_tree(start, goal)
          self.rrt_goal.start_tree(goal, start)
     def clear tree (self):
          self.rrt_start.clear_tree()
          self.rrt_goal.clear_tree()
     def extend_tree(self):
          goalFound_g, new_goal_nodes =
```

```
self.rrt goal.extend tree(stopAtGoal=True)
     if len(new_goal_nodes) != 0:
           target = new\_goal\_nodes[-1].state
     else:
           target = None
     goalFound_s, new_start_nodes =
         self.rrt_start.extend_tree(des_state=target, stopAtGoal=True)
     treesMet, meet_point = self.tree_meet(new_start_nodes,
         new_goal_nodes)
\begin{array}{c} \textbf{return TreesMet}\,,\,\,\, \textbf{meet\_point}\,,\,\,\, \textbf{new\_start\_nodes}\,,\,\,\, \textbf{new\_goal\_nodes}\\ \textbf{def grow\_tree}\,(\,\textbf{self}\,\,,\,\,\,\textbf{start}\,\,,\,\,\,\textbf{goal}\,)\,: \end{array}
     self.start_tree(start, goal)
treesMet = False
     while not treesMet:
          return meet_point
def plan(self, start, goal):
    meet_point = self.grow_tree(start, goal)
found_path, path_cost = self.find_path(meet_point)
    return found_path, path_cost
def tree_meet(self, new_start_nodes=None, new_goal_nodes=None):
     for s in self.rrt_start.tree:
for g in new_goal_nodes:
                if self.env.distance(s.state, g.state) < self.step_size:
                     return True, (s, g)
     for g in self.rrt_goal.tree:
           for s in new_start_nodes:
                if self.env.distance(s.state, g.state) < self.step_size:
                     return True, (s, g)
return False, (None, None)
def find_path(self, meet_point):
     start\_meet = meet\_point[0]
     goal_meet = meet_point[1] start_path = self.rrt_start.path_to_root(start_meet)
     goal_path = self.rrt_goal.path_to_root(goal_meet)
     path\_cost = meet\_point[0].cost + meet\_point[1].cost +
         self.env.distance(meet_point[0].state, meet_point[1].state)
     goal_path.reverse()
     return goal_path + start_path, path_cost
```

img_env.py

```
# 环境搭建
import maths
import rrt_birrt
import numpy as np
# 前进
def forward (destination, curstate, stepsize):
    diffstate = maths.combinevector(destination, '-', curstate)
unitvector = maths.vectoroperator(diffstate, '/',
     unit vector = maths. vector operator (diffstate,
        np.linalg.norm(diffstate))
    newstate = maths.combinevector(curstate, '+',
        maths.vectoroperator(unitvector, '*', stepsize))
    return newstate
class ImgEnv:
    # 初始化2维图像
          _{\rm init}_{\rm (self, img)}:
         self.state_dim = 2
self.img = img
         self.FREE = 0
         self.state\_limits = [(0, 0), (img.shape[0]-1, img.shape[1]-1)]
    # 距离用欧氏距离
    def distance (self, s1, s2, metric='euclidean'):
```

```
return rrt birrt.distance(s1, s2, metric)
# 要在图的边界内
def limits(self, point):
    limit = self.state_limits
    for i in range(self.state_dim):
        if point[i] < limit[0][i] or point[i] > limit[1][i]:
            return False
     return True
# 检查该点可走还是不可走
def collision (self, point):
    # print(type(point))
     pointlist = list(point)
    # print(point)
    # print (pointlist)
'''由于角标限制,要将tuple内元素取整,方法是tuple->list,将list内每个元素强制转换为int后再将list->tuple.'''
     for i in range (self.state_dim):
         pointlist[i] = int(pointlist[i])
     point = tuple (pointlist)
     # print(type(pointlist[i]))
    # print(type(pointlist))
    # for i in range (self.state_dim):
            pointlist[i] = int(pointlist[i])
        self.limits(point) and self.img[point] = self.FREE:
         return False
     else:
         return True
# 前进过程中一些参量
def paraofforward (self, destination, curstate, stepsize):
     newstate = forward (destination, curstate, stepsize)
     newstate = maths.vectoroperator(newstate,
     cost = self.distance(newstate, curstate)
     if self.collision(newstate):
         flag = False
         flag = True
     return flag, newstate, cost
```

rrt_example.py

```
\# -*- coding: UTF-8 -*-
# RRT测 例
import time
import rrt_birrt
 import pygame
from img_env import ImgEnv
   Parameter

\begin{array}{l}
\text{RED} = (255, 0, 0) \# \text{Red color} \\
\text{GREEN} = (0, 255, 0) \# \text{Green color}
\end{array}

BLUE = (0, 0, 255) # Blue color
START = (50, 150) # Start position

GOAL = (450, 350) # Goal position

step_size = 10 # RRT Step Size

# 载入的图像文件

IMG_NAME = "images/easy.png"
# IMG_NAME = "images/medium.png"
# IMG_NAME = "images/hard.png"
# 图形界面显示
pygame.init()
clock = pygame.time.Clock()
img = pygame.image.load (IMG_NAME)
pixacc = pygame.surfarray.array2d(img)
screen = pygame.display.set_mode(img.get_size())
```

```
screen. fill ((0, 0, 0))
screen. blit (img, (0, 0))
pygame.draw.circle(screen, RED, START, 5)
pygame.draw.circle(screen, GREEN, GOAL, 5)
env = ImgEnv(pixacc)
start = time.time()
# RRT
rrt = rrt_birrt.RRT(env, step_size, 'extend')
rrt.start_tree(START, GOAL)
samples = 0
don\underline{\hat{e}} = False
goalFound = False
drawPath = True
while not done:
    for event in pygame.event.get():
         if event.type == pygame.QUIT:
done = True
    if not goalFound:
         goalFound, new_nodes = rrt.extend_tree(stopAtGoal=True)
         samples += 1
         # 将生成树的过程显示在图上
         for node in new_nodes:
    pygame.draw.line(screen, RED, node.state, node.parent.state)
    if goalFound and drawPath:
         time\_taken = time.time() - start
         path, cost = rrt.find_path(GOAL)
        # 将路线标记在图上
for node in path:
    if node.parent != None:
                  pygame.draw.line(screen, GREEN, node.state,
                     node.parent.state, 3)
         drawPath = False
    pygame.display.update()
tree_size = len(rrt.tree)
pygame.quit()
# 打印相关量
print ("抽得样本数 =", samples)
print ("执行时间 =", time_taken, "s")
print ("路径长度 =", len(path)-1)
print("开销 =", cost)
print ("树的总规模 ="´, tree_size)
```

birrt_example.py

```
# -*- coding: UTF-8 -*-
# Bi-RRT测例文件
import time
import rrt_birrt
import pygame
from img_env import ImgEnv
# Parameter
RED = (255, 0, 0) # Red color
GREEN = (0, 255, 0) # Green color
BLUE = (0, 0, 255) # Blue color
START = (50, 150) # Start position
GOAL = (450, 350) # Goal position
step_size = 10 # RRT Step Size
# 载入的图像文件
IMG_NAME = "images/easy.png"
# IMG_NAME = "images/medium.png"
# IMG_NAME = "images/hard.png"
# 图形界面显示
pygame.init()
clock = pygame.time.Clock()
```

```
img = pygame.image.load (IMG NAME)
pixacc = pygame.surfarray.array2d(img)
screen = pygame.display.set_mode(img.get_size())
screen.fill((0, 0, 0))
screen.blit(img, (0, 0))
pygame.draw.circle(screen, RED, START, 5)
pygame.draw.circle(screen, GREEN, GOAL, 5)
env = ImgEnv(pixacc)
# Bi-RRT
birrt = rrt_birrt.BiRRT(env, step_size, 'ce')
birrt.start_tree(START, GOAL)
samples = 0
done = False
goalFound = False
drawPath = True
while not done:
     for event in pygame.event.get():
         if event.type == pygame.QUIT:
done = True
     if not goalFound:
         goalFound, meet_point, start_nodes, goal_nodes =
             birrt.extend_tree()
         samples += 1
# 在图上标出生成树
for node in start_nodes + goal_nodes:
              pygame.draw.line(screen, RED, node.state, node.parent.state)
     if goalFound and drawPath:
          time\_taken = time.time() - start
         path , cost = birrt.find_path(meet_point)
         # 在图上标记出路径
for node in path:
    if node.parent != None:
                   pygame.draw.line(screen, GREEN, node.state,
                       node.parent.state, 3)
         drawPath = False
     pygame.display.update()
tree_size = len(birrt.rrt_goal.tree) + len(birrt.rrt_start.tree)
pygame.quit()
# 打印相关量
print("抽得样本数 =", samples)
print("执行时间 =", time_taken, "s")
print("路径长度 =", len(path)-1)
print("开销 =", cost)
print ("树的总规模 =", tree_size)
```

5 实验结果

实验结果的衡量主要看如下参量: 执行时间, 抽样数, 路径长度, 开销, 树的规模 (Bi-RRT 为两棵树的总规模). 本表格只列出每种情况在测试 50 次的情况下最好和最坏的情况.

| 测例难度 | 方法 | 最好/坏 | 执行时间 | 抽样数 | 路径长度 | 开销 | 树的规模 |
|--------|--------|------|---------|------|------|-----|------|
| easy | RRT | 好 | 0.300s | 470 | 80 | 758 | 293 |
| easy | RRT | 坏 | 6.779s | 2413 | 95 | 901 | 2033 |
| medium | RRT | 好 | 0.253s | 395 | 64 | 97 | 293 |
| medium | RRT | 坏 | 13.273s | 5425 | 67 | 631 | 2176 |
| hard | RRT | 好 | 0.790s | 995 | 68 | 638 | 487 |
| hard | RRT | 坏 | 16.031s | 6732 | 75 | 709 | 1779 |
| easy | Bi-RRT | 好 | 0.047s | 50 | 86 | 812 | 155 |
| easy | Bi-RRT | 坏 | 0.227s | 191 | 87 | 827 | 322 |
| medium | Bi-RRT | 好 | 0.042s | 61 | 67 | 654 | 113 |
| medium | Bi-RRT | 坏 | 11.633s | 3739 | 69 | 672 | 2092 |
| hard | Bi-RRT | 好 | 0.126s | 147 | 69 | 655 | 208 |
| hard | Bi-RRT | 坏 | 25.830s | 4931 | 61 | 597 | 2843 |

可以看到,在 easy 难度下,Bi-RRT 可以说完胜 RRT;在 medium 难度下,Bi-RRT 和 RRT 在很多指标都很接近;只看最好的情况下 Bi-RRT 比 RRT 要快很多. 但如果只看最坏的情况下,在 medium 难度下与 RRT 差不多,而在 hard 难度下要比 RRT 要慢. 个人认为在 hard 难度下,图中可通过的缝隙过于狭窄,即便 Bi-RRT 的两颗树虽然会互相提供给对方信息,但两棵树在缝隙附近相遇就很难,而 RRT 只需要起点的生成树穿过该缝隙即可,因此 RRT 比 Bi-RRT 要快.至于机器人的大小对于本问题的影响,如果机器人的半径为 r,我们可以将周边障碍物扩展 r,机器人的大小不变,起到的效果是一样的. 经过实验发现:在 medium(hard)难度下,当机器人半径 >4(2)时,无论如何也不能通过障碍物(easy 情况就不讨论了因为可行区域比较开阔).当然,由于实现过程中存在一些类型转换,导致数据可能损失一定的精度,这也是需要注意的问题.

6 附录

代码运行后可以观察到输出信息如下:

```
 \hbox{C:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox{$C$:\scalebox
```

Hello from the pygame community. https://www.pygame.org/contribute.html

抽得样本数 = 1568

执行时间 = 1.3977222442626953 s

路径长度 = 72

开销 = 676.2123055599062

树的总规模 = 563

Figure 6

这里放几张效果图, 红色线为随机生成树, 绿色线为找到的路径, 红色点为起点, 绿色点为终点:

